Domain-wall drift in weak ferromagnets

V. S. Gerasimchuk and A. L. Sukstanskiĭ*

Makeev Engineering and Construction Institute

(Submitted 21 April 1992)
Zh. Eksp. Teor. Fiz. **103**, 151–162 (January 1993)

Domain-wall drift in weak ferromagnets in an oscillating external magnetic field is considered. The dependences of the drift velocity on the amplitude, frequency, and polarization of the field are obtained. The possibility of drift of a stripe domain structure is considered.

Principal attention is paid, in both theoretical and experimental studies of the dynamic properties of domain walls (DW) in various magnetically ordered crystals, to two main types of DW motions: 1) translational DW motion in a constant external magnetic field and 2) vibrational DW motion in an oscillating external field. The theoretical dependences of the DW velocity on the external field, the maximum steady velocities of DW, and the nonlinear regime of wall oscillations have been found for all principal magnet types (ferro-, antiferro-, weak ferro-, and ferromagnets).

Experiment[1,2] has revealed one more type of DW motion—wall drift, i.e., the onset of a constant DW velocity component, in an oscillating external magnetic field. A similar effect was observed in Refs. 3 and 4 for another topological-soliton form, viz., a Bloch line.[3,4]

Domain-wall drift in a ferromagnet (FM) was predicted from energy considerations in Refs. 5 and 6. A more consistent analysis of DW drift in FM, based on the solution of the equations of motion averaged over the period of the oscillations, was carried out in Ref. 7 and an analogous method was used[8] to analyze the drift of Bloch lines in DW. The results in Refs. 7 and 8, however, are valid only for frequencies ω substantially higher than the ferromagnetic-resonance frequency.

The drift of Bloch lines was studied in Ref. 9 by using the most adequate approach for this class of problems, a specific perturbation theory for solitons. A similar approach was used in Ref. 10 to study DW drift in FM at various polarizations of the external magnetic fields.

The present paper is devoted to a study of DW drift in another class of FM—weak ferromagnets (WFM). The nonlinear dynamics of WFM differs substantially from the dynamics in a single-sublattice FM.[11,12] In particular, the velocity limit of steady DW motion, which is determined only by exchange interactions, and the DW mobility in an external magnetic field, greatly exceed the corresponding values in FM. One should therefore expect the DW drift velocity in an oscillating field to be also substantially higher than in an FM.

We use as an example a WFM of the type of rare-earth orthoferrites (REO), the DW dynamics of which has been studied in detail both theoretically and experimentally (see, e.g., the review in Ref. 12 and the literature cited there).

1. GENERAL EQUATIONS

As shown in Refs. 11 and 13, the nonlinear macroscopic dynamics of a two-sublattice WFM can be described on the basis of a Lagrange function L expressed in terms of the antiferromagnetism unit vector $\mathbf{l}$ with $\mathbf{l}^2 = 1$. For WFM REO, characterized by a symmetry $2_x^- 2_z^-$ (the Cartesian axes x, y, and z are oriented respectively along the a, b, and c axes of the crystal), the Lagrange function $L\{\mathbf{l}\}$ can be written in the form

$$L\{\mathbf{l}\}=M_0^2\int d\mathbf{r}\left\{\frac{\alpha}{2c^2}\dot{\mathbf{l}}^2-\frac{\alpha}{2}(\nabla\mathbf{l})^2-\frac{\beta_1}{2}l_z^2-\frac{\beta_2}{2}l_y^2\right.$$
$$\left.-\frac{2}{\delta}(\mathbf{l}\mathbf{h})^2+\frac{4}{\delta g M_0}(\mathbf{h}[\dot{\mathbf{l}}\mathbf{l}])+\frac{2d}{\delta}(h_x l_z-h_z l_x)\right\},\qquad(1)$$

where M_0 is the modulus of the sublattice-magnetization vector, δ and α are respectively the homogeneous- and inhomogeneous-exchange constants, β_1 and β_2 are the anisotropy constants, g is the gyromagnetic ratio,

$$c=(gM_0)(\alpha\delta)^{1/2}/2$$

is the characteristic velocity and coincides with the minimum spin-wave phase velocity, d is the constant of the exchange-relativistic Dzyaloshinskiĭ interaction

$$\mathbf{h}=\mathbf{H}/M_0,$$

and $\mathbf{H}=\mathbf{H}(t)$ is the external magnetic field; a superior dot marks a derivative with respect to time.

Since the components of the vector $\mathbf{l}$ are connected by the relation $\mathbf{l}^2 = 1$, it is convenient to rewrite the Lagrange function (1) in terms of two independent angle variables θ and φ which parametrize the unit vector $\mathbf{l}$:

$$l_y=\cos\theta,\quad l_x+il_z=\sin\theta\, e^{i\varphi},\qquad(2)$$

$$L\{\theta,\varphi\}=M_0^2\int d\mathbf{r}\left\{\frac{\alpha}{2c^2}(\dot\theta^2+\sin^2\theta\dot\varphi^2)\right.$$
$$-\frac{\alpha}{2}[(\nabla\theta)^2+\sin^2\theta(\nabla\varphi)^2]$$
$$-\frac{\beta_1}{2}\sin^2\theta\sin^2\varphi-\frac{\beta_2}{2}\cos^2\theta$$
$$+\frac{2d}{\delta}(h_x\sin\theta\sin\varphi-h_z\sin\theta\cos\varphi)$$
$$-\frac{2}{\delta}(h_x\sin\theta\cos\varphi+h_y\cos\theta+h_z\sin\theta\sin\varphi)^2$$
$$+\frac{4}{\delta g M_0}[-h_x\theta\sin\varphi$$
$$+\dot\varphi\sin\theta\cos\theta\cos\varphi)+h_y\dot\varphi\sin^2\theta$$
$$\left.+h_z(\dot\theta\cos\varphi-\dot\varphi\sin\theta\cos\theta\sin\varphi)]\right\}.\qquad(3)$$

