

Nonlinear electrical conductivity of a pure metal at low temperature

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The possibility of finite electrical resistance in a pure metal at $T = 0$ due to phonon emission by electrons is studied. In the limits of small and large fields the kinetic equation is solved and the current-voltage characteristic and electron temperature are found.

1. INTRODUCTION

Consider a metal which remains nonsuperconducting at $T = 0$. If there are no impurities, the residual resistance equals zero, and the electric current should not damp out. In an electric field electrons are accelerated until the energy difference (on the Fermi surface) for the electrons moving parallel and antiparallel to the field is large enough for the onset of spontaneous phonon emission. The latter is possible if the drift velocity $v = j/en$ is larger than the sound velocity u . The expected current-voltage characteristic should have the form shown in Fig. 1.

This effect was discovered in 1982 by Bogod *et al.*¹ in bismuth, whose carrier density and critical current j_c are small. The curve shown in Fig. 1 is obtained by subtracting the contribution of residual resistance from the experimental curve.¹ Twenty years before that Esaki² had measured magnetoresistance in bismuth in a strong quantizing magnetic field and found the current-voltage characteristic shown in Fig. 2.

Esaki was right to connect his effect with the onset of phonon emission when the drift velocity v_d of electron orbits in crossed electric and magnetic fields becomes larger than the sound velocity.

Thus, though the current-voltage characteristics shown in Figs. 1 and 2 are opposite to each other, the crucial role of the sign of the difference $v_d - u$ in both effects indicates their common intrinsic cause. Theoretical reviews of the Esaki effect are discussed at length in Ref. 3.

In the present paper we give the theory of the BVGG-effect (Bogod, Valeev, Gitsu, Grozav) (see Ref. 1).

Using the kinetic equation describing the electron scattering accompanied by phonon emission 1) the boundaries of nondissipative electron motion as a function of the phonon dispersion law and Fermi surface shape have been found, and 2) for an isotropic model with the Debye phonon spectrum the exact form of the electron distribution function has been obtained in two limiting cases, $E \ll E_0$ and $E \gg E_0$, where E_0 is the field in which an electron increases its velocity by u in the time between two spontaneous phonon emission events. 3) In a weak field ($E \ll E_0$) the exact solution of the kinetic equation for a more realistic model with an ellipsoidal Fermi surface has also been found. The theory developed below describes well the current-voltage characteristic of Fig. 1.

In the conclusion we direct our attention to the fact that the current-voltage characteristic in a magnetic field (Fig. 2) can be related to the one without the field (Fig. 1) by

means of usual theory of galvanomagnetic phenomena in a compensated metal.

2. THE KINETIC EQUATION; THE MAXIMUM NONDISSIPATIVE CURRENT; THE τ APPROXIMATION

Consider a metal which remains normal at zero temperature. If there are no impurities, the only possible mechanism for electron scattering at $T = 0$ is phonon emission. Then electron conductivity in a constant uniform electric field E is determined by the kinetic equation

$$eE_\alpha \frac{\partial f}{\partial p_\alpha} = St f, \quad St f = Y_{in} - Y_{out}, \quad (2.1)$$

$$Y_{in} = \int d\Gamma' W f' (1 - f) \delta(\epsilon' - \epsilon - \omega),$$

$$Y_{out} = \int d\Gamma' W f (1 - f') \delta(\epsilon - \epsilon' - \omega).$$

We will consider mainly an isotropic model 1* in which

$$\epsilon_p = p^2/2, \quad \omega_q = uq, \quad \mathbf{q} = \mathbf{p} - \mathbf{p}', \quad (2.2)$$

$$W = (q/2p_F)W_0, \quad W_0 = u\pi^2,$$

$$d\Gamma = v_F d\epsilon d\omega / 4\pi, \quad v_F = p_F/\pi^2.$$

Let electrons be in a state with a current $j_z = v_d n$ (n is the electron density and v_d is the drift velocity) and given by the distribution

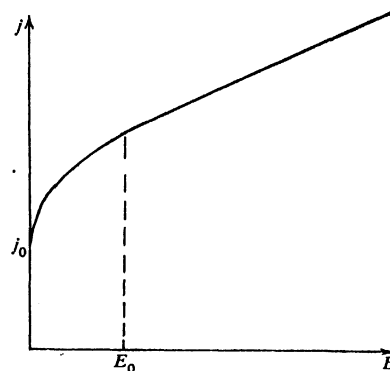


FIG. 1.

