# Theory of smectic *C* liquid crystals: kinks, vortices, monopoles, and periodic structures

S. S. Rozhkov

*Institute of Physics of the Academy of Sciences of Ukraine*

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The free energy density of the smectic *C* phase is constructed for an arbitrary equilibrium configuration of the smectic layers. Kinks, vortices, and monopoles in the order-parameter field are studied. Two-dimensional periodic structures induced by stretching of a smectic *C* sample are investigated. It is shown that very diverse periodic structures can appear near the smectic *A*—smectic *C* phase transition. The theory is compared with other theories and with experiment.

## 1. INTRODUCTION

Smectic *C* liquid crystals were first studied theoretically by Saupe,[1] the Orsay group,[2] and de Gennes.[3,4] In describing the smectic *A*—smectic *C* phase transition (*AC* transition) de Gennes[4] noted an analogy between the smectic order parameter and the wave function of the condensate in He II According to this analogy the condensate density corresponds to the cant angle $\vartheta$ of the molecules in the smectic layer and the phase of the wave function corresponds to the angle $\alpha$ giving the cant direction of the molecules. In these works, as well as in Ref. 5, the smectic *C* free energy, which describes elastic deformations of the smectic layers and the field of the direction **n** indicating the direction of the long axes of the molecules, was constructed for the case of small angles $\alpha$ and $\vartheta = \text{const}$. The theory of the smectic *C* phase was further elaborated by Kats and Lebedev,[6] who in describing the *AC* transition took into account the interaction of the smectic order parameter with the displacements of the layers. In Ref. 6 an expression not containing these restrictions on $\alpha$ and $\vartheta$ was proposed for the free energy. A more detailed description of the dynamics of smectics is given in Ref. 7. The following, however, should be noted. In spite of the significant attention given to the smectic *C* phase, there is no universally accepted expression for the free energy, such as, for example, the Frank energy for nematics. This fact is indicated, in particular, in the recent work of Leslie, Stewart, and Nakagawa,[8] where a theory of the smectic *C* phase is given. In Ref. 8 the free energy of the *C* phase is constructed and the equations of hydrodynamics are derived. Comparing the works cited above shows that there are disagreements between them (some of them will be discussed below). These discrepancies probably arise for the following reason. In the works cited a flat equilibrium configuration of smectic layers is considered, and this makes it impossible for the order parameter of the *C* phase of the smectic to manifest a global structure. In order to explain this straightforwardly, we consider the exotic example of a smectic *C* film in the form of Möbius strip. In such a film the order-parameter field contains a domain wall in which the director **n** turns by 180°. Nothing like this happens in the field of the phase of the order parameter of He-II. In other words, the "helium analogy" for the *C* phase holds only locally at each point of the smectic layer—the global properties of the order parameter depend on the global topology of the smectic layers. The formation of monopolar structures—observed by Kurik and Lavrentovich[9] (see also Ref. 10) in drops of the *C* phase where the smectic layers formed concentric spheres—is connected precisely with the global topology of the layers.

In the present paper the smectic *C* free energy density is constructed for an arbitrary equilibrium configuration of the smectic latyers (in the absence of dislocations in the system of layers) and extended objects in the order-parameter field (kinks, vortices, and monopoles) as well as periodic structures induced by stretching the smectic sample are studied.

## 2. ORDER PARAMETER

A general property of smectic liquid crystals is their layered structure. Different phases of smectics are distinguished by the structure of the layers consisting of elongated molecules. In the *A* and *C* phases the centers of mass of the molecules are arranged randomly in the layer, i.e., each layer consists of a two-dimensional liquid, and in the *B* phase (and other phases) some crystal order exists within the layer. We are concerned with *A* and *C* smectics, which are distinguished by the cant of the long axis of the molecules relative to the normal direction to the layer: In the *A* phase the long axes of the molecules are perpendicular to the plane of the layer and in the *C* phase the long axes of the molecules are tilted with respect to the plane of the layer, and long-range orientational order of the elongated molecules, which is described by the nematic director **n**, such that $\mathbf{n}^2 = 1$ and **n** and $-$ **n** are equivalent, exists.

It is convenient to describe the orientational order in the smectic *C* phase by a triplet of mutually orthogonal unit vectors **N**, **c**, and **m**, where $\mathbf{N} = [\mathbf{mc}]$ is the normal to the smectic layer and the vector **c** (the *C* director[4]) lies in the plane of the normal **N** and the director **n**, which at a fixed temperature make an angle $\vartheta$. Since **n** and $-$ **n** are equivalent, for definiteness we assume that the angle $\vartheta$ is acute. Note that **c** is an ordinary vector, i.e., **c** and $-$ **c** are not equivalent. It is here that the smectic *C* phase differs significantly from a nematic: For example, only disclinations of integer strength can form in the field of the *C* director.[4] Thus orientational order in the *C* phase at a fixed temperature ($\vartheta = \text{const}$) is described locally by a two-dimensional vector **c** (or **m**), lying in planes tangent to the smectic layers at each point of the sample, i.e., locally the space of the order parameter (the range of values of the order parameter) is a circle $S^1$ or the group $SO_2$. If the configuration of the layers is

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not two-dimensional, then the properties of the smectic depend strongly on the topology of the smectic surfaces. A smectic layer of the $C$ phase is an example of tangential stratification with the stratification group $G = SO_2$ and base $M$, where $M$ is some surface. For example, it can be the plane $R^2$, the cylinder $S^1 \times R^1$, or the sphere $S^2$.

In order to take into account changes in the cant angle $\vartheta$ (for example, in an $AC$ transition or in the core of a disclination), we shall study together with the vectors $\mathbf{m}$ and $\mathbf{c}$, the vectors $\boldsymbol{\psi} = \psi\mathbf{m}$ and $\boldsymbol{\varphi} = \psi\mathbf{c}$, where $\psi = \sin\vartheta$ $(0 \leqslant \psi \leqslant 1)$. We describe the smectic layers by the equation

$$W(\mathbf{r}, t) = \text{const}, \tag{1}$$

and then

$$\mathbf{N} = \nabla W / |\nabla W|. \tag{2}$$

We represent the function $W$ in the form

$$W(\mathbf{r}, t) \equiv F(\mathbf{r}) - u(\mathbf{r}, t), \tag{3}$$

where $F$ describes the equilibrium configuration, assumed to be given, of the layers (for example, flat, cylindrical, or spherical), and $u$ is the displacement. With the equilibrium configuration of the layers we associate a local coordinate system $\{\xi,\eta,\zeta\}$ with basis vectors $\mathbf{m}_0$, $\mathbf{c}_0$, and $\mathbf{v}_0$, where the subscript "0" indicates the equilibrium state and $\mathbf{N} \equiv \mathbf{v}_0 + \mathbf{v}$. We also assume that the number of layers is constant[4]

$$\operatorname{rot} \mathbf{v}_0 = 0, \tag{4}$$

i.e., at equilibrium $(u \equiv 0)$ the layers are equidistant and there are no dislocations in the system of layers.