The dynamic stopping of the DW, due to dissipative processes, will be taken into account by using the dissipative function Q:

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$$Q=\frac{\lambda M_0}{2g}\int d\mathbf{r}(\dot\theta^2+\sin^2\theta\dot\varphi^2), \qquad (4)$$

where λ is the relaxation constant.

The equation of motion for the angle variables θ and φ take, with allowance for the relaxation terms, the form

$$\alpha\left(\Delta\theta-\frac{1}{c^2}\ddot\theta\right)+\sin\theta\cos\theta\left(\frac{\alpha}{c^2}\dot\varphi^2-\alpha(\nabla\varphi)^2-\beta_1\sin^2\varphi+\beta_2\right)$$
$$-\frac{4}{\delta}(h_x\sin\theta\cos\varphi+h_y\cos\theta+h_z\sin\theta\sin\varphi)$$
$$\times(h_x\cos\theta\cos\varphi-h_y\sin\theta$$
$$+h_z\cos\theta\sin\varphi)+\frac{4}{\delta gM_0}[\dot h_x\sin\varphi-\dot h_z\cos\varphi$$
$$+2\dot\varphi\sin^2\theta(h_x\cos\varphi$$
$$+h_z\sin\varphi)+h_y\dot\varphi\sin2\theta]$$
$$+\frac{2d}{\delta}(h_x\sin\varphi-h_z\cos\varphi)\cos\theta=\frac{\lambda}{gM_0}\dot\theta, \qquad (5)$$

$$\alpha\nabla(\nabla\varphi\sin^2\theta)-\frac{\alpha}{c^2}(\dot\varphi\sin^2\theta)^{\cdot}-\beta_1\sin^2\theta\sin\varphi\cos\varphi$$
$$-\frac{4}{\delta}(h_x\sin\theta\cos\varphi$$
$$+h_y\cos\theta+h_z\sin\theta\sin\varphi)(-h_x\sin\varphi+h_z\cos\varphi)\sin\theta$$
$$+\frac{4}{\delta gM_0}[(\dot h_x\cos\varphi$$
$$+\dot h_z\sin\varphi)\sin\theta\cos\theta-\dot h_y\sin^2\theta-h_y\dot\theta\sin2\theta$$
$$-2\dot\theta\sin^2\theta(h_x\cos\varphi$$
$$+h_z\sin\varphi)]+\frac{2d}{\delta}\sin\theta(h_x\cos\varphi+h_z\sin\varphi)$$
$$=\frac{\lambda}{gM_0}\dot\varphi\sin^2\theta. \qquad (6)$$

2. PERTURBATION THEORY. FIRST APPROXIMATION

If $\beta_2,\beta_1>0$, the vector $\mathbf{l}$ in the absence of an external field is collinear with the x axis in the homogeneous ground state. Two types of 180-degree DW can exist then in an REO far from the spin-reorientation region. The vector $\mathbf{l}$ rotates in the xz plane in one of them and in the xy plane in the other. If $\beta_2>\beta_1$, the stable DW is the one with rotation of $\mathbf{l}$ in the xz plane.[11,14] This DW corresponds to $\theta=\theta_0=\pi/2$, and the angle variable $\varphi=\varphi_0(y)$ satisfies the equation

$$\alpha\varphi_0''-\beta_1\sin\varphi_0\cos\varphi_0=0. \qquad (7)$$

(we shall assume that the magnetization distribution in the DW wall is not uniform along the Y axis; a prime denotes differentiation with respect to this coordinate). A static 180-degree DW in which the functions $\varphi_0(y)$ satisfy the boundary conditions $\varphi_0(-\infty)=0$, $\varphi_0(+\infty)=\pi$, and $\varphi_0'(\pm\infty)\to0$, are described by the relations

$$\varphi_0'=\frac{1}{y_0}\sin\varphi_0=\frac{1}{y_0}\mathrm{ch}^{-1}\left(\frac{y}{y_0}\right),\quad \cos\varphi_0=-\mathrm{th}\left(\frac{y}{y_0}\right), \qquad (8)$$

where $y_0=(\alpha/\beta_1)^{1/2}$ is the wall thickness.

We consider now the solutions of the equations of motion in an external magnetic field.

A DW moves in a constant field of definite orientation (in our case along the z axis) with a fixed velocity determined by the balance of the magnetic pressure acting on the DW and the dynamic stopping force.[11] In an oscillating field, the wall oscillates at the field frequency[15] and we shall show below that its center drifts with a certain definite velocity. In addition, the presence of the field distorts the shape of the DW.

