Aharonov–Casher effect in the Hubbard model with repulsion

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The effect of Aharonov–Bohm (AB) and Aharonov–Casher (AC) fields on a Hubbard chain with repulsion is studied theoretically. The dependence of the ground-state energy on the filling of the band is found. In particular, for half filling of the band there is no dependence of the energy on the AB phase while the response to the AC field is a maximum. This is analogous to the energy of a one-dimensional antiferromagnet with spin 1/2 under conditions such that the AC phase changes.

In 1984 Aharonov and Casher[1] showed that the interaction of a magnetic moment with the electric field of a uniformly charged filament has a topological character and leads to the appearance of a universal phase when the particle possessing the magnetic moment passes around a closed contour containing the filament. In essence, in $(2+1)$-dimensional space-time the Aharonov–Casher (AC) effect is the exact (dual) analog of the Aharonov–Bohm (AB) effect. The phase advance under the conditions of the AB and AC experiments has the form

$$\Delta\varphi_{\rm AB}=2\pi(\Phi/\Phi_0),\ \Delta\varphi_{\rm AC}=2\pi(F/F_0). \tag{1}$$

Here, Φ is the magnetic flux, $\Phi_0=hc/e$ is the quantum of flux, e is the particle charge, $F=4\pi\tau$ is the flux of the electric field through the contour of the trajectory (τ is the linear charge density on the filament), $F_0=hc/\mu_s$ is the "quantum" of electric flux, and μ_s is the projection of the magnetic moment along the direction of the filament. The formulas (1) display the complete equivalence of the effects under the replacements $\Phi\leftrightarrow F$ and $e\leftrightarrow\mu_s$.

In condensed media the AB effect is manifested in the form of flux oscillations of various characteristics of mesoscopic multiply connected samples (see, e.g., Ref. 2). This type of oscillation (but this time with change of the charge on the filament) should also be expected under the conditions of the AC "solid experiment." In Ref. 3 this problem was studied for the example of mesoscopic metallic rings. From a general point of view, magnetic materials are more natural for the investigation of AC oscillations. Topological quantum oscillations in small magnetic rings were first considered in Ref. 4, in which the oscillating part of the free energy of a one-dimensional integer-spin antiferromagnet was calculated and a phenomenological Lagrangian was proposed for the description of oscillations in ferromagnetic rings with a topologically nontrivial magnetic structure.

In the present paper we continue the study of AC oscillations in magnetic systems, using as the initial model the exactly solvable one-dimensional Hubbard model. For our purposes an obvious advantage of this model is the fact that in it the microscopic introduction of interaction with an electromagnetic field is easy since the initial Hamiltonian is constructed from electron operators. On the other hand, in the limit of strong repulsion on the sites, the Hubbard model with half filling describes, as is well known, a Heisenberg antiferromagnet with spin $S=1/2$. This gives the possibility of using a microscopic approach to study the response of an antiferromagnet chain to the AC field (F). In view of this, it seems to us that it is interesting to compare the coherence properties of mesoscopic rings of integer and half-integer spins.

The nonforce topological interaction of an electron spin with the electric field of a charged filament can be taken into account easily if on the wave function of the electrons on the ring we impose quasiperiodic boundary conditions

$$\Psi_\sigma(\varphi+2\pi)=\exp(i\alpha_\sigma)\Psi_\sigma(\varphi). \tag{2}$$

In Eq. (2) the subscript $\sigma=\pm1$ labels the projection of the electron spin along the quantization axis. In the conditions of the AC experiment it is natural to choose as this axis the direction of the charged filament (perpendicular to the plane of the ring). This situation is easily realized physically by switching on a weak (orienting) magnetic field parallel to the filament. In this case the sum and difference of the phases $\alpha_{\pm1}$ are none other than the AB (c) phase and AC (s) phase:

$$2\alpha_c=\alpha_1+\alpha_{-1}=2\Delta\varphi_{\rm AB};\quad 2\alpha_s=\alpha_1-\alpha_{-1}=2\Delta\varphi_{\rm AC}.$$

We note that for free electrons the magnetic moment in the definition of the AC phase coincides with the Bohr magneton μ_B, and, therefore, for (e.g.) a metallic ring, the period of the AC oscillations is equal to $F_0=hc/\mu_B$ (Ref. 3).

Since the quasiperiodic boundary conditions are equivalent to the presence of a phase advance α_σ/N_a (N_a is the number of sites in the chain) accompanying transitions of electrons between sites that depends on the projection of the electron spin, the Hamiltonian of the Hubbard model takes the form

$$H=-\sum_{j,\sigma}\{a^+_{j,\sigma}a_{j+1,\sigma}\exp(i\alpha_\sigma/N_a)+\text{h.c.}\}+4U\sum_j n_{j,1}n_{j,-1}, \tag{3}$$

where $a^+_{j,\sigma}$ ($a_{j,\sigma}$) is the creation (annihilation) operator for an electron with spin projection σ on site j; $n_{j,\sigma}=a^+_{j,\sigma}a_{j,\sigma}$; $4U$ is the Hubbard-repulsion energy ($U>0$) on a site, and, in the chosen units, the hopping integral $t=1$.