## 3. FREE ENERGY

In its simplest form the energy of stretching (compression) of the smectic layers can be written thus:

$$F_L = \frac{B}{2}\tilde{\gamma}^2. \tag{5}$$

Here the deformation $\tilde{\gamma}$ is a scalar quantity and has the form

$$\tilde{\gamma} = \mathbf{N}\nabla u \simeq u_\zeta - (\nabla_\perp u)^2 \left(1 + \frac{3}{2} u_\zeta\right) + \ldots, \tag{6}$$

where in the approximate equality the fact that

$$\mathbf{N} \simeq \left\{-u_\xi, -u_\eta, 1 - \frac{1}{2}(\nabla_\perp u)^2\right\},$$

$$\nabla = \{\partial_\xi, \partial_\eta, \partial_\zeta\} \equiv \{\nabla_\perp, \partial_\zeta\} \tag{7}$$

was taken into account. We note that in the local coordinate system we have $\mathbf{v}_0 = \{0,0,1\}$. The form of the expansion of the deformation $\tilde{\gamma}$ in terms of the gradients of the displacement (6) makes it possible to generalize the expression for $\tilde{\gamma}$ and $F_L$ as follows:

$$\tilde{\gamma} = u_\zeta - \frac{1}{2} b_1 u_\xi^2 - \frac{1}{2} b_2 u_\eta^2 + \ldots, \tag{8}$$

$$F_L = \frac{1}{2} B_3 u_\zeta^2 - \frac{1}{2}(B_{31} u_\xi^2 + B_{32} u_\eta^2) u_\zeta + \frac{1}{8} B_1 u_\xi^4 + \frac{1}{4} B_{12} u_\xi^2 u_\eta^2 + \frac{1}{8} B_2 u_\eta^4 + \ldots. \tag{9}$$

In the case of smectic $A$ we have $B_{31} = B_{32}$ and $B_1 = B_{12} = B_2$.[11] Note also that usually a flat configuration of layers with deformation

$$\tilde{\gamma}_p = u_z - \frac{1}{2}\mathbf{N}_\perp^2 \simeq u_z - \frac{1}{2}(\nabla_\perp u)^2 (1 + 2u_z)$$

$(\mathbf{N}_\perp = \{N_x, N_y\})$ in the formula (5) is considered.[4,5] In Ref. 6 the expression $\tilde{\gamma}_W = u_z - \frac{1}{2}(\nabla u)^2$ is employed for the deformation. The quantity $\tilde{\gamma}_W$ differs from $\tilde{\gamma}$ and $\tilde{\gamma}_p$ by the presence of the term $u_z^2$ and the absence of terms of the form $u_z(\nabla_\perp u)^2$, etc.

Next, we must include in the free energy of the smectic terms which determine the cant of the molecules in the layer (the angle $\vartheta$) and the relationship of the cant of the molecules to the deformation of the layers.[5,6] This part of the free energy has the form

$$F_s = U\tilde{\gamma}\psi^2 - \frac{A}{2}\psi^2 + \frac{D}{4}\psi^4, \tag{10}$$

where $U$, $A$, and $D$ are positive constants, and $\tilde{\gamma}$ is given by the formula (8). In the $A$ phase of the smectic we have $A < 0$ and $\psi = 0$ $(\vartheta = 0)$. We assume that near the point of the $AC$ transition $A \propto |T_c - T|$ holds,[6] where $T_c$ is the temperature of the $AC$ transition.

It remains to add to the free energy the elastic energy of the smectic order parameter: the triplet of vectors $\mathbf{N}$, $\boldsymbol{\psi}$, and $\boldsymbol{\varphi}$. As has already been mentioned, in the literature there is no universally accepted expression for this energy. (This can be seen by comparing Refs. 4–8.) As in Ref. 6, we start from the Frank energy for a nematic

$$F_n = \frac{K_1}{2}(\operatorname{div}\mathbf{n})^2 + \frac{K_2}{2}(\mathbf{n}\operatorname{rot}\mathbf{n})^2 + \frac{K_3}{2}[\mathbf{n}\operatorname{rot}\mathbf{n}]^2 \tag{11}$$

and the representation of $\mathbf{n}$ in the form

$$\mathbf{n} = \mathbf{N}\sqrt{1 - \psi^2} + [\mathbf{N}\boldsymbol{\psi}] \tag{12}$$

or

$$\mathbf{n} = \mathbf{N}\sqrt{1 - \psi^2} + \boldsymbol{\varphi}, \tag{13}$$

where, as above, a local coordinate system $\{\xi,\eta,\zeta\}$ is employed and $\mathbf{N} = \mathbf{v}_0 + \mathbf{v}$. Substituting Eqs. (12) and (13) into Eq. (11) gives two equivalent expressions for $F_n$, which, to lowest order in $\psi$ and the gradients of the fields $\mathbf{N}$, $\boldsymbol{\varphi}$, and $\boldsymbol{\psi}$ we represent as follows:

$$F_n = F_1 + F_2 + F_3, \tag{14}$$

$$F_1 = \frac{\varkappa_1}{2}(\operatorname{div}\mathbf{v})^2, \tag{15}$$

$$F_2 = \varkappa_1 \operatorname{div}\mathbf{v}\operatorname{div}\boldsymbol{\varphi} = \varkappa_1 \operatorname{div}\mathbf{v}(\mathbf{v}_0 \operatorname{rot}\boldsymbol{\psi}), \tag{16}$$

$$F_3 = \frac{\varkappa_1}{2}(\operatorname{div}\boldsymbol{\varphi})^2 + \frac{\varkappa_2}{2}(\mathbf{N}\operatorname{rot}\boldsymbol{\varphi})^2 + \frac{\varkappa_3}{2}[\mathbf{N}\operatorname{rot}\boldsymbol{\varphi}]^2, \tag{17}$$

or

$$F_3 = \frac{\varkappa_2}{2}(\operatorname{div}\boldsymbol{\psi})^2 + \frac{\varkappa_1}{2}(\mathbf{N}\operatorname{rot}\boldsymbol{\psi})^2 + \frac{\varkappa_3}{2}[\mathbf{N}\operatorname{rot}\boldsymbol{\psi}]^2, \tag{18}$$

where, so as not to confuse the constants of the nematic and smectic $C$, the new constants $\varkappa_1$, $\varkappa_2$, and $\varkappa_3$ were introduced for the elastic constants $K_1$, $K_2$, and $K_3$, respectively. Note that the expression obtained for $F_n$ differs from the analogous energy given in Refs. 6 and 8. In Ref. 6 the term of the form $F_2$ is not taken into account. In Ref. 8 the case $\psi = \text{const}$ is considered, the expression for the free energy does not contain a term of the form $(\mathbf{N}\,\text{rot}\;\mathbf{c})^2$, and three terms [containing the factor $(\mathbf{Nc})\,\text{rot}\;\mathbf{c}$], which do not follow from the approach considered above, are included.

