Electrons in double quantum wells with junctions

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Equations describing the propagation of electron waves in double quantum wells or wires with nonuniform parameters are derived and employed for investigating the passage of waves through smooth junctions in these structures. Quasiclassical solutions of the derived equations are presented. Both the case of no classical turning points and cases in which turning points are present in one of the wells of the well pair are studied. It is demonstrated that at a turning point the passing wave generates a new wave. A junction with linear variation of the asymmetry parameter of the double well is studied and it is demonstrated that the electron transmission coefficient oscillates as a function of the junction parameters.

1. INTRODUCTION

Double quantum wells (DQWs), i.e., pairs of parallel quantum wells separated by a tunnel-permeable barrier, became an object of study in the modern physics of heterostructures comparatively recently. In earlier works the transverse-field-effect-controlled dissipative electric conductivity of DQWs was studied theoretically[1-3] and experimentally.[4] Ballistic transport of electrons in tunnel-coupled quantum wires and wells[6-11] was first studied in Ref. 5. The idea of an analogy between electron waves in coupled quantum wires and electromagnetic waves in coupled optical waveguides[12-14] is elaborated in Refs. 5–8. This analogy makes it possible to propose an electronic coupler similar to an optical coupler (specific "designs" of such devices are illustrated in Refs. 5 and 6). We note that finite fragments of double quantum wells are not studied theoretically in these works, i.e., these works contain only fundamental wave solutions for an infinite uniform DQW.

In Refs. 9 and 10, in contrast to the works indicated above, results obtained for specific finite DQW structures are described. Three such structures are considered there: 1) an overlapping tunnel junction,[9] 2) a tunnel "hole,"[9] and 3) a tunnel reflector.[10] The overlapping tunnel junction consists of two separate semi-infinite quantum wells which overlap on a finite strip $|x| \leqslant d$, where they form a finite DQW. The tunnel "hole" is an infinite DQW with a potential barrier whose tunnel penetrability is finite (i.e., nonzero) only in the strip $|x| < d$. Finally, the tunnel reflector is a single quantum well covered by another quantum well only in the strip $|x| < d$. In all three cases an effective DQW arises only in the strip $|x| < d$; outside this strip there are either only isolated wells (in the case of the overlapping junction and the reflector) or only pairs of uncoupled wells, i.e., also single wells (in the case of a tunnel hole).

All three structures are characterized by the fact that their properties change abruptly at the boundaries $x = \pm d$. This sharpness is associated with the extreme range of variation: one well—two wells or zero transmission—nonzero transmission. It is also assumed that the properties change one at a time. In realizable heterostructures (see, for example, Ref. 15) these conditions are not satisfied: The junctions are smooth, the values of the parameters change by a finite amount, and, as a rule, several parameters describing the DQW change simultaneously. (Here and below a junction in a DQW is a region where the parameters of DQW change.)

In this paper a more general procedure is developed for describing junctions in DQWs and electron transport in DQWs with junctions. The procedure is described in Sec. 2. In subsequent sections this procedure is employed for describing DQW structures with smooth junctions. The behavior of the solutions near a classical turning point in one of the wells of the DQW is studied.

2. EQUATIONS FOR AN ELECTRON IN A DQW

In this paper we confine our attention to a nearly symmetric DQW and assume a single constant effective mass $m = \text{const}$. There are no fundamental difficulties in extending the model to more general cases. We begin with a symmetric quantum well determined by the even potential

$$U_0(z) = U_0(-z). \tag{1}$$

The motion of an electron in this potential is characterized by a system of states $\psi_1(z)$, $\psi_2(z)$, $\psi_3(z)$, ... with energies $\varepsilon_1, \varepsilon_2, \varepsilon_3, \ldots$, where

$$2\delta\varepsilon \equiv \varepsilon_2 - \varepsilon_1 \ll \varepsilon_3 - \varepsilon_1, \tag{2}$$

$\psi_1(z) = \psi_1(-z)$, $\psi_2(z) = -\psi_2(-z)$, and $\psi_1(z)$ and $\psi_2(z)$ are real normalized functions corresponding to the lowest states ε_1 and $\varepsilon_2 > \varepsilon_1$:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi_{1,2}}{dz^2} + U_0(z)\psi_{1,2} = \varepsilon_{1,2}\psi_{1,2}. \tag{3}$$

Together with the states $\psi_{1,2}(z)$ in the DQW we consider "single-well" states

$$\psi^{(\pm)}(z) = \frac{1}{\sqrt{2}}[\psi_1(z) \pm \psi_2(z)], \tag{4}$$

where

$$\psi_{1,2}(z) = \frac{1}{\sqrt{2}}[\psi^{(+)}(z) \pm \psi^{(-)}(z)], \qquad \psi^{(+)}(z) = \psi^{(-)}(-z).$$

