Production of a positron and a bound electron by a high-energy photon in a strong Coulomb field

A. I. Mil'shteĭn and V. M. Strakhovenko

G. I. Budker Nuclear Physics Institute, Siberian Branch of the Russian Academy of Sciences, Novosibirsk
(Submitted 18 November 1992)
Zh. Eksp. Teor. Fiz. **103**, 1584–1592 (May 1993)

We use the quasiclassical approach to calculate exactly in the Coulomb field the cross section of electron-positron pair production induced by a high-energy photon for the case when the final electron is in an arbitrary state of the discrete spectrum of the hydrogenlike atom being formed. We find the dependence of the cross section on the nuclear charge for large quantum numbers of the electron, making it possible to calculate the process cross section summed over all the states of the discrete spectrum.

1. INTRODUCTION

In late years there has been extensive research into QED processes occurring in a strong Coulomb field. One reason is related to projects for high-energy heavy-ion accelerators. Building such accelerators will make it possible, along with solving other problems, to verify the QED predictions for processes in ultrahigh external fields. Special attention is being paid to electron-positron pair production in collisions of heavy ions for the case when the final electron is captured by an ion, thus forming a bound state. This process has a profound effect on the lifetime of the beam of heavy ions in the accelerator, hence the interest.

It has proved expedient to study the physics of this phenomenon in the reference frame linked with the ion that brings the electron into the bound state (we call this ion the target ion). For very high ion energies the equivalent-photons approximation holds true. In this approximation the cross section of pair production by the incident ions can be expressed in terms of the cross section of pair production by a photon in the target-ion field. In this case the cross section of the process of interest to us is proportional to the square of the charge and to the logarithm of the relativistic factor of the incident ion that creates the equivalent photons. As noted in recent papers,[1,2] when the energies are not too high, there is a mechanism that can prove to be important, namely, one that is not described by the equivalent-photons method and has a markedly different dependence on the charge of the incident ion. However, the contribution of this mechanism to the total cross section does not increase logarithmically in the ultrarelativistic limit.

Use of the equivalent photons method requires knowing the cross section of the respective process involving photons. In heavy-ion collisions the total cross section of pair production with the electron in a bound state is determined by equivalent-photon energies of the order of several electron masses (in this paper we use a system of units in which $\hbar=1$ and $c=1$). This cross section has been calculated by many researchers for the K- and L-shells (see reviews in Refs. 3 and 4 and the literature cited). The differential (in the outgoing-positron energies) cross sections are also of unquestionable interest. But we are interested in the case of relativistic positrons. We must, therefore, find the cross section of pair production by a high-energy photon in a Coulomb field ($\omega \gg m$, where m is the electron mass and ω the photon energy.) This cross section has been studied in detail for the case of K- and L-shells in Refs. 5 and 6, where the reader can also find references to earlier literature. As noted by Pratt,[5] for $\omega \gg m$ the pair-production cross section coincides with the photoelectric-effect cross section and with the radiative-recombination cross section (to within a constant factor related to summation or averaging over the polarizations of the particles participating in the process.) In Refs. 5 and 6 the result was obtained by directly calculating the matrix elements sandwiched by the wave functions taken in the Sommerfeld-Maue approximation.[7]

The present paper is devoted to calculating exactly in the Coulomb field the cross section of photon-induced electron-positron pair production for the case when the final electron is in an arbitrary state of the discrete spectrum. The study is based on an earlier convenient integral representation for the Green function of an electron in a Coulomb field, a representation valid in the entire complex energy plane,[8] and on the quasiclassical Green function of an electron.[9,10] We have analyzed in detail the case of large quantum numbers of the electrons. The resulting asymptotic expression has a high accuracy. This made it possible to calculate the cross section of the respective process summed over all states of the discrete spectrum.

