Quasitransition radiation of a charged particle reflected by a crystal surface

M. I. Ryazanov and A. N. Safronov

Moscow Institute of Physical Engineering
(Submitted 17 June 1992)
Zh. Eksp. Teor. Fiz. **103**, 547–554 (February 1993)

The radiation of a charged relativistic particle undergoing mirror reflection from a plane interface is considered. It is shown that the frequency-angular distribution of radiation depends on the relative orientation of the effective planes of electromagnetic wave and particle reflection.

1. It is known that transition radiation arises when a charged particle traverses the interface between two media due to surface polarization. On the other hand, a fast charged particle which intersects a crystal surface at a grazing angle ζ (between the particle velocity and crystal surface) undergoes mirror reflection if $\zeta<\vartheta_L=(U/E)^{1/2}$ (here ϑ_L is the Lindhard angle, U is the surface potential barrier, and E is the particle energy).[1] The change in the particle velocity under reflection gives rise to bremsstrahlung. However in such a process the particle comes close to the surface and polarizes the latter so that the same processes are induced which give rise to transition radiation, when a charge intersects a surface. Therefore in this case radiation similar to transition radiation may be expected. As a result of the simultaneous action of the mechanisms responsible for bremsstrahlung and transition radiation we have combined radiation which in what follows will be called, for short, quasitransition radiation.

The estimate of the intensity of quasitransition radiation is of interest for two reasons. First, it is interesting to know how the contributions of bremsstrahlung and transition radiation depend on the charge velocity and the properties of the medium. The transition radiation is a unique process whose intensity depends on the particle Lorentz factor rather than the velocity. Is this still true for quasitransition radiation? Second, the effective crystal surface that reflects electrons may not, generally speaking, coincide with the effective surface that reflects electromagnetic field. Indeed, electrons are reflected by surface atoms and light by the electrons of the medium. In metals the density of conduction electrons experiences small Friedel oscillations near the surface and vanishes on the outside of the surface ion layer at the distance $z=\pi h/p_F$, where h is the Planck constant, p_F is the Fermi momentum, and the $z=0$ plane coincides with the surface ion layer.[3,4] Thus, the effective crystal surfaces defined by the particle and light reflection are, generally speaking, displaced by a certain distance b. The quantity b contains definite information about surface properties. To find this distance in experiment it is convenient to use processes involving the interaction of both charged particles and field with the surface. The quasitransition radiation under reflection of fast electrons from a plane monocrystal surface is one such process.

Note that for $\zeta>\vartheta_L$ a charged particle is not reflected by the surface atomic layer but penetrates into the substance, after which it can leave the crystal due to scattering. In what follows we will not consider this process, which is analyzed in Ref. 2.

2. When considering the quasitransition radiation, one should take into account the fact that radiation is a wave process which occurs in a time of order $\tau=(\omega-\mathbf{kv})^{-1}$, where ω and $\mathbf{k}$ are the frequency and wave vector of radiation, and $\mathbf{v}$ is the particle velocity, rather than instantaneously. In a time τ the particle moves closer to the surface ionic layer by $\tau v\zeta$. It is possible to assume that the surface is different for particles and field only if the b distance between the reflecting surfaces is larger than $\tau v\zeta$. If, on the contrary,

$$\pi h/p_F \ll v\zeta/(\omega-\mathbf{kv}), \tag{1}$$

there is no point in distinguishing these surfaces. When the condition (1) is valid, the surfaces coincide for particles and field.

Near the interface of two media there is always a thin layer near the surface that differs in its properties from both media due to surface reconstruction, microinhomogeneities, impurities, surface electron states, etc. If the radiation wavelength λ is large in comparison with the thickness h of the surface layer, we can neglect the existence of such a layer, considering the properties of each medium unchanged up to the interface representing an ideal plane. For ultrarelativistic particles and large frequencies such approximation can be used even in the x-ray region, since in this case

$$\tau\sim E^2/\omega m^2c^4,\quad v\zeta\tau\gg h$$

for

$$\omega\lesssim(mc^2)(e^2/\hbar c)^2$$

(see Ref. 5).