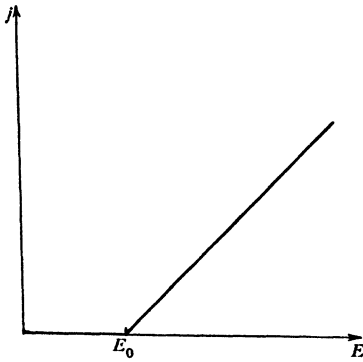


FIG. 2.

$$f_p = \theta(-\xi), \quad \xi = \varepsilon - \varepsilon_F^*, \quad \theta(x) = (1 + \text{sign } x)/2 \quad (2.3)$$

with a shifted Fermi surface

$$\varepsilon_F^* = \varepsilon_F + v_d p_z. \quad (2.4)$$

For this distribution we have

$$Y_{in} = \int d\Gamma' W \theta(-\xi') \theta(\xi) \delta(\xi' - \xi + \psi), \quad (2.5)$$

$$Y_{out} = \int d\Gamma' W \theta(-\xi) \theta(\xi') \delta(\xi - \xi' + \psi),$$

$$\psi = v_d q_z - u q. \quad (2.6)$$

For $v_d < u$, the function ψ is negative for all transferred momenta q . The integrals Y_{in} and Y_{out} vanish, since the energy conservation law forbids emission of a phonon.

This means that the distribution (2.3) for $v_d < u$ is the solution of the kinetic equation with $E = 0$. Thus, in the isotropic model (2.2) with a sound spectrum the electric current is not damped until the drift velocity v_d is smaller than the sound velocity u .

If we allow for dispersion of the short wavelength fraction of acoustic phonons (model 2*)

$$\omega(q), \quad 0 < d\omega/dq < u, \quad d^2\omega/dq^2 < 0,$$

we can increase the nondissipative current by choosing a deformed Fermi surface in the form

$$\varepsilon_F^* = \varepsilon_F + \frac{1}{2} (1 + \mu) \omega(2p_z) \text{sign } p_z. \quad (2.7)$$

The greatest nondissipative current corresponds to $\mu = 0$. In the model 2* the role of (2.6) is played by the expression

$$\psi = \frac{1}{2} (1 + \mu) [\omega(2p_z) \text{sign } p_z - \omega(2p'_z) \text{sign } p'_z] - \omega(q), \quad (2.8)$$

which, like (2.6), is largest when a phonon is emitted in the direction of the Z axis:

$$\max \psi = \mu \omega(2p_z). \quad (2.9)$$

The energy conservation law allows phonon emission for $\mu > 0$. The critical current corresponding to the limit $\mu = +0$ equals ($\langle \dots \rangle$ stands for an average over the Fermi surface)

$$j_c = \frac{1}{2} v_F \langle |p_z| \omega(2p_z) \rangle. \quad (2.10)$$

The corresponding critical drift velocity has the limits

$$U(2p_F) < v_{dc} < U(0), \quad (2.11)$$

where U is the phonon phase velocity with momentum q .

Note that, according to (2.10), the main contribution to the critical current is made by short-wavelength phonons.

At last, having in mind primarily bismuth, we consider a model 3* with an ellipsoidal Fermi surface. We will restrict the discussion to a linear phonon spectrum, which is reasonable since for bismuth $p_F \ll 1$ holds:

$$\varepsilon_p = \sum_{\alpha} p_{\alpha}^2 / 2m_{\alpha}, \quad \omega_q = u(\mathbf{e}_q) q. \quad (2.12)$$

Here $\alpha = 1, 2, 3$ are the directions of the main crystal axes, and $u(\mathbf{e}_q)$ is the sound velocity in the direction $\mathbf{e}_q = \mathbf{q}/q$.

Choosing a deformed Fermi surface in the form (2.4) for which $j \parallel z$ and $j_z = v_d n e$, we find

$$\psi = v_d q - u(\mathbf{e}_q) q. \quad (2.13)$$

It follows from this expression that the critical drift velocity equals the sound velocity in the direction of the current.

