Electrohydrodynamic oscillations in a nematic with permittivity anisotropy

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The behavior of the oscillation mode of the electrohydrodynamic (EHD) effect in a uniaxial nematic liquid crystal (NLC) with permittivity anisotropy ε_a is investigated on the basis of a linear theory of low-frequency EHD instability in a NLC in the presence of an auxiliary magnetic field. It is found that the oscillatory EHD-effect, in contrast to the stationary effect competing with it, is highly sensitive to the magnitude and sign of the anisotropy. Only one of the two oscillation branches can be observed. EHD oscillations are virtually unobservable in a planar NLC. EHD oscillations in a homeotropic NLC with small permittivity anisotropy can be observed in the absence of a magnetic field in layers of NLC of thickness L with upper limit L_f and lower limit L_u; the lower limit L_{u+} for $\varepsilon_a > 0$ is several times greater than the limit L_{u-} for $\varepsilon_a < 0$. It is pointed out that the uniaxial nematic PAA is a promising material for observing EHD oscillations.

INTRODUCTION

Since the second half of the 1980s, after an almost 10-year hiatus, interest in electrohydrodynamic (EHD) instability in liquid crystals (LC) is reviving. This is largely due to the widespread occurrence of electrooptic properties employing this phenomena, and it is also due to the more accurate experiments that are now possible and more powerful computers which have become available. As a rule, the main area of investigation in the last few years has been the nonlinear EHD instability in the nematic phase of a liquid crystal (NLC) far from[1,2] and near[3–5] the critical value of the external magnetic field. However, in spite of the significant success achieved in the nonlinear region, unsolved problems still remain in the linear region of the low-frequency EHD effect. One such problem is the oscillatory EHD effect, predicted in 1974 by Penz[6] and still not observed experimentally, in the linear theory, admitting this effect together with the well-known stationary EHD effect (Williams domains). The fact that there is no experimental proof that EHD oscillations of the director **n** of an NLC arise from the unperturbed state of the nematic in a low-frequency field **E** destroys the analogy between the EHD effect and the thermoconvective (TC) effect in NLC, both of which were investigated simultaneously. It was for the TC effect that success was achieved in the 1970s with respect to the prediction[7] and observation[8,9] of the oscillatory mode of the TC effect, and later it was shown[10] that homeotropic orientation of the director **n** at the plane-parallel boundaries of the LC layer is necessary in order to observe TC oscillations in an NLC.

Penz's results[6] were later obtained with slight modifications by a number of authors.[11,12] As in Ref. 6, however, they employed a rough approximation for solving the Navier–Stokes equations (the hydrodynamic mechanism of relaxation of the director **n** of the NLC was neglected),[1)] which resulted in unphysical features appearing in the spectrum of the critical field $E_o(q)$ of the oscillatory EHD effect in a dielectrically isotropic NLC ($\varepsilon_a = 0$). There is, however, an objection to this result: The case of permittivity anisotropy in the qualitative theory of the EHD effect does not result in removal of any of the relaxation mechanisms—elasto-orientational, hydrodynamic, or Maxwellian—operating in a liquid crystal. For this reason, as in the stationary EHD effect, the case $\varepsilon_a = 0$ should result in a finite critical field E_o. This was later pointed in Ref. 14. A systematic linear theory of EHD instability in an NLC, including a description of the stationary and oscillation modes, was also constructed in Ref. 14. A condition was found for the existence of the EHD effect in terms of the characteristic relaxation times of the director **n**. For the case of permittivity anisotropy in an NLC this condition is simple: The destabilization time of the undisturbed structure owing to external fields and hydrodynamic flows should not exceed the characteristic stabilization time of the structure owing to the elastic properties of the NLC. In other words, the processes which destroy the structure should be faster than the processes which stabilize it.

The case $\varepsilon_a = 0$, not described by Penz's theory, was investigated in detail in Ref. 14. The main result obtained in Ref. 14 is that EHD oscillations are virtually unobservable in a planar NLC, but EHD oscillations can arise in a homeotropic NLC, even in the absence of an auxiliary magnetic field, if the ratio of the Leslie viscosities α_2/α_3 in the plane-parallel layers of thickness L, exceeding some critical value L_*, is sufficiently large:

$$\frac{\alpha_2}{\alpha_3} > \left(\frac{\alpha_2}{\alpha_3}\right)_* = \zeta\left(\frac{B_2}{\nu B_3}\right)^{1/2},$$

$$L > L_* = \frac{L_c}{\zeta^{1/2}}\left[\left(\frac{\alpha_2}{\alpha_3}\right)^2\left(\frac{\alpha_3}{\alpha_2}\right)_*^2 - 1\right]^{-1/2},$$

where $\zeta = 1 + \alpha_2/B_2$, $L_0 \approx (\varepsilon K/\sigma\alpha)^{1/2}$ is the characteristic length in the NLC, B_i are the Miesowicz viscosity coefficients, which are linear combinations of the Leslie viscosity coefficients α_i,[14] and ε and σ are the characteristic values of the permittivity and electric conductivity of the NLC, respectively. This is related to the fact that in a homeotropic NLC the threshold field of the stationary EHD effect is $E_s \propto \alpha_2/\alpha_3$, while the threshold field of EHD oscillations does not depend on this ratio,[14] i.e., as the ratio α_2/α_3 in-

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creases the oscillational EHD effect can compete more easily with the stationary EHD effect. It was pointed out that a layer of p-azoxyanisole (PAA) not more than 50 μm thick is a promising material for observing EHD oscillations in a homeotropic NLC.

Six years after Ref. 14 was published, no progress has been made in the experimental arena. Oscillations of domain structures,[15-18] which have been known since the beginning of the 1970s, including those mentioned by Penz[6] and other authors[11,12] as confirming their theory, form when the stationary Williams domains break down in the substantially nonlinear region of the EHD effect and have no relation to the phenomenon under consideration, which must appear from the undisturbed state of the NLC instead of Williams domains. I do not know of any special experiments concerned with the search for linear EHD oscillations in NLC. For this reason, one can expect that such EHD oscillations arise in real dielectrically anisotropic NLC under very specific conditions. Indeed, a recent report[19] shows that the development of EHD instability in NLC via the oscillatory type cannot be observed largely because of the high sensitivity, discovered by numerical methods, of the oscillatory mode of the EHD effect to the magnitude and sign of the anisotropy of the dielectric properties of the NLC.

The present paper is devoted to a detailed exposition of the results announced previously in Ref. 19.

LINEAR THEORY OF THE EHD EFFECT

In the present section the formulation of the problem and the solution of the system of equations for the low-frequency EHD effect in NLC with planar (p) and homeotropic (h) orientations are briefly presented (see Ref. 14 for a more detailed exposition).

Consider an infinite plane-parallel layer of an NLC with free boundaries. The unsteady flow of the LC medium is described by a system of four differential equations supplemented by free boundary conditions on the surface of the LC layer:

1) equation of continuity of an incompressible liquid;
2) equation of conservation of the volume charge;
3) Navier–Stokes equation in the NLC; and,
4) equation of motion of the director **n** neglecting the small specific moment of inertia of the liquid.