2. THE GREEN FUNCTION AND THE PAIR-PRODUCTION CROSS SECTION

In accordance with the usual Feynman rules, the cross section of electron-positron pair production by a photon in the field of the nucleus, with the electron in one of the states of the discrete spectrum, is

$$\sigma_\gamma=\frac{\alpha}{2\pi\omega}\int d\mathbf{p}\,\delta(\omega-\varepsilon_p-\varepsilon_n)\,|M|^2, \tag{1}$$

where $\alpha=e^2=1/137$ is the fine-structure constant, summation over positron polarizations is assumed, and the matrix element M is given by the following formula:

 1063-7761/93/050775-05$10.00

$$M=\int d\mathbf{r}\bar{\psi}_n^{(+)}(\mathbf{r})\hat{e}\psi_p^{(+)}(\mathbf{r})\exp(i\mathbf{q}\mathbf{r}).$$

Here $\psi_n^{(+)}(\mathbf{r})$ is the discrete-spectrum wave function, $\psi_p^{(-)}(\mathbf{r})$ the negative-frequency continuous-spectrum wave function corresponding to the positron, e_μ the photon polarization vector, and $\mathbf{q}$ the photon wave vector. In accordance with the usual definition [see, e.g., Eq. (109.19) in Ref. 11], the Green function of an electron in an external field is

$$G^{\pm}(\mathbf{r},\mathbf{r}'|\varepsilon)=\sum_n \frac{\psi_n^{(+)}(\mathbf{r})\bar{\psi}_n^{(+)}(\mathbf{r}')}{\varepsilon-\varepsilon_n\pm i0} +\int \frac{d\mathbf{p}}{(2\pi)^3}\left[\frac{\psi_p^{(+)}(\mathbf{r})\bar{\psi}_p^{(+)}(\mathbf{r}')}{\varepsilon-\varepsilon_p\pm i0} +\frac{\psi_p^{(-)}(\mathbf{r})\bar{\psi}_p^{(-)}(\mathbf{r}')}{\varepsilon+\varepsilon_p\mp i0}\right], \tag{2}$$

where G^+ defines the Green function G in the upper half-plane of the complex variable ε, and G^- defines it in the lower half-plane. Equation (2) yields

$$\delta G(\mathbf{r},\mathbf{r}'|-|\varepsilon|)=G^+-G^-=2i\pi\int\frac{d\mathbf{p}}{(2\pi)^3}\psi_p^{(-)}(\mathbf{r})\bar{\psi}_p^{(-)} \times(\mathbf{r}')\delta(|\varepsilon|-\varepsilon_p), \tag{3}$$

$$\psi_n^{(+)}(\mathbf{r})\bar{\psi}_n^{(+)}(\mathbf{r}')\equiv\rho_n(\mathbf{r},\mathbf{r}')=\lim_{\varepsilon\to\varepsilon_n}(\varepsilon-\varepsilon_n)G(\mathbf{r},\mathbf{r}'|\varepsilon).$$

Substituting (3) into (1), we arrive at for the cross section a formula expressed solely in terms of the Green function:

$$\sigma_\gamma=\frac{i\pi\alpha}{\omega}\int\int d\mathbf{r}d\mathbf{r}'\mathrm{Sp}\{\rho_n(\mathbf{r},\mathbf{r}')\gamma_\mu\delta G(\mathbf{r}',\mathbf{r}|\varepsilon_n -\omega)\gamma^\mu\}\exp[i\mathbf{q}(\mathbf{r}-\mathbf{r}')]. \tag{4}$$

Here we have averaged over photon polarizations ($e_\mu e\nu\to-\frac{1}{2}g_{\mu\nu}$), since the total cross section is polarization-independent. Note that in the case of degeneracy the matrix $\rho_n(\mathbf{r},\mathbf{r}')$ is the sum over the degenerate states, and the cross section σ_γ given by (4) is the sum of cross sections of electron production in states with fixed energies.