The distribution in the form (2.3) with the Fermi surface (2.4) or (2.7) is naturally chosen as an approximate solution of the kinetic equation (2.1) in connection with the moment method (τ approximation). We multiply the kinetic equation (2.1) by $v_{\alpha} (p_{\alpha}$ or $p_{\alpha}/m_{\alpha})$ and integrate over all momenta:

$$E_{\alpha} n = \langle \langle (p_{\alpha} - p'_{\alpha}) \psi \theta(\psi) W \rangle \rangle v_F^2. \quad (2.14)$$

Here ($\langle \dots \rangle$) indicates an average over the Fermi surfaces of the initial and final electron states.

This averaging is done most simply in the model 2* without phonon dispersion ($\omega_q = \omega_0$, $W_q = W_0$). In this model [see (2.10)]

$$\psi = \mu \omega_0, \quad v_d = (1 + \mu) v_{dc}, \quad E = \mu E_0,$$

$$E_0 = W_0 v_F v_{dc}, \quad v_{dc} = 3\omega_0 / 4p_F \quad (2.15)$$

and the current-voltage characteristic has the form

$$j = v_{dc} e N [1 + (E/E_0)]. \quad (2.16)$$

The same behavior of the current-voltage characteristic in the limits $E \ll E_0$ and $E \gg E_0$ is conditioned by the fact that in this model, for $v_d > v_{dc}$, the scattering from any point on the upper part of the Fermi surface (2.7) to any point on the lower hemisphere has the same probability.

In contrast, in the model 1*, near the threshold scattering is allowed only into a narrow cone about the direction opposite that of the current. The integral on the right-hand side of Eq. (2.14) is proportional to $(v_d - u)^2$ and, for a large drift velocity when there is no restriction on the scattering direction, is proportional to v_d . As a result, we find a current-voltage characteristic of the form

$$j = neu \begin{cases} (1 + \alpha_1 (E/E_0)^{1/2}), & E \ll E_0, \\ \alpha_2 E/E_0, & E \gg E_0, \end{cases} \quad (2.17)$$

where $E_0 = W_0 \nu_F u$ and has the same order of magnitude as in (2.15), and $\alpha_{1,2} \sim 1$.

If we allow for anisotropy of electron and phonon spectra, that will not alter the qualitative dependence of the field magnitude on the current magnitude, but the field direction does not coincide with the current direction.

3. "EXACT SOLUTION". ISOTROPIC MODEL

Only very rarely does the kinetic Boltzmann equation allow an analytic solution. Such a solution exists for the Maxwell gas and elastic electron scattering by impurities (residual resistance). The kinetic equation (2.1) for the model (2.2), in spite of the inelastic character of the scattering, has an exact solution (in terms of perturbation theory) in the limits of small and large field values. This is mainly because the "exact" distribution function for electrons in a field is close to the sample function of the τ approximation (2.3), i.e., it is a function of one variable (energy) in a coordinate system moving with the drift velocity.

We rewrite the kinetic equation (2.1) for $E \parallel z$ in the form

$$A \cos \theta \frac{\partial f}{\partial \varepsilon} = -f \langle (q/2p_F) [1 - f(\varepsilon - \omega, \theta')] \rangle' + (1 - f) \langle (q/2p_F) f(\varepsilon + \omega, \theta') \rangle'. \quad (3.1)$$

Here, in the left-hand side of the kinetic equation, we have replaced $\partial f / \partial p_z$ by $v_z \partial f / \partial \varepsilon$, which is always possible for metals, where $v_d \ll v_F$ and $T \ll \varepsilon_F$. The parameter $A = e E v_F / \nu_F W_0$ is the work done by the field in the time between two acts of phonon emission. In the right-hand side we have integrated over ε' and introduced a notation $\langle \dots \rangle'$ for averaging over the Fermi surface: $\langle \dots \rangle' = \int \dots d\omega' / 4\pi$.

In the small-field limit the problem is simplified because the electron scattering is restricted to a narrow cone in the direction opposite the field. The cone solid angle is the small parameter of the problem. Under these circumstances emission is possible only from the front ($v_z > 0$) to the back hemisphere ($v_z < 0$). This means that in (3.1) for $\cos \theta > 0$ there is only "outgoing" and for $\cos \theta < 0$ only "incoming". We change over to the hole representation on the back hemisphere and introduce the notation

$$f(\varepsilon, \theta) = \begin{cases} f^+(\xi, \theta), & 0 < \theta < \pi/2, \\ 1 - f^-(\xi, \pi - \theta), & \pi/2 < \theta < \pi, \end{cases} \quad (3.2)$$