Assuming the external-field amplitude to be small enough, we determine the drift velocity of the DW and the distortion of its form, following Refs. 9 and 10, by one of the perturbation-theory versions for solitons. To this end we introduce a collective variable $Y(t)$, which has the meaning of the coordinate of the DW center at the instant t, and seek a solution of Eqs. (5) and (6) in the form

$$\theta=\pi/2+\vartheta(\xi,t),\quad \varphi=\varphi_0(\xi)+\psi(\xi,t), \qquad (9)$$

where $\xi=y-Y(t)$. The function $\varphi_0(\xi)$ describes the motion of an undistorted DW [the structure of $\varphi_0(\xi)$ is the same as that of $\varphi_0(y)$ in the static solution (8)]. The wall drift velocity is defined as the instantaneous DW velocity

$$V(t)=\dot Y(t)$$

averaged over the oscillation period

$$V_{\mathrm{dr}}=\overline{V(t)}$$

(the bar denotes averaging over the external-field oscillation period).

We represent the functions $\vartheta(\xi,t)$ and $\psi(\xi,t)$, describing the distortion of the DW shape, as well as the wall velocity $V(t)$, by series in powers of the field amplitude, recognizing that we are interested only in stimulated DW motion:

$$\psi=\psi_1+\psi_2+\ldots,\quad \vartheta=\vartheta_1+\vartheta_2+\ldots,\quad V=V_1+V_2+\ldots, \qquad (10)$$

where the subscripts $n=1,2,\ldots$ denote the order of smallness of the quantity relative to the field amplitude ψ_n, ϑ_n, $V_n\sim h^n$.

We substitute the expansions (10) in Eqs. (5)–(6) and separate terms of different orders of smallness. Obviously, in the zeroth approximation we obtain Eq. (7), which describes a DW at rest.

The first-order perturbation-theory equation can be written in the form

$$\left(\hat L+\sigma+\frac{1}{\omega_0^2}\frac{d^2}{dt^2}+\frac{\omega_r}{\omega_0^2}\frac{d}{dt}\right)\vartheta_1$$
$$=\frac{4}{\beta_1\delta gM_0}(\dot h_x\sin\varphi_0(\xi)-\dot h_z\cos\varphi_0(\xi)), \qquad (11)$$

$$\left(\hat L+\frac{1}{\omega_0^2}\frac{d^2}{dt^2}+\frac{\omega_r}{\omega_0^2}\frac{d}{dt}\right)\psi_1=\frac{2d}{\beta_1\delta}h_x\cos\varphi_0(\xi)-\frac{4\dot h_y}{\beta_1\delta gM_0}$$
$$+\frac{1}{\beta_1}\left[\frac{\alpha}{y_0c^2}\dot V_1+\frac{\lambda V_1}{y_0gM_0}+\frac{2d}{\delta}h_z\right]\sin\varphi_0(\xi), \qquad (12)$$

where

$$\omega_0=c/y_0=gM_0(\beta_1\delta)^{1/2}/2$$

is the activation frequency of the lower spin-wave mode,

$$\omega_r=\lambda\delta gM_0/4$$

is the characteristic relaxation frequency, and

$$\sigma=(\beta_2-\beta_1)/\beta_1.$$

The operator $\hat{L}$ takes the form of a Schrödinger equation with a nonreflecting potential:

$$\hat{L}=-y_0^2\frac{d^2}{d\xi^2}+1-\frac{2}{\mathrm{ch}^2\,\xi/y_0}. \tag{13}$$

The spectrum and the wave functions of the operator $\hat{L}$ (13) are well known. It has one discrete level with eigenvalue $\lambda_0=0$ corresponding to a localized wave function

$$f_0(\xi)=\frac{1}{(2y_0)^{1/2}}\mathrm{ch}^{-1}\frac{\xi}{y_0}, \tag{14}$$

and also to a continuous spectrum

$$\lambda_k=1+k^2y_0^2,$$

corresponding to the eigenfunctions

$$f_k(\xi)=\frac{e^{ik\xi}}{b_kL^{1/2}}\left(\mathrm{th}\frac{\xi}{y_0}-iky_0\right), \tag{15}$$

where

$$b_k=(1+k^2y_0^2)^{1/2},$$

and L is the crystal length.

The functions $\{f_0, f_k\}$ form a complete orthonormalized set, and it is natural to seek the first-approximation solutions of Eqs. (11) and (12) in the form of an expansion in this set. For a monochromatic external field of frequency ω we put

$$\vartheta_1(\xi,t)=\mathrm{Re}\left\{\left[\sum_k c_kf_k(\xi)+c_0f_0(\xi)\right]e^{i\omega t}\right\}, \tag{16}$$

$$\psi_1(\xi,t)=\mathrm{Re}\left\{\left[\sum_k d_kf_k(\xi)+d_0f_0(\xi)\right]e^{i\omega t}\right\}. \tag{17}$$