Solving the system of equations with the help of the Fourier transform, we find that the spatial modes separate and we obtain a cubic characteristic equation for the damping rate μ, appearing in the basis U functions for the electric and hydrodynamic variables, for the perturbation:

$$U \propto \exp[\mu t + i(q_{s,o}x + kz)],$$

where $q_{s,o}$ are the wave numbers of the periodic EHD structure, which are determined at the point of the minimum of the lower branches of the spectrum of the stationary mode $E_s(q)$ and the oscillatory mode $E_o(q)$, $k = \pi/L$, and L is the thickness of the LC layer. The Routh-Hurwitz criterion for the roots μ_0 of the characteristic equation gives an expression for the critical field of one branch of the stationary ($\mathrm{Im}\,\mu_0 = 0$) instability (Fig. 1a):

$$E_s^2(q) = \frac{\Gamma_1\Gamma_2\Gamma_\sigma}{(q^2+k^2)(\psi\sigma_s\Gamma_1 - q\Gamma_3\Gamma_4)}, \tag{1}$$

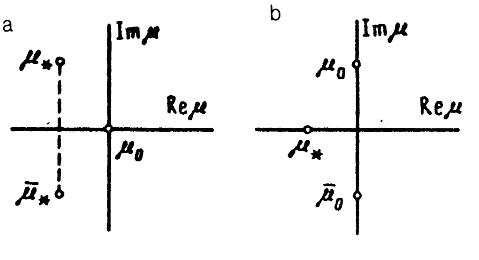

FIG. 1. Position of the roots of the characteristic equation in the complex plane of the perturbation decrement μ with development of stationary (a) and oscillational (b) EHD instabilities in NLC ($\bar{\mu}$ and μ are complex-conjugate quantities).

and two branches of the oscillatory ($\mathrm{Im}\,\mu_0 = \omega_0 \neq 0$) instability (Fig. 1b):

$$Q_4E_o^4 + Q_2E_o^2 + Q_0 = 0 \tag{2}$$

with the threshold frequency ω_0 of the oscillations given by

$$\omega_0 = (D_1/D_3)^{1/2}, \tag{3}$$

where

$$Q_0 = d_1d_2 - \rho\gamma_1\Gamma_1\Gamma_2\Gamma_\varepsilon\Gamma_\sigma(q^2+k^2),$$

$$Q_2 = \psi\left\{\rho\frac{\varepsilon_s}{4\pi}d_1(q^2+k^2) + \left[\frac{\varepsilon_s}{4\pi}\Gamma_1 + \rho\sigma_s(q^2+k^2)\right]d_2\right\}(q^2+k^2)$$
$$+ \rho\gamma_1\Gamma_\varepsilon(q^2+k^2)^2(\psi\sigma_s\Gamma_1 - q\Gamma_3\Gamma_4),$$

$$Q_4 = \psi^2\rho\frac{\varepsilon_s}{4\pi}\left[\frac{\varepsilon_s}{4\pi}\Gamma_1 + \rho\sigma_s(q^2+k^2)\right](q^2+k^2)^3,$$

$$D_1 = \psi E^2(q^2+k^2)\left[\frac{\varepsilon_s}{4\pi}\Gamma_1 + \rho\sigma_s(q^2+k^2)\right] + d_1,$$

$$D_3 = -\rho\gamma_1\Gamma_\varepsilon(q^2+k^2),$$

$$d_1 = \Gamma_\sigma(\Gamma_3^2 - \gamma_1\Gamma_1) - \Gamma_2[\Gamma_1\Gamma_\varepsilon + \rho(q^2+k^2)\Gamma_\sigma], \quad \Gamma_\sigma = \sigma_c q^2 + \sigma_s k^2,$$

$$d_2 = \Gamma_\varepsilon(\Gamma_3^2 - \gamma_1\Gamma_1) - \rho(q^2+k^2)(\Gamma_2\Gamma_\varepsilon + \gamma_1\Gamma_\sigma),$$

$$\Gamma_\varepsilon = \frac{1}{4\pi}(\varepsilon_c q^2 + \varepsilon_s k^2),$$

$$\Gamma_1 = B_2k^4 + (B_1+B_4)k^2q^2 + B_3q^4, \quad \Gamma_4 = \frac{1}{4\pi}(\varepsilon_a\sigma_\perp - \sigma_a\varepsilon_\perp)q\cos 2\varphi,$$

$$\Gamma_2 = (K\cos^2\varphi + K_3\sin^2\varphi)k^2$$
$$+ (K_1\sin^2\varphi + K_3\cos^2\varphi)q^2 + \chi_a H^2\cos 2(\varphi - \lambda),$$

$$\Gamma_3 = \frac{1}{2}(\gamma_1 + \gamma_2\cos 2\varphi)k^2 + \frac{1}{2}(\gamma_1 - \gamma_2\cos 2\varphi)q^2,$$

$$\psi = \frac{1}{4\pi}\varepsilon_a\cos 2\varphi,$$

$$\varepsilon_a = \varepsilon_\parallel - \varepsilon_\perp, \quad \chi_a = \chi_\parallel - \chi_\perp, \quad \sigma_a = \sigma_\parallel - \sigma_\perp, \quad \varepsilon_c = \varepsilon_\perp + \varepsilon_a\cos^2\varphi,$$
$$\varepsilon_s = \varepsilon_\perp + \varepsilon_a\sin^2\varphi, \quad \sigma_c = \sigma_\perp + \sigma_a\cos^2\varphi, \quad \sigma_s = \sigma_\perp + \sigma_a\sin^2\varphi,$$
$$\gamma_1 = \alpha_3 - \alpha_2, \quad \gamma_2 = \alpha_6 - \alpha_5 = \alpha_3 + \alpha_2. \tag{4}$$

In the formulas presented above the following notation was introduced: K_1 and K_3 are the Frank elasticity constants; $\varepsilon_\parallel$, $\varepsilon_\perp$, $\chi_\parallel$, $\chi_\perp$, $\sigma_\parallel$, and $\sigma_\perp$ are the principal values of the diagonalized permittivity tensor ε_{ij}, diamagnetic susceptibility tensor χ_{ij}, and electric conductivity tensor σ_{ij}, respectively. The auxiliary magnetic field **H** is oriented in a planar fashion

($\lambda = 0$) or homeotropically ($\lambda = \pi/2$). The threshold character of the breakdown of the undisturbed structure[20] for the p-orientation ($\varphi = 0$) of h-orientation ($\varphi = \pi/2$) of the director $\mathbf{n}$ at the boundaries of the liquid crystal layer remains.

In order that the fields E_s and E_o describe the physically observable appearance of EHD instability in an NLC, the remaining roots μ_* (besides μ_0) of the characteristic equation must lie on the left-hand side of the complex plane of the damping rate μ of the disturbance, i.e., $\mathrm{Re}\,\mu_* < 0$ (Fig. 1). It can be shown[21] that the last requirement, together with the expression (1) for instability of the stationary type or the expression (2) for an instability of the oscillational type, leads to $D_1 < 0$. If this condition is not satisfied, then stable hydrodynamic phenomena are impossible in NLC, but relaxational hydrodynamic processes can nonetheless exist. In particular, for sufficiently strong $\mathbf{E}$ and $\mathbf{H}$ fields an orientational instability can develop in the NLC: the director $\mathbf{n}$ is stabilized by the external fields. In this case the equations of the EHD instability describe the relaxational process in the Fréderiksz effect taking into account hydrodynamic flows. This situation was discussed previously in Ref. 22. It should be noted that the Fréderiksz effect can compete with the EHD effect even if $D_1 < 0$. For this reason, for EHD oscillations to develop the threshold Fréderiksz field E_F must exceed the threshold E_o of the oscillatory EHD effect. It is convenient to use for E_F the expressions found in Ref. 20 for the p-orientation of the NLC:

$$E_{Fp}^2 = E_1^2[1 + (H/H_1)^2 \cos 2\lambda], \tag{5}$$

and for the h-orientation of the NLC:

$$E_{Fh}^2 = E_3^2[(H/H_3)^2 \cos 2\lambda - 1], \tag{6}$$

where[2)]

$$E_i^2 = k^2 \frac{4\pi K_i}{\varepsilon_a}, \quad H_i^2 = k^2 \frac{K_i}{\chi_a}, \quad i = 1, 3.$$

The next necessary condition for the existence of EHD oscillations with $\varepsilon_a \neq 0$ is $Q_2^2 > 4Q_4Q_0$. There is, however, one other, stronger, necessary condition for the existence of EHD oscillations: $E_o^2 > 0$. This condition includes the previous condition. Finally, in order to observe EHD oscillations the minimum E_s of the stationary branch must lie of above the minimum E_o of one of the oscillatory branches of the EHD instability. Thus in order for the oscillatory type of EHD effect to exist the threshold fields E_o, E_s, and E_F must satisfy the following criteria:

$$1)\ D_1 < 0, \quad 2)\ E_o^2 > 0, \quad 3)\ E_o^2 < E_s^2, \quad 4)\ E_o^2 < E_F^2. \tag{7}$$