In the derivation of Ref. 14, it was assumed that the condition (4) is satisfied and the average radius of curvature of the layers and the characteristic scale of the nonuniformities along the normal $\mathbf{v}_0$ are significantly greater than the dimensions of the transverse gradient of the fields $\mathbf{v}$, $\boldsymbol{\varphi}$, and $\boldsymbol{\psi}$ (in particular, it is assumed that rot $\mathbf{N} \simeq 0$). When necessary, the dropped terms can be easily included in $F_n$. The expression for $F_n$ is still not the final one. The energies $F_1$ and $F_2$, containing the term div $\mathbf{v}$, must be generalized as was done in the case of the energy (5), containing the quantity $\mathbf{N}\nabla u$. Thus, instead of $F_1$ and $F_2$ we have, respectively

$$F_1 = \frac{C_1}{2} u_{\xi\xi}^2 + C_{12} u_{\xi\xi} u_{\eta\eta} + \frac{C_2}{2} u_{\eta\eta}^2, \tag{19}$$

$$F_2 = -(L_1 u_{\xi\xi} + L_2 u_{\eta\eta})\,\text{div}\;\boldsymbol{\varphi}, \tag{20}$$

and we rewrite the formulas (17) and (18) once more in a different form:

$$F_3 = \frac{\varkappa_1}{2}(\text{div}\;\boldsymbol{\varphi})^2 + \frac{\varkappa_3}{2}(\text{rot}\;\boldsymbol{\varphi})^2 + \frac{1}{2}(\varkappa_2 - \varkappa_3)(\mathbf{N}\;\text{rot}\;\boldsymbol{\varphi})^2, \tag{21}$$

$$F_3 = \frac{\varkappa_2}{2}(\text{div}\;\boldsymbol{\psi})^2 + \frac{\varkappa_3}{2}(\text{rot}\;\boldsymbol{\psi})^2 + \frac{1}{2}(\varkappa_1 - \varkappa_3)(\mathbf{N}\;\text{rot}\;\boldsymbol{\psi})^2. \tag{22}$$

In the case of smectic $A$ we have[11] $C_1 = C_{12} = C_2 \equiv C$. The transformation in the expressions (21) and (22) [as well as in Eqs. (17) and (18)] from the field $\boldsymbol{\varphi}$ to the field $\boldsymbol{\psi}$ is made with the help of the formulas

$$\begin{aligned} \text{div}\;\boldsymbol{\varphi} &\simeq -\mathbf{N}\;\text{rot}\;\boldsymbol{\psi}, \quad \text{div}\;\boldsymbol{\psi} \simeq \mathbf{N}\;\text{rot}\;\boldsymbol{\varphi}, \\ (\text{rot}\;\boldsymbol{\varphi})^2 &\simeq (\text{div}\;\boldsymbol{\psi})^2 + (\text{rot}\;\boldsymbol{\psi})^2 - (\mathbf{N}\;\text{rot}\;\boldsymbol{\psi})^2 \\ &= (\text{div}\;\boldsymbol{\psi})^2 - [\mathbf{N}\;\text{rot}\;\boldsymbol{\psi}]^2. \end{aligned} \tag{23}$$

So, the smectic $C$ free energy density $F$ consists of the terms $F_L$, $F_s$, and $F_n$, given by the formulas (9), (10), (14), and (19)–(22):

$$F = F_L + F_s + F_n.$$

## 4. MONOPOLES IN A DROP

The energy $F_3$, associated with elastic deformations of the order parameter $\boldsymbol{\varphi}$ or $\boldsymbol{\psi}$, assumes a simpler form in the single-constant approximation: $\varkappa_1 = \varkappa_2 = \varkappa_3 \equiv \varkappa$. In this case we have

$$F_3 = \frac{\varkappa}{2}(\nabla\boldsymbol{\varphi})^2 = \frac{\varkappa}{2}(\nabla\boldsymbol{\psi})^2 = \frac{\varkappa}{4}[(\nabla\boldsymbol{\varphi})^2 + (\nabla\boldsymbol{\psi})^2]. \tag{24}$$

Recall that $\boldsymbol{\varphi}\boldsymbol{\psi} = 0$, $\boldsymbol{\varphi} = \psi\mathbf{c}$, $\boldsymbol{\psi} = \psi\mathbf{m}$ and $[\mathbf{mc}] = \mathbf{N}$. If $\psi = \text{const}$ (the cant angle of the molecules $\vartheta = \text{const}$), then

$$F_3 = \frac{\tilde{\varkappa}}{2} A_\mu^2, \tag{25}$$

where

$$\tilde{\varkappa} = \psi^2\varkappa, \quad A_\mu = \mathbf{m}\partial_\mu \mathbf{c}.$$

The vector $\mathbf{A}$ is related to the normal $\mathbf{N}$ by the Mermin–Ho relation:[12]

$$(\text{rot}\;\mathbf{A})_\mu = \frac{1}{2}\varepsilon_{\mu\alpha\beta}\mathbf{N}[\partial_\alpha \mathbf{N}\partial_\beta \mathbf{N}]. \tag{26}$$

In the case of the $A$ phase of superfluid $^3$He the superfluid velocity $\mathbf{v}_s$ corresponds to the vector $\mathbf{A}$ and the quantization axis of the orbital angular momentum $\mathbf{l}$ of Cooper pairs corresponds to the vector $\mathbf{N}$.

