Spectroscopic method for determining the polarization state of oscillating electric fields in plasmas

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Analytic expressions are derived for the intensities of the satellites of dipole-forbidden spectral lines of non-hydrogen-like atoms in a quasimonochromatic oscillating electric field. These expressions are valid for an arbitrary polarization of the emitted photons and for an arbitrary polarization state of the field. On the basis of these expressions, a spectroscopic method is proposed for determining the polarization state of an oscillating electric field in a plasma.

1. INTRODUCTION

Oscillating electric fields play an important role in many processes which occur in plasmas. Such fields may be the fields of electromagnetic radiation penetrating into a plasma or the fields of "natural" plasma vibrations (e.g., a plasma turbulence). Intense oscillating electric fields in plasmas are characteristic of many experimental situations, including those in which intense charged-particle beams pass through plasmas, magnetic field lines reconnect, shock waves are excited, and intense laser or microwave beams are applied to plasmas.

Spectroscopic methods based on modifying the emission spectra of atoms or ions by means of oscillating electric fields are widely used for the diagnostics of oscillating electric fields in plasmas (see Refs. 1–4 and the papers cited there). One of the most important of these methods is based on the appearance of satellites of dipole-forbidden spectral lines in the emission spectrum of non-hydrogen-like radiators on account of the oscillating electric field. This method was proposed in Ref. 5 and has since then been used in various modifications in many experiments, in particular, in Refs. 6–14. On the other hand, this "satellite method," like many other spectroscopic methods for the diagnostics of the oscillating electric fields in plasmas, is oriented primarily toward measurements of the strengths of these fields, and it requires certain *a priori* information on the polarization state of these fields. For example, by using the results of Ref. 5 one can measure the mean square strength of oscillating electric fields in plasmas under the assumption that the oscillating fields are an isotropic high-frequency noise with uncorrelated components in different directions. If the oscillating fields are linearly polarized, the results of Ref. 7 can be used for the diagnostics.[7]

In this paper we examine the polarization of the satellites of forbidden spectral lines of nonhydrogen-like atoms as a function of the polarization characteristics of the oscillating electric field acting on these atoms. Using the analytic expressions derived below, we propose a method for determining the polarization state of an oscillating electric field in a plasma. We wish to stress that a determination of the polarization characteristics of an oscillating field also makes it possible to correctly measure the mean square strength of the oscillating field, by making use of the intensities of satellites of forbidden spectral lines. Another important feature of the present paper is that the three-level scheme proposed in Ref. 5 is used for an analytic study of all possible types of relations among the angular-momentum quantum numbers J of the atomic shell, corresponding to the three "working" energy levels of the nonhydrogen-like atoms. Previous studies of satellites of forbidden spectral lines of nonhydrogen-like atoms (e.g., Refs. 5–15) have generally used the Coulomb approximation for the wave functions of the atomic levels. In that approximation the wave functions are characterized by the principal quantum number n, the orbital quantum number l, and the magnetic quantum number m. This approximation substantially shortens the list of emitting atoms which can be used for the diagnostics of oscillating electric fields.

In the present paper we take the oscillating electric field to be a quasimonochromatic electric field of arbitrary form

$$\mathbf{E}(t) = \mathbf{E}_0(t)\exp(-i\omega t) + \mathbf{E}_0^*(t)\exp(i\omega t), \tag{1}$$

where the asterisk means complex conjugation. In Eq. (1) it is assumed that the complex amplitude $\mathbf{E}_0(t)$ varies much more slowly than $\exp(\pm i\omega t)$ as a function of the time. The Cartesian (x, y, and z) components of the vector $\mathbf{E}_0(t)$ can be arbitrary low-frequency functions of the time. If the field $\mathbf{E}(t)$ in (1) is the field of a plane wave, Stokes parameters used to describe its polarization state (see, for example, §50 in Ref. 16). The Stokes parameters are generalized to waves or arbitrary shape in Ref. 17.