The functions $\psi^{(\pm)}(z)$ are normalized and orthogonal to $\psi_n(z)$, $n \geqslant 3$, and to one another; they describe mixed states with energy $\bar{\varepsilon} = (\varepsilon_1 + \varepsilon_2)/2$. It is also easy to show that

$$-\frac{\hbar^2}{2m}\frac{d^2}{dz^2}\psi^{(\pm)} + U_0(z)\psi^{(\pm)} = \bar{\varepsilon}\psi^{(\pm)} - \delta\varepsilon\psi^{(\mp)}. \tag{5}$$

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Suppose that the total potential of the problem $U(x,z)$ contains, besides $U_0(z)$, the perturbation $\delta U(x,z)$:

$$U(x, z) = U_0(z) + \delta U(x, z). \tag{6}$$

We divide the perturbation into even and odd components:

$$\delta U(x, z) = \delta U_1(x, z) + \delta U_2(x, z), \tag{7}$$

$$\delta U_1(x, z) = \delta U_1(x, -z), \qquad \delta U_2(x, z) = -\delta U_2(x, -z), \tag{8}$$

and we separate from the even component $\delta U_1(x,z)$ the z-independent "constant" component $\delta U_0(x)$ according to the rule introduced below [see Eq. (12)].

We seek the wave function of the complete problem with the potential (6) in the following form:

$$\psi(x, z) = C^{(+)}(x)\psi^{(+)}(z) + C^{(-)}(x)\psi^{(-)}(z) + \varphi(x, z), \tag{9}$$

where $\varphi(x,z)$ can be chosen in the form $\varphi(x,z) = \Sigma_{n=3}^{\infty} C_n(x)\psi_n(z)$, i.e., it is orthogonal to $\psi^{(\pm)}(z)$. Substituting Eq. (9) into the Schrodinger equation with the potential $U(x,z)$, we obtain in first-order perturbation theory

$$-\frac{\hbar^2}{2m}\frac{d^2C^{(\pm)}}{dx^2} + C^{(\pm)}[\bar{\varepsilon} - \varepsilon + U^{(\pm)}(x)]$$

$$- C^{(\mp)}[\delta\varepsilon - w(x)] = 0, \tag{10}$$

where

$$U^{(\pm)}(x) = \int_{-\infty}^{\infty} dz|\psi^{(\pm)}(z)|^2\delta U(x, z), \tag{10'}$$

$$w(x) = \int_{-\infty}^{\infty} dz\psi^{(-)}(z)\psi^{(+)}(z)\delta U(x, z), \tag{10''}$$

and ε is a fixed energy. Now, as we did for $\delta U(x,z)$ (7), we divide $U^{(\pm)}(x)$ and $w(x)$ each into two components:

$$U^{(\pm)}(x) = U_1^{(\pm)}(x) + U_2^{(\pm)}(x), \qquad w(x) = w_1(x) + w_2(x). \tag{11}$$

Since $U_1^{(+)}(x) = U_1^{(-)}(x)$, it is convenient to introduce the constant component $\delta U_0(x)$ in the form

$$\delta U_0(x) = \int_{-\infty}^{\infty} dz|\psi^{(\pm)}(z)|^2\delta U_1(x, z). \tag{12}$$

The functions introduced above also have the properties

$$U_2^{(+)}(x) = -U_2^{(-)}(x) \equiv U_2(x), \qquad w_2(x) = 0,$$

$$w_1(x) = \int_{-\infty}^{\infty} dz\psi^{(-)}(z)\psi^{(+)}(z)[\delta U_1(x, z) - \delta U_0(x)],$$

so that Eq. (10) can be rewritten in the form

$$-\frac{\hbar^2}{2m}\frac{d^2C^{(\pm)}}{dx^2} + C^{(\pm)}[\bar{\varepsilon}(x) - \varepsilon \pm U_2(x)] - C^{(\mp)}\delta\varepsilon(x) = 0, \tag{13}$$

where we have introduced the functions

$$\bar{\varepsilon}(x) = \bar{\varepsilon} + \delta U_0(x), \tag{13'}$$

$$\delta\varepsilon(x) = \delta\varepsilon - w_1(x). \tag{13''}$$

Together with $C^{(\pm)}(x)$ we introduce and employ the functions

$$C_{1,2}(x) = \frac{1}{2}[C^{(+)}(x) \pm C^{(-)}(x)], \tag{14}$$

for which we obtain from Eq. (13)

$$-\frac{\hbar^2}{2m}\frac{d^2C_{1,2}}{dx^2} + C_{1,2}[\bar{\varepsilon}(x) \mp \delta\varepsilon(x) - \varepsilon] + C_{2,1}U_2(x) = 0. \tag{15}$$