We demonstrate the usefulness of the formula (26) by the following example. Consider the equilibrium spherical configuration of layers:

$$u \equiv 0, \quad \mathbf{N} = \mathbf{v}_0 = \mathbf{r}/r. \tag{27}$$

Then, according to Eq. (26), for $\mathbf{r} \neq 0$

$$\text{rot}\;\mathbf{A} = \frac{\mathbf{r}}{r^3}. \tag{28}$$

The formula (28) makes it possible to draw an analogy with the Dirac magnetic monopole described by the vector potential $\mathbf{A}$.[13] The following form of the potential $\mathbf{A}$ satisfies Eq. (28):

$$\mathbf{A} = \frac{s - \cos\theta}{r\sin\theta}\hat{\boldsymbol{\Phi}}, \tag{29}$$

where $\hat{\boldsymbol{\Phi}}$ is a unit basis vector of the spherical coordinate system $\{r,\theta,\Phi\}$, and $s = 0, \pm 1, \pm 2, \ldots$. For $s = 0$

$$\mathbf{A} = -\frac{1}{r}\,\text{ctg}\,\theta\,\hat{\boldsymbol{\Phi}}, \tag{30}$$

and for $s = 1$

$$\mathbf{A} = \frac{1}{2r}\,\text{tg}\,\frac{\theta}{2}\,\hat{\boldsymbol{\Phi}}. \tag{31}$$

For $s = 1$ the vector potential $\mathbf{A}$ is singular on the half-axis $z < 0$ and for $s = 0$ it is singular on the entire $z$ axis, i.e., in the first case we have a monopole with one Dirac string and in the second case we have a monopole with two Dirac strings. For the field $\mathbf{c}$ (or $\mathbf{m}$) such distributions of the field $\mathbf{A}$ indicate that the field of the $C$ director contains disclinations emanating from the coordinate origin: For $s = 0$ there is one disclination of strength 2 and for $s = 1$ there are two disclinations of unit strength. By analogy with magnetic monopoles the structures considered above can be called Schwinger monopoles (see, for example, Ref. 13). Such configurations of the field of the $C$ director have been observed by Kurik and Lavrentovich[9] in spherical drops of smectic $C$. In order to describe more accurately monopole structures in the smectic $C$ phase the corresponding equations of motion must be solved.

## 5. VORTICES

We now consider the simpler situation of vortices in the field $\boldsymbol{\varphi}$ (or $\boldsymbol{\psi}$) for a flat equilbrium configuration of layers: $u = 0$, $\mathbf{N} = \mathbf{v}_0 = \hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ is a unit basis vector of the cylindrical coordinate system $\{\rho,\Phi,z\}$. We assume that $\boldsymbol{\psi} = \psi\mathbf{m}$,

$\mathbf{m} = \{\cos\alpha, \sin\alpha\}$, $\alpha = \alpha(\Phi)$, and $\psi = \psi(\rho)$. Then the free energy density is

$$F = -\frac{A}{2}\psi^2 + \frac{D}{4}\psi^4 + \frac{1}{2}\left[\frac{\partial\psi}{\partial\rho} + \frac{1}{\rho}\psi\frac{\partial\alpha}{\partial\Phi}\right] \times [\varkappa_1\sin^2(\alpha - \Phi) + \varkappa_2\cos^2(\alpha - \Phi)]. \quad (32)$$

The system of equations for $\alpha$ and $\psi$ is quite complicated. For this reason we give the function $\alpha = \alpha(\Phi)$ in cases corresponding to radial (tangential) $\alpha = \Phi$ and tangential (radial) $\alpha = \Phi + \pi/2$ disclinations of unit strength in the field $\mathbf{m}$ (in the field $\mathbf{c}$). Then (for given $\alpha$) we find the following equation for $\psi$:

$$\frac{\partial^2\psi}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial\psi}{\partial\rho} - \frac{1}{\rho^2}\psi + \mu_i^2\psi - d_i^2\psi^3 = 0, \quad (33)$$

where $i = 2$ corresponds to $\alpha = \Phi$ and $i = 1$ corresponds to $\alpha = \Phi + \pi/2$,

$$\mu_i^2 = \frac{A}{\varkappa_i}, \quad d_i^2 = \frac{D}{\varkappa_i}. \quad (34)$$

The equation (33) is called the Gross–Pitaevskii equation. It describes a single-quantum vortex in a Bose gas. A numerical solution of this equation is given in Ref. 14. For $\rho \ll \rho_i$ we have $\psi \propto \rho$, and for $\rho \gg \rho_i$ we have $\psi \to \psi_0$, where $\rho_i = \mu_i^{-1}$ is the characteristic length and $\psi_0 = \mu_i/d_i = (A/D)^{1/2}$. A solution of this form means that the $A$ phase fills the vortex core, and the $C$ phase (condensate) is restored as the distance from the center of the vortex increases. The character of the solution for $\psi$ is the same as for disclinations of strength $|s| > 1$ $(\alpha = s\Phi)$ in the field $\mathbf{m}$. We note that for $\varkappa_1 = \varkappa_2$ the solution of the equation for $\alpha$ has the form $\alpha = s\Phi + \Phi_0$, where $s = 0, \pm 1, \pm 2, \ldots$ and $\Phi_0$ is a constant. In this case the equation for $\psi$ has the form (33), in which the third term is multiplied by $s^2$. Tangential disclinations of unit strength for the $C$ director (i.e., $\alpha = \Phi$) were observed in Ref. 5. This means that for the smectics investigated in Ref. 5 $\varkappa_1 > \varkappa_2$.

## 6. PERIODIC STRUCTURES AND KINKS

We now consider wave-like instabilities[4] induced by smectic $C$ by stretching the smectic sample perpendicular to the smectic layers. Wave-like instability was observed in the $A$ phase in Ref. 15. Modulated structures have also been observed in other smectic phases: the $C$ phase[5] and the $B$ and $H$ phases.[16] Wave-like modulation in the $C$ phase with $\vartheta = \text{const}$ was described in Ref. 5. One-dimensional periodic structures in smectic $C$ with modulation of the cant angle of the molecules were studied in Ref. 17.

In this section we investigate two-dimensional modulated structures in smectic $C$ taking into account changes in the cant angle $\vartheta$ of the molecules. This is especially important near the point of the $AC$ transition, when changes in the angle $\vartheta$ can be significant. The region of the $AC$ transition is of interest both from the standpoint of studying the phase transition itself and the possibility of observing a new state of the smectic—coexistence of the $A$ and $C$ phases.[17] This state is analogous to the mixed state of a superconductor: The $A$ phase corresponds to the normal state of the metal and the $C$ phase corresponds to the superconducting state.

We consider a planar equilibrium configuration of the layers for a smectic $C$ sample of thickness $d$ along the equilibrium normal $\mathbf{v}_0 = \{0,0,1\}$ and we determine how uniform stretching of the sample perpendicular to the smectic layers affect the state of the smectic. In this case the displacement $u$ has the form

$$u = \gamma z, \quad (35)$$

where $\gamma > 0$ is the deformation, and variation of the energy $F_s$ [the formula (10)] gives the equilibrium value for the magnitude of the order parameter $\psi_0$:

$$\psi_0^2 = \frac{2U}{D}(\gamma_0 - \gamma), \quad (36)$$

where

$$\gamma_0 = \frac{A}{2U}. \quad (37)$$

One can see from the formula (36) that for

$$\gamma \geq \gamma_0 \quad (38)$$

uniform stretching of the sample induces a transition of the $C$ phase into the $A$ phase: $\psi_0^2 = 0$ $(\vartheta = 0)$. Thus we observe an effect opposite to that described in Ref. 18: A transition of the $A$ phase into the $C$ phase under compression: $|\gamma| > |\gamma_0|$ $(A, \gamma < 0)$.