2. CALCULATION OF THE POLARIZATION AND INTENSITY OF THE SATELLITES OF FORBIDDEN SPECTRAL LINES

In the energy spectrum of the atom we distinguish a system of three levels, 0, 1, and 2. Levels 1 and 2 constitute a pair of closely spaced upper levels, while 0 is a distant lower level, to which a radiative transition occurs (Fig. 1). We denote by $\varphi_{pJ_pM_p}$ the wave function of the state corresponding by level p, while ε_p is the energy of level p. Here J_p is the quantum number of the total angular momentum of the atomic shell, and M_p is the quantum number of the z projection of the angular momentum of the shell. We assume that the only nonzero matrix elements for the electric dipole moment d in level system 0, 1, 2 are those between the 1 and 2 states and between the 0 and 2 states:

$$\langle 1|\mathbf{d}|2\rangle \neq 0, \quad \langle 0|\mathbf{d}|2\rangle \neq 0.$$

We assume that a quasimonochromatic oscillating elec-

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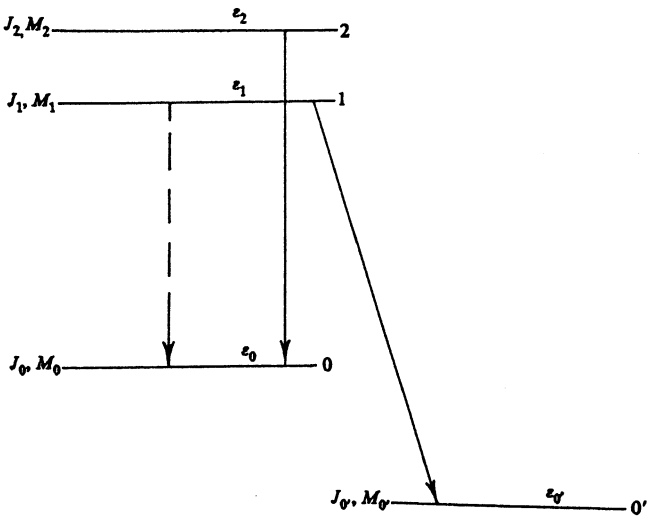

FIG. 1. Energy-level diagram of the non-hydrogen-like atom. The two closely spaced upper levels, 1 and 2, are coupled by a dipole matrix element. The solid arrows are allowed electric dipole transitions to the distant lower levels 0 and 0'. The dashed arrow is a transition which is forbidden in the dipole approximation in the absence of an external electric field.

tric field $\mathbf{E}(t)$ as in (1) acts on the two-level system 1,2. Under the assumption that the operator representing the electric dipole interaction of the atom with the field in (1) is of the form $V = -\mathbf{dE}(t)$, we can easily derive the following expression for the wave function $\Psi_{1J_1M_1}(t)$ of level 1 in the field $\mathbf{E}(t)$ in (1) from the Schrödinger equation in first-order time-varying perturbation theory (here and below we are using atomic units: $\hbar = m_e = e = 1$):

$$\Psi_{1J_1M_1}(t) = \varphi_{1J_1M_1}\exp(-i\varepsilon_1 t) + \sum_{M_2=-J_2}^{J_2} \varphi_{2J_2M_2}\left\{\frac{\langle\varphi_{2J_2M_2}|\mathbf{dE}_0(t)|\varphi_{1J_1M_1}\rangle}{\varepsilon_{21}-\omega} \times\exp[-i(\varepsilon_1+\omega)t] + \frac{\langle\varphi_{2J_2M_2}|\mathbf{dE}_0^*(t)|\varphi_{1J_1M_1}\rangle}{\varepsilon_{21}+\omega} \times\exp[-i(\varepsilon_1-\omega)t]\right\}, \qquad (2)$$

where $\varepsilon_{kk'} \equiv \varepsilon_k - \varepsilon_{k'}$. Expression (2) is valid under the condition

$$|\langle\psi_{2J_2M_2}|\mathbf{dE}_0(t)|\varphi_{1J_1M_1}\rangle/(|\varepsilon_{21}|-\omega)| \ll 1. \qquad (3)$$