If $U_2(x) = 0$ holds, i.e., if the perturbation $\delta U(x,z)$ does not contain an odd component $[\delta U_2(x,z) = 0]$, then the equations (15) decouple:

$$-\frac{\hbar^2}{2m}\frac{d^2C_{1,2}}{dx^2} + C_{1,2}[\bar{\varepsilon}(x) \mp \delta\varepsilon(x) - \varepsilon] = 0, \tag{16}$$

i.e., we obtain two independent electron waves—even and odd—which do not mix in any way if only even perturbations are considered.

The junction in a DQW is determined by prescribing three functions: $\bar{\varepsilon}(x)$, $\delta\varepsilon(x)$, and $U_2(x)$. A change in $\bar{\varepsilon}(x)$ leads to the usual nonspecific quantum reflection. Changes in $\delta\varepsilon(x)$ and $U_2(x)$ lead to more specific phenomena, characteristic of a DQW. Before studying some of them, for simplicity we introduce or replace some notation. We replace x by the new variable $\xi = (2m)^{1/2}x/\hbar$, which makes it possible to write Eqs. (13) and (16) in a simpler form, and we introduce the following new notation

$$\varepsilon - \bar{\varepsilon}(x) = \varepsilon(\xi), \qquad \delta\varepsilon(x) = \delta(\xi), \qquad U_2(x) = \eta(\xi).$$

3. QUASICLASSICAL APPROXIMATION

If the "potentials" $\varepsilon(\xi)$, $\delta(\xi)$, and $\eta(\xi)$ vary sufficiently smoothly, then the quasiclassical approximation can be used for solving Eqs. (13) and (16). This gives the following:

$$C^{(\pm)}(\xi) = \sqrt{1 \mp \frac{\eta}{\sqrt{\eta^2 + \delta^2}}}\{A_1\exp[i\int^{\xi} \varkappa_1(\xi')d\xi']$$

$$+ B_1\exp[-i\int^{\xi} \varkappa_1(\xi')d\xi']\}/\sqrt{\varkappa_1(\xi)} \pm \sqrt{1 \pm \frac{\eta}{\sqrt{\eta^2 + \delta^2}}}$$

$$\times\{A_2\exp[i\int^{\xi} \varkappa_2(\xi')d\xi'] + B_2\exp[-i\int^{\xi} \varkappa_2(\xi')d\xi']\}/\sqrt{\varkappa_2(\xi)}, \tag{17}$$

where $A_{1,2}$ and $B_{1,2}$ are integration constants and

$$\varkappa_{1,2}^2(\xi) = \varepsilon \pm \sqrt{\eta^2 + \delta^2}. \tag{18}$$

The solutions $C_{1,2}(\xi)$ are constructed from Eq. (17) with the help of the formulas (14).

Let a wave be incident from the left and propagate to the right. We consider first the case when the potentials vary so that $\varkappa_1^2(\xi)$ and $\varkappa_2^2(\xi)$ are everywhere positive. In this case, in the quasiclassical approximation reflection does not occur anywhere, i.e., $B_{1,2} = 0$. In order to specify the values of A_1 and A_2 we assume that for $\xi = -\infty$ the electron wave propagates only into the "plus" well, i.e., $C^{(-)}(-\infty)=0$. Physically, this condition can be imposed only if the wells are isolated from one another at $\xi=-\infty$, i.e.,

$$\delta(-\infty) = 0. \tag{19}$$

For $\eta(-\infty)>0$ the conditions $C^{(-)}(-\infty)=0$ and $\delta(-\infty)=0$ imply $A_1=0$, i.e.,

$$C^{(\pm)}(\xi) = \pm \sqrt{1 \pm \frac{\eta}{\sqrt{\eta^2 = \delta^2}}} A_2 \exp[i\int_{-\infty}^{\xi} \varkappa_2(\xi')d\xi'] / \sqrt{\varkappa_2(\xi)} \tag{20}$$

a single wave with the single wave number $\varkappa_2(\xi)$ propagates at each point. In this problem the fraction of the flux remaining in the "plus" well after passage through the junction and the fraction of the flux transferred into the "minus" well are of interest. In order to formulate this problem it must be assumed that the wells are also isolated at $\xi=\infty$, i.e.,