The wave-like modulation of layers in smectic $A$, predicted by Clark and Meyer,[19] is well known:[4,11] When the sample is stretched perpendicular to the smectic layers, the uniform state of the smectic may become unstable with respect to periodic distortions of the layers. An instability with period

$$L_A = 2(\pi\lambda_A d)^{1/2}, \quad (39)$$

where $\lambda_A = (C/B_3)^{1/2}$ is the characteristic length, appears when the deformation $\gamma$ reaches the critical value

$$\gamma_A = \frac{2\pi}{d}\left(\frac{CB_3}{B_{31}^2}\right)^{1/2} \equiv 2\pi\frac{B_3}{B_{31}}\frac{\lambda_A}{d}. \quad (40)$$

For typical sample thicknesses $d = 100\ \mu\text{m}$ the modulation period satisfies $L_A \ll d$ [$\lambda_A = 2.2 \cdot 10^{-7}$ cm (Ref. 15)]. Periodic structures are found to be metastable: An applied stress gives rise to breakage of the layers and the dislocations formed begin to move, which reduces the stress.[19]

The existence of an order parameter $\boldsymbol{\varphi} = \psi\mathbf{c}$ in the smectic $C$ phase indicates that a greater diversity of spatially nonuniform structures should be expected in the $C$ phase than in the $A$ phase. For example, wave-like modulation of the layers can result in periodic configurations of the $\psi$ and $\mathbf{c}$ fields. The character of such structures will depend significantly on the ratio of the threshold deformation of the formation of the $A$ phase $\gamma_0 = A/2U$ $(A \propto T_c - T)$ and the threshold of wave-like modulation (a quantity of the form $\gamma_A$).

We now continue our description of the $C$ phase when the equilibrium configuration of the layers is planar. We assume that $\boldsymbol{\psi} = \psi\mathbf{m} \equiv \psi\{\cos\alpha, \sin\alpha, 0\}$, $\boldsymbol{\varphi} = \psi\mathbf{c} \equiv \{-\sin\alpha, \cos\alpha, 0\}$, $\mathbf{N} = \{-u_x, -u_y, 1\}$, where $\psi = \psi(\mathbf{r})$, $\alpha = \alpha(\mathbf{r})$

and $u = u(\mathbf{r})$. We assume that the angle $\alpha$ describes deflections of the vectors $\mathbf{m}$ and $\mathbf{c}$ from their equlibrium orientation $\mathbf{m}_0 = \{1,0,0\}$ and $\mathbf{c}_0 = \{0,1,0\}$ in the Cartesian coordinate system $\{x,y,z\}$.

As has already been mentioned, wavelike modulation in smectic $C$ crystals was observed in Ref. 5. When the sample was cooled (which corresponds to stretching) wavelike modulation usually appeared in the direction of the vector $\mathbf{m}_0$, and under further cooling periodic structure arose in the direction $\mathbf{c}_0$. This resulted in the formation of a rectangular grid for which the period of the secondary structure was 1.6 times greater than the period of the primary structure. Sometimes the secondary structure appeared as the first structure (at the periphery of the sample). In this case, the periods were in the ratio 2:1. The difference in the ratios of the periods is apparently associated with the difference in the thickness of the sample at the center and at the periphery, and the appearance of orthogonal periodic structures at different temperatures (different stretchings) and in different sequence indicates that the periodic structure along $\mathbf{m}_0$ and $\mathbf{c}_0$ arise independently.

The latter fact makes it possible to regard periodic structures in the $C$ phase as a superposition of two one-dimensional structures: along $\mathbf{m}_0$ and along $\mathbf{c}_0$. This supposition is based on the fact that the directions $\mathbf{m}_0$ and $\mathbf{c}_0$ are not equivalent and quantities of the type $L_A$ and $\gamma_A$ [the formulas (39) and (40)] for these directions differ appreciably from one another. Thus we give expressions for the energies $F_2$ and $F_3$ describing nonuniform states along the directions $\mathbf{m}_0$ and $\mathbf{c}_0$ for small deviations from equilibrium ($\alpha \ll 1$):

$$F_{2\mathrm{m}} = L_1 u_{xx}(\alpha_x\psi + \alpha\psi_x), \tag{41}$$

$$F_{3\mathrm{m}} = \frac{\varkappa_1}{2}\psi^2\alpha_x^2 + \frac{\varkappa_2}{2}\psi_x^2 + \frac{\varkappa_3}{2}\psi_z^2 + \frac{1}{2}(\varkappa_1 - \varkappa_2)[\alpha^2\psi_x^2 + 2\alpha\alpha_x\psi\psi_x] + \frac{\varkappa_3}{2}[\psi^2\alpha_z^2 + u_x^2\psi_x^2 - 2u_x\psi_x\psi_z], \tag{42}$$

$$F_{2\mathrm{c}} = -L_2 u_{yy}(\psi_y + \alpha\alpha_y\psi), \tag{43}$$

$$F_{3\mathrm{c}} = \frac{\varkappa_2}{2}\psi^2\alpha_y^2 + \frac{\varkappa_1}{2}\psi_y^2 + \frac{\varkappa_3}{2}(\psi_z^2 + u_y^2\psi_z^2) + \frac{1}{2}(\varkappa_2 - \varkappa_1)\alpha^2\psi_y^2 + (\varkappa_1 + \varkappa_2)\alpha\alpha_y\psi\psi_y + \frac{\varkappa_3}{2}[\psi^2\alpha_z^2 + u_y^2\psi_y^2 - 2u_y\psi_y\psi_z]. \tag{44}$$

We recall that the total free energy density $F$ consists of $F_L$, $F_s$, and $F_n = F_1 + F_2 + F_3$. Correspondingly, we also assume that the energies $F_L$, $F_s$, and $F_1$ (the fields $u$ and $\psi$) depend either on $x$ or $y$ as well as on $z$.

Thus, we have two expressions for the free energy of a smectic which in equilibrium has a planar configuration of layers. These energies make it possible to describe spatially nonuniform states along the directions $\mathbf{m}_0$ and $\mathbf{c}_0$, the superposition of the states representing one or another two-dimensional structure. These structures are described in terms of the displacement $u$, the modulus of the order parameter $\psi$ (cant angle $\vartheta$ of the molecules), and the Goldstone variable $\alpha$.