The spectrum of the electric dipole radiation for the $1\to 0$ transition can be written in the form (see, for example, Chap. VII of Ref. 18)

$$I^{(\mathbf{e})}(\Delta\omega) = \sum_{M_0=-J_0}^{J_0}\sum_{M_1=-J_1}^{J_1}\lim_{T\to\infty}\frac{1}{2\pi T} \times\left|\int_0^T \exp(-i\Delta\omega t)\langle\Psi_{1J_1M_1}(t)|\mathbf{de}|\varphi_{0J_0M_0}\exp(-i\varepsilon_0 t)\rangle dt\right|^2, \qquad (4)$$

where $\mathbf{e}$ is the unit polarization vector of the emitted photons. In (4) we are assuming that the Zeeman states of level 1 with various values of M_1 are equally populated. Since there are terms in (2) which contain the expression

$$\varphi_{2J_2M_2}\exp[-i(\varepsilon_1\pm\omega)t],$$

and since the spectral width of the function $\mathbf{E}_0(t)$ is considerably smaller than ω, we find that in the case $\mathbf{E}_0\neq 0$ the emission spectrum corresponding to the $1\to 0$ transition contains two satellites of the forbidden $1\to 0$ spectral line, which are at distances $\pm\omega$ from the frequency of the $1\to 0$ transition. Satellites of this sort are shown schematically in Fig. 2. Our problem is to determine the polarization characteristics of these satellites as a function of the polarization state of the field $\mathbf{E}(t)$ in (1).

Substituting (2) into (4), we easily find the following expressions for the satellite intensities $S_{1,2}^{(\mathbf{e})}$:

$$\hat{S}_1^{(\mathbf{e})} = \frac{1}{(\varepsilon_{21}-\omega)^2} \times\sum_{M_0=-J_0}^{J_0}\sum_{M_1=-J_1}^{J_1}\left\{\left|\sum_{M_2=-J_2}^{J_2}\langle\varphi_{1J_1M_1}|\mathbf{dE}_0^*(t)|\varphi_{2J_2M_2}\rangle \times\langle\varphi_{2J_2M_2}|\mathbf{de}|\varphi_{0J_0M_0}\rangle\right|^2\right\}_{av}, \qquad (5)$$

$$\hat{S}_2^{(\mathbf{e})} = \frac{1}{(\varepsilon_{21}+\omega)^2}\sum_{M_0=-J_0}^{J_0}\sum_{M_1=-J_1}^{J_1}\left\{\left|\sum_{M_2=-J_2}^{J_2}\langle\varphi_{1J_1M_1}|\mathbf{dE}_0(t)|\varphi_{2J_2M_2}\rangle \times\langle\varphi_{2J_2M_2}|\mathbf{de}|\varphi_{0J_0M_0}\rangle\right|^2\right\}_{av}, \qquad (6)$$

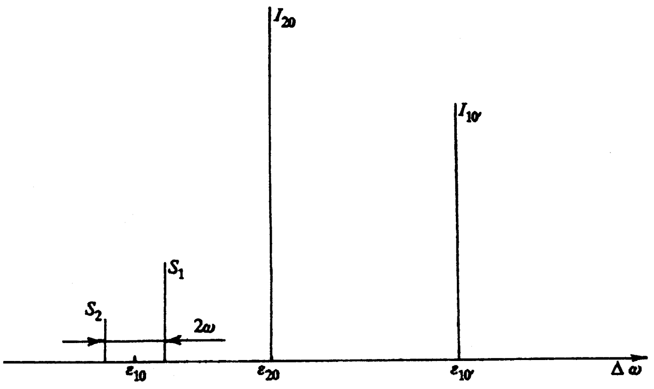

FIG. 2. Emission spectrum of a non-hydrogen-like atom in a quasimonochromatic electric field $\mathbf{E}(t)$ as in (1). The spectral components at the frequencies $\Delta\omega = \varepsilon_{20}$ and $\Delta\omega = \varepsilon_{10}$ correspond to allowed electric dipole transitions $2\to 0$ and $1\to 0'$. The satellites S_1 and S_2 arise at frequencies $\pm\omega$ with respect to the dipole-forbidden spectral line $1\to 0$ as a result of the field $\mathbf{E}(t)$ in (1).

where $\{...\}_{av}$ means the time average

$$\{f(t)\}_{av}=\lim_{T\to\infty}\frac{1}{T}\int_0^T f(t)dt,$$

and the caret in $\hat{S}_p^{(e)}$ means the intensity of the satellite $S_p^{(e)}$. The satellite $S_1^{(e)}$ is emitted at the frequency $\Delta\omega=\varepsilon_{10}+\omega$, and $S_2^{(e)}$ at $\Delta\omega=\varepsilon_{10}-\omega$. Below we will be concerned primarily with $\hat{S}_1^{(e)}$ alone, since the expression for $\hat{S}_2^{(e)}$ can be found from that for $\hat{S}_1^{(e)}$ by making the substitutions $\omega\to-\omega$, $\mathbf{E}_0^*\to\mathbf{E}_0$, $\mathbf{E}_0\to\mathbf{E}_0^*$, according to (5) and (6).