$$\delta(\infty) = 0. \tag{21}$$

This formulation of the problem corresponds to the constructions considered in Refs. 5 and 6. It is evident from the solution (20) that we have $C^{(+)}(\infty)=0$ for $\eta(\infty)<0$ and $C^{(-)}(\infty)=0$ for $\eta(\infty)>0$. Thus in the quasiclassical approach the transmission coefficient for "its own" well [in this case the coefficient $T(+,+)$] is equal to unity if $\eta(-\infty)$ and $\eta(\infty)$ have the same signs, $\eta(-\infty)\eta(\infty)>0$; the same coefficient is zero if $\eta(-\infty)$ and $\eta(\infty)$ have different signs, $\eta(-\infty)\eta(\infty)<0$. In the latter case, the entire electron flux is transferred into the "foreign" well, so that the junction transmission coefficient $T(+,-)$ is equal to unity. This result can be written in the form

$$T(+,+) = \frac{1}{2}[1 + \text{sign}(\eta(\infty)\eta(-\infty))],$$
$$T(+,-) = \frac{1}{2}[1 - \text{sign}(\eta(\infty)\eta(-\infty))]. \tag{22}$$

The result obtained above does not depend on the absolute values of $\eta(\infty)$ and $\eta(-\infty)$, but only on their signs.

A completely different result follows from Eq. (17) if the limits $\eta(\pm\infty)\to 0$ and $\varepsilon(\pm\infty)\to 0$ are taken in different order. Assume first that

$$\eta(-\infty) = \eta(\infty) = 0. \tag{23}$$

Then the condition $C^{(-)}(-\infty)=0$ combined with the condition $\delta(-\infty)\to 0$ leads to

$$A_1 = A_2 = A, \tag{24}$$

so that

$$C^{(\pm)}(\xi) = A\left\{\sqrt{1 \mp \frac{\eta}{\sqrt{\eta^2+\delta^2}}} \exp[i\int_{-\infty}^{\xi}\varkappa_1(\xi')d\xi']/\sqrt{\varkappa_1(\xi)} \pm \sqrt{1 \pm \frac{\eta}{\sqrt{\eta^2+\delta^2}}} \exp[i\int_{-\infty}^{\xi}\varkappa_2(\xi')d\xi']/\sqrt{\varkappa_2(\xi)}\right\}. \tag{25}$$

If $\delta(\infty)\to 0$ also, then [taking into account Eq. (23)]

$$C^{(\pm)}(\infty) = A\frac{\exp[i\int_{-\infty}^{\infty}\bar{\varkappa}(\xi)d\xi]}{\sqrt{\varkappa(\infty)}}\{\exp[i\int_{-\infty}^{\infty}\delta\varkappa(\xi)d\xi] \pm \exp[-i\int_{-\infty}^{\infty}\delta\varkappa(\xi)d\xi]\}, \tag{26}$$

where $\varkappa(\infty)=\varkappa_1(\infty)=\varkappa_2(\infty)=\sqrt{\varepsilon(\infty)}$, $\bar{\varkappa}(\xi)=\frac{1}{2}(\varkappa_1+\varkappa_2)$, $\delta\varkappa(\xi)=\frac{1}{2}(\varkappa_1-\varkappa_2)$.

For the passing and transfer transmission coefficients $T(+,+)$ and $T(+,-)$ we have instead of Eq. (22)

$$T(+,+) = \frac{1}{2}[1 + \cos(2\int_{-\infty}^{\infty}\delta\varkappa(\xi)d\xi)],$$
$$T(+,-) = \frac{1}{2}[1 - \cos(2\int_{-\infty}^{\infty}\delta\varkappa(\xi)d\xi)], \tag{27}$$

i.e., these coefficients oscillate as functions of $2\int_{-\infty}^{\infty}\delta\varkappa(\xi)d\xi$.

Thus, for $\eta(\infty)$, $\eta(-\infty)\neq 0$ we have the result (22). If, however, $\eta(\infty)$, $\eta(-\infty)\to 0$ we have either (22) or (27), depending on the order in which the limits are taken [together with $\delta(\infty)$, $\delta(-\infty)\to 0$]. The question of the experimental realization of (27) remains open. Recall that both Eqs. (22) and (27) are consequences of the quasiclassical approximation. In our work Ref. 9 a result similar to Eq. (27) was obtained for the case of a tunnel "hole" with sharp edges.

4. BEHAVIOR NEAR A TURNING POINT

The results of the preceding section were obtained under the assumption that $\varkappa_1^2(\xi)>\varkappa_2^2(\xi)>0$ holds everywhere, i.e., there are no classical turning points anywhere. We now consider the situation in which, together with regions where these inequalities are satisfied, there exist other regions where $\varkappa_1^2(\xi)>0>\varkappa_2^2(\xi)$. On the boundary of such regions at the point $\xi=\xi_c$ we have

$$\varkappa_2^2(\xi_c) = 0, \tag{28}$$

i.e., for electrons with wave vectors $\pm\varkappa_2$ this is a classical turning point. For electrons with the wave vectors $\pm\varkappa_1$ the point $\xi=\xi_c$ is a point of general position.