Following Ref. 14, we introduce the characteristic length L_0 in the NLC and the dimensionless parameters ν and δ:

$$L_0 = \pi \left(\tau_m \frac{K}{\alpha}\right)^{1/2}, \quad \nu = \frac{\rho K}{\alpha B_2}, \quad \delta = \frac{\varepsilon_a \sigma}{\varepsilon \sigma_a},$$

and the characteristic Maxwellian τ_m, hydrodynamic τ_h, and elasto-orientational τ_o relaxation times of the director:

$$\tau_m = \frac{\varepsilon}{4\pi\sigma}, \quad \tau_h = \frac{\rho}{B_2 k^2}, \quad \tau_o = \frac{\alpha}{K k^2}. \tag{8}$$

In experiments the inequalities $\tau_o, \tau_m \gg \tau_h$ usually hold. It is convenient to represent the relaxation times in the form $\tau_o = \tau_m l^2$, and $\tau_h = \nu \tau_m l^2$, where $l = L/L_0$ is the reduced thickness of the LC layer. We also introduce the magnetic field scale $m = H/H_F$, where $H_F = k(K_\varphi/\chi_a)^{0.5}$ is the threshold magnetic field in the Fréderiksz effect, and $K_\varphi = K_1 \cos^2\varphi + K_3 \sin^2\varphi$. It is convenient to take the dimensionless physical parameters l^2 and m^2 as the generatrices of the two-dimensional parameter space in which the conditions (7) describe the region of observation Δ_0 of the oscillatory EHD effect.

Numerical calculations[19] of the boundaries of the region Δ_0 for NLC with dielectric anisotropy $|\varepsilon_a/\varepsilon| = 10^{-5}$ show that these boundaries are appreciably different from the case of NLC with isotropic permittivity. This high sensitivity to the dielectric anisotropy is not accidental; it is related to the restructuring, occurring with small values of $\varepsilon_a/\varepsilon$, of the spectrum of the oscillatory mode of the EHD instability in contrast to the stationary mode. In order to verify this, we examine the shift occurring in the singularities of the spectra of the stationary mode $E_s(q)$ and oscillatory modes $E_{o1}(q)$ and $E_{o2}(q)$ modes when dielectric anisotropy of the NLC is introduced.

Investigation of the spectra $E_s(q)$ and $E_o(q)$ for the p- and h-orientations of NLC with isotropic permittivity showed[14] that in the case of the p-orientation singularities exist for the stationary and oscillational branches:

$$(\overset{0}{q}_{op1})^2 = 0, \quad (\overset{0}{q}_{op2})^2 = (q_{sp}^0)^2 = k^2\alpha_3/\alpha_2,$$

and in the case of the h-orientation

$$(\overset{0}{q}_{oh1})^2 = 0, \quad (\overset{0}{q}_{oh2})^2 = (q_{sh}^0)^2 = k^2\alpha_2/\alpha_3.$$

The weak anisotropy of the dielectric properties of the NLC results in different shifts of these singularities. In the linear δ-approximation we find, with the help of Eqs. (1), (2), and (4), for the p-orientation

$$q_{op1}^2 = k^2\delta \frac{B_2}{\alpha_3}\left(1 + \frac{1 + m^2\cos 2\lambda}{l^2} + \frac{1}{\nu l^2}\right),$$

$$q_{op2}^2 = k^2 \frac{\alpha_3}{\alpha_2}\left(1 - \delta \frac{\alpha_2 B_2}{\alpha_3^2} \frac{1}{\nu l^2}\right), \quad q_{sp}^2 = k^2 \frac{\alpha_3}{\alpha_2}\left(1 + \delta \frac{\alpha_2 B_2}{\alpha_3^2}\right) \tag{9}$$

and for the h-orientation

$$q_{oh1}^2 = -k^2\delta \frac{B_3}{\alpha_2}\left(\zeta + \frac{1 - m^2\cos 2\lambda}{l^2} + \frac{1}{\nu l^2}\right), \quad \zeta = 1 + \frac{\alpha_2}{B_2},$$

$$q_{oh2}^2 = k^2 \frac{\alpha_2}{\alpha_3}\left(1 + \delta \frac{\alpha_2 B_3}{\alpha_3^2} \frac{1}{\nu l^2}\right), \quad q_{sh}^2 = k^2 \frac{\alpha_2}{\alpha_3}\left(1 - \delta \frac{B_3}{\alpha_3}\right). \tag{10}$$

As follows from Eq. (4), $Q_4 \sim \delta^2$, and for this reason the solutions of the dispersion equation (2) for the oscillatory EHD-effect, in the first order of perturbation theory, have the form

$$E_{o1}^2 \approx -\frac{Q_0}{Q_2}, \quad E_{o2}^2 \approx -\frac{Q_2}{Q_4} + \frac{Q_0}{Q_2}, \tag{11}$$

where in the limit $\delta \to 0$ the first branch E_{o1} assumes the form investigated in Ref. 14 for NLC with $\varepsilon_a = 0$, while the second branch E_{o2} vanishes ($E_{o2} \to \infty$). Since the polynomial

$Q_4(q)$ is positive-definite, all poles of both oscillatory branches (11) coincide with the zeros of the polynomial $Q_2(q)$. Analysis of the latter polynomial with the help of Descartes' theorem shows that q_{op1} and q_{op2} from Eq. (9) and q_{oh1} and q_{oh2} from Eq. (10) exhaust the list of singularities of the branches $E_{op}(q)$ and $E_{oh}(q)$, respectively.

Since for real NLC $\nu \approx 10^{-6}-10^{-4}$, it is easy to show that the shifts of the singularities of the oscillatory branches q_{op} and q_{oh} are several orders of magnitude greater than the shifts of the singularities of the branches q_{sp} and q_{sh}. The minima of the oscillatory and stationary branches, of interest to us, behave similarly. From this physical standpoint, such behavior of the oscillatory mode of the EHD effect accompanying switching-on of the permittivity anisotropy ε_a in the liquid crystal is connected with the change in the destabilizing mechanism of the instability from hydrodynamic to Maxwellian, which corresponds to the appearance of the factor $1+\delta\tau_m/\tau_h$, while in the stationary EHD effect the Maxwellian mechanism predominates irrespective of the anisotropy.

We call attention to the behavior of the singularities of the oscillatory branch $E_{o1}(q)$, assuming that $\alpha_2, \alpha_3 < 0$, as a function of the sign of the anisotropy δ. In a p-NLC with $\delta > 0$ and in an h-NLC with $\delta < 0$ the minimum of this branch can be expected to shift into the region $q^2 < 0$ and, as a consequence, the oscillatory EHD mechanism will be disrupted or a different local minimum will be physically realized on this branch. Abrupt changes of the oscillatory structure should not be observed in an h-NLC with $\delta > 0$ and in a p-NLC with $\delta < 0$.