$$\xi = (\mathbf{p} - u\mathbf{e}_z)^2/2 - \varepsilon_F$$

The kinetic equations for the functions f^+ and f^- have absolutely symmetric form

$$A \cos \theta \frac{\partial f^+}{\partial \xi} = -f^+ \int (d\omega' / 4\pi) (q/2p_F) f^-(\xi - \psi),$$

$$A |\cos \theta| \frac{\partial f^-}{\partial \xi} = -f^- \int (d\omega' / 4\pi) (q/2p_F) f^+(\xi - \psi), \quad (3.3)$$

$$\psi = u(|q_z| - q).$$

When scattering occurs backwards into a narrow cone we can write where

$$q/2p_F = |\cos \theta|, \quad q - q_z = q\theta^2/2,$$

$$d\omega' / 4\pi = q^2 d\vartheta / (2p_F^2 \cos \theta) = d\psi / (up_F),$$

where θ is a small angle between q and the z axis, and reduce the problem to a one-dimensional equation

$$f^+(\xi) = f^-(\xi) = \Phi(x), \quad (3.4)$$

$$\frac{d\Phi(x)}{dx} = -\Phi(x) \int_0^\infty dy \Phi(y - x), \quad \Phi(-\infty) = 1, \quad \Phi(\infty) = 0.$$

Here we have introduced the notations

$$x = \xi/T_e, \quad y = -\psi/T_e, \quad T_e = up_F(E/E_0)^{1/2}, \quad E_0 = u\nu_F W_0. \quad (3.5)$$

Equations f^+ and f^- results in the correct normalization of the total distribution function (3.2).

The function ϕ tends to its asymptotic values at $e^{-1/2x^2}$. However in the region $x < 10$ it is given by the Fermi distribution

$$\Phi(x) = \{\exp[(x - 0,42)/0,57] + 1\}^{-1}. \quad (3.6)$$

Thus, for small fields the stationary electron distribution is

$$f(\mathbf{p}) = \{\exp[(\varepsilon - up_z - 0,42T_e \text{sign } p_z - \varepsilon_F)/0,57T_e] + 1\}^{-1}. \quad (3.7)$$

The discontinuity near the "equator" is smoothed out in the belt $|\cos \theta| \leq (E/E_0)^{1/2}$, where Y_{in} and Y_{out} cancel each other out. Comparison between (3.7) and (2.3) shows a qualitative difference between the exact solution and the τ approximation.

In the time between two phonon emission events an electron is accelerated by the field, and for $E \gg E_0$ an effective electron temperature T_e builds up, which is large in comparison with the energy lost in a single scattering event. A new small parameter, the ratio up_F/T_e , arises, and we can use the expansion

$$f(\varepsilon + \omega) = f(\varepsilon) + \omega \frac{\partial f}{\partial \varepsilon}. \quad (3.8)$$

The kinetic equation (3.1) acquires the form

$$A \cos \theta \frac{\partial f}{\partial \varepsilon} = -f \langle (q/2p_F) \rangle' + \langle (q/2p_F) \rangle' + (1 - 2f) \langle \frac{\partial f'}{\partial \varepsilon} (\omega q/2p_F) \rangle'. \quad (3.9)$$

Averaging over the \mathbf{p}' directions gives

$$\langle (q/2p_F) \rangle' = 2/3, \quad \langle (q/2p_F) \cos \theta \rangle' = -(2/15) \cos \theta,$$

$$\langle (q/2p_F) (\cos^2 \theta - 1/3) \rangle' = (2/105) (\cos^2 \theta - 1/3),$$

$$\langle q\omega/2p_F \rangle' = up_F. \quad (3.10)$$

Using these relations and assuming that

$$\frac{A}{T_e} \sim \frac{up_F}{A} \sim \frac{E_0}{E} \ll 1, \quad (3.11)$$

we find the solution of (3.9) in the form of a series in the small parameter (3.11):

$$f = f_0 - \frac{5}{4} A \frac{\partial f_0}{\partial \varepsilon} \cos \theta + \frac{525}{272} A^2 \frac{\partial^2 f_0}{\partial \varepsilon^2} \left(\cos^2 \theta - \frac{1}{3} \right), \quad (3.12)$$

where f_0 satisfies the equation

$$T_e \frac{\partial^2 f_0}{\partial \varepsilon^2} = (2f_0 - 1) \frac{\partial f_0}{\partial \varepsilon}, \quad T_e = \frac{5A^2}{12up_F} = \frac{5up_F}{12} \left(\frac{E}{E_0} \right)^2. \quad (3.13)$$

It follows from (3.13) that f_0 is the Fermi distribution with temperature T_e .