One important remark is in order here. The first-approximation equation (11)–(12) describes excitation of linear spin waves against a DW background. The last term in the expansion of the function $\psi_1(\xi,t)$ corresponds to the Goldstone mode, i.e., to DW motion as a whole. The corresponding degree of freedom of the system, however, has already been taken into account by introducing the collective coordinate $Y(t)$ into the definition of the variable ξ. The Goldstone mode should therefore be left out of the expansion (17), i.e., one must put

$$d_0=0$$

(for a detailed discussion of this question see Rajaraman's book[16]). This condition leads to the requirement that the right-hand side of (12) be orthogonal to the function f_0, which determines in turn the equation for the DW velocity $V_1(t)$ in the approximation linear in the field:

$$\dot{V}_1+\frac{\lambda c^2}{\alpha gM_0}V_1=-\frac{2dc^2y_0}{\alpha\delta}h_z+\frac{2\pi c^2y_0}{\alpha\delta gM_0}\dot{h}_y. \tag{18}$$

The solution of Eq. (18) describes the DW oscillations in an oscillating external field and, as can be easily seen, does not lead to a DW drift, i.e.,

$$\overline{V_1(t)}=0.$$

If $h_y=0$, the equation agrees, apart from the notation, (in the limit of low velocities $V\ll c$) with the equation obtained for the DW velocity in Ref. 15 by a somewhat different method in the framework of the adiabatic approximation. The presence in the right-hand side of (18) of a second term not connected with the Dzyaloshinskii interaction attests to the possibility of exciting stimulated DW oscillations in a "pure" antiferromagnet in which $d=0$. This effect was first noted in Ref. 17.

The coefficients c_k, c_0, and d_k in the expansions (16) and (17) can be found in standard fashion multiplying the right-hand sides of (11) and (12) by f_k^* and f_0^* and integrating with respect to the variable ξ.

For a monochromatic external field of frequency ω, with all three components different from zero and with arbitrary phase shifts,

$$h_x=h_{0x}\cos\omega t,\quad h_y=h_{0y}\cos(\omega t+\chi_1),\quad h_z=h_{0z}\cos(\omega t+\chi), \tag{19}$$

we obtain from Eqs. (11) and (12)

$$\begin{aligned}\vartheta_1(\xi,t)&=2\mathrm{Re}\{a_1(t)\sin\varphi_0(\xi)+a_2(t)\cos\varphi_0(\xi)\},\\ \psi_1(\xi,t)&=2\mathrm{Re}\{a_3(t)\cos\varphi_0(\xi)+a_4(t)G(\xi)\}.\end{aligned} \tag{20}$$

We have introduced here the notation

$$a_1(t)=\frac{2i\omega}{\beta_1\delta gM_0}\frac{h_{0x}e^{i\omega t}}{(\sigma-q_1+iq_2)},$$

$$a_2(t)=-\frac{2i\omega}{\beta_1\delta gM_0}\frac{h_{0z}e^{i(\omega t+\chi)}}{(1+\sigma-q_1+iq_2)},$$

$$a_3(t)=\frac{d}{\beta_1\delta}\frac{h_{0x}e^{i\omega t}}{1-q_1+iq_2},$$

$$a_4(t)=-\frac{2i\omega}{\beta_1\delta gM_0}h_{0y}e^{i(\omega t+\chi_1)},$$

$$G(\xi)=\frac{y_0}{2}\int_{-\infty}^{+\infty}dk\frac{(\mathrm{th}(\xi/y_0)\sin k\xi-ky_0\cos k\xi)}{b_k^2(\lambda_k-q_1+iq_2)\,\mathrm{sh}(\pi ky_0/2)}, \tag{21}$$

where

$$q_1=(\omega/\omega_0)^2,\quad q_2=(\omega\omega_r/\omega_0^2).$$

It follows from (20) and (21) that the external-field components h_x and h_z excite bulk oscillations only with $k=0$, whereas the presence of the field component h_y makes possible excitation of bulk spin waves with $k\neq0$.

3. SECOND APPROXIMATION. DW DRIFT

We analyze now the second-approximation equations for the amplitude of the external magnetic field.

We shall not write down the pertinent equations in general form, since they are extremely cumbersome, but only an equation, averaged over the period of the oscillations, which follows from Eq. (6):

$$\hat{L}\Phi_2(\xi) = \frac{\lambda}{gM_0}\varphi_0'(\xi)V_2 + \overline{N(\xi,t)}, \qquad (22)$$

where

$$\Phi_2(\xi) = \overline{\psi_2(\xi,t)},$$

and the function $N(\xi,t)$ is defined as

$$N(\xi,t) = \left(\frac{\alpha}{c^2}\dot{V}_1 + \frac{\lambda}{gM_0}V_1\right)\psi_1' - \frac{\alpha}{c^2}V_1^2\varphi_0''$$

$$-2\alpha\varphi_0'\vartheta_1\vartheta_1' + \beta_1 \sin 2\varphi_0\psi_1^2$$

$$+\frac{2d}{\delta}(h_z\cos\varphi_0 - h_x\sin\varphi_0)\psi_1$$

$$-\frac{4}{\delta}[(h_z^2 - h_x^2)\sin\varphi_0\cos\varphi_0 + h_x h_z\cos 2\varphi_0]$$

$$-\frac{4}{\delta q M_0}[(h_x\cos\varphi_0 + h_z\sin\varphi_0)\dot{\vartheta}_1$$

$$+2(h_x\cos\varphi_0 + h_z\sin\varphi_0)\dot{\vartheta}_1]. \qquad (23)$$

The second equation of the system, which follows from Eq. (5) and defines the function $\vartheta_2(\xi,t)$, has a similar structure, but contains no second-order term in the expansion of the DW velocity (V_2) and will therefore be of no interest.