We now consider the case illustrated in Fig. 1. Let a wave with wave vector $\varkappa_1(\xi)$ be incident from the left:

$$C^{(\pm)}(\xi) = \sqrt{1 \mp \frac{\eta}{\sqrt{\eta^2+\delta^2}}} A_1 \exp[i\int^{\xi}\varkappa_1(\xi')d\xi']/\sqrt{\varkappa_1(\xi)}. \tag{29}$$

Suppose also that at least near a turning point $\eta(\xi)\gg\delta(\xi)$ holds, so that the ratio of the amplitudes of the waves in the "plus" and "minus" wells is

$$\frac{C^{(+)}(\xi)}{C^{(-)}(\xi)} \approx \frac{\delta(\xi)}{2\eta(\xi)} \ll 1. \tag{30}$$

Near the turning point $\xi=\xi_c$ the quasiclassical solution (29) is no longer applicable; this could be reflected in

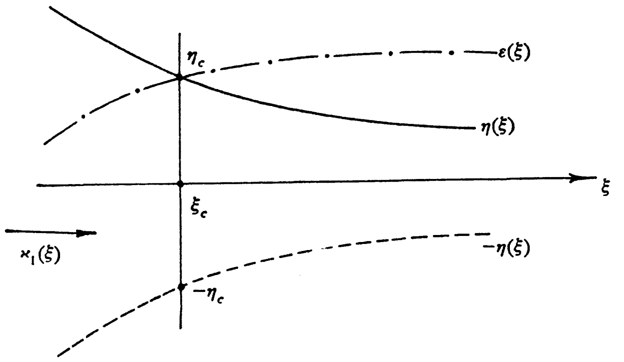

FIG. 1. $\varepsilon(\xi)$ and $\pm\eta(\xi)$ near the turning point $\xi=\xi_c$.

the appearance of additional transmitted waves to the right of the turning point (for $\xi>\xi_c$) and reflected waves on the left (for $\xi<\xi_c$). In the vicinity of a turning point we rewrite Eq. (13) in the form

$$\frac{d^2C^{(+)}}{d\xi^2}+\alpha(\xi-\xi_c)C^{(+)}=-\delta_cC^{(-)}, \tag{31}$$

$$\frac{d^2C^{(-)}}{d\xi^2}+2\eta_cC^{(-)}=-\delta_cC^{(+)}, \tag{32}$$

where $\eta_c=\eta(\xi_c)$, $\delta_c=\delta(\xi_c)$, $\varepsilon-\eta\approx\alpha(\xi-\xi_c)$. In what follows we measure ξ from the point ξ_c, i.e., instead of $\xi-\xi_c$ we write simply ξ. Since δ_c is small compared with η_c and we have the relation (30), in the zeroth approximation we set the right-hand side of Eq. (32) equal to zero and we write $C^{(-)}$ in the form

$$C^{(-)}(\xi)\approx\sqrt{\frac{2}{\varkappa_{1c}}}A_1\exp(i\varkappa_{1c}\xi), \tag{33}$$

where $\varkappa_{1c}=\sqrt{2\eta_c}$. The solution (33) is identical to the expression for $C^{(-)}(\xi)$ that follows from Eq. (29) (to within an insignificant phase factor). The general solution of Eq. (31) [with the expression for $C^{(-)}(\xi)$ from Eq. (33) substituted into its right-hand side] that does not diverge as $\xi\to-\infty$ has the form

$$C^{(+)}(\zeta)=a\int_{-\infty}^{\zeta}e^{i\beta\zeta'}\mathrm{Ai}(-\zeta')d\zeta'\,\mathrm{Bi}(-\zeta)-\Big(B+a\int_{0}^{\zeta}e^{i\beta\zeta'}\mathrm{Bi}(-\zeta')d\zeta'\Big)\mathrm{Ai}(-\zeta), \tag{34}$$

where $\mathrm{Ai}(-\zeta)$ and $\mathrm{Bi}(-\zeta)$ are Airy functions of the first and second kind,[16] $a=\sqrt{2/\varkappa_{1c}}\,(\pi\delta_c/\alpha^{2/3})A_1$, $\zeta=\alpha^{1/3}\xi$, $\beta\zeta=\varkappa_{1c}\xi$; and, B is an integration constant, found below from the condition that there is no reflected wave as $\zeta\to\infty$. After B is found from this condition we have

$$C^{(+)}(\zeta)=a\int_{-\infty}^{\infty}\exp(i\beta\zeta')\mathrm{Ai}(-\zeta')d\zeta'\,[\mathrm{Bi}(-\zeta)+i\,\mathrm{Ai}(-\zeta)]-a\int_{\zeta}^{\infty}\exp(i\beta\zeta')d\zeta'\,[\mathrm{Ai}(-\zeta')\mathrm{Bi}(-\zeta)-\mathrm{Bi}(-\zeta')\mathrm{Ai}(-\zeta)]. \tag{35}$$