Consider radiation arising when a charge e is reflected by a plane surface of a semi-infinite ($z<0$) cubic crystal with a dielectric constant ε_1, assuming that the charged particle moves in an isotropic medium with a dielectric constant ε_2 and the plane $z=0$ is the reflection plane both for the particle and the field. For wavelengths larger than the lattice constant the crystal can be considered uniform, therefore the problem is uniform in the x and y directions and stationary, so that the field frequency and x and y projections of the wave vectors are conserved. It is convenient to use the Fourier transformation:

$$\mathbf{E}(q_x,q_y,z,\omega)=(2\pi)^{-3}\int\int\int dxdydt\,\mathbf{E}(\mathbf{r},t)\exp(-iq_xx-iq_yy+i\omega t). \tag{2}$$

 1063-7761/93/020276-04$10.00

The particle moves in the region $z > 0$ and is reflected by the crystal. The field outside the crystal is the sum of the particle intrinsic field $\mathbf{E}_0$ and the field $\mathbf{E}_2$ of the transverse waves, propagating from the interface. The field $\mathbf{E}_1$ inside the crystal should consist only of transverse plane waves propagating from the surface.

The fields $\mathbf{E}_1$ and $\mathbf{E}_2$ can be written as

$$\mathbf{E}_{1(2)}(\mathbf{q}, z, \omega) = \mathbf{E}_{1(2)}(\mathbf{q}, \omega)\exp(\mp)i\varkappa_{1(2)}z), \qquad (3)$$

where

$$\varkappa_{1(2)} = [(\omega^2/c^2)\varepsilon_{1(2)} - q^2]^{1/2}.$$

The fields $\mathbf{E}_{1(2)}$ and $\mathbf{H}_{1(2)}$ being transverse, the boundary conditions at the surface $z = 0$ give the following relations for the normal components of the radiation fields

$$(\varepsilon_1\varkappa_2 + \varepsilon_2\varkappa_1)E_{1(2)z}(\mathbf{q}, \omega)$$

$$= \binom{+}{-}\varepsilon_{2(1)}\varkappa_{2(1)}E_{0z}(\mathbf{q}, 0, \omega) + \varepsilon_{2(1)}(\mathbf{q}, \mathbf{E}_0(\mathbf{q}, 0, \omega)),$$

$$(\varkappa_1 + \varkappa_2)H_{1(2)z}(\mathbf{q}, \omega) = \binom{+}{-}\varkappa_{2(1)}H_{0z}(\mathbf{q}, 0, \omega) + \mathbf{q}\mathbf{H}_0(\mathbf{q}, 0, \omega). \qquad (4)$$

Using the Maxwell equations, we can find the other field components. For example, with the help of the equation

$$\mathrm{rot}\,\mathbf{H}_{1(2)} = -i(\omega/c)\mathbf{D}_{1(2)}$$

we get

$$q^2H_{1(2)x} = q_y\varepsilon_{1(2)}(\omega/c)E_{1(2)z}\binom{+}{-}\varkappa_{1(2)}q_xH_{1(2)z},$$

$$q^2H_{1(2)y} = -q_x\varepsilon_{1(2)}(\omega/c)E_{1(2)z}\binom{+}{-}\varkappa_{1(2)}q_yH_{1(2)z}. \qquad (5)$$

The equations $\mathrm{div}\,\mathbf{D}_0 = 4\pi\rho_0$ and $\mathrm{div}\,\mathbf{H}_0 = 0$ yield

$$\mathbf{q}\mathbf{H}_0(\mathbf{q}, 0, \omega) = i\frac{\partial}{\partial z}H_{0z}(\mathbf{q}, 0, \omega),$$

$$\varepsilon_1\mathbf{q}\mathbf{E}_0(\mathbf{q}, 0, \omega) = i\varepsilon_1\frac{\partial}{\partial z}E_{0z}(\mathbf{q}, 0, \omega) - i4\pi\rho_0. \qquad (6)$$

As follows from (4) and (6), to find the radiation field it is sufficient to find the Fourier-transform of the normal components of the intrinsic field of the charge

$$E_{0z}(\mathbf{q}, 0, \omega),$$

$$H_{0z}(\mathbf{q}, 0, \omega).$$

The Fourier transform of the current density of the charge experiencing mirror reflection from the surface $z = 0$ has the form

$$\mathbf{j}(\mathbf{q}, z, \omega) = \frac{e}{4\pi^3u}\left\{\mathbf{v}\cos\left[(\omega - \mathbf{q}\mathbf{v})\frac{z}{u}\right]\right.$$