Just as in the first-approximation equation (12), we must stipulate that the expansion of the function $\Phi_2(\xi)$ in terms of the eigenfunction of the operator $\hat{L}$ contains no shear mode, i.e., it is necessary that the right-hand side of (22) be orthogonal to $f_0(\xi)$ (14). This yields an expression for the DW drift velocity $V_{dr} = V_2$:

$$V_{dr} = -\frac{gM_0y_0}{2\lambda}\int_{-\infty}^{+\infty} d\xi\,\overline{N(\xi,t)}\varphi_0'(\xi). \qquad (24)$$

Substituting the functions $\psi_1(\xi,t)$ and $\vartheta_1(\xi,t)$ (20)–(21) calculated in the preceding section in (23), and integrating over the oscillation period and integrating in (24), we obtain for the drift velocity V_{dr}

$$V_{dr} = \nu_{xz}(\omega;\chi)H_{0x}H_{0z} + \nu_{xy}(\omega;\chi_1)H_{0x}H_{0z}, \qquad (25)$$

where $\nu_{xz}(\omega:\chi)$ and $\nu_{xy}(\omega:\chi_1)$ are certain functions of the frequency and of the phase shifts, which we shall call nonlinear mobilities (NM) of the domain wall (their structure will be given below).

It follows from (25) that the DW drift occurs only if at least two components of the magnetic field differ from zero—either h_x and h_z or h_x and h_y. This fact can be interpreted in the following manner: the z or y component of the field, as follows from (18), cause DW oscillations, while the x component ensures different values of the wall's linear mobility as it moves in the positive and negative y direction. If, however, the field is oriented in the yz plane, there is no DW drift.

We consider next DW drift in a field polarized separately in the xz or xy plane.

A. Field in xz plane

The nonlinear mobility ν_{xz} which determines the drift velocity in an oscillating field polarized (in general, elliptically) in the xz plane [see (19) for $h_y = 0$) is of the form

$$\nu_{xz}(\omega;\chi) = \nu_0[D(\omega;\chi) + A(\omega;\chi)], \qquad (26)$$

where

$$\nu_0 = \pi g^2 y_0/4\omega_r,$$

$$D(\omega;\chi) = \frac{d^2}{\beta_1\delta}\frac{[(q_1-1)\cos\chi + q_2\sin\chi]}{(q_1-1)^2+q_2^2}$$

$$A(\omega;\chi) = q_1q_2\frac{q_2\cos\chi + [(q_1-\sigma)(q_1-\sigma-1)+q_2^2]\sin\chi}{[(q_1-\sigma)^2+q_2^2][(q_1-\sigma-1)^2+q_2^2]}. \qquad (27)$$

We see from (26) and (27) that the nonlinear mobility ν_{xz} is determined by two terms of different type. The first $[D(\omega;\chi)]$ is connected with the presence Dzyaloshinskii interaction in the WFM, while the second term $[A(\omega;\chi)]$ differs from zero in a "pure" AFN.

To compare the contributions of both terms at different values of the frequency and phase shift χ and to obtain a numerical estimate of the drift velocity we use the values of the parameters indicative of the typical and well-investigated WFM of REO type—yttrium orthoferrite $YFeO_3$ (see, e.g., Ref. 12):

$$\sigma\approx 2,\ y_0\approx 10^{-6}\,\text{cm},\ g\approx 1.76\cdot 10^7\,\text{s}^{-1}\cdot\text{Oe}^{-1}.$$

$$\omega_0 = c/y_0 \approx 2\cdot 10^{12}\,\text{s}^{-1},\ d^2/\beta_1\delta \sim 1.$$

The relaxation frequency ω_r can be calculated from the experimentally known linear mobility μ of a DW in a static field[11,12]

$$\omega_r = \frac{g^2y_0H_d}{\mu},$$

where H_d is the Dzyaloshinskii field. For $YeFeO_3$ we have[12]:

$$H_d = 1{,}4\cdot 10^5\,\text{Oe},\ \mu\approx 6{,}2\cdot 10^3\,\text{cm/s}\cdot\text{Oe},$$

whence[1)]

$$\omega_r = 0{,}7\cdot 10^{10}\,\text{s}^{-1}.$$

This yields an estimate of the characteristic nonlinear mobility ν_0:

$$\nu_0 = \frac{\pi}{4}\frac{\mu}{H_d} \approx 3{,}5\cdot 10^{-2}\,\text{cm/s}\cdot\text{Oe}^2. \qquad (28)$$

It follows from (28) that the characteristic DW drift velocity

$$V_0 \sim \nu_0 H_0^2$$

is much lower than the DW stationary velocity in a static field of the same strength (by an approximate factor H_0/H_d). For an oscillating field of amplitude ~ 10 Oe we obtain $V_0 \sim 3.5$ cm/s.