HOMEOTROPIC NEMATIC

In the linear δ-approximation the coordinates of the minima $q_{sh(\min)}$ and $q_{oh1(\min)}$ of the stationary $E_{sh}(q)$ and first oscillatory $E_{oh1}(q)$ branches, respectively, as well as the Fréderiksz threshold E_{Fh} have the form

$$E_{sh}^2 \approx \frac{B_3}{\sigma_a}\left(\frac{\alpha_2}{\alpha_3}\right)^2 \frac{1}{\tau_m\tau_o}\left[1-\frac{\alpha_3}{\alpha_2}m^2\cos 2\lambda\right]\left(1-\frac{B_3-\alpha_2}{\alpha_2}\delta\right)^{-1},$$

$$q_{sh(min)}^2 \geqslant k^2\frac{\alpha_2}{\alpha_3}; \tag{12}$$

$$E_{oh1}^2 \approx \frac{B_2}{\sigma_a}\zeta\omega_{oh1}^2\left[1-\frac{\zeta}{1-\zeta}\delta\left(\zeta+\frac{1-m^2\cos 2\lambda}{l^2}+\frac{1}{\nu l^2}\right)\right]^{-1},$$

$$\omega_{oh1}^2 \approx \frac{1}{\tau_m\tau_h}\left[\zeta+\frac{1-m^2\cos 2\lambda}{l^2}+\left(A_h+\frac{K_1}{K_3}l^{-2}\right)\frac{\zeta}{1-\zeta}\frac{\delta}{\nu l^2}\right],$$

$$A_h = \zeta\frac{\sigma_\perp}{\sigma_\parallel}+\frac{B_1+B_4}{B_2}-2\frac{\alpha_3}{B_2}, \tag{13}$$

$$q_{oh1(min)}^2 \approx \frac{4}{5}k^2\frac{\alpha_2}{\alpha_3}\left[1+\frac{\delta}{\nu l^2}\frac{B_3\alpha_2}{\alpha_3^2}\left(A_h+\frac{K_1}{K_3}l^{-2}\right)\right];$$

$$E_{Fh}^2 = -\frac{\alpha_2}{\sigma_a}\frac{1}{\delta}\frac{1}{\tau_m\tau_o}(m^2\cos 2\lambda-1). \tag{14}$$

The second oscillatory branch has a characteristic minimum only for $\delta < 0$. In this case, the criteria (7) of its existence, as will be shown below, make the EHD oscillations corresponding to it virtually unobservable. We present the coordinates of its minimum in the absence of a magnetic field:

$$E_{oh2}^2 \approx \frac{\alpha_2}{\sigma_a}\frac{\alpha_2}{\alpha_3}\frac{1}{\delta}\frac{1}{\tau_m\tau_h}\left[1-\frac{\nu}{\delta}\frac{\alpha_3^2}{\alpha_2 B_3}l^2 + \delta\frac{B_3}{\alpha_3}\left(1+\frac{\delta}{\nu}\frac{\alpha_2 B_3}{\alpha_3^2}l^{-2}\right)\left(1+\frac{\alpha_2}{\alpha_3}l^{-2}\right)\right],$$

$$q_{oh2(min)}^2 \geqslant k^2\frac{\alpha_2}{\alpha_3}\left[1+\delta\frac{\alpha_2 B_3}{\alpha_3^2}\frac{1}{\nu l^2}\right]. \tag{15}$$

FIRST OSCILLATORY BRANCH

We now turn to the criteria (7) for the existence of EHD oscillations of the first type, introducing the notation $v=\zeta\delta/(1-\zeta)\nu$:

1) $$m^2\cos 2\lambda < 1+\zeta l^2+\left(A_h+\frac{K_1}{K_3}l^{-2}\right)v, \tag{16}$$

2) $$m^2\cos 2\lambda > v^{-1}+1-\left(\frac{1-\zeta}{\zeta}\delta^{-1}-\zeta\right)l^2, \tag{17}$$

3) $$m^2\cos 2\lambda > 1+\left[\zeta-\frac{B_3\alpha_2^2\nu}{B_2\alpha_3^2(1-\zeta)+B_3\alpha_2^2\nu\delta}\frac{1-\zeta}{\zeta}\right]l^2 + \left[A_h+\frac{K_1}{K_3}l^{-2}+\frac{B_3}{B_2}\left(\frac{\alpha_2}{\alpha_3}\right)^2\frac{\nu}{\zeta}\right]v, \tag{18}$$

4) $$(m^2\cos 2\lambda-1)\delta<0,$$

$$\nu(m^2\cos 2\lambda-1)^2+\frac{l^2}{v}(m^2\cos 2\lambda-1) -\zeta l^2-\left(A_h+\frac{K_1}{K_3}l^{-2}\right)v>0. \tag{19}$$

It is convenient to give the last inequality (19) in an explicit form in two cases:

4.1) $\delta>0$, $m^2\cos 2\lambda<1$,

$$m^2\cos 2\lambda > 1-\frac{l^2}{2\nu v}+\frac{1}{2}\left[\frac{l^4}{\nu^2 v^2}+4\frac{\zeta}{\nu}l^2 +4\left(A_h+\frac{K_1}{K_3}l^{-2}\right)\frac{v}{\nu}\right]^{1/2}, \tag{19.1}$$

4.2) $\delta<0$, $m^2\cos 2\lambda>1$,

$$m^2\cos 2\lambda < 1-\frac{l^2}{2\nu v}-\frac{1}{2}\left[\frac{l^4}{\nu^2 v^2}+4\frac{\zeta}{\nu}l^2 +4\left(A_h+\frac{K_1}{K_3}l^{-2}\right)\frac{v}{\nu}\right]^{1/2}. \tag{19.2}$$

In the limit $\delta \to 0$, the inequalities (16)–(18) assume the form of the well-known conditions[14] for a dielectrically isotropic NLC, while the conditions (19.1) and (19.2) do not limit the possibility of observing EHD oscillations in any region of the (m^2, l^2) plane.

Figures 2 and 3 show the results of the numerical calculations of the region Δ_0 of existence of EHD oscillations for the classical nematic PAA, whose dielectrically isotropic variant is promising for observing oscillations.[14] The values of the physical parameters of PAA were taken from the review Ref. 24. Calculations were performed for planar ($\lambda=0$) and homeotropic ($\lambda=\pi/2$) orientations of the magnetic field **H**. For the calculation we used the exact ex-

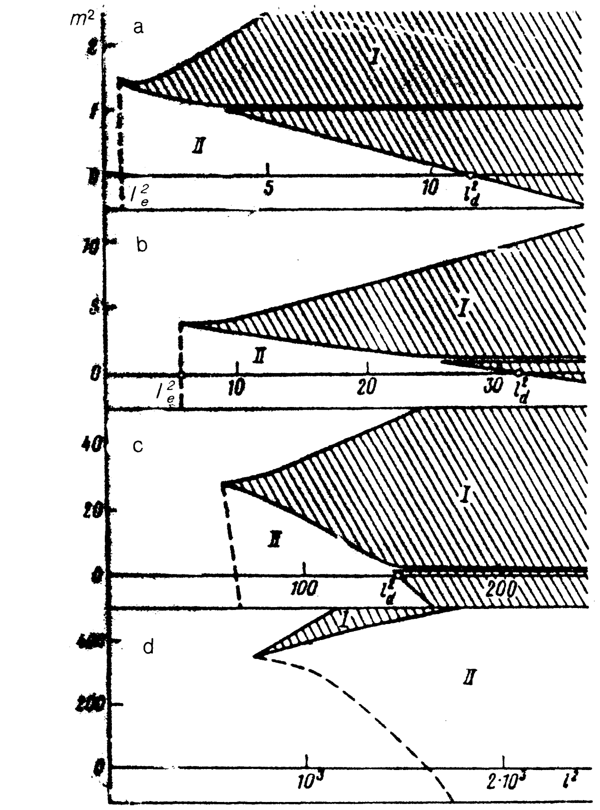

FIG. 2. Range of observation of the oscillation EHD instability in an *h*-NLC with positive permittivity anisotropy $\varepsilon_a/\varepsilon = 10^{-5}$ (a), 10^{-4} (b), 10^{-3} (c), and 10^{-2} (d). The direction of the magnetic field $\mathbf{H}\lambda = 0$ in the upper half-plane and $\lambda = \pi/2$ in the lower half-plane. Here and in Figs. 3 and 4 the hatched region I corresponds to the condition $E_o < E_s$, the upper limit corresponds to $\omega_o = 0$, and the region corresponds to $E_o > E_s$. The region I is divided by the gap β_+.