As already mentioned, wavelike modulation of the layers of a smectic sample can arise when the sample is stretched uniformly perpendicular to the smectic layers. It is obvious that the interaction of the displacement $u$ with the fields $\psi$ and $\alpha$ can initiate spatially nonuniform configurations of the fields $\psi$ and $\alpha$. Periodic structures for $\psi$ and $\alpha$ can arise in many possible ways even in the case of uniform stretching of the sample. For this reason, we confine our attention to some typical cases. We assume that the periods of $u$, $\psi$, and $\alpha$ modulations are comparable. Specifically, we assume that the periods of the modulations of $u$ and $\alpha$ along $\mathbf{c}_0$ or $\mathbf{m}_0$ are the same, and the period of modulation of $\psi$ is half of the periods of $u$ and $\alpha$. The latter circumstance is connected with the fact that the quantity $\psi$, which describes the cant of the director relative to the normal to the layers, does not depend on the side to which (left or right) the director cants for a given direction. This means also that under some conditions a domain wall perpendicular to a fixed direction and separating the part of the sample with different canting of the molecules (to the right and left), can appear. The director does not turn in the wall, but rather the cant angle of the director changes from $-\vartheta_0$ to $\vartheta_0$. In order to describe such a wall, it is sufficient to redefine the quantity $\psi$, making the assumption that $\psi$ can be both positive and negative (previously $\psi$ was defined as a positive quantity). Then the domain wall will be the well-known static kink of the $\psi^4$ model:[20]

$$F_{\mathrm{m}} = \frac{\varkappa_2}{2}\psi_x^2 - \frac{A}{2}\psi^2 + \frac{D}{4}\psi^4, \tag{45}$$

$$\psi_{xx} + \mu_2^2\psi - d_2^2\psi^3 = 0, \tag{46}$$

$$\psi = \pm\psi_0 \operatorname{th}\frac{x}{2\rho_2}. \tag{47}$$

It is evident from the solution (47) that $\psi$ (for the upper sign) varies from $-\psi_0$ to $\psi_0$ ($\psi_0^2 = A/D$) as $x$ varies from $-\infty$ to $+\infty$ over a characteristic distance $\rho_2 = (A/\varkappa_2)^{1/2}$, i.e., the domain wall is an $A$ phase separating the "left-hand" and "right-hand" $C$ phases. After this digression, we now return to the periodic structures and the previous definition of $\psi$: $\psi > 0$.

Thus we consider the following modulated structures:

$$u = \gamma z + u_0 \cos qx_i \sin q_3 z, \tag{48}$$

$$\psi = \psi_0 - \psi_1 \sin^2 qx_i \sin q_3 z, \tag{49}$$

$$a)\ \alpha = \alpha_0 \sin qx_i \sin q_3 z, \tag{50}$$

$$b)\ \alpha = \alpha_0 \cos qx_i \sin q_3 z, \tag{51}$$

where the index $i = 1$ and 2 designates the directions along $\mathbf{m}_0$ and $\mathbf{c}_0$, respectively; $x_1 \equiv x$, $x_2 \equiv y$, and $q_3 = \pi/d$; and the cases $a$ and $b$ differ from one another by the phase.

The free energy $\mathscr{F}_i$, by varying which it is possible to obtain the corresponding amplitudes and periods of the modulations, is found by substituting $u$, $\psi$, and $\alpha$ given by the formulas (48)–(51) (for the directions $\mathbf{m}_0$ or $\mathbf{c}_0$) into the free energy density $F_i$ and averaging $F_i$ over the sample thickness and the modulation period $\lambda_i = 2\pi/q_i$:

$$\mathcal{F}_i = \frac{1}{d\lambda_i} \int_0^d dz \int_0^{\lambda_i} dx_i F_i. \tag{52}$$

In spite of the restrictions imposed on the form of the modulations of the $u$, $\psi$, and $\alpha$ fields [the formulas (48)–(51)], periodic structures can still appear in many ways. For this reason, we confine our attention to the most typical cases: A) $\psi_1 = 0$ and B) $\psi_1 = \psi_0$. The case A corresponds to a uniform distribution of the field $\psi$ and the case B corresponds to complete modulation of $\psi$, i.e., alternation of $A$ and $C$ phases. In these cases the energy $\mathcal{F}_i$ has the form

$$\mathcal{F}_i = \mathcal{F}_{Li} + \mathcal{F}_{si} + \mathcal{F}_{ni}, \tag{53}$$

$$\mathcal{F}_{Li} = \frac{B_3}{2}\left(\gamma^2 + \frac{1}{4} q_3^2 u_0^2\right) + \frac{B_i}{8}\frac{9}{64} q^4 u_0^4 - \frac{B_{3i}}{8} \gamma q^2 u_0^2, \tag{54}$$

$$\mathcal{F}_{si} = \tilde{U}\gamma\psi_0^2 - \tilde{U}_i q^2 u_0^2 \psi_0^2 - \frac{\tilde{A}}{2}\psi_0^2 + \frac{\tilde{D}}{4}\psi_0^4, \tag{55}$$

$$\mathcal{F}_{ni} = \frac{\tilde{C}_i}{8} q^4 u_0^2 + \frac{\tilde{k}_i}{8} q^2 \psi_0^2 + \frac{\tilde{k}_3}{8}\frac{3}{8} q^4 u_0^2 \psi_0^2. \tag{56}$$

Here and below we neglect terms of order $q_3^2/q^2 \ll 1$. In the formulas (54)–(56) the index $i$ in the quantities $q$, $u_0$, and $\psi_0$ is dropped as a simplification, and the fact that

$$\alpha_{01} = \frac{\tilde{L}_1}{\tilde{\varkappa}_1}\frac{q u_0}{\psi_0}, \quad \alpha_{02} = 0 \tag{57}$$

is taken into account [in the formula (56)]. The latter condition means that $\alpha$ is modulated only in the direction $\mathbf{m}_0$, and for modulations of $a$ and $b$ [the formulas (50) and (51)] we have

$$a)\ \tilde{C}_1 = C_1\left(1 - \frac{\tilde{L}_1^2}{\tilde{\varkappa}_1 C_1}\right), \tag{58}$$

$$b)\ \tilde{C}_1 = C_1. \tag{59}$$

The constants appearing in the formulas (55)–(58) are different for the cases A and B:

$$\begin{aligned} &\text{A: } \tilde{U} = U, \quad \tilde{U}_i = b_i U/8, \quad \tilde{A} = A, \quad \tilde{D} = D, \\ &\qquad \tilde{C}_2 = C_2, \quad \tilde{L}_1 = L_1, \quad \tilde{\varkappa}_1 = \varkappa_1, \quad \tilde{k}_i = \tilde{k}_3 = 0; \end{aligned} \tag{60}$$

$$\begin{aligned} &\text{B: } \tilde{U} \simeq 0{,}55U, \quad \tilde{U}_i \simeq 0{,}34 b_i U, \quad \tilde{A} \simeq 0{,}55A, \\ &\qquad \tilde{D} \simeq 0{,}73D, \quad \tilde{k}_1 = \varkappa_2, \quad \tilde{k}_2 = \varkappa_1, \quad \tilde{k}_3 = \varkappa_3, \\ &\qquad \tilde{C}_2 = C_2, \quad \tilde{L}_1 \simeq 0{,}36L_1, \quad \tilde{\varkappa}_1 \simeq 0{,}57\varkappa_1 + 0{,}1\varkappa_2. \end{aligned} \tag{61}$$

When necessary we label these quantities with the indices A or B.