The DW drift velocity increases substantially at definite resonance frequencies. Consider, for example, the drift in a linearly polarized field ($\chi = 0$). From (37) we have for $\chi = 0$:

$$D(\omega;0)=\frac{d^2}{\beta_1\delta}\frac{(\omega^2/\omega_0^2-1)}{(\omega^2/\omega_0^2-1)^2+(\omega\omega_r/\omega_0^2)^2},$$

$$A(\omega;0)=\frac{\omega^2}{\omega_0^2}$$

$$\times\frac{(\omega\omega_r/\omega_0^2)^2}{[(\omega^2/\omega_0^2-\sigma)^2+(\omega\omega_r/\omega_0^2)^2][(\omega^2/\omega_0^2-1-\sigma)^2+(\omega\omega_r/\omega_0^2)^2]}. \tag{29}$$

Numerical estimates of the frequencies ω_0 and ω_r yield for all frequencies, up to optical, a value

$$q_2=(\omega\omega_r/\omega_0^2)\sim10^{-15}\omega\ll1.$$

so that the contribution of the term

$$A(\omega;\ 0)\sim q_2^2$$

to (26) is small at practically all frequencies, and the principle role is assured by the term $D(\omega;0)$ due to the Dzyaloshinskii interaction.

In the limiting case of low frequencies ($\omega\ll\omega_0$) the drift velocity is

$$V_{\rm dr}\approx-V_0=-\nu_0H_{0x}H_{0z}.$$

A negative $V_{\rm dr}$ means that the DW moves in the negative Y direction. At high frequencies ($\omega\gg\omega_0$) we have

$$V_{\rm dr}(\omega)\approx V_0\ (d^2/\beta_1\delta)(\omega_0/\omega)^2>0.$$

In the frequency region near ω_0 (activation frequency of the lower mode of the bulk spin waves) the function $D(\omega;0)$ has a singularity of the "resonance–antiresonance" type. The maximum (in absolute value) drift velocity is realized at

$$\omega=\omega_0\pm\omega_r/2$$

and reaches a value of the order of

$$(\omega_0/\omega_r)\,V_0\sim3\cdot10^2V_0.$$

Therefore even in relatively weak (~10 Oe) fields the drift velocity at resonance is of the order of 10 m/s.

The second term $A(\omega;0)$ has the two usual resonances at the frequencies

$$\omega_1=\omega_0\sigma^{1/2}$$

and

$$\omega_2=\omega_0(1+\sigma)^{1/2},$$

which coincide respectively with the activation frequencies of the localized and upper bulk spin-wave modes. At the resonances we have

$$A(\omega_{1,2};\ 0)\sim1,$$

which is comparable with $D(\omega_{1,2};0)$.

The dependence of the drift velocity on the frequency at $\chi=0$ (normalized to the characteristic value of V_0) is shown schematically in Fig. 1a.

At phase shifts $\chi(0<\chi<\pi/2)$ that differ from zero but are not too close to $\pi/2$ the frequency dependence of $V_{\rm dr}$ remains approximately the same as for $\chi=0$. The function $D(\omega;\chi)$ has as before a resonance–antiresonance behavior

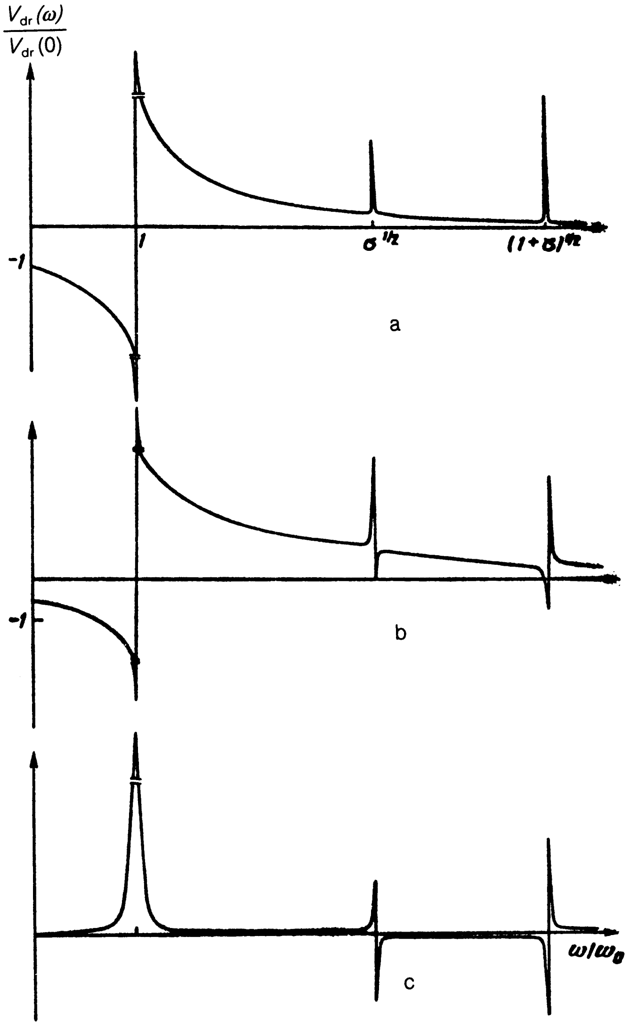

FIG. 1. Frequency dependence of domain-wall drift velocity at various values of the phase shift χ: $\chi=0$ (a), $\pi/3$ (b), $\pi/2$ (c).

near $\omega=\omega_0$, but it becomes asymmetric: the negative amplitude

$$-(\omega_0/2\omega_r)(1-\sin\chi)$$

decreases with increase of χ, but the positive

$$(\omega_0/2\omega_r)(1+\sin\chi)$$

increases.