Using the Wigner–Eckart theorem (§14 in Ref. 19), we find the following expression for the intensity $\hat{S}_1^{(e)}$ from (5):

$$\hat{S}_1^{(e)}=(\varepsilon_{21}-\omega)^{-2}|(1,J_1\|d\|2,J_2)(2,J_2\|d\|0,J_0)|^2\times\sum_{M_0=-J_0}^{J_0}\sum_{M_1=-J_1}^{J_1}\times\left\{\left|\sum_{M_2=-J_2}^{J_2}\sum_{\nu,\tilde{\nu}=-1}^{1}(-1)^{M_2}\begin{pmatrix}J_1 & 1 & J_2\\ -M_1 & \nu & M_2\end{pmatrix}\times\begin{pmatrix}J_2 & 1 & J_0\\ -M_2 & \tilde{\nu} & M_0\end{pmatrix}(\varkappa_\nu^*\mathbf{E}_0^*(t))(\varkappa_{\tilde{\nu}}^*\mathbf{e})\right|^2\right\}_{av}, \qquad (7)$$

where $(pJ\|d\|p'J')$ is a reduced matrix element, $\begin{pmatrix}j_1 & j_2 & j\\ m_1 & m_2 & m\end{pmatrix}$ is the Wigner 3-j symbol, and $\varkappa_\nu$ are the unit vectors of the spherical basis. From the properties of the 3-j symbols (§13 of Ref. 19, for example) we know that the intensity $\hat{S}_1^{(e)}$ is nonzero in only the following nine cases:

$$\begin{aligned}&1)\ J_2=J_1+1=J_0-1,\quad 2)\ J_2=J_1+1=J_0,\\ &3)\ J_2=J_1+1=J_0+1,\quad 4)\ J_2=J_1=J_0-1,\\ &5)\ J_2=J_1=J_0,\quad 6)\ J_2=J_1=J_0+1,\\ &7)\ J_2=J_1-1=J_0-1,\quad 8)\ J_2=J_1-1=J_0,\\ &9)\ J_2=J_1-1=J_0+1.\end{aligned} \qquad (8)$$

In the fourth, fifth, and sixth of these cases we have $J_1\neq0$, and in the eight we have $J_1\neq1$.

We assume that the vectors $\mathbf{E}_0(t)$ and $\mathbf{e}$ are of the form

$$\mathbf{E}_0(t)=\sum_{k=1}^{3}E_{0k}(t)\mathbf{e}_k,\quad \mathbf{e}=\sum_{k=1}^{3}\alpha_k\mathbf{e}_k,$$

where $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ are unit vectors along the x, y, z axes, respectively. From expression (7), we then find a universal formula[1)] for the intensity $\hat{S}_1^{(e)}$ i.e., one which is valid for all nine possible relations among the quantum numbers J_0, J_1, J_2 [see (8)]:

$$\hat{S}_1^{(e)}=\frac{|(1,J_1\|d\|2,J_2)(2,J_2\|d\|0,J_0)|^2}{30(\varepsilon_{21}-\omega)^2\Delta}\sum_{k,k'=1}^{3}\{\{|E_{0k}(t)|^2\}_{av}\times[a_1|\alpha_k|^2\delta_{kk'}+a_2|\alpha_{k'}|^2(1-\delta_{kk'})]-\{E_{0k}(t)E_{0k'}^*(t)\}_{av}(1-\delta_{kk'})\times(a_3\alpha_k\alpha_{k'}^*+a_4\alpha_k^*\alpha_{k'})\}, \qquad (9)$$

where $\delta_{kk'}$ is the Kronecker delta.