$$\left.+ iu\sin\left[(\omega - \mathbf{q}\mathbf{v})\frac{z}{u}\right]\right\} \text{ for } z > 0, \qquad (7)$$

for $z > 0$, where v and u are the charge velocity components tangent and normal to the surface respectively. Therefore the partial solution of the Maxwell equation for the particle intrinsic field is

$$E_{0z}(\mathbf{q}, z, \omega) = \frac{e}{\pi^2\varepsilon_2c^2}\frac{\omega\varepsilon_2u^2 - c^2(\omega - \mathbf{q}\mathbf{v})}{u^2\varkappa_2^2 - (\omega - \mathbf{q}\mathbf{v})^2}\sin\left[(\omega - \mathbf{q}\mathbf{v})\frac{z}{u}\right],$$

$$H_{0z}(\mathbf{q}, z, \omega) = i\frac{eu}{\pi^2c}\frac{[\mathbf{q}\mathbf{v}]_z}{u^2\varkappa_2^2 - (\omega - \mathbf{q}\mathbf{v})^2}\cos\left[(\omega - \mathbf{q}\mathbf{v})\frac{z}{u}\right]. \qquad (8)$$

To find the energy distribution of the quasitransition radiation in angle and frequency, we calculate the radiation energy flux through the plane $z = L > 0$:

$$\Delta\varepsilon = \int\int\int dxdydt\frac{c}{4\pi}[\mathbf{E}_2(\mathbf{r}, t)\mathbf{H}_2(\mathbf{r}, t)]_z. \qquad (9)$$

Then the energy in the frequency range $d\omega$ and angle range $d\Omega$ is

$$d^2\varepsilon(\mathbf{n}, \omega) = (2\pi)^2k^2c[\mathbf{E}_2(\mathbf{q}, \omega)\mathbf{H}_2^*(\mathbf{q}, \omega)]_z\cos\theta d\Omega d\omega, \qquad (10)$$

where θ and φ are the angles of the spherical coordinate system with axis in the z direction,

$$\mathbf{k} = \mathbf{q} + \mathbf{1}_z\varkappa_2, \quad |\mathbf{k}| = (\omega/c)\sqrt{\varepsilon_2},$$

and

$$q_x = k\sin\theta\cos\varphi, \quad q_y = k\sin\theta\sin\varphi, \quad \varkappa_2 = k\cos\theta.$$

Equation (10) yields

$$d^2\varepsilon(\mathbf{n}, \omega)$$

$$= (4\pi^2\omega^4\varepsilon_2\sqrt{\varepsilon_2}\cos^2\theta/q^2c^3)\{\varepsilon_2|E_{2z}|^2 + |H_{2z}|^2\}d\Omega d\omega. \qquad (11)$$

Substituting (6) and (8) into (4) and then into (11), we find the distribution of radiation energy in angle and frequency in the medium where the particle moves:

$$\frac{d^2\varepsilon_2}{d\omega d\Omega} = \frac{4e^2}{\pi^2c^3}\frac{u^2\varepsilon_2\sqrt{\varepsilon_2}\cos^2\theta}{|(1 - (v/c)\sqrt{\varepsilon_2}\sin\theta\cos\varphi)^2 - (u/c)^2\varepsilon_2\cos^2\theta|^2}$$

$$\times\left\{\frac{|\varepsilon_1 - \varepsilon_2\sin^2\theta|(v/c)^2\sin^2\varphi}{|\sqrt{\varepsilon_2}\cos\theta + \sqrt{\varepsilon_1 - \varepsilon_2\sin^2\theta}|^2}\right.$$

$$\left.+\frac{|\varepsilon_1|^2|\sin\theta - (v/c)\varepsilon_2\cos\varphi|^2}{|\varepsilon_1\sqrt{\varepsilon_2}\cos\theta + \varepsilon_2\sqrt{\varepsilon_1 - \varepsilon_2\sin^2\theta}|^2}\right\}. \qquad (12)$$

Here we have used the assumption that the radiation is detected in a transparent medium ($\mathrm{Im}\,\varepsilon_2 = 0$). Note that in (12) we have allowed both for the transition radiation and bremsstrahlung reflection from the interface. For $\varepsilon_1 = \varepsilon_2$ the distribution (12) coincides with the bremsstrahlung distribution in a medium associated with a sudden deviation in the motion of the charge.