If $\chi\neq0$ the function $A(\omega;\chi)$ changes at $\omega=\omega_{1,2}$ changes from resonant to resonant-antiresonant. The amplitude values of the function $A(\omega;\chi)$ at frequencies close to $\omega_1=\omega_0\sigma^{1/2}$ are equal to

$$\sigma(\cos\chi\pm1)/2\sim1,$$

which is much less than the amplitude of the function $D(\omega;\chi)$ at $\omega\approx\omega_0$. These values, however are comparable with and may even exceed the value $D(\omega_1;\chi)$, which decreases with increase of χ:

$$D(\omega_1,\ \chi)=D(\omega_1;\ 0)\cos\chi.$$

At sufficiently large χ the contribution of the antiferromagnetic term $A(\omega;\chi)$ can therefore make the DW drift velocity negative in a narrow frequency interval, of order ω_r near $\omega = \omega_1$.

The picture is similar also at frequencies close to

$$\omega_2=\omega_0(1+\sigma)^{1/2}.$$

A typical dependence of the drift velocity on the frequency for elliptic polarization of the magnetic field in the *xz* plane at $\chi = \pi/3$ is shown in Fig. 1b.

If the phase shift is equal to $\pi/2$ the behavior of the function $D(\omega;\pi/2)$ becomes purely resonant (the negative peak vanishes), and the amplitude at the maximum is double the corresponding value for $\chi = 0$. The function $A(\omega;\pi/2)$ exhibits a symmetric resonance–antiresonance behavior with amplitudes of order 1. The two terms of (25) are of the same order outside the resonance regions and are small:

$$D\sim A\sim q_2\ll 1.$$

A plot of $V_{\rm dr}(\omega)$ for $\chi = \pi/2$ is shown in Fig. 1c.

B. Field in *xy* plane

The nonlinear mobility $v_{xy}(\omega;\chi)$ in the case of an oscillating field (19) is given for $h_z = 0$ by

$$v_{xy}(\omega;\chi_1)=-v_0\frac{\pi}{4}\left(\frac{d}{(\beta_1\delta)^{1/2}}\right)\frac{\omega}{\omega_0}\mathrm{Im}\left\{e^{i\chi_1}\left(\frac{1+P_2}{1-q_1+iq_2}+P_1\right)\right\}, \tag{30}$$

where the functions $P_n = P_n(\omega)$, $n = 1, 2$, are defined by the integrals

$$P_n(\omega)=\frac{2}{\pi}\int_0^{+\infty}\frac{x^{2n}\,dx}{(1+x^2)(1+x^2-q_1+iq_2)\,\mathrm{sh}^2(\pi x/2)}. \tag{31}$$

Using the approximation

$$P_n(\omega)\approx\eta_n/(1-q_1+iq_2),$$

where η_n are certain constants of the order of unity, we obtain from (30):

$$v_{xy}(\omega;\chi_1)\approx-v_0\frac{\pi}{4}\left(\frac{d}{(\beta_1\delta)^{1/2}}\right)\frac{\omega}{\omega_0}\times\left[\frac{\cos\chi_1(1-\eta_1)q_2+\sin\chi_1[(1+\eta_1)(1-q_1)+\eta_2]}{(q_1-1)^2+q_2^2}\right]. \tag{32}$$

It follows from (32) that the frequency dependence of the nonlinear mobility v_{xy} differs somewhat from $v_{xz}(\omega)$ [Eqs. (26) and (27)]. First, $v_{xy}\sim\omega$ and vanishes as $\omega\to 0$; second, at high frequencies we have $v_{xy}\sim\omega^{-1}$ in place of $v_{xz}\sim\omega^{-2}$. The resonant properties of $v_{xy}(\omega;\chi_1)$ are similar to the corresponding properties of the function $A(\omega;\chi)$ (27). In the case of a linearly polarized field ($\chi_1 = 0$) we have at the frequency $\omega = \omega_0$ the usual resonance with amplitude of the order of $v_0\omega_0/\omega_r$ at the maximum. In an elliptically polarized field ($\chi_1\neq 0$ the function $v_{xy}(\omega;\chi_1)$ has an asymmetric resonance–antiresonance behavior, which becomes symmetric at $\chi_1 = \pi/2$. The maximum amplitudes of the drift velocity in fields of the order of 10 Oe are ~ 10 m/s, just as in the previously considered field in the *xz* plane.