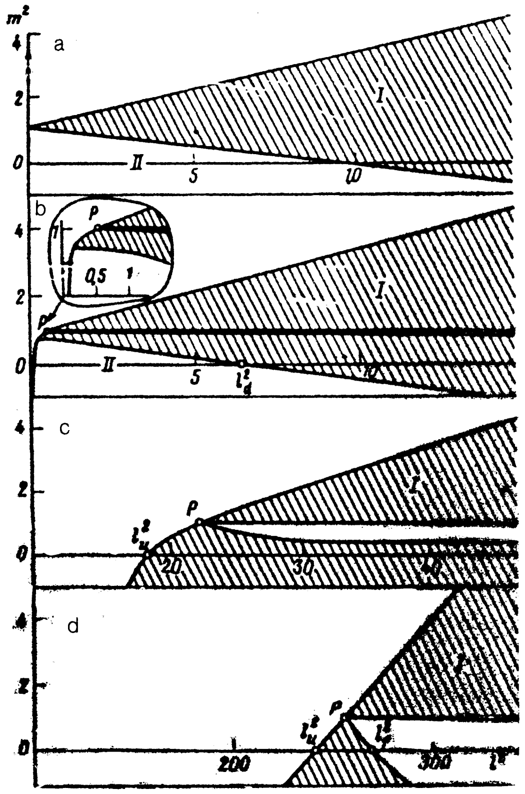

FIG. 3. Region of observation of the oscillational EHD instability in an *h*-NLC with permittivity isotropy (a) and negative permittivity anisotropy $\varepsilon_a/\varepsilon = 10^{-5}$ (b), -10^{-3} (c), and -10^{-2} (d). The direction of the magnetic field $\mathbf{H}\lambda = 0$ in the upper half-plane and $\lambda = \pi/2$ in the lower half-plane. The region I is separated by the gap β_-.

pressions for $E_{sh}(q)$, $E_{oh1}(q)$, and E_{Fh} from Eqs. (1), (2), (4), and (6). As analysis shows, the resulting analytical expressions (16)–(19) describe well the numerical results.

For $\nu > \delta > 0$ the limit of existence of the EHD effect (17) is the vertical line with the coordinate $l_e^2 = v$ in the (m^2, l^2) plane (Fig. 2a). The sign of the anisotropy of the dielectric properties of the NLC is significant for observing EHD oscillations. For NLC with positive anisotropy $\delta > 0$ in the absence of a magnetic field ($m = 0$) the oscillatory type of EHD effect can be observed in LC layers with thickness $l > l_d$,

$$l_d^2 = \frac{1}{2C}\left\{\left[(1+v(A_h+C+\zeta))^2+4vC\frac{K_1}{K_3}\right]^{1/2} +1+v(A_h+C+\zeta)\right\}, \tag{20}$$

where[3)]

$$C = \frac{B_3}{B_2}\left(\frac{\alpha_2}{\alpha_3}\right)^2\frac{\nu}{\zeta} - \zeta > 0.$$

For $v \ll 1$

$$l_d^2 \approx v\frac{K_1}{K_3} + \frac{1}{C}[1+v(A_h+C+\zeta)], \tag{21.1}$$

while for $v \gg 1$

$$l_d^2 \approx v\left(1+\frac{A_h+\zeta}{C}\right). \tag{21.2}$$

A numerical calculation shows that there probably exists a finite positive value $\delta \gg \nu$ (Fig. 2d) for which $l_d^2 = \infty$, i.e., EHD oscillations cannot be achieved in the absence of a magnetic field.

Due to the competition between the hydrodynamic mechanism of EHD oscillations and the orientational mechanism of the Fréderiksz effect, for $m^2 \cos 2\lambda > 1$ the region Δ_0 decomposes into two disconnected parts (Fig. 2), separated by a gap β_+ where EHD oscillations are forbidden:

$$1 < m^2 \cos 2\lambda < G_+(l). \tag{22}$$

As follows from Eq. (19.1), the upper limit $G_+(l)$ has the following asymptotic behavior:

$$1) \qquad v < \frac{K_1}{K_3}\frac{\zeta}{A_h^2}, \quad l^2 < \left(4v\frac{K_1}{K_3}\right)^{1/2} v;$$

$$v > \frac{1}{4\zeta^2}\frac{A_h}{\nu}, \quad l^2 < \frac{K_1}{K_3}\frac{1}{A_h},$$

$$G_+(l) \sim \left(\frac{K_1}{K_3}\frac{v}{\nu}\right)^{1/2}\frac{1}{l}, \tag{23.1}$$

2) $$v<\frac{K_1}{K_3}\frac{\zeta}{A_h^2},\quad l^2>\left(4\nu\frac{K_1}{K_3}\right)^{1/2}v;$$

$$v>\frac{1}{4\zeta^2}\frac{A_h}{\nu},\quad l^2>4\zeta\nu v^2,$$

$$G_+(l)\sim 1+\zeta v+A_h v^2\frac{1}{l^2}. \qquad (23.2)$$

For negative anisotropy $\delta<0$ and no magnetic field the oscillatory type of EHD effect should be observed in LC layers with thickness $l>l_u$,

$$l_u^2=\frac{1}{2\zeta}\left\{\left[(1+vA_h)^2-4\zeta v\frac{K_1}{K_3}\right]^{1/2}-1-vA_h\right\}. \qquad (24)$$

It is interesting to note the behavior of the frequency of the oscillations $\omega_{oh1}(l)$:

$$\omega_{oh1}(l_u)=0;\quad \omega_{oh1}\sim 1/l,\quad l\to\infty.$$

The maximum value of the frequency ω_{oh1} is reached at $l=l_m$,

$$l_m^2=\frac{1}{\zeta}\left\{\left[(1+vA_h)^2-3\zeta v\frac{K_1}{K_3}\right]^{1/2}-1-vA_h\right\}, \qquad (25)$$

and ω_{oh1} can reach several kHz. The criterion (19.2), associated with the possible Fréderiksz effect in an h-oriented NLC with $\delta<0$, results in the appearance of a gap β_-, where EHD oscillations are forbidden, in the region Δ_0 with $m^2\cos 2\lambda<1$ (Fig. 3):

$$G_-(l)<m^2\cos 2\lambda<1. \qquad (26)$$

The gap β_- divides the region Δ_0 into two disconnected parts, which join at the point $P(m^2\cos 2\lambda=1,\ l^2=l_s^2)$ in the (m^2,l^2) plane,

$$l_s^2=\frac{1}{2\zeta}\left\{\left[A_h^2v^2-4\zeta v\frac{K_1}{K_3}\right]^{1/2}-A_hv\right\}, \qquad (27)$$

so that for $l<l_s$ there is no gap β_-. The lower limit $G_-(l)$ exhibits the following behavior for $l\gtrsim l_s$:

$$G_-(l)\approx 1+\frac{v}{l_s^2}\left(\zeta-\frac{K_1}{K_3}\frac{v}{l_s^4}\right)(l^2-l_s^2), \qquad (28)$$

and the following asymptotic behavior for large l:

$$|v|<\frac{K_1}{K_3}\frac{\zeta}{A_h^2},\quad l^2>|v|\left(4\nu\frac{K_1}{K_3}\right)^{1/2};$$

$$|v|>\frac{1}{4\zeta^2}\frac{A_h}{\nu},\quad l^2>4\zeta\nu v^2,$$

$$G_-(l)\sim 1-\zeta|v|+A_hv^2\frac{1}{l^2}. \qquad (29)$$

We now present approximations for l_u^2, l_m^2, l_s^2, and $G_-(l)$ from Eq. (28) for $|v|\ll\frac{1}{2}(A_h-2\zeta K_1/K_3)^{-1}$:

$$l_u^2\approx|v|\frac{K_1}{K_3},\quad l_m^2={}^3/_2 l_u^2,\quad l_s^2\approx\left(\frac{K_1}{K_3}\frac{|v|}{\zeta}\right)^{1/2},$$

$$G_-(l)\approx 1-2\left(\frac{K_3}{K_1}\zeta^3|v|\right)^{1/2}(l^2-l_s^2), \qquad (30.1)$$

and for $|v|\gg 2A_h^{-2}(A_h-2\zeta K_1/K_3)$:

$$l_u^2\approx|v|\frac{A_h}{\zeta}+\frac{K_1}{K_3}\frac{1}{A_h}-\zeta^{-1},$$

$$l_m^2=2l_u^2,\quad l_s^2\approx|v|\frac{A_h}{\zeta}+\frac{K_1}{K_3}\frac{1}{A_h},$$

$$G_-(l)\approx 1-\frac{\zeta^2}{A_h}(l^2-l_s^2). \qquad (30.2)$$

The asymptotic expansion (29) shows that for $v>\zeta^{-1}$ the region where EHD oscillations are observed in the absence of a magnetic field also has an upper limit:

$$l_u^2<l^2<l_f^2, \qquad (31)$$

where

$$l_f^2=\frac{[(\nu+|v|A_h)^2+4(\zeta|v|-1)K_1/K_3]^{1/2}+\nu+|v|A_h}{2(\zeta-|v|^{-1})}. \qquad (32)$$