Since the particle moves only in one medium, the radiation in the first medium will markedly differ from that in the second medium. The radiation in the medium from which the charge is reflected is found in the same way as Eq. (12). Thus,

$$\frac{d^2\varepsilon_1}{d\omega d\Omega}$$

$$=\frac{4e^2}{\pi^2c^3}\frac{u^2\varepsilon_1\sqrt{\varepsilon_1}\cos^2\theta'}{|(1-(v/c)\sqrt{\varepsilon_1}\sin\theta'\cos\varphi)^2-(u/c)^2(\varepsilon_2-\varepsilon_1\sin^2\theta')|^2}$$

$$\times\left\{\frac{|\varepsilon_2-\varepsilon_1\sin^2\theta'|(v/c)^2\sin^2\varphi}{|\sqrt{\varepsilon_1}\cos\theta'+\sqrt{\varepsilon_2-\varepsilon_1\sin^2\theta'}|^2}\right.$$

$$\left.+\frac{|\varepsilon_1\sin\theta'-(v/c)\sqrt{\varepsilon_1\varepsilon_2}\cos\varphi|^2}{|\varepsilon_2\sqrt{\varepsilon_1}\cos\theta'+\varepsilon_1\sqrt{\varepsilon_2-\varepsilon_1\sin^2\theta'}|^2}\right\}, \qquad (13)$$

where θ' is the angle formed by the z axis and the radiation direction. It is assumed that $\mathrm{Im}\,\varepsilon_1=0$.

Consider the radiation polarization. The energy of radiation polarized in the plane defined by the normal to the interface and the wave vector can be found as Eq. (12)

$$\left(\frac{d^2\varepsilon_2}{d\omega d\Omega}\right)_\parallel$$

$$=\frac{4e^2}{\pi^2c^3}\frac{u^2\varepsilon_2\sqrt{\varepsilon_2}\cos^2\theta}{|(1-(v/c)\sqrt{\varepsilon_2}\sin\theta\cos\varphi)^2-(u/c)^2\varepsilon_2\cos^2\theta|^2}$$

$$\times\frac{|\varepsilon_1|^2|\sin\theta-(v/c)\sqrt{\varepsilon_2}\cos\varphi|^2}{|\varepsilon_1\sqrt{\varepsilon_2}\cos\theta+\varepsilon_2\sqrt{\varepsilon_1-\varepsilon_2\sin^2\theta}|^2}. \qquad (14)$$

The energy of radiation polarized perpendicular to the plane defined by the normal to the interface and the wave vector is

$$\left(\frac{d^2\varepsilon_2}{d\omega d\Omega}\right)_\perp$$

$$=\frac{4e^2}{\pi^2c^3}\frac{u^2\varepsilon_2\sqrt{\varepsilon_2}\cos^2\theta}{|(1-(v/c)\sqrt{\varepsilon_2}\sin\theta\cos\varphi)^2-(u/c)^2\varepsilon_2\cos^2\theta|^2}$$

$$\times\frac{|\varepsilon_1-\varepsilon_2\sin^2\theta|(v/c)^2\sin^2\varphi}{|\sqrt{\varepsilon_2}\cos\theta+\sqrt{\varepsilon_1-\varepsilon_2\sin^2\theta}|^2}. \qquad (15)$$

Note that the transverse polarization disappears when the radiation is detected in the plane of incidence ($\varphi=0$). In the nonrelativistic limit Eq. (13) reduces to

$$\frac{d^2\varepsilon_1}{d\omega d\Omega}=\frac{4e^2}{\pi^2c^3}\frac{u^2\varepsilon_1^3\sqrt{\varepsilon_1}\cos^2\theta'\sin^2\theta'}{|\varepsilon_2\sqrt{\varepsilon_1}\cos\theta'+\varepsilon_1\sqrt{\varepsilon_2-\varepsilon_1\sin\theta'}|^2}. \qquad (16)$$