As is well known,[5] the Schrödinger problem for the Hamiltonian of a Hubbard chain can be solved exactly by means of the Bethe ansatz. We are interested in that part of the ground-state energy which depends on the topological phases. In the conditions of the AB experiment, when the phase advance associated with hopping does not depend on

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the spin projection, this problem was solved in Ref. 6. The principal assertion of Ref. 6 is that for half filling ($N=N_a$, where N is the number of electrons in the chain), and also for empty and completely filled bands, the energy of the ground state of the model (3), to within exponentially small corrections [on the order of $\exp(-N_a)$], does not depend on the flux. A nonzero response to an AB field arises for partial filling of the band. In this case, for a large Hubbard-repulsion constant $U\gg 1$, it was found in Ref. 6 that the main contribution to the flux-induced diamagnetic moment of the ring (to the current) is made by practically free electrons.

This conclusion is in complete agreement with general ideas about AB oscillations in pure metals and dielectrics (see, e.g., Refs. 7 and 8). In fact, for half filling the Hubbard model describes a dielectric in which all charge excitations have a gap Δ (Ref. 5). The amplitude of the AB oscillations in dielectrics always contains an exponential factor (of order $\exp(-L/\xi)$, where $\xi=\hbar v_F/\Delta$ is the characteristic coherence length and L is the length of the ring[7]). Therefore, in the approximation of Ref. 6, which makes it possible to take only corrections of order $1/N_a$ into account, the phenomenon of oscillations is absent. (We note that the contribution of the AB phase to the ground-state energy has also been found recently through numerical analysis.[9,10]) The same is true, of course, for an empty ($N=0$) or completely filled ($N=2N_a$) band. For other than half filling the Hubbard model with repulsion describes a metal; the spectrum of the charge carriers if gapless,[5] and their contribution to the AB oscillations becomes decisive.

Unlike the spectrum of charge excitations, the spectrum of spin excitations of the Hubbard chain with repulsion in the absence of a magnetic field remains gapless even for half filling.[11] In particular, for $N=N_a$ and $U\to\infty$ the Hubbard model with repulsion describes an isotropic ($S=1/2$) Heisenberg antiferromagnet. It may be expected, therefore, that the maximum response to an AC field will occur precisely when the band is half-filled.

The system of equations of the Bethe ansatz for the rapidities (spin quantum numbers) λ_β ($\beta=1, ..., M$) and quasimomenta (charge quantum numbers) k_j ($j=1, ..., N$) for a Hubbard chain with quasiperiodic boundary conditions has the form (this system was first obtained in Ref. 12)

$$\exp(iN_ak_j-i\alpha_1)=\prod_{\beta=1}^{M}\left[\frac{(\sin k_j-\lambda_\beta+iU)}{(\sin k_j-\lambda_\beta-iU)}\right], \tag{4}$$

$$\prod_{j=1}^{N}\left[\frac{(-\sin k_j+\lambda_\alpha+iU)}{(-\sin k_j+\lambda_\alpha-iU)}\right]\exp(-2i\alpha_s)=\prod_{\beta=1}^{M}\left[\frac{(\lambda_\alpha-\lambda_\beta+2iU)}{(\lambda_\alpha-\lambda_\beta-2iU)}\right]. \tag{5}$$

The energy of the system is equal to

$$E=-2\sum_{j=1}^{N}\cos k_j. \tag{6}$$

In the conditions of the AB (or AC) experiment the phases α_c (α_s) must be regarded as variable external parameters. Equation (4) is periodic in the fluxes of the magnetic and electric fields, with periods $\Phi_0=hc/e$ and $F_0=hc/\mu_B$, respectively. In Eq. (5) only the AC phase appears explicitly, and the smallest period, as is easily seen, is smaller by a factor of two: $F_s=F_0/2$. It is physically obvious that the spectrum as a whole and the various thermodynamic characteristics should oscillate as the fluxes change. As follows from Eqs. (4) and (5), in the general case (for arbitrary filling and arbitrary magnitude of the coupling constant U) the smallest common period of the AC oscillations is equal to F_0. We show below, however, that the anomalous periodicity (F_s) also arises in the Hubbard model in the limit of strong repulsion ($U\gg 1$) for half filling, when the model describes a Heisenberg antiferromagnet with spin $S=1/2$. This periodicity (which is, of course, a manifestation of the gauge invariance in multiply connected spaces) makes it possible, when obtaining the dependence of the ground-state energy on the fluxes, to replace the phases α_1 and $2\alpha_s$ by their fractional parts (to the nearest integer)

$$\alpha_1\to\{\{\alpha_1\}\},\quad 2\alpha_s\to\{\{2\alpha_s\}\}.$$

Therefore, at zero temperature the response of the system to a topological perturbation (on the fluxes Φ and F) will be depicted by a piecewise-linear function of the "sawtooth" type. Inclusion of a nonzero temperature should smooth the discontinuities at points corresponding to integer and half-integer parts of the periods of the AB and AC oscillations (extrema of the amplitude of the oscillations), and reduce the amplitude of the oscillations. This qualitative picture is easily reconstructed starting from general ideas about the destruction of the oscillations by temperature (see, e.g., Ref. 2).