For $|v|\gg 2A_h^{-2}(\nu A_h+2\zeta K_1/K_3)$ we obtain the approximation

$$l_f^2\approx|v|\frac{A_h}{\zeta}+\frac{K_1}{K_3}\frac{1}{A_h}+\frac{\nu}{\zeta}. \qquad (33)$$

Comparing Eqs. (30.2) and (33), we obtain an estimate for the width of the region where EHD oscillations are observed with $m=0$:

$$\frac{1+\nu}{\zeta}<l_f^2-l_u^2<\infty. \qquad (34)$$

It is interesting to note that if for a dielectrically isotropic NLC EHD oscillations can be observed without a magnetic field only for nematics with a large ratio of the Leslie viscosities α_2/α_3 (or $C>0$),[14] then when dielectric anisotropy is introduced this result remains valid only for positive ε_a; for negative anisotropy, as follows for Eq. (34), EHD oscillations can be observed for $m=0$ for any nematics. In this case, instead of l_f, the thickness of the LC layer for which the threshold of the oscillational instability E_{o1} is equal to the threshold of the stationary instability E_s can play the role of the upper limit of observation, just as in MBBA.

SECOND OSCILLATORY BRANCH

We now consider the conditions (7) under which the second type of EHD oscillations are observed: $E_{sh}^2>E_{oh2}^2>0$. These conditions can be written in the form of a double inequality for a bicubic polynomial:

$$0>\Phi_h(l)>-\frac{\alpha_2}{\alpha_3}\nu^2u^2l^4, \qquad (35)$$

where we have introduced the notation

$$u=\frac{\delta}{\nu}\frac{B_3}{\alpha_3}>0,\quad \delta<0,$$

$$\Phi_h(l)=l^6-\frac{\alpha_2}{\alpha_3}u(1+\nu u)l^4-\left(\frac{\alpha_2}{\alpha_3}\right)^2\nu u(1+u)l^2-\left(\frac{\alpha_2}{\alpha_3}\right)^3u^2. \qquad (36)$$

The solutions of the double inequality (35) lie in a narrow neighborhood Δl_h of the positive zero l_h of the polynomial $\Phi_h(l)$:

$$\eta_h=\Delta l_h/l_h\approx\nu u^{1/2}, \tag{37}$$

and the value of l_h can be found analytically in different regions of the values of the parameter:

1) $$0<u<1,\quad l_h^2=\frac{\alpha_2}{\alpha_3}u^{2/3},\quad \eta_h<\nu, \tag{38.1}$$

2) $$1<u<(2/\nu)^{2/3},\quad \nu<\eta_h<(2\nu^2)^{1/3},$$

$$l_h^2=\frac{1}{3}\frac{\alpha_2}{\alpha_3}u\left\{1+\left[1+3\left[3\left(u^{-1}-\frac{u^2\nu^2}{4}\right)\right]^{1/2}\right]^{1/3}+\left[1-3\left[3\left(u^{-1}-\frac{u^2\nu^2}{4}\right)\right]^{1/2}\right]^{1/3}\right\}, \tag{38.2}$$

3) $$(2/\nu)^{2/3}\leqslant u<\nu^{-1},\ (2\nu^2)^{1/3}<\eta_h<\nu^{1/2},$$

$$l_h^2=\frac{1}{3}\frac{\alpha_2}{\alpha_3}u\left\{1+2\left[1+27\left(\frac{u^2\nu^2}{4}-\frac{1}{u}\right)\right]^{1/6}\times\cos\left[\frac{1}{3}\operatorname{arctg}\left[27\left(\frac{u^2\nu^2}{4}-\frac{1}{u}\right)\right]^{1/2}\right]\right\}, \tag{38.3}$$

4) $$\nu^{-1}<u,\quad l_h^2=\frac{\alpha_2}{\alpha_3}\nu u^2,\quad \eta_h>\nu^{1/2}. \tag{38.4}$$

The condition $D_1<0$ admits wide possibilities for the existence of EHD oscillations, but the second type of EHD oscillations nonetheless cannot be observed. Indeed, the case (38.1) requires that the thickness of the LC layer be known to within $\Delta l_h\sim\nu(\alpha_2/\alpha_3)^{1/2}$, which is equal to 10^{-3}–10^{-4}. Since $L_0=22.5\,\mu$m for PAA and $L_0=6.3\,\mu$m for MBBA,[14] we find that the thickness of a layer of these nematics must be determined to within a fraction of a micron. First of all, this exceeds the technical capabilities of experiments and, second, it results in fluctuation smearing of the oscillation effect when the noise effect of the external field **E** (Ref. 25) and the hydrodynamic fluctuations themselves in the liquid crystal (Ref. 26) in the neighborhood of the thresholds E_s and E_o of the EHD effect are taken into account. The case (38.4) requires LC layer thicknesses of not less 3–5 mm, which also falls outside the range of the technical possibilities in the production of uniform LC layers.[8] The cases (38.2) and (38.3) give comparatively acceptable requirements for the thickness L of the LC layer and accuracy with which the value of L must be determined, but in this case, as it is easy to show, the threshold of the first oscillatory branch lies below the threshold of the second branch. Indeed, for $u=(2/\nu)^{2/3}$

$$\frac{E^2_{oh1}}{E^2_{oh2}}\approx\frac{1}{2}\zeta^2\left(\frac{\alpha_3}{\alpha_2}\right)^2\frac{B_2}{B_3}\left(\frac{2}{\nu}\right)^{2/3}. \tag{39}$$

For PAA and MBAA this is equal to 10^{-2} and 10^{-1}, respectively. Thus for the h-orientation of the NLC the first oscillatory branch can be observed, in contrast to the second branch, in a dielectrically anisotropic nematic in the absence of a magnetic field. As the value of δ decreases, irrespective of the sign of the anisotropy, it becomes easier to observe EHD oscillations.

PLANAR NEMATIC

In the δ-approximation the coordinates of the minimum $q_{op(\min)}$, $q_{op1(\min)}$, and $q_{op2(\min)}$ of the stationary branch $E_{sp}(q)$ and the two oscillatory branches $E_{op1}(q)$ and $E_{op2}(q)$ as well as the Fréderiksz threshold E_{Fp} have the form

$$E_{sp}^2\approx\frac{B}{\sigma_a}\frac{\sigma_\parallel+\sigma_\perp}{\sigma_\perp}\frac{1}{\tau_m\tau_0}\left[\frac{K_3}{K_1}+1+m^2\cos 2\lambda\right]\left(1-\frac{B+\alpha_2}{\alpha_2}\delta\right)^{-1},$$

$$q^2_{sp(\min)}\approx k^2,\quad B=\sum_{i=1}^{4}B_i; \tag{40}$$

$$E^2_{op1}\approx\frac{B_2}{\sigma_a}\left(\frac{\alpha_2}{\alpha_3}\right)^2\omega^2_{op1}\left[1+\delta\frac{B_2\alpha_2}{\alpha_3^2}\times\left(1+\frac{1+m^2\cos 2\lambda}{l^2}+\frac{1}{\nu l^2}\right)\right]^{-1},$$

$$\omega^2_{op1}\approx\frac{1}{\tau_m\tau_h}\left\{1+\frac{1+m^2\cos 2\lambda}{l^2}-\left(A_p+\frac{K_3}{K_1}l^{-2}\right)\frac{B_2}{\alpha_3}\frac{\delta}{\nu l^2}\right\},$$

$$A_p=\frac{\sigma_\parallel}{\sigma_\perp}+\frac{B_1+B_4}{B_2}-2\frac{\alpha_3}{B_2},$$

$$q^2_{op1(\min)}\approx\frac{1}{2}k^2\frac{\alpha_3}{\alpha_2}\left[1-\frac{\delta}{\nu l^2}\frac{B_2}{\alpha_3}\left(A_p+\frac{K_3}{K_1}l^{-2}\right)\right]; \tag{41}$$

$$E_{Fp}^2=-\frac{\alpha_2}{\sigma_a}\frac{1}{\delta}\frac{1}{\tau_m\tau_o}(1+m^2\cos 2\lambda). \tag{42}$$