Varying the energy (53) gives the following expressions for the amplitudes $u_0$ and $\psi_0$:

$$u_0^2 = \frac{32}{9q^2 B_i}\frac{(B_{3i} - w\tilde{D}_1 E)\gamma - \tilde{B} + a\tilde{D}_1 E}{1 - \tilde{D}_1 E^2/B_i}, \tag{62}$$

$$\psi_0^2 = \frac{a - E\tilde{B} - (w - \beta E)\gamma}{1 - \tilde{D}_1 E^2/B_i}, \tag{63}$$

where

$$w = \frac{2\tilde{U}}{\tilde{D}}, \quad \tilde{D}_1 = \frac{9}{8}\tilde{D}, \quad \beta = \frac{B_{3i}}{B_i},$$

$$\tilde{B} = B_3\frac{q_3^2}{q^2} + \tilde{C}_i q^2, \quad E = \frac{w}{\delta}\left(1 - \frac{3\tilde{\varkappa}_3}{64\tilde{U}_i} q^2\right),$$

$$a = w\left(\gamma_0 - \frac{\tilde{\varkappa}_i}{8\tilde{U}} q^2\right), \quad \delta = \frac{9}{32}\frac{\tilde{U}}{\tilde{U}_i}, \quad \gamma_0 = \frac{\tilde{A}}{2\tilde{U}} = \frac{A}{2U}.$$

All quantities containing constants labeled with the index $i$ are also labeled with this index, but for simplicity it is omitted. Next, we assume

$$\frac{U^2}{BD} \ll 1, \quad \frac{UA}{BD} \ll 1 \tag{64}$$

($B = B_i, B_{3i}$) and also, as previously, $q_3^2/q^2 \ll 1$. Then we obtain for the amplitude of the wavelike modulation the formula

$$u_0^2 = \frac{64\beta}{9q^2}(\gamma - \gamma_t), \tag{65}$$

where the threshold stress $\gamma_t$ has the form

$$\gamma_t = B_3\frac{q_3^2}{q^2} + \bar{K}q^2 - \bar{A}, \tag{66}$$

$$\bar{K} = \tilde{C}_i + \frac{2\tilde{U}_i\tilde{k}_i}{\tilde{D}} + \frac{3}{8}\frac{\tilde{A}\tilde{k}_3}{\tilde{D}}, \quad \bar{A} = \frac{8\tilde{U}_i\tilde{A}}{\tilde{D}}.$$

The quantity $\gamma_t$ has a minimum for $q = q_c$ given by

$$q_c = q_3^{1/2}\left(\frac{B_3}{\bar{K}}\right)^{1/4}, \tag{67}$$

or for the period of modulation $\lambda_c$:

$$\lambda_c = \frac{2\pi}{q_c} = 2(\pi d\lambda_3)^{1/2}, \tag{68}$$

where $\lambda_3 = (\bar{K}/B_3)^{1/2}$ is a characteristic length. The modulation amplitude $u_0$, corresponding to the period $\lambda_c$ is

$$u_c = \frac{8}{3}\lambda_i\left(\frac{\gamma - \gamma_c}{\gamma_c}\right)^{1/2}, \tag{69}$$

where $\lambda_i = (\bar{K}/B_i)^{1/2}$ are characteristic lengths, and the critical stress $\gamma_c$ is given by the formula ($\bar{A}/B_{3i} \ll 1$)

$$\gamma_c = \frac{2\pi}{d}\left(\frac{\bar{K}B_3}{B_{3i}^2}\right)^{1/2} \equiv 2\pi\frac{\lambda_4}{d}. \tag{70}$$

All characteristic lengths $\lambda_j$, $j = 1, 2, 3$, and 4, are of the same order of magnitude and

$$\lambda_j \ll d. \tag{71}$$

The quantities $u_c$, $\lambda_c$, and $\gamma_c$ are analogous to the corresponding quantities for the $A$ phase.[11,19] A specific feature of the $C$ phase is that modulation of the $\alpha$ and $\psi$ fields is possible. Based on what was said above, the critical amplitude $\psi_c$ is obtained by substituting $q_c$ into the formula (63):

$$\psi_c^2 = \frac{2\widetilde{U}}{\widetilde{D}}\,[\gamma_0 - \varepsilon(1+\sigma)\gamma_c - (1-\varepsilon)\gamma], \tag{72}$$

where

$$\varepsilon = \frac{\beta}{\delta} = \frac{32}{9}\frac{\widetilde{U}_i}{\widetilde{U}}\frac{B_{3i}}{B_i}, \quad \sigma = \frac{9}{2^{10}}\frac{\widetilde{k}_i}{\bar{K}}\frac{B_i}{\widetilde{U}_i}.$$

A formula for the critical amplitude $\alpha_{c1}$ is obtained by substituting into the formula (7) for $\alpha_{01}$ ($\alpha_{02} = 0$) the quantities $q_c$, $u_c$, and $\psi_c$ ($\psi_c = 0$) instead of $q$, $u_0$ and $\psi_0$, respectively:

$$\alpha_{c1} = \frac{\widetilde{L}_1}{\bar{\varkappa}_1}\frac{q_{c1}u_{c1}}{\psi_{c1}}. \tag{73}$$

We recall that two types of modulations are possible for $\alpha$: $a$) and $b$), if $\widetilde{L}_1^2/\bar{\varkappa}_1 C_1 < 1$ [see the formula (58)]. According to the experiment of Ref. 5, the modulation period is 1.6–2 times longer in the $\mathbf{c}_0$ direction than in the $\mathbf{m}_0$ direction. The ratio of these periods, according to the formula (68), is

$$\frac{\lambda_{c2}}{\lambda_{c1}} = \left(\frac{\bar{K}_2}{\bar{K}_1}\right)^{1/4} \simeq \left(\frac{C_2}{\widetilde{C}_1}\right)^{1/4}. \tag{74}$$