We can now write expressions for the coefficients a_p ($p=1,...,4$) and Δ for the possible relations among J_0, J_1, J_2 specified in (8) [we assume $J_1=J$ everywhere in (10)]:

$$\begin{aligned}&1)\ J_2=J+q=J_0-q\quad(q=\pm1):\\ &\Delta=2(J+q)+1,\quad a_1=2a_4=4,\quad a_2=-a_3=3;\\ &2)\ J_2=J+q=J_0+q\quad(q=\pm1):\\ &\Delta=2^{-1}[(2J+1)(2J+1+q)(2J+1+2q)],\\ &a_1=2[(2J+1+q)^2+1],\\ &a_2=2^{-1}(2J+1-q)(6J+3+4q),\\ &a_3=(2J+3+q)(2J-1+q),\\ &a_4=-2^{-1}(6J+3+2q)(2J+1+3q);\\ &3)\ J_2=J+q=J_0+r\quad(q=0,\pm1;r=0,\pm1;q^2+r^2=1):\\ &\Delta=2^{-1}[(2J+q-r+1)(2J+2q+1)],\\ &a_1=2a_4=2J-q-3r+1,\quad a_2=4J+3q-r+2,\\ &a_3=2^{-1}(2J+9q+7r+1);\\ &4)\ J_2=J=J_0:\\ &\Delta=J(J+1)(2J+1),\\ &a_1=2(3J^2+3J-1),\quad a_2=-a_4=2J^2+2J+1,\\ &a_3=-2(J^2+J-2).\end{aligned} \qquad (10)$$

In order to measure the absolute intensity of the oscillating electric field we need to compare the satellite intensities $S_{1,2}^{(e)}$ with the intensity of the dipole-allowed spectral lines. These dipole-allowed lines may have as upper level either level 2 (the $2\to0$ transition is shown in Fig. 1) or level 1 (an arrow in Fig. 1 shows the transition $1\to0'$, where $0'$ is a distant lower energy level which is not level 0). Ignoring the effect of the field $\mathbf{E}(t)$ in (1) on the intensities of the dipole-allowed spectral lines under condtion (3), and assuming that the Zeeman states of levels 2 and 1 are equally populated, we find the following expressions for the intensities of the allowed $2\to0$ and $1\to0'$ spectral lines, respectively:

$$\begin{aligned}&I_{20}^{(e)}=3^{-1}|(2,J_2\|d\|0,J_0)|^2,\\ &I_{10'}^{(e)}=3^{-1}|(1,J_1\|d\|0',J_{0'})|^2.\end{aligned} \qquad (11)$$

The reduced dipole matrix elements in (9) and (11) are calculated by standard methods; the particular type of coupling of the angular momenta in the atoms is taken into account (see, for example, §31 in Ref. 19).

To compare the results of the present paper with existing results,[5,7,15] we consider some particular cases which follow from the general relations (9)–(11). We first note that the hydrogen-like functions φ_{nlm} were used for the wave functions of the atomic energy levels in Refs. 5, 7, and 15. The selection rule in terms of l for the electric dipole transitions in $\Delta l = \pm 1$. We can thus make a comparison with the results of Refs. 5, 7, and 15 only for cases 1), 3), 7), and 9) in (8).

The ratio ($\mathscr{S}$) of the intensity of the satellites of the forbidden spectral line to the intensity of an allowed spectral line was calculated in Ref. 5 for the situation in which the oscillating electric field has an isotropic directional pattern with uncorrelated components in different directions. In this case we have

$$\{E_{0k}(t)E^*_{0k'}(t)\}_{av} = 6^{-1}\langle E_p^2\rangle\delta_{kk'}, \qquad (12)$$

where $\langle E_p^2\rangle$ is the time average of the square strength of the oscillating electric field (this notation was introduced in Ref. 5). Substituting (12) into (9), and using (10) and (11), we easily find expression (1) of Ref. 5 for the ratio $\mathscr{S}$ under the assumption that levels 1 and 2 are populated in accordance with their statistical weights.[2] The linear polarization of satellites of forbidden spectral lines of nonhydrogen-like atoms in a linearly polarized oscillating electric field was analyzed in Ref. 7. Two cases were considered there: $l' = l \pm 2$. Let us assume that the field $\mathbf{E}(t)$ in (1) is directed along the z axis and that the satellites with polarizations $\mathbf{e}_1$ and $\mathbf{e}_3$ are observed along the y axis. For the parameter

$$P = [\hat{S}_q^{(\mathbf{e}_3)} - \hat{S}_q^{(\mathbf{e}_1)}]/[\hat{S}_q^{(\mathbf{e}_3)} + \hat{S}_q^{(\mathbf{e}_1)}] \quad (q = 1, 2),$$

which is a measure of the degree of linear polarization of the satellites along the direction of $\mathbf{E}(t)$, we then find $P = 1/7$ from (9) and (10). This result agrees with the result of Ref. 7. Finally, from (9)–(11) we can easily find the results of Ref. 15, where a study was made of the linear polarization of the satellites of forbidden spectral lines of non-hydrogen-like atoms in an elliptically polarized oscillating electric field.