The second oscillatory branch has a characteristic minimum only for $\delta>0$. In the absence of a magnetic field its coordinates will be

$$E^2_{op2}\approx-\frac{\alpha_2}{\sigma_a}\frac{\zeta}{2\delta}\frac{1}{\tau_m\tau_h}\left\{1-2\frac{\nu}{\delta}\frac{1-\zeta}{\zeta}l^2+\frac{\delta}{1-\zeta}\left(1+\frac{\delta}{\nu}\frac{\zeta}{2(1-\zeta)}l^{-2}\right)\left[1+\left(1+\frac{K_3}{K_1}\right)l^{-2}\right]\right\},$$

$$q^2_{op2(\min)}\gtrsim k^2\frac{\alpha_3}{\alpha_2}\left[1-\delta\frac{\alpha_2B_2}{\alpha_3^2}\frac{1}{\nu l^2}\right]. \tag{43}$$

FIRST OSCILLATORY BRANCH

The criteria (7) for the existence of EHD oscillations of the first type in a p-NLC have the form

1) $$-m^2\cos 2\lambda<1+l^2-\left(A_p+\frac{K_3}{K_1}l^{-2}\right)u, \tag{44}$$

2) $$-m^2\cos 2\lambda>\nu^{-1}+1+\left(1+\frac{\alpha_3^2}{\alpha_2B_2}\frac{1}{\delta}\right)l^2, \tag{45}$$

3) $$-m^2\cos 2\lambda>1+l^2\left(1-\xi-\xi\frac{\sigma_\parallel+\sigma_\perp}{\sigma_\perp}\frac{B}{\alpha_2}\frac{K_3}{K_1}\nu\delta\right)-\left(A_p+\frac{K_3}{K_1}l^{-2}+\xi\frac{\alpha_2}{\alpha_3}\right)u, \tag{46}$$

4)
$$(1+m^2\cos 2\lambda)\delta<0,$$
$$\nu(1+m^2\cos 2\lambda)^2+\left(2+\nu l^2+\frac{\alpha_3}{\alpha_2}u^{-1}l^2\right)(1+m^2\cos 2\lambda)$$
$$+l^2-\left(A_p+\frac{K_3}{K_1}l^{-2}\right)u<0. \qquad (47)$$

where the parameter u can be seen in Eq. (35), and

$$\xi \doteq \nu\frac{B}{B_2}\left(\frac{\alpha_3}{\alpha_2}\right)^2\frac{\sigma_\parallel+\sigma_\perp}{\sigma_\perp}\frac{K_3}{K_1}.$$

As $\delta\to 0$ the inequalities (44)–(46) assume the form of the well-known inequalities for a dielectrically isotropic NLC with p-orientation,[14] while the condition (47) makes it possible to observe EHD oscillations in the entire (m^2,l^2) plane. Numerical analysis shows that the resulting analytical expressions accurately describe the computational results.

The general picture of the behavior of the region Δ_0 where EHD oscillations are observed in the (m^2,l^2) plane is as follows (Fig. 4): As the dielectric anisotropy δ increases the narrow strip Δ_0 of width $\xi(1+\xi B_2\alpha_2\delta/\alpha_3^2)$ shifts in the (m^2,l^2) plane away from its isotropic dielectric position. The directions of displacement, depending on the sign of the anisotropy, are the same as in an h-NLC. In contrast to the homeotropic situation, however, in a p-NLC the physical parameters of the liquid crystal ($\xi = 10^{-8}$–10^{-6}) virtually exclude the observation of EHD oscillations for real NLC, just as in dielectrically isotropic p-NLC.

SECOND OSCILLATORY BRANCH

The conditions for the observation of the second type of EHD oscillations, just as for h-orientation (35), also reduce to a double inequality for a bicubic polynomial:

$$0>\Phi_p(l)>-\vartheta s^2v^2l^4, \qquad (48)$$

where $v>0$ is determined in Eq. (16), and in addition

$$s=\nu/\zeta,\ \vartheta=4B/B_2,\ \varkappa=1+K_3/K_1,\ \varkappa s\ll 1,$$
$$\Phi_p(l)=l^6-{}^1/_2vl^4-{}^1/_2sv^2(\varkappa+{}^1/_2v)l^2-{}^1/_4\varkappa sv^3. \qquad (49)$$

The solutions of the double inequality (48) lie in a narrow neighborhood $\Delta l_p = \eta_p l_p$ of the positive zero l_p of the polynomial $\Phi_p(l)$, whose value can be found analytically in different regions of values of the parameter:

1) $0<v\leqslant 2/s$,
$$l_p^2={}^1/_4v[1+(1+4sv)^{1/2}]\approx{}^1/_2v(1+sv),\quad \eta_p={}^1/_4\vartheta s^2v,$$
$$0<l_p<s^{-1/2},\quad 0<\Delta l_p<{}^1/_2\vartheta s^{1/2}, \qquad (50.1)$$

2) $2/s\leqslant v<1/s^2$,
$$l_p^2={}^1/_4v(4sv+1)^{1/2}\approx{}^1/_2s^{1/2}v^{3/2},\quad \eta_p={}^1/_{16}\vartheta s,$$
$$s^{-1/2}<l_p<s^{-5/4},\ {}^1/_{16}\vartheta s^{1/2}<\Delta l_p<{}^1/_{16}\vartheta s^{-1/4}. \qquad (50.2)$$

The region $v\geqslant s^{-2}$ is of no practical interest, since it corresponds to anomalously large positive dielectric anisotropy of the nematic:

$$\frac{\varepsilon_a}{\varepsilon}\geqslant 2\frac{\zeta(1-\zeta)}{\nu}\frac{\sigma_a}{\sigma}\approx(10^3-10^5)\frac{\sigma_a}{\sigma},$$

where the coefficients of σ_a/σ correspond to the physical parameters of PAA and MBBA. Apparently, even in liquid-crystalline cyanophenyl ethers, for which the maximum positive value of the dielectric anisotropy is obtained,[27] the last relation is not satisfied.

The analysis of the conditions (50.1) and (50.2), as a whole, is identical to the analogous analysis of the conditions (38) for an h-NLC. The conditions (50.1) require that for a material of the type PAA (with positive $\varepsilon_a/\varepsilon$) in a layer of thickness 200 μm, the layer thickness must be determined to within tenths of a micron. On the other hand, the conditions (50.2), for which the required accuracy of the thickness of the LC layer is technically acceptable, give a thickness of several millimeters, which, just as in the preceding case, makes it virtually impossible to observed EHD oscillations of the second type for the same reason as in an NLC of the h-type. The intermediate condition $v\approx s^{-1}$ gives acceptable requirements for the thickness of the LC layer and the accuracy of the value of L, but, in this case, as in Eq. (39), the threshold of the first oscillation branch falls below the threshold of the second branch:

$$\frac{E_{op1}^2}{E_{op2}^2}\approx\frac{1}{2(1+\zeta)}\approx 0.4.$$

Thus EHD oscillations of both types are practically unobservable in p-type NLC.