Here the approximate equality is written under the assumption $D \gg U, A$: $\bar{K}_2 \simeq \widetilde{C}_2 = C_2$ and $\bar{K}_1 \simeq \widetilde{C}_1$, where $\widetilde{C}_1$ is given either by Eq. (58) or (59). Comparing the experiments suggests that $C_2 \gg \widetilde{C}_1$. In addition, according to Ref. 5, modulation in the $\mathbf{c}_0$ direction appears at higher critical stresses than in the $\mathbf{m}_0$ direction. Their ratio is

$$\frac{\gamma_{c2}}{\gamma_{c1}} = \left(\frac{\bar{K}_2}{\bar{K}_1}\frac{B_{31}}{B_{32}}\right)^{1/2} \simeq \left(\frac{C_2}{\widetilde{C}_1}\frac{B_{31}}{B_{32}}\right)^{1/2}. \tag{75}$$

Since $\gamma_{c2}$ cannot significantly exceed $\gamma_{c1}$ ($\gamma_{c2} \sim \gamma_{c1}$), we have $B_{13} \sim B_{23}$. Modulation of the cant angle of the molecules was not discussed and apparently not observed in Ref. 5. Probably the case A ($\psi_1 = 0$), corresponding to a spatially uniform distribution of $\psi$, was realized in the experiment of Ref. 5.

As one can see from the formula (72), under the conditions of wavelike modulation of the layers ($\gamma > \gamma_c$) $\psi_c$ as a function of $\gamma$ is determined by the relations between the elastic constants and their temperature dependence: In particular, near the $AC$ transition we have $\gamma_0 \propto A \propto T_c - T$. We also have $\varepsilon, \sigma \sim 1$. For $\varepsilon < 1$ the amplitude $\psi_c$ decreases with increasing $\gamma$, while for $\varepsilon > 1$ it increases. Far from the point of the $AC$ transition we have $\gamma_0 \gg \gamma_c$, and the case A—a spatially uniform distribution of $\psi$—obtains. Near the $AC$ transition, when $\gamma_0 \sim \gamma_c$ holds, wave-like modulation of the smectic layers can induce a mixed state of the smectic, which we call the $M$ phase (total modulation or case B). As already mentioned, the $M$ phase is a state in which the $A$ and $C$ phases coexist.[1)]

The $M$ phase can form by different paths (for different smectics), depending on the ratios of the elastic constants. For example, for $\varepsilon_{\rm B} > 1$ and $\gamma_0 < \gamma_c^{\rm A}$ the following transformations can occur when the sample is stretched: $C$ phase—$A$ phase—wavelike modulation—$M$ phase. For $\varepsilon_{\rm B} < 1$ and $\gamma_0 > \gamma_c^{\rm A}$ the following path is possible: $C$ phase—wavelike modulation (case A)—$M$ phase. Other possibilities also exist. It is obvious that for a given sample (specific smectic of definite thickness) the energetically most favorable configuration obtains. We give an expression for the energy $\mathscr{F}_i$ in the case $\gamma = \gamma_c$:

$$\mathscr{F}_{ic} - \frac{B_3}{2}\gamma_c^2 = \frac{\widetilde{A}^2}{4\widetilde{D}}\left\{-1 + 2\frac{\gamma_c}{\gamma_0} + \left[1 - (1+\varepsilon\sigma)\frac{\gamma_c}{\gamma_0}\right]\frac{\widetilde{k}_i q_c^2}{4\widetilde{U}\gamma_0}\right\}, \tag{76}$$

and in addition $\tilde{k}_i^{\rm A} \equiv 0$ and

$$\bar{K}_{\rm B} > \bar{K}_{\rm A}, \quad \lambda_j^{\rm B} > \lambda_j^{\rm A}, \quad \gamma_c^{\rm B} > \gamma_c^{\rm A}. \tag{77}$$

The formulas (76) and (77) give an idea of how the free energy near the threshold of wavelike instability depends on the parameters of the system. We note also that in the mixed state described above (for one-dimensional modulation) the $C$ phase is either the "left-hand" or "right-hand" phase. However, structures in which a domain wall separates the left- and right-handed $C$ phases (see above) can form. Such a configuration is reminiscent of a domain wall in polyacetylene.[21]

## 7. CONCLUSIONS

As one can see from the preceding exposition, smectics $C$ have very diverse properties. An entire series of questions has remained outside the scope of this paper. It is important to describe dislocations in a system of smectic layers. Dislocations in a smectic play an important role in both statics and dynamics. For example, relaxation of stresses that appear with the formation of the periodic structures considered above occurs as a result of the motion of dislocations.[4,5,19] A large class of phenomena is associated with the motion of the liquid (including taking into account the motion of dislocations as well as in the presence of external fields). A general approach—based on the method of Poisson brackets—to the derivation of the equations of hydrodynamics for smectics is developed in Ref. 7, but the problem of the hydrodynamics of the $C$ phase cannot be considered as completely resolved, both from the standpoint of constructing a general theory and analyzing specific effects. In a nematic, for example, motion of domain walls was observed in the field of the director $\mathbf{n}$ in the presence of flow of a nematic liquid.[22] As shown in Ref. 23, the existence of such domain walls is associated with the orienting action of the flow on the director $\mathbf{n}$, similar to the orienting action of a magnetic field. A similar effect is also possible in the $C$ phase for the Goldstone variable $\alpha$.

Finally, we note some experimental aspects of the questions considered in this paper. Vortices in the field $\boldsymbol{\varphi}$ (disclinations of strength ± 1 in the $C$-director field) were observed in Ref. 5. The formulas (34) make it possible to find the constants $A$ and $D$ (and their temperature dependence) from measurements of the radius of the vortex core (and from independent measurements of the elastic constants $\varkappa_1$ and $\varkappa_2$). Just as in He-II or in a superconductor, it is possible that vortex lattices can form in the field of the vector $\boldsymbol{\varphi}$. The distance between the vortices can be estimated from the photographs displayed in Ref. 5: It is on the order of the thickness of the sample. A qualitative comparison of this theory with the experiment of Ref. 5 was made above. A quantitative comparison, however, requires more detailed experiments, especially near the $AC$ transition for $|T_c - T| \sim 1$–$10^{-3}$ K. This is also important for investigation of the $AC$ transition itself.

[1)] In order to avoid misunderstandings, we note that we are not talking about some phase transition or phase interfaces. The $M$ phase is a modulated structure in which the cant angle $\vartheta$ of the molecules varies sinusoidally from some value $\vartheta_0$ ($C$ phase) to 0 ($A$ phase).

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