In Ref. 8 a convenient integral representation was obtained for the Green function of an electron in a Coulomb field. Using Eqs. (19)–(22) of that paper, we arrive at the following expression for $\rho_n(\mathbf{r},\mathbf{r}')$:

$$\rho_n(\mathbf{r},\mathbf{r}')=\frac{Z\alpha(-1)^n}{8\pi^2rr'[(\gamma+n)^2+(Z\alpha)^2]} \times\int_{-\pi/2}^{\pi/2}d\tau\exp\left\{i\left[k(r+r')\mathrm{tg}\tau -2Z\alpha\tau\frac{E_n}{k}\right]\right\}T, \tag{5}$$

where n is the radial quantum number, Z the atomic number, $\gamma=[l^2-(Z\alpha)^2]^{1/2}$, with $l=j+\frac{1}{2}$ and j the total angular momentum of the state, E_n is the bound-state energy,

$$E_n=m(\gamma+n)/[(\gamma+n)^2+(Z\alpha)^2]^{1/2},$$

$k=[m^2-E_n^2]^{1/2}$, with m the electron mass, and the T-matrix is given by the formula

$$T=[1+\mathbf{n}\mathbf{n}'+i\mathbf{\Sigma}[\mathbf{n}\mathbf{n}']]\left[\frac{y}{2}J'_{2\gamma}(y)(\gamma^0E_n+m) -iZ\alpha J_{2\gamma}(y)\gamma^0k\,\mathrm{tg}\tau)\right]B +[1-\mathbf{n}\mathbf{n}'-i\mathbf{\Sigma}[\mathbf{n}\mathbf{n}']](\gamma^0E_n+m)J_{2\gamma}(y)A +imZ\alpha\gamma^0(\gamma,\mathbf{n}+\mathbf{n}')J_{2\gamma}(y)B +\left[\frac{ik^2(r-r')}{2\cos^2\tau}(\gamma,\mathbf{n}+\mathbf{n}')B-k\,\mathrm{tg}\tau(\gamma,\mathbf{n} -\mathbf{n}')A\right]J_{2\gamma}(y), \tag{6}$$

$$A=l\frac{d}{dx}[P_l(x)+P_{l-1}(x)],\quad B=\frac{d}{dx}[P_l(x)-P_{l-1}(x)],$$

$$x=\mathbf{n}\mathbf{n}',\quad \mathbf{n}=\frac{\mathbf{r}}{r},\quad \mathbf{n}'=\frac{\mathbf{r}'}{r'},\quad y=\frac{2k\sqrt{rr'}}{\cos\tau},$$

with $J_{2\gamma}$ a Bessel function. The integral with respect to τ in (5) can easily be evaluated by the method of the theory of residues if we expand the Bessel functions in a series, go over to the variable $v=\tan\tau$, and close the contour of integration with respect to v in the upper half-plane. The result is

$$\int_{-\pi/2}^{\pi/2}d\tau\exp\left\{i\left[k(r+r')\mathrm{tg}\,\tau-2Z\alpha\tau\frac{E_n}{k}\right]\right\}J_{2\gamma}\left(\frac{2k\sqrt{rr'}}{\cos\tau}\right) =2\pi(-1)^n\sum_{M=0}^{n}\frac{(k^2rr')^{\gamma+M}}{M!(n-M)!\Gamma(2\gamma+M+1)} \times\left(\frac{d}{dv}\right)^{n-M}[(1+v)^{2\gamma+n+M-1}e^{-k(r+r')v}]_{v=1}. \tag{7}$$

The main contribution to the cross section in (4) is determined by distances r and r' of the order of the Compton wavelength of the electron, $1/m$, and by angles between the vectors $\mathbf{k}$, $\mathbf{r}$, and $\mathbf{r}'$ of the order of unity. The contributions of small distances r, $r'\sim\omega^{-1}$ and of small angles ($\sim m/\omega$) are suppressed in a power-like manner in the parameter m/ω. The positron energy for $\omega\gg m$ is of the order of ω. Hence, the characteristic positron orbital angular momenta l_p providing the main contribution to the cross section are of the order of $\omega r\gg1$. In Eq. (4), therefore, instead of the exact Green-function jump corresponding to positrons we can use the quasiclassical Green function. This function was obtained in Refs. 9 and 10. Using formula (5) of Ref. 10, we get