4. DRIFT OF A STRIPE-DOMAIN STRUCTURE

We consider now the possibility of a drift in an external alternating magnetic field with a stripe-domain structure (SDS), consisting of 180-deg DW. It must be borne in mind here that neighboring DW in the DS have opposite topological charges determined by the boundary conditions of Eq. (7). In addition, the rotation of the vector **l** in various DW can be about either a positive or a negative direction of the *z* axis. These two factors determine the DW drift direction in a field of fixed frequency ω and a phase shift χ (or χ_1). An SDS drift is possible, naturally, only when neighboring DW move in one and the same direction.

We define the topological charge $R = \pm 1$ of the DW and the parameter $\rho = \pm 1$ that describes the rotation of the vector **l** in a DW as follows:

$$l_x(\pm\infty)=\mp R,\quad l_z(y=0)=\pm\rho.$$

The domain walls considered in the preceding section, having a magnetization (8), correspond to $R=\rho= +1$. In the general case we have in lieu of (8)

$$\varphi_0'=\frac{1}{y_0}R\sin\varphi_0=\frac{1}{y_0}R\rho\,\mathrm{ch}^{-1}\frac{y}{y_0},\qquad \cos\varphi_0=-R\,\mathrm{th}\frac{y}{y_0}. \tag{33}$$

Analysis shows that in the general case the drift velocity of DW with the given values of the parameters R and ρ is determined by an equation similar to (25)

$$V_{\rm dr}=B\rho v_{xz}(\omega;\chi)H_{0z}H_{0z}+Rv_{xy}(\omega;\chi_1)H_{0x}H_{0y}, \tag{34}$$

where the nonlinear mobilities v_{xz} and v_{xy} are described as before by (26) and (30).

We see thus from (33) that SDW drift in a field polarized in the *xy* plane is altogether impossible, since the topological charges R of neighboring DW are different. In a field polarized in the *xz* plane, SDW drift is possible, but provided that neighboring DW have different values of the parameter ρ and of the topological charge, i.e., the rotation of the vector **l** in neighboring DW must be in opposite directions. A similar situation (possibility of SDW drift in a field polarized in the DW plane) obtains also in ferromagnets.[10]

*Donetsk Physico-Technical Institute of the Ukrainian Academy of Sciences.

[1)] This value of relaxation frequency corresponds to a dimensionless relaxation constant $\lambda\sim 10^{-3}$.

[1] Yu. N. Dragoshanskiĭ, E. B. Khan, and V. A. Zaikova, Fiz. Met. Metallogr. **39**, 289 (1975).
[2] V. K. Vlasko-Vlaskov, L. M. Dedukh, V. I. Nikitenko, and L. S. Uspenskaya, Fiz. Tverd. Tela (St. Petersburg) **24**, 1255 (1982) [Sov. Phys. Solid State **24**, 714 (1982)].
[3] V. S. Gornakov, L. M. Dedukh, and V. I. Nikitenko, Zh. Eksp. Teor. Fiz. **86**, 1505 (1984) [Sov. Phys JETP **59**, 881 (1984)].
[4] L. M. Deduch, V. S. Gornakov, and V. I. Nikitenko, Phys. Stat. Sol. **75**, 117 (1983).
[5] E. Schlomann and J. D. Milne, IEE Trans. Mag-10, 791 (1974).
[6] E. Schlomann, *ibid.* Mag-11, 1051 (1975).
[7] V. M. Eleonskii, A. K. Zvezdin, and V. G. Redko, Fiz. Met. Metallov. **43**, 7 (1977).
[8] V. S. Gerasimchuk, Fiz. Tverd. Tela (St. Petersburg) 32, 391 (1990) [Sov. Phys. Solid State **32**, 225 (1990)].
[9] S. V. Iordanskii and V. I. Marchenko, Zh. Eksp. Teor. Fiz. **91**, 1867 (1986) [Sov. Phys. JETP **64**, 1105 (1986)].
[10] V. G. Bar'yakhtar, Yu. I. Gorobets, and S. I. Denisov, *ibid.* **98**, 1345 (1990) [**71**, 751 (1991)].
[11] V. I. Bar'yakhtar, B. A. Ivanov, and A. L. Sukstanskiĭ, *ibid.* **78**, 15 (1980) [*sic*].

[12] V. G. Bar'yakhtar, B. A. Ivanov, and M. V. Chetkin, Usp. Fiz. Nauk **146**, 417 (1985) [Sov. Phys. Usp. **28**, 563 (1985)].
[13] A. F. Andreev and V. I. Marchenko, *ibid.* **130**, 39 (1980) [**23**, 21 (1980)].
[14] A. L. Sukstanskiĭ, Fiz. Tverd. Tela (St. Petersburg) **27**, 3509 (1985) [Sov. Phys. Solid State **27**, 2119 (1985)].
[15] V. G. Bar'yakhtar, V. A. Ivanov *et al.*, Pis'ma Zh. Eksp. Teor. Fiz. **37**, 35 (1983) [JETP Lett. **37**, 41 (1983)].
[16] R. Rajraman, *Solitons and Instantons in Quantum Theory*, (Russ. transl.) Mir, 1985.
[17] V. G. Bar'yakhtar, V. A. Ivanov, V. F. Lapchenko, and V. A. L'vov, Fiz. Nizk. Temp. **13**, 551 (1987) [Sov. J. Low Temp. Phys. **13**, 312 (1987)].

Translated by J. G. Adashko