For $\zeta\gg1$ this solution becomes the quasiclassical solution:

$$C^{(+)}(\xi)\approx\frac{\delta(\xi)}{\eta(\xi)}A_1\exp\Big[i\int_0^{\xi}\varkappa_1(\xi')d\xi'\Big]\Big/\sqrt{2\varkappa_1(\xi)}+\delta(\xi)\sqrt{\frac{2\pi}{\alpha\varkappa_{1c}\varkappa_2(\xi)}}A_1\exp\Big[i\frac{\pi}{4}+i\frac{\beta^3}{3}+i\int_0^{\xi}\varkappa_2(\xi')d\xi'\Big], \tag{36}$$

which differs from the expression (29) for $C^{(+)}(\xi)$ by the presence of the second term in Eq. (36)—a wave propagating with wave vector $\varkappa_2(\xi)$. This wave is generated by the turning point because it is not quasiclassical in the vicinity of this point). The electron flux in the plus-well is given by the expression

$$j^{(+)}(\xi)\approx\frac{2\delta^2|A_1|^2}{(2m)^{1/2}}\left[\frac{1}{2\eta^2}+\frac{2\pi}{\alpha\varkappa_{1c}}+\frac{1}{\eta}\sqrt{\frac{\pi}{\alpha\varkappa_2}}\left(1+\frac{\varkappa_2}{\varkappa_{1c}}\right)\times\cos\left(\frac{\pi}{4}+\frac{\beta^3}{3}-\int_0^{\xi}(\varkappa_1(\xi')-\varkappa_2(\xi'))d\xi'\right)\right]. \tag{37}$$

For

$$\alpha^{2/3}\ll2\eta_c. \tag{38}$$

the solution (35) becomes the quasiclassical solution (36). In this case the fraction of the flux associated with the second term on the right-hand side of Eq. (36) clearly predominates over the flux in the "plus" well, propagating for $\xi<\xi_c$. The smaller α, i.e., the smoother the change in $\varepsilon(\xi)-\eta(\xi)$ near the turning point, the stronger the generation of electrons at this point from the "minus" the "plus" well is. On the other hand, according to the conditions of the approximation adopted, the electron flux passing into the "plus" well should be small compared with the flux remaining in the "minus" well. This requirement imposes a lower limit on α, limiting the possible variation in this quantity near the turning point by the inequalities

$$(2\eta_c)^{3/2}\gg\alpha\gg\frac{2\pi\delta_c^2}{(2\eta_c)^{1/2}}. \tag{39}$$

The existence of this range is due to the condition (30) adopted above.

The generation of a wave with the wave vector $\varkappa_2(\xi)$ results in spatial oscillations of electron fluxes in each of the two wells of the DQW, but these oscillations are relatively small not only in the "minus" well, through which the main electron flux passes, but also in the "plus" well, due to the fact that when (38) is satisfied the first term on the right-hand side of Eq. (36) is small compared to the second term.

5. LINEAR η-JUNCTION

In the final section of this paper we consider the case opposite to the cases studied in the two preceding sections. Let $\eta(\xi)$ vary quite rapidly, so that the quasiclassical approximation is inapplicable. Assume that $\varepsilon(\xi)=\text{const}$, $\delta(\xi)=\text{const}$, and $\eta(\xi)$ is an odd function:

$$\eta(-\xi)=-\eta(\xi). \tag{40}$$

We call such junctions in DQW symmetric η-junctions. Assume, as before, that $\varepsilon(\xi) \ll \varepsilon_3 - \bar{\varepsilon}$ and also that the range of variation of $\eta(\xi)$ exceeds ε, so that

$$|\eta(\pm \infty)| > \varepsilon. \quad (41)$$

Due to this condition, in the absence of tunneling between the wells the electrons within each of the two cells would be confined in the half-spaces $\xi > -\xi_c$ and $\xi < \xi_c$, where $\pm \xi_c$ are classical turning points:

$$|\eta(\pm \xi_c)| = \varepsilon. \quad (42)$$

It is obvious that $\xi_c > 0$ if $\varepsilon > 0$ and $\xi_c < 0$ if $\varepsilon < 0$ (see Fig. 2). For $\varepsilon > 0$ there exists an interval $(-\xi_c, \xi_c)$ of combined classical motion in which an effective DQW is realized. For $\varepsilon < 0$ there is no such interval, and tunneling occurs from one well into the other through an extended triangular barrier with the base $(\xi'_c, -\xi'_c)$ (see Fig. 2).