The first two terms in (3.12) can be represented in the form

$$f = f_0(\varepsilon - v_d p_z), \quad v_d = \frac{5}{4} u \frac{E}{E_0}. \quad (3.14)$$

It is interesting that (3.12) is a series in $1/E$, while in the τ approximation a function is usually expanded in the small field E . Nevertheless, the expression (3.14) is close to the test function of the τ approximation, (2.3). Both distributions give the same current value, since the latter is not affected by smearing of the distribution in energy. Formally, Eq. (3.12) is the usual expansion in field. Only at $T \neq 0$ is it an expansion in E/T , in our case in powers of E/T_e .

Thus, the exact solution of the kinetic equation (2.1) leads to the same current-voltage characteristic as the τ approximation (2.17). A new feature is that the electron temperature departs from the lattice temperature ($T = 0$), as in the case of hot electrons in semiconductors. We have

$$T = up_F \begin{cases} (E/E_0)^{1/2}, & E \ll E_0, \\ (5/12)(E/E_0)^2, & E \gg E_0. \end{cases} \quad (3.15)$$

The calculations made above are easily generalized to the model, where both the phonon spectrum and the probability of phonon emission are arbitrary scalar functions of the phonon momentum q . In large fields the generalization reduces to changing the numerical coefficients in such relations as (3.10). In small fields, if the phonon spectrum does not contain portions with zero group velocity, changes reduce to the parametric dependence in (3.5). In this case the current-voltage characteristic (2.7) does not change qualitatively. However, if there is a noticeable portion without dispersion in the short wavelength region of the spectrum, the idea of phonon emission confined to a narrow cone becomes irrelevant, and we have only the result (2.16) at the τ approximation.

4. "EXACT SOLUTION" FOR AN ELLIPSOIDAL FERMI SURFACE

It is most interesting, both from the methodological point of view and that of applying the theory to bismuth, to

find the solution of the kinetic equation (2.1) for an anisotropic model [see (2.12)]. We restrict the discussion to the weak-field limit. Let the phonon spectrum be isotropic acoustic, $W = (q/2p_F)W_0$ and $p_F = \sqrt{2\varepsilon_F}$. Consider phonon emission with momentum $\mathbf{q}_0 \parallel \mathbf{e}$ from the point \mathbf{p} of the Fermi surface accompanied by electron transition to another point of the latter. Allowing for $u \ll v$, we find

$$q_0 = 2m_e(\mathbf{e}\mathbf{v}_p) = -2m_e(\mathbf{e}\mathbf{v}_p) > 0. \quad (4.1)$$

Here $(m_e)^{-1} = \sum_\alpha e_\alpha^2/m_\alpha$. We see that upon phonon emission in the \mathbf{e} direction the electron moves from the front part of the ellipsoid on which $(\mathbf{e}\mathbf{v}_p) > 0$ to its back part where $(\mathbf{e}\mathbf{v}_p) < 0$. By analogy with (3.2), we write

$$f(\varepsilon, \theta) = \begin{cases} f^+(\xi, \mathbf{v}_p), & (\mathbf{e}\mathbf{v}_p) > 0, \\ 1 - f^-(-\xi, -\mathbf{v}_p), & (\mathbf{e}\mathbf{v}_p) < 0, \end{cases} \quad (4.2)$$

$$\xi = \varepsilon_p - u(\mathbf{e}\mathbf{p}) - \varepsilon_F.$$

The same reasoning, as in Sec. 3, leads to two similar equations (the field direction coincides with the unit vector \mathbf{e}).