3. METHOD FOR DETERMINING THE POLARIZATION STATE OF THE ELECTRIC FIELD

From (9)–(11), we can find all nine elements of the tensor

$$\sigma_{kk'} = \{E_{0k}(t)E^*_{0k'}(t)\}_{av}, \qquad (13)$$

which determines the polarization state and energy density of the oscillating field. For this purpose we must measure $S_1^{(\mathbf{e})}$ (or $S_2^{(\mathbf{e})}$), the intensity ratio of a satellite to one of the allowed[3] spectral lines ($1-0'$ or $2\to0$), for the nine different polarization vectors $\mathbf{e}$ of the emitted photons, recording the emission spectrum along three noncoplanar axes. Here is one possible set of polarization vectors $\mathbf{e}$:

$$\begin{gathered}\mathbf{e}_{\mathrm{I}} = \mathbf{e}_1, \quad \mathbf{e}_{\mathrm{II}} = \mathbf{e}_2, \quad \mathbf{e}_{\mathrm{III}} = \mathbf{e}_3, \quad \mathbf{e}_{\mathrm{IV}} = 2^{-1/2}(\mathbf{e}_1 + \mathbf{e}_2),\\ \mathbf{e}_{\mathrm{V}} = 2^{-1/2}(\mathbf{e}_1 + i\mathbf{e}_2), \quad \mathbf{e}_{\mathrm{VI}} = 2^{-1/2}(\mathbf{e}_1 + \mathbf{e}_3),\\ \mathbf{e}_{\mathrm{VII}} = 2^{-1/2}(\mathbf{e}_1 + i\mathbf{e}_3), \quad \mathbf{e}_{\mathrm{VIII}} = 2^{-1/2}(\mathbf{e}_2 + \mathbf{e}_3),\\ \mathbf{e}_{\mathrm{IX}} = 2^{-1/2}(\mathbf{e}_2 + i\mathbf{e}_3).\end{gathered} \qquad (14)$$

The vectors $\mathbf{e}_{\mathrm{I}}, \mathbf{e}_{\mathrm{II}}, \mathbf{e}_{\mathrm{III}}, \mathbf{e}_{\mathrm{IV}}, \mathbf{e}_{\mathrm{VI}}, \mathbf{e}_{\mathrm{VIII}}$ correspond to linear polarization of the emitted photons, while the vectors $\mathbf{e}_{\mathrm{V}}, \mathbf{e}_{\mathrm{VII}}, \mathbf{e}_{\mathrm{IX}}$ correspond to circular polarization. The spectrum of emission with the polarization vectors $\mathbf{e}_{\mathrm{I}}, \mathbf{e}_{\mathrm{II}}, \mathbf{e}_{\mathrm{IV}}, \mathbf{e}_{\mathrm{V}}$ can be observed along the z axis; the spectrum with $\mathbf{e}_{\mathrm{III}}, \mathbf{e}_{\mathrm{VI}}, \mathbf{e}_{\mathrm{VII}}$ can be observed along the y axis; and that with $\mathbf{e}_{\mathrm{VIII}}$ and $\mathbf{e}_{\mathrm{IX}}$ can be observed along the x axis. To find $\sigma_{kk'}$ in (13), we should first measure the emission spectrum with $\mathbf{e}_{\mathrm{I}}, \mathbf{e}_{\mathrm{II}}, \mathbf{e}_{\mathrm{III}}$; we would then be in a position to determine the quantities $\{|E_{0k}(t)|^2\}_{av}$ ($k = 1,2,3$), which characterize the energy density of the oscillations of the oscillating field along the x, y, z axes. Furthermore, by measuring the spectrum with $\mathbf{e}_{\mathrm{IV}}, \mathbf{e}_{\mathrm{V}}$, we can determine the quantities $\mathrm{Re}\{E_{01}(t)E^*_{02}(t)\}_{av}$, $\mathrm{Im}\{E_{01}(t)E^*_{02}(t)\}_{av}$. Correspondingly, by measuring the spectrum with the polarization $\mathbf{e}_{\mathrm{VI}}, \mathbf{e}_{\mathrm{VII}}$ and $\mathbf{e}_{\mathrm{VIII}}, \mathbf{e}_{\mathrm{IX}}$, we can determine the quantities $\mathrm{Re}\{E_{01}(t)E^*_{03}(t)\}_{av}$, $\mathrm{Im}\{E_{01}(t)E^*_{03}(t)\}_{av}$, and $\mathrm{Re}\{E_{02}(t)E^*_{03}(t)\}_{av}$, $\mathrm{Im}\{E_{02}(t)E^*_{03}(t)\}_{av}$, respectively.