The radiation in the second medium has the same form. In the ultrarelativistic case the range of high frequencies and grazing-angle particle incidence on the interference is of interest. For $\omega\gg\omega_p$ the dielectric constant is close to unity and equals

$$\varepsilon_{1(2)}(\omega)=1-(\omega_{p1(2)}/\omega)^2.$$

For grazing incidence we have $u\ll v$ and $u/v\sim\zeta\ll1$. Consider the range $\varphi\ll1$ and $|\pi/2-\theta|\equiv\psi\ll1$. Assuming also that $\omega_{p1(2)}/\omega\ll1$ and $mc^2/E\ll1$, we get from (13)

$$\frac{d^2\varepsilon_1}{d\omega d\Omega}=\frac{64e^2}{\pi^2c}\frac{\psi^2\zeta^2}{\left|\left[\left(\frac{mc^2}{E}\right)^2+\zeta^2+\psi^2+\varphi^2+\left(\frac{\omega_{p1}}{\omega}\right)^2\right]^2-4\zeta^2\left[\psi^2+\frac{(\omega_{p1}^2-\omega_{p2}^2)}{\omega^2}\right]\right|^2}$$

$$\times\frac{\varphi^2\left|\psi^2+\frac{(\omega_{p1}^2-\omega_{p2}^2)}{\omega^2}\right|+\frac{1}{4}\left|\left(\frac{mc^2}{E}\right)^2+\zeta^2-\psi^2+\varphi^2+\frac{(2\omega_{p2}^2-\omega_{p1}^2)}{\omega^2}\right|^2}{\left|\psi+\left(\psi^2+\frac{\omega_{p2}^2-\omega_{p1}^2}{\omega^2}\right)^{1/2}\right|^2} \qquad (17)$$

and from (12)

$$\frac{d^2\varepsilon_2}{d\omega d\Omega}=\frac{64e^2}{\pi^2c}\frac{\psi^2\zeta^2}{\left|\left[\left(\frac{mc^2}{E}\right)^2+\zeta^2+\psi^2+\varphi^2+\left(\frac{\omega_{p2}}{\omega}\right)^2\right]^2-4\zeta^2\psi^2\right|^2}$$

$$\times\frac{\varphi^2\left|\psi^2+\frac{\omega_{p1}^2-\omega_{p2}^2}{\omega^2}\right|+\frac{1}{4}\left|\left(\frac{mc^2}{E}\right)^2+\zeta^2+\varphi^2-\psi^2+\left(\frac{\omega_{p2}}{\omega}\right)^2\right|^2}{\left|\psi+\left(\psi^2+\frac{\omega_{p1}^2-\omega_{p2}^2}{\omega^2}\right)^{1/2}\right|^2}. \qquad (18)$$

It is seen that the main part of the radiated energy is within the range $\psi, \varphi \lesssim mc^2/E, \omega_p/\omega$. Note also that in the ultrarelativistic case the intensity ratio for different polarizations in the medium where the charge moves is

$$\frac{d^2\varepsilon_2(\mathbf{n},\omega)_{\parallel}}{d^2\varepsilon_2(\mathbf{n},\omega)_{\perp}} \approx \frac{\psi^2}{|\psi^2 + (\omega_{p2}^2 - \omega_{p1}^2)/\omega^2|}$$

This ratio is small for $\psi^2 \ll |\omega_{p2}^2 - \omega_{p1}^2|/\omega^2$.

We emphasize, in conclusion, that in all examples considered the transition radiation cannot be separated from the bremsstrahlung and both radiation processes should be considered together.

4. In the case when the particle reflects from a metal the difference in the location of the effective surfaces for particle and field reflection can be of interest.

Consider the interface of vacuum and ideal conductor ($\mathrm{Im}\, \varepsilon \gg 1$) filling the half-space $z<0$. In this case we can make use of the image method.[6] Let the electromagnetic field undergo reflection and refraction at the surface $z=0$. The charge $+e$ is reflected by the plane $z=b$.

When the effective planes of particle and field reflection coincide, i.e., for $b=0$ the motion of the charge $+e$ has the form

$$\mathbf{r}_+(t) = (\mathbf{v}+\mathbf{u})\theta_f(t)t + (\mathbf{v}-\mathbf{u})\theta_f(-t)t, \tag{19}$$

where $\theta_f(t)$ is the Heaviside step function, and $\mathbf{v}$ and $\mathbf{u}$ are the tangent and normal components of charge velocity respectively. The image charge $-e$ moves along the mirror trajectory

$$\mathbf{r}_-(t) = (\mathbf{v}-\mathbf{u})\theta_f(t)t + (\mathbf{v}+\mathbf{u})\theta_f(-t)t. \tag{20}$$

The distribution of radiation energy in angle and frequency has the form

$$\frac{d^2\varepsilon}{d\omega d\Omega}(b=0) = \frac{4e^2}{\pi^2 c^3} \frac{u^2[(v^2/c^2)\cos^2\theta \sin^2\varphi + [\sin\theta - (v/c)\cos\varphi]^2]}{\{[1-(v^2/c^2)\sin\theta\cos\varphi]^2 - [(u/c)\cos\theta]^2\}^2}, \tag{21}$$

where θ and φ are spherical angles determining the radiation direction.