We now consider the case of an actual linear η-junction, illustrated in Fig. 2,

$$\eta = \alpha\xi = eFx, \quad (43)$$

where $F = \alpha(2m)^{1/2}/\hbar e$ has the dimension of an electric field. We call attention to the fact that here the sign of α is different from that in the preceding section.

Then Eqs. (13) acquire the form

$$\frac{d^2C^{(\pm)}}{d\zeta^2} + C^{(\pm)}(\zeta_c \mp \zeta) = -\frac{\delta C^{(\mp)}}{\alpha^{2/3}}, \quad (44)$$

where $\zeta_c = \varepsilon/\alpha^{2/3}$. If an electron wave is incident on such a junction from the left through the "plus" well, then for $\delta = 0$ the wave is completely reflected, forming a standing wave described by an Airy function of the first kind[16]

$$C^{(+)} = \mathrm{Ai}(\zeta - \zeta_c). \quad (45)$$

For $\delta \neq 0$ a transmitted wave appears in the "minus" well. This wave is induced by tunneling transitions from the "plus" well and in the limit $\zeta \to \infty$ it has the form

$$C^{(-)} \approx D \exp\left\{i\left[\frac{2}{3}(\zeta + \zeta_c)^{3/2} - \frac{\pi}{4}\right]\right\}/2\sqrt{\pi}(\zeta + \zeta_c)^{1/4}, \quad (46)$$

and the transmission coefficient T is

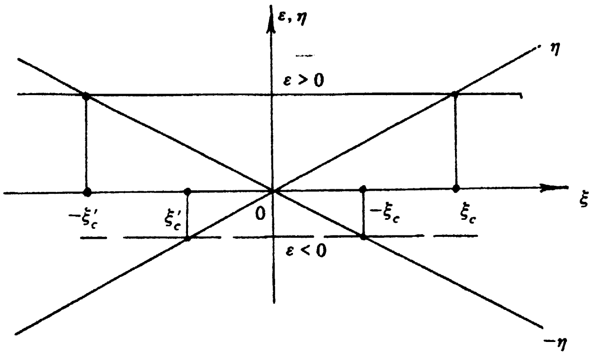

FIG. 2. Energy diagram of an η-junction for a linear function $\eta = \alpha\xi$. The turning points $\pm\xi_c$ correspond to $\varepsilon > 0$ and $\mp\xi'_c$ correspond to $\varepsilon < 0$.

$$T = |D|^2 = \left(\frac{2\pi\delta}{\alpha^{2/3}}\right)^2 \left[\int_{-\infty}^{\infty} \mathrm{Ai}(\zeta - \zeta_c)\mathrm{Ai}(-\zeta - \zeta_c)d\zeta\right]^2$$

$$= 2^{4/3}\left(\frac{\pi\delta}{\alpha^{2/3}}\right)^2 \left[\mathrm{Ai}\left(-\frac{2^{2/3}\varepsilon}{\alpha^{2/3}}\right)\right]^2. \quad (47)$$

It is natural to conjecture that this result is valid only for small values of T, i.e., in the case $\varepsilon > 0$; then the factor $[\pi\delta(2/\alpha)^{2/3}]^2$ is small, which requires that $\delta/\alpha^{2/3}$ be small:

$$\alpha^{2/3} \gg \delta, \quad (48)$$

i.e., the junction must be quite sharp. The strong inequality (48) is consistent with the inequality (38): there exist values of α which satisfy both inequalities. But such a discrepancy should not have been expected because the corresponding problems are formulated differently.

The transmission coefficient T (for $\varepsilon > 0$) is characterized by oscillations as a function of $\varepsilon/\alpha^{2/3}$. The oscillations appear for $2^{2/3}\varepsilon/\alpha^{2/3} > 1$, i.e., in addition to the requirement that T be small, the condition $\varepsilon \gg \delta$ must also be satisfied. Note that in this case we are dealing with oscillations of the transmission coefficient as a function of the characteristic electron wavelengths, i.e., these oscillations are somewhat different from the oscillations with long spatial period which were studied in Refs. 9 and 10. This is because the changes in the function $\eta(\xi)$ are not small, and they lead to changes in $\varkappa_{1,2}(\xi)$ which are not small, i.e., here the condition $\varkappa_1 - \varkappa_2 \ll \varkappa_1 + \varkappa_2$, imposed on the wave vector in the works cited, is far from being satisfied.