The system of equations (4), (5) can be solved analytically in the thermodynamic limit N_a, N, $M\gg 1$ for fixed filling numbers $\nu_c=N/N_a$ and $\nu_s=M/N_a$. An explicit expression for the dependence of the ground-state energy on ν_c can be obtained for large $U\gg 1$ (the answer for the particular case $\nu_c=1$ and $\nu_s=1/2$ was obtained in Ref. 13). Calculations analogous to those performed in Ref. 14 lead to the following expression for the shift in the energy of the ground state of a Hubbard chain with repulsion in zero magnetic field for $\nu_s=\nu_c/2$:

$$\begin{aligned}\Delta E(\nu_c)=&\frac{4\pi\sin(\pi\nu_c)}{N_a}\\&\times\left[\left(1+\frac{\ln 2}{U}\left[\frac{2}{\pi}\sin(\pi\nu_c)-\nu_c\cos(\pi\nu_c)\right]\right)\right.\\&\times\left(\left\{\left\{\frac{\Phi}{\Phi_0}+\frac{F}{F_0}\right\}\right\}-\frac{1}{2}\left\{\left\{\frac{F}{F_s}\right\}\right\}\right)^2\\&\left.-\frac{1}{12}\left(1-\frac{\ln 2}{U}\nu_c\cos(\pi\nu_c)\right)\right]\\&-\frac{\pi^2}{2UN_a}\left(1-\frac{\sin(2\pi\nu_c)}{2\pi\nu_c}\right)\\&\times\left[\left\{\left\{\frac{F}{F_s}\right\}\right\}^2-\frac{1}{6}\right]+o\left(\frac{1}{N_a}\right).\end{aligned} \tag{7}$$

According to (7), for half filling ($\nu_c=1$) the dependence of the ground-state energy on the AB flux drops out, while the response to the AC field, on the contrary, becomes a maximum:

$$\Delta E(\nu_c=1)=\frac{\pi^2}{2UN_a}\left[\left\{\left\{\frac{F}{F_s}\right\}\right\}^2-\frac{1}{6}\right]. \tag{8}$$

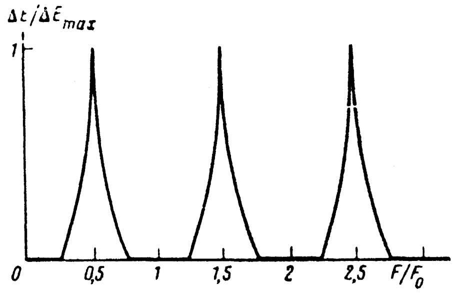

FIG. 1. Oscillating charge-related part of the ground-state energy of a Hubbard chain with repulsion versus the electric field flux (the AC phase).

In essence, Eq. (8) describes the response of a one-dimensional Heisenberg antiferromagnet of half-integer spin (with exchange constant $J = 1/2U$) to the electric field of a charged filament. In this case the appearance of twice the Bohr magneton ($2\mu_B$) in the period of the AC oscillations ($F_s = hc/2\mu_B$) is physically justified.

In fact, the magnetic moment $2\mu_B$ in our case coincides with the magnetic moment of the elementary excitations (magnons) in an antiferromagnet. Although at zero temperature there are no real excitations in the system, the interaction of the electromagnetic field with the zero-point fluctuations (virtual magnons) leads to "superconducting" oscillations in the spin subsystem.

To conclude, we shall compare the AC oscillations in metals[3] and antiferromagnets with integer[4] and half-integer spins. In normal metals the magnetic moment of the elementary excitations (conduction electrons) is equal to the Bohr magneton, and, therefore, the period of the oscillations is double the period in antiferromagnets with half-integer and integer spins. We note that for other than half filling the Hubbard model describes a metal. According to (7), the response of the "charge" sector to the AC field (for a fixed AB phase) oscillates with period F_s, as in a normal metal, but with a rather specific form of oscillation amplitude. For illustration, Fig. 1 shows the dependence of the energy on the electric-field flux F in zero magnetic field ($\Phi = 0$).

Both in a mesoscopic metallic ring and in an antiferromagnetic ring with half-integer spin at low temperatures ($T \to 0$) the amplitude of the oscillations is inversely proportional to the size of the system. We recall that such a weak dependence on the length is characteristic for one-dimensional systems with a gapless excitation spectrum. In one-dimensional antiferromagnetic chains with integer spin the magnon spectrum has a gap[15] and, therefore, the amplitude of the AC oscillations is exponentially small in the thermodynamic limit.

Analogous conclusions concerning the periods of the AB and AC oscillations can also be reached by studying the one-dimensional supersymmetric $t - J$ model, which also admits an exact solution.[16]

The "anomalous" periodicity discovered here for the AC oscillations in the Hubbard model with half filling (the doubling of the Bohr magneton in the "quantum" of electric-field flux) is, of course, normal from the point of view of magnetic systems, since the operators that change the magnitude of a site spin are quadratic in the electron second-quantization operators. This property, of course, is valid for spaces of any dimensionality.

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