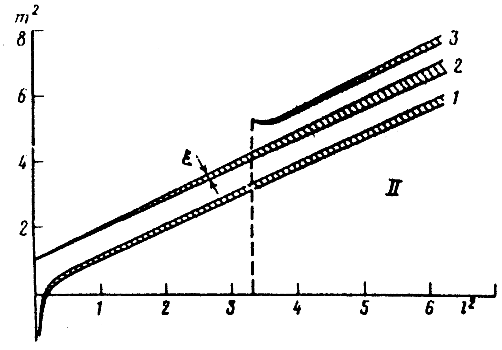

FIG. 4. Region of observation of the oscillational EHD-instability in a p-type NLC for different values of $E_a/\varepsilon = -10^{-6}$ (1), 0 (2), and 10^{-6} (3). The direction of the magnetic field $\mathbf{H}\lambda = \pi/2$ in the upper half-plane and $\lambda = 0$ in the lower half-plane. The region I is separated by the gap β_-.

CONCLUSIONS

The behavior of the oscillatory mode of the EHD effect in a uniaxial nematic with dielectric anisotropy ε_a was investigated on the basis of the linear theory of EHD instability in a low-frequency electric field. It was found that the EHD effect, in contrast to the stationary EHD effect which competes with it, is highly sensitive to the magnitude and sign of the anisotropy. This is connected with the change in the destabilizing mechanism of the oscillatory EHD instability from hydrodynamic to Maxwellian, while in the stationary EHD effect the Maxwellian mechanism predominates irrespective of whether or not dielectric anisotropy is present in the NLC.

A specific feature of the development of EHD oscillations in a dielectrically anisotropic NLC is the presence of two oscillatory modes of the EHD effect, the second of which E_{o2} should be virtually unobservable in an experiment against of the background of the first mode E_{o1}.

TABLE I.

NLC	$\tau_h L^{-2}$, sec/mm²	$\tau_0 L^{-2}$ sec/mm²	τ_m, sec	L_0, μm	A_h	C	ν	ς	ξ	$\varkappa$	$\sigma_a/\sigma_\perp$
MBBA	$1{,}8\cdot10^{-3}$	10^3	$4\cdot10^{-2}$	6,3	1,45	−0,19	$5\cdot10^{-6}$	0,25	10^{-8}	2,25	0,3
PAA	$4{,}5\cdot10^{-2}$	10^2	$3{,}5\cdot10^{-2}$	22,5	1,58	1,03	$5\cdot10^{-4}$	0,26	10^{-6}	3,1	0,3

The regions Δ_0 where the EHD oscillations in p- and h-oriented nematics are observed were found in the (m,l) plane, where m is the reduced magnitude of the auxiliary magnetic field and l is the thickness of the liquid crystal layer. The calculation was performed for the classical nematic PAA, whose dielectrically isotropic variant is promising for observation of EHD oscillations.[14]

In a p-type NLC the region Δ_0 is a narrow strip of width $\xi = 10^{-6}$–10^{-8}, irrespective of the sign and magnitude of the anisotropy ε_a. This virtually precludes observation of EHD oscillations for a real p-NLC, just as in the dielectrically isotropic p-type NLC.

In a h-NLC the region Δ_0 consists of two disconnected parts, separated by a gap β, where, owing to competition with the Fréderiksz effect, EHD oscillations are forbidden. The sign of the anisotropy ε_a is significant for observing the oscillatory EHD effect. For a dielectrically isotropic NLC, EHD oscillations can be observed without a magnetic field for only nematics with a quite large ratio of the Leslie viscosities α_2/α_3,[14] while in the presence of dielectric anisotropy this is true only for positive ε_a; for negative anisotropy the EHD oscillations can be observed with $m = 0$ in any nematics.

We now discuss the possibility of observing EHD oscillations with $m = 0$ in real classical NLC with negative anisotropy—PAA and MBAA, whose physical parameters were taken from Ref. 24 and are presented in Table I. For h-PAA with anisotropy $\varepsilon_a/\varepsilon = -3\cdot10^{-2}$ at $T = 125$ °C oscillations can be observed in a liquid crystal whose thickness L lies in the range $L_u = 496\,\mu m < L < L_f = 498\,\mu m$. Analogous calculations for a layer of the other nematic MBAA with anisotropy $\varepsilon_a/\varepsilon = -0.14$ at $T = 25$ °C give a lower limit of the thickness $L_u = 2.65\,\mu m$, which falls outside the range of technical capabilities for producing uniform homeotropic liquid crystal layers.[8]

Decreasing the parameter δ makes it easier to observe EHD oscillations in these nematics. We shall briefly discuss the technical possibilities for varying the value of δ. The simplest method is to vary continuously the temperature of the liquid crystal phase. However, as the temperature of the liquid crystal approaches the bleaching point [$T_{\rm MBAA} = 46$ °C, $T_{\rm PAA} = 135$ °C (Ref. 28)], $\varepsilon_a \to 0$ and $\sigma_a \to 0$, simultaneously. In addition, since in our case ε_a and σ_a are proportional to the first power of the magnitude of the order parameter of the liquid crystal, at the bleaching point δ can differ appreciably from zero. There exists, however, a different method for varying δ—doping with dielectric and ionic additives in order to change ε_a and σ_a, respectively. For example, tetrabutyl ammonium bromide is used to change the sign of σ_a in MBBA, and 4 cyano-benzylidene-4-octyloxyanilene is used to change the sign of ε_a in MBBA.[12] This method, however, is limited in that δ can only be varied by appreciable steps. For this reason, a combination of both methods can make it possible to achieve continuous variation of δ sufficient for our purposes.

When the anisotropy $\varepsilon_a/\varepsilon = 1.38\cdot10^{-3}$ with constant $\sigma_a/\sigma = 0.3$ is achieved in PAA, the range of thicknesses L of the liquid crystal layer can be infinitely expanded; more accurately, $L_u = 98\,\mu m$ and $L_f \to \infty$. The lower limit of $\varepsilon_a/\varepsilon$ is, as a rule, 10^{-3} for the other known nematics also;[4)] this is apparently connected with the limits of the accuracy of measuring instruments. When $\varepsilon_a/\varepsilon = -1.8\cdot10^{-5}$ ($\sigma_a/\sigma = 0.3$) is achieved in MBBA, the Fréderiksz effect no longer competes with EHD oscillations in h-MBBA with $m = 0$; in this case, $L_u = 27.6\,\mu m$. However, the range of thicknesses of MBBA has an upper limit L_f; this is connected, as pointed in Eq. 20, with the negative sign of the constant C for MBBA. Calculations give $L_f = 36\,\mu m$. However, in MBBA the range of thicknesses of the liquid crystal layer can be extended to infinity (L_u, $L_f \to \infty$) by increasing the ratio α_2/α_3 and changing the sign of C. The latter can be achieved by dissolving in MBBA complexes of macrocyclic ether MCPE-18-crown-6 and KCl salts with total concentration $c = 0.15$–0.2%; this results in a sharp reduction (by an order of magnitude or even with a change in the sign with $c = 0.25\%$) of the Leslie viscosity α_3,[29] and in addition the other physical properties of the NLC (the Franck elasticity constants, the other Leslie viscosities $\alpha_i, i \neq 3$, and the anisotropies σ_a, ε_a) do not change significantly.

This range of parameters of liquid crystal layers in real nematics that is required to observe EHD oscillations and the specific methods employed to make these parameters approach the experimental capabilities explain why there have never been any accidental experimental observations of electrohydrodynamic oscillations in real nematics, and they indicate a real possibility for observing EHD oscillations in NLC with specially prepared liquid-crystal samples, primarily PAA.

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[1)] This approximation is justified in Ref. 6 by the smallness of the dimensionless parameter $\nu = \rho K/\alpha^2$, which for known NLC is indeed equal to 10^{-6}–10^{-4} (ρ is the density of the NLC, α and K are the characteristic values of the isotropic shear viscosity and Frank's elasticity constant). As shown in Ref. 13, however, the expressions describing the nonstationary EHD effect contain the combination $\nu(\alpha_2/\alpha_3)^2$, which is by no means small (α_2, α_3 are the Leslie viscosity coefficients).

[2)] The threshold fields E_i and H_i in the Fréderiksz effect do not depend on the anisotropy of the electric conductivity σ_a of the NLC.[23]

[3)] In contrast to PAA, the other classical nematic MBBA belongs to the class of liquid crystals with $C < 0$.

[4)] S. V. Belyaev, private communication.

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