At this point we assume that the experimental conditions are such that we can measure the spectrum only along some single axis, say the z axis. In this case, by measuring the spectrum with the polarizations $\mathbf{e}_{\mathrm{I}}$ and $\mathbf{e}_{\mathrm{II}}$, we can determine the quantities

$$\begin{aligned} f_1 &= \varepsilon^2 + \{|E_{03}(t)|^2\}_{av}(a_2 - a_1)/(a_1 + a_2),\\ f_2 &= \{|E_{01}(t)|^2\}_{av} - \{|E_{02}(t)|^2\}_{av},\end{aligned} \qquad (15)$$

where $\varepsilon^2 = \Sigma_{k=1}^3\{|E_{0k}(t)|^2\}_{av}$ characterizes the energy density of the oscillations of the oscillating field, and the quantities a_1, a_2 are given by (10). By virtue of the inequality $|(a_2 - a_1)/(a_1 + a_2)| < 1$, expression (15) can be thought of as an estimate of the quantity ε^2. When we measure the spectrum with the polarization $\mathbf{e}_{\mathrm{IV}}$ and $\mathbf{e}'_{\mathrm{IV}} = 2^{-1/2}(\mathbf{e}_1 - \mathbf{e}_2)$ we can determine the quantity $\mathrm{Re}\{E_{01}(t)E^*_{02}(t)\}_{av}$; when we measure the spectrum with $\mathbf{e}_{\mathrm{V}}$ and $\mathbf{e}'_{\mathrm{V}} = 2^{-1/2}(\mathbf{e}_1 - i\mathbf{e}_2)$, we can determine $\mathrm{Im}\{E_{01}(t)E^*_{02}(t)\}_{av}$.

We now consider the case in which we know at the outset that the oscillating field vector $\mathbf{E}(t)$ is in the xy plane (i.e., $E_{03} = 0$). In this case the polarization state of field $\mathbf{E}(t)$ can be specified by means of the Stokes parameters ξ_1, ξ_2, ξ_3 (§50 in Ref. 16). In the case $E_{03} = 0$, the Stokes parameters are related to the tensor elements $\sigma_{kk'}$ in (13) by

$$\frac{\sigma_{kk'}}{\sigma} = \frac{1}{2}\begin{pmatrix} 1 + \xi_3 & \xi_1 - i\xi_2 \\ \xi_1 + i\xi_2 & 1 - \xi_3 \end{pmatrix}, \qquad (16)$$

where $\sigma = \sigma_{11} + \sigma_{22}$. According to (5), (6), (9), and (10), by observing the satellites $S_{1,2}^{(\mathbf{e})}$ of the forbidden $1\to0$ with various polarizations $\mathbf{e}$ along the z axis we can find all the Stokes parameters for the field $\mathbf{E}(t)$, using the relations