$$\delta G(\mathbf{r}',\mathbf{r}|\varepsilon)=\frac{i\nu}{4\pi}\int_{-\infty}^{\infty}ds\exp\left\{i\left[\nu(r+r')\operatorname{cth}s+2Z\alpha\frac{s\varepsilon}{\nu}\right]\right\}\left\{J_0(w)\left[\gamma^0\varepsilon+m+\frac{\nu}{2}(\gamma,\mathbf{n}-\mathbf{n}')\operatorname{cth}s\right]+\frac{iJ_1(w)}{w}\left[(\gamma\mathbf{n}+\mathbf{n}')\times\left(\frac{\nu^2(r'-r)}{2\operatorname{sh}^2 s}+Z\alpha m\gamma^0\right)-Z\alpha\nu\gamma^0(1-(\gamma\mathbf{n}')(\gamma\mathbf{n}))\operatorname{cth}s\right]\right\}, \tag{8}$$

where

$$\nu=(\varepsilon^2-m^2)^{1/2},\quad w=[2rr'(1+\mathbf{nn}')]^{1/2}/\operatorname{sh}s.$$

We now substitute (8) and (5) into (4) and calculate the trace. It is convenient at this point to make several transformations that simplify further calculations. Since the total cross section is independent of the direction of the incident-photon momentum. It is convenient in Eq. (4) to average over these directions:

$$\exp[iq(\mathbf{r}-\mathbf{r}')]\to\frac{\sin(\omega|\mathbf{r}-\mathbf{r}'|)}{\omega|\mathbf{r}-\mathbf{r}'|}.$$

As a result the integrand in (4) depends only on r, r', and $x=\mathbf{n}\cdot\mathbf{n}'$. We introduce new variables $r=\frac{1}{2}\rho(1+t)$ and $r'=\frac{1}{2}\rho(1-t)$. The above reasoning implies that the main contribution to the cross section is provided by the region where $\rho\sim 1/m$ and $x\sim 1$, in which the arguments of the exponential functions in (4) and (8) and the argument of the Bessel function in (8) are large ($\sim\omega/m$). Hence, to evaluate the integral with respect to t we can use the asymptotic formula for Bessel functions and employ the stationary-phase method, the point of stationary phase being

$$t_0=\left[1-\frac{2\nu^2}{(1+x)(\nu^2+\omega^2\operatorname{sh}^2 s)}\right]^{1/2}.$$

Using the value of the integral with respect to τ given by (7), we can evaluate the integral with respect to ρ, which can be reduced to Euler's integral of the second kind, the gamma function Γ. It has also proved expedient to go from x to the variable $y=[\cosh^2 s-2(1+x)^{-1}]^{1/2}/\sinh s$, shift the contour of integration with respect to s, namely, $s\to s-i\pi/2$, and then go over to the variable $u=\tanh s$. Performing these transformations, we arrive at the final expression for the pair-production cross section with the electron in an arbitrary state of the discrete spectrum:

$$\sigma_\gamma=\frac{2\pi\alpha l\beta^{2\gamma+1}e^{-\pi Z\alpha}}{\omega m(\gamma+n)\sqrt{1+\beta^2}}\sum_{M=0}^{n}\times\frac{\Gamma(2\gamma+2M+2)\beta^{2M}}{M!(n-M)!\Gamma(2\gamma+M+1)\cdot 2^{2\gamma+2M}}\left(\frac{d}{dv}\right)^{n-M}\times(1+v)^{2\gamma+n+M-1}\int_0^1 dy\int_{-1}^{1}\times du\left(\frac{1-u}{u+1}\right)^{iZ\alpha}\frac{(y^2-u^2)^{\gamma+M}}{g^{2(\gamma+M+1)}(v)}\times\left\{\left[\beta^2(1-v^2)(\gamma+M+1)y^2\frac{1}{g(v)}+\gamma+M+\beta Z\alpha v\right]b+l\,ag(v)\right\}\Bigg|_{v=1}. \tag{9}$$