It is easy to show that the neighborhood of the resonance point $\xi = 0$ makes the main contribution to the tunneling between the wells. The size of the active neighborhood depends on the smoothness of the junction, and this is responsible for the strong inequality (48).

The inequality (48) makes it possible to answer one other question. In order to realize experimentally the macroscopic quantum effects, indicated in the Introduction and generated by a long interwell tunneling length $\lambda = \pi/\delta k$, where $2\delta k$ is the difference of the wave numbers of the Fermi electron waves in tunnel-split states with energies ε_1 and ε_2 [see Eqs. (26) and (27), and Ref. 9 for a more detailed discussion], quite sharp transitions from a DQW to a single QW in Refs. 9 and 10 or to a pair of spatially separated QWs in Refs. 5, 6, and 9 are required. As already mentioned in the Introduction, for the most attractive method of preparation[15] such junctions cannot be extremely sharp. The condition (48) is the criterion for admissable smoothness: A junction is sharp when the condition (48) is satisfied. However, it should be kept in mind that the transitional region with the maximum "field" $F \gg (2m\delta^3)^{1/2}/e\hbar$ must also pass quite abruptly into the region of quasiclassical behavior. The interval where the field F drops from some maximum value to zero must be significantly shorter than the length λ.

6. CONCLUSIONS

The propagation of electron waves in nonuniform DQW structures was studied on the basis of equations of the form (10) or (13) derived in this paper. The cases of quasiclassical behavior as well as the case (specific to these structures) in which a classical turning point exists only for one of the resulting pair of levels were investigated. In the latter case it was shown that a wave is generated by the turning point. The effect is small because the ratio (30) was initially assumed to be small. The effect increases with increasing ratio δ/η_c as well as with the smoothness of the junction. But as the effect becomes stronger, it must be analyzed by a different method.

In Sec. 5 the case of nonquasiclassical transport was studied and a new example of oscillating (as a function of the parameters x and ε) transmission coefficients T was obtained.

The problem of the criteria for a quasiclassical analysis arises in connection with these limiting cases. It is obvious that the standard criteria of the form

$$\left|\frac{d\varkappa_{1,2}}{d\xi}\right| \ll \varkappa_{1,2}^2, \quad \left|\frac{d^2\varkappa_{1,2}}{d\xi^2}\right| \ll \varkappa_{1,2}^3 \tag{49}$$

are inadequate in this case. An example of such inadequacy is the solution (47). In this solution, for $(2/\alpha)^{2/3}\varepsilon > 1$ the Airy functions on the right-hand side can be represented entirely legitimately by their asymptotic approximations. This indicates that the conditions (49) are satisfied. However, the formula (7) itself cannot be derived from the quasiclassical solution (17). The inadequacy of the criteria (49) is connected with the fact that in our case not only the wave vectors $\varkappa_{1,2}$ themselves but also their small difference play an important role. For this reason, together with the conditions (49) the much more delicate condition

$$\left|\frac{d}{d\xi}(\varkappa_1 - \varkappa_2)\right| \ll |\varkappa_1^2 - \varkappa_2^2|. \tag{50}$$

must also be satisfied.

In addition, another criterion for the validity of the solution (17), representing the first few terms of the quasiclassical asymptotic expansion, is that interwell transitions must be adiabatic, i.e., the preexponential factors in Eq. (17) must change much over the distance λ:

$$\left|\frac{d}{d\xi}\frac{\eta}{\sqrt{\eta^2+\delta^2}}\right| \ll |\varkappa_1 - \varkappa_2|. \tag{51}$$

We now apply the criteria (50) and (51) written out above to the linear junction of Sec. 5 with $\varepsilon = \text{const}$, $\delta = \text{const}$, and $\eta = \alpha\xi$. The criterion (50) gives near the point $\xi = 0$ the condition

$$\alpha \ll 4\sqrt{\varepsilon\delta}, \tag{52}$$

which by no means contradicts (48), since $\varepsilon \gg \delta$.

From the condition (51) we obtain

$$\alpha \ll \delta^2/\sqrt{\varepsilon}, \tag{53}$$

which always contradicts the condition (48) and indicates that the quasiclassical approach is not applicable in the situation considered in Sec. 5.

The inequality (53), combined with the inequality (48), shows that there exists and interval of intermediate slopes α

$$\delta^{3/2} > \alpha > \delta^2/\sqrt{\varepsilon}, \tag{54}$$

where the transmission coefficient T oscillates with large amplitude ($T_{\max} \sim 1$) as a function of ε and α.

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