$$\begin{aligned} \xi_1 &= \left(\frac{a_1 + a_2}{a_3 + a_4}\right)\left(\frac{\hat{S}_p^{(\mathbf{e}'_{\mathrm{IV}})} - \hat{S}_p^{(\mathbf{e}_{\mathrm{IV}})}}{\hat{S}_p^{(\mathbf{e}_{\mathrm{IV}})} + \hat{S}_p^{(\mathbf{e}'_{\mathrm{IV}})}}\right),\\ \xi_2 &= (-1)^p\left(\frac{a_1 + a_2}{a_4 - a_3}\right)\left(\frac{\hat{S}_p^{(\mathbf{e}_{\mathrm{V}})} - \hat{S}_p^{(\mathbf{e}'_{\mathrm{V}})}}{\hat{S}_p^{(\mathbf{e}_{\mathrm{V}})} + \hat{S}_p^{(\mathbf{e}'_{\mathrm{V}})}}\right),\\ \xi_3 &= \left(\frac{a_1 + a_2}{a_1 - a_2}\right)\left(\frac{\hat{S}_p^{(\mathbf{e}_{\mathrm{I}})} - \hat{S}_p^{(\mathbf{e}_{\mathrm{II}})}}{\hat{S}_p^{(\mathbf{e}_{\mathrm{I}})} + \hat{S}_p^{(\mathbf{e}_{\mathrm{II}})}}\right), \quad p = 1,2.\end{aligned} \qquad (17)$$

To determine ξ_2 we could (instead of measuring the same satellite with two different polarizations $\mathbf{e}_V$ and $\mathbf{e}'_V$) take the approach of measuring the intensities of both satellites, $S_1^{(e)}$, $S_2^{(e)}$ for one polarization (say $\mathbf{e} = \mathbf{e}_V$) using the relation

$$(\varepsilon_{21} - \omega)^2 \hat{S}_1^{(e'_V)} = (\varepsilon_{21} + \omega)^2 \hat{S}_2^{(e_V)}.$$

4. CONCLUSION

We have calculated the polarizations of satellites of dipole-forbidden lines which appear in the emission spectrum of non-hydrogen-like atoms in a quasimonochromatic oscillating electric field. We have used the results to propose a spectroscopic method for determining the polarization state of this oscillating field in a plasma. We have shown that in order to determine all elements of the tensor $\{E_{0k}(t), E^*_{0k'}(t)\}_{av}$ in (13) we need to measure the emission spectrum with circular polarization in addition to the emission spectrum with linear polarization. We have also shown how the energy density of the oscillations of the oscillating field along selected axes can be determined (and thus how the total energy density of the field oscillations can be determined). For example, to find the energy density of the oscillations of the field along the x axis, we can determine the quantity $S_p^{(e)}$ ($p = 1,2$) the intensity ratio of the satellite of the forbidden $1 \to 0$ line to the corresponding allowed line ($1 \to 0'$ or $2 \to 0$), for three orthogonal linear polarizations $\mathbf{e} = \mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ and then use Eqs. (9)–(11).

Since the different types of waves in a plasma have different polarization characteristics (Refs. 20 and 21, for example), a detailed polarization analysis of the satellites of dipole-forbidden spectral lines would make it possible to determine the type of wave propagating in the plasma. For example, the wave propagating in a magnetized plasma may include (among others) ordinary and extraordinary waves with elliptical polarizations with electric vectors rotating in opposite directions. The polarization spectroscopic method proposed in this paper can be used to determine the basic properties of these waves, including the direction in which their electric vector is rotating and the mean square strength of the electric field. That is an important problem for monitoring the propagation and absorption of the microwaves used for auxiliary heating of plasmas in magnetic-confinement devices. Often, the waves used for this purpose are superpositions of ordinary and extraordinary waves at the electron cyclotron frequency.[22] The results of the present paper can also be used for a polarization spectroscopic determination of the directional pattern of high-frequency turbulent noise in a plasma.

[1)]In the derivation starting from (7) we made use the circumstance that the 3-j symbols in (7) are nonzero only if $-M_1 + \nu + M_2 = 0$, $-M_2 + \tilde{\nu} + M_0 = 0$.

[2)]Here is the correspondence between the notation of the present paper and that of Ref. 5: $J_0 \leftrightarrow l_0$, $J_1 \leftrightarrow l'$, $J_2 \leftrightarrow l$. We might also note the correspondence between the notation of the present paper and that of Refs. 7 and 15: $J_0 \leftrightarrow l'$, $J_1 \leftrightarrow l$, $J_2 \leftrightarrow l''$.

[3)]If the $2 \to 0$ spectral line is used as the allowed line, we also need information on the relative populations of levels 1 and 2.

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