In the case when the particle is reflected without reaching the effective field-reflection surface, i.e., for $b>0$, the laws of motion for the charge and image charge are

$$\begin{aligned} \mathbf{r}_+(t) &= \mathbf{b} + (\mathbf{v}+\mathbf{u})\theta_f(t)t + (\mathbf{v}-\mathbf{u})\theta_f(-t)t, \\ \mathbf{r}_-(t) &= -\mathbf{b} + (\mathbf{v}-\mathbf{u})\theta_f(t)t + (\mathbf{v}+\mathbf{u})\theta_f(-t)t. \end{aligned} \tag{22}$$

The corresponding distribution of the radiation in angle and frequency has the form

$$\frac{d^2\varepsilon}{d\omega d\Omega} = \cos^2\left(\frac{\omega}{c} b\cos\theta\right)\frac{d^2\varepsilon}{d\omega d\Omega}(b=0). \tag{23}$$

In the case when the particle is reflected by "interior" layers of the material, i.e., for negative b ($b=-|b|$), the motion of the charge and its image is given by the following expressions:

$$\begin{aligned} \mathbf{r}_+(t) &= (\mathbf{v}+\mathbf{u})\theta_f(t-|b|/u)t + (\mathbf{v}-\mathbf{u})\theta_f(-t-|b|/u)t, \\ \mathbf{r}_-(t) &= (\mathbf{v}-\mathbf{u})\theta_f(t-|b|/u)t + (\mathbf{v}+\mathbf{u})\theta_f(-t-|b|/u)t. \end{aligned} \tag{24}$$

The distribution in angle and frequency is

$$\frac{d^2\varepsilon}{d\omega d\Omega} = \cos^2\left[(\omega - \mathbf{kv})\frac{|b|}{u}\right]\frac{d^2\varepsilon}{d\omega d\Omega}(b=0). \tag{25}$$

As seen from (23) and (25), the angular-frequency distribution depends on the reciprocal position of effective surfaces for charge and field reflection. In particular, for $b<0$, $\mathbf{kv}=0$ and $1 \gg |b|/\lambda \gg u/c$ the angular-frequency distribution can vanish when the particle penetration depth b_0 is equal to

$$|b_0| = \frac{\pi u}{\omega}\left(m+\frac{1}{2}\right), \quad m=0,1,2\ldots. \tag{26}$$

This corresponds to the radiation fields arriving out of phase to the observation point from the particle trajectory before the particle enters the ideal conductor (for $t<-|b|/u$) and after leaving it (for $t>|b|/u$). Thus, the angular distribution of the radiation considered gives the magnitude and sign of the distance between the effective planes for light and electron reflection and the interface.

When we compare (21) with the expression for the energy of the bremsstrahlung in vacuum for the same particle deviation, it is not difficult to see that the bremsstrahlung gives only 1/4 of the total intensity.

[1]I. Lindhard, Usp. Fiz. Nauk **99**, 243 (1969).
[2]D. B. Rogozkin and V. V. Frolov, *Poverkhnost' i effekty neodnorodnosti tvyordogo tela* (*Surface and Inhomogeneity Effects in Solid States*), Moscow, Energoatomizdat (1987), p. 28.
[3]M. F. Partenskkii, Usp. Fiz. Nauk **128**, 69 (1979) [Sov. Phys. Usp. 22, 330 (1979)].
[4]N. D. Lang and W. Kohn, Phys. Rev. B **3**, 1215 (1971).
[5]M. L. Ter-Mikaelyan, *Vliyanie sredy na elektromagnitnye protsesy pri vysokikh energiyakh* (*High-Energy Electromagnetic Processes in Condensed Media*, Wiley-Interscience, New York, 1972), Yerevan, Armenian Academy of Sciences Press (1969).
[6]V. A. Ginzburg and I. M. Frank, Zh. Eksp. Teor. Fiz. **16**, 15 (1946) [*sic*].

Translated by E. Khmelnitski