The equation for f^+ is

$$A_e \frac{\partial f^+}{\partial \xi} = -f^+ \sqrt{-\xi - \psi},$$

$$A_e = Ep_F/W_0 v_F m_e, \quad v_F = (m_1 m_2 m_3)^{1/2} p_F / \pi^2, \quad (4.3)$$

$$\psi = u[|(\mathbf{e}\mathbf{q})| - q] = -uq\vartheta^2/2.$$

The average over the surface of final states, $(4\pi^3 v_F)^{-1} \int dS' / v'_z$, reduces to integration over a small part of the surface near the base of the vector \mathbf{q}_0 : $(m_e / \pi^2 u v_F) \int_{-\infty}^0 d\psi$. As a result, we again arrive at Eq. (3.4) and an effective electron temperature equal to

$$T_e = \omega_e(E/E_0)^{1/2}, \quad \omega_e = up_F(m_1 m_2 m_3)^{1/4} m_e^{-1}. \quad (4.4)$$

The energy ω_e has the order of magnitude of the Debye temperature and depends on the \mathbf{e} direction as m_e^{-1} . Recall that in the atomic system of units masses are dimensionless.

Thus, in the case of an anisotropic Fermi surface a weak electric field directed along \mathbf{e} forms a distribution

$$f(\mathbf{p}) = \left\{ \exp \left[\left(\sum_\alpha (p_\alpha - um_\alpha e_\alpha)^2 / 2m_\alpha - 0,42T_e \text{sign}(\mathbf{v}_p \mathbf{e}) - \varepsilon_F \right) / (0,57T_e) \right] + 1 \right\}^{-1}. \quad (4.5)$$

The current created by this distribution is

$$\delta j_\alpha = \int d\Gamma f p_\alpha / m_\alpha = j_c e_\alpha + \int d\Gamma (p_\alpha / m_\alpha) \times \left\{ \exp \left[(\varepsilon_p - \varepsilon_F - 0,42T_e \text{sign}(\mathbf{v}_p \mathbf{e})) / (0,57T_e) \right] + 1 \right\}. \quad (4.6)$$

The first term is the critical current. Extra current is easily found in terms of $p'_\alpha = p_\alpha / \sqrt{m_\alpha}$:

$$\delta j_\alpha = 0,63 j_c (E/E_0)^{1/2} (m_1 m_2 m_3)^{1/4} m_e^{-1/2} m_\alpha^{-1} a_\alpha, \quad (4.7)$$

$$a_\alpha = m_\alpha e_\alpha / m_\alpha, \quad m_\alpha^{-2} = \sum_\alpha (e_\alpha / m_\alpha)^2.$$

Thus, in the model with isotropic sound velocity the fraction of the current independent of the field is directed along the field and corresponds to the drift velocity $v_d = u$. The part of the current proportional to \sqrt{E} is directed along \mathbf{a} , and the cosine of the angle between \mathbf{a} and \mathbf{e} equals m_*/m_e , while the anisotropy of δj is such that

$$\max \delta j / \min \delta j = \max(m_\alpha)^{3/2} / \min(m_\alpha)^{3/2}. \quad (4.8)$$

The sound velocity anisotropy in bismuth is negligible, so we might think that the model considered above can be used to explain the current-voltage characteristic of bismuth. Since the latter is a compensated metal with one hole Fermi surface along the three-fold axis and three electron ellipsoids in the perpendicular plane,⁴ electrons and holes give the same contribution to the almost isotropic critical current, and in large fields $E \gg E_0$ the bismuth electroconductivity at $T = 0$ has the same character as at high temperatures. If we subtract the contribution of residual resistance from the experimental curve found by Bogod *et al.*, we will get a current-voltage characteristic well described by Eq. (2.17) with $E_0 \approx 1$ V/cm. Note that the field E_0 is associated with the temperature $T_e \approx 10$ K, while the experiment was carried out at the lattice temperature $T_{\text{ion}} = 4.2$ K. We do not know any experiments in which a "hot electron" temperature was observed.

As is well known, in a strong magnetic field B the current in a compensated metal is

$$j_B = [\rho_e(R_e B)^{-2} + \rho_h(R_h B)^{-2}]E. \quad (4.9)$$

Here R_e and R_h are the Hall constants for electrons and holes respectively, and ρ_e and ρ_h are the electron and hole resistivities in zero magnetic field corresponding to the current equal to the Hall current $j_H = E/(RB)$.

This formula explains the difference between the current-voltage characteristic without magnetic field (Fig. 1), when $j_0 \sim \rho^{-1}$, from the current-voltage characteristic of the compensated metal bismuth, for which $j_B \sim \rho$. We have

$$j_0/j_B = (RB/\rho)^2 = (\Omega\tau)^2 \gg 1. \quad (4.10)$$

This inequality shows that to observe a bend on the current-voltage curve in the absence of magnetic field is much more difficult than under the conditions of the Esaki experiment.

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