In this formula

$$a=P_{l-1}(w)-P_l(w),\quad b=P_l(w)+P_{l-1}(w),$$

$$w=\frac{2-u^2-y^2}{y^2-u^2},\quad g(v)=1-ivu\beta,\quad \beta=\frac{k}{E_n}=\frac{Z\alpha}{n+\gamma},$$

and the contour of integration with respect to u passes above the real axis. Differentiation with respect to v in (9) can be done explicitly. We have retained the differential notation for reasons of compactness. Note also that for all values of l and n we can, via the formula

$$P_l(w)=(-1)^l F\left(-l,l+1;1;\frac{1-u^2}{y^2-u^2}\right),$$

with $F(a,b,c;x)$ the hypergeometric function, reduce the integral with respect to y in (9) to

$$\int_0^1 dy(y^2-u^2)^\lambda=\frac{1}{2}\exp[-i\pi\lambda\operatorname{sign}(u)](u^2)^{\lambda+1/2}\times B\left(\lambda+1,\frac{1}{2}\right)+\frac{(1-u^2)^{\lambda+1}}{2(\lambda+1)}\times F\left(\frac{1}{2},1;\lambda+2;1-u^2\right), \tag{10}$$

where we have allowed for the rule of bypassing in variable u. At $n=0$ the formula for the cross section simplifies considerably:

$$\sigma_\gamma(n=0)=\frac{\pi\alpha(2\gamma+1)le^{-\pi Z\alpha}}{\omega m\gamma}\left(\frac{Z\alpha}{\gamma}\right)^{2\gamma+1}\times\int_0^1 dy\int_{-1}^{1}du\left(\frac{1-u}{u+1}\right)^{iZ\alpha}\times\frac{(y^2-u^2)^\gamma}{[1-iZ\alpha u/\gamma]^{2\gamma+2}}[(\gamma-iZ\alpha u)a+lb]. \tag{11}$$

At $l=1$ (the ground state) this formula transforms into the one obtained in Ref. 5 if we introduce the change of variable $u=1/x$ and distort the contour of integration with respect to x in such a way that it goes from zero to unity.

Formula (9) is extremely convenient for numerical calculations and for obtaining the asymptotes, which we discuss in the next section.

3. CROSS-SECTION ASYMPTOTICS

Let us consider the behavior of the cross section as a function of the radial quantum number n, the angular momentum j (recall that the parameter l in (9) is equal to $j+\frac{1}{2}$), and the value of $Z\alpha$.

We start with the case when $l\gg 1$ and $n\sim 1$. Using the standard integral representation for the Legendre polynomials for values of the argument greater than unity (Ref. 12), we easily find that for $l\gg 1$,

$$P_l(w)\approx\frac{2^{2l}\exp\left[-(u^2+y^2)l/2\right]}{\sqrt{\pi l}(y^2-u^2)^l}. \tag{12}$$

This implies that the main contribution to the integral in (9) is provided by the variable region $y\sim u\sim l^{-1/2}$. Substituting (12) in (9), expanding the integrand for small values of u, and evaluating the integrals, we find that

$$\sigma_\gamma(l\gg 1)=\frac{\pi\alpha(2l)^n\sqrt{l\pi}}{8\omega mn!}\left(\frac{2Z\alpha}{l+n}\right)^{2l+3}e^{-\pi Z\alpha}. \tag{13}$$

We see that even for $Z\alpha\sim 1$ the cross section with $l\gg 1$ is numerically suppressed. Comparison of (13) with the results of Pratt's numerical calculations[6] performed for $l=2$ and $n=0$ (the $2p_{3/2}$ state) shows that already for $l=2$ formula (13) provides an accuracy better than 5% in the $Z\alpha\leqslant 0.7$ ($Z<96$) range. But if we allow for the fact (see Ref. 6) that the contribution of the $2p_{3/2}$ state to the cross section with the capture of the electron by the L-shell, $\sigma_L=\sigma_\gamma(n=1,l=1)+\sigma_\gamma(n=0,l=2)$, does not exceed 11%, the use of the asymptotic formula (13) for determining σ_L ensures an accuracy better than 1%.

Now let us study the case when $n\gg 1$ and $l=1$. The leading term in the expansion of (9) in $1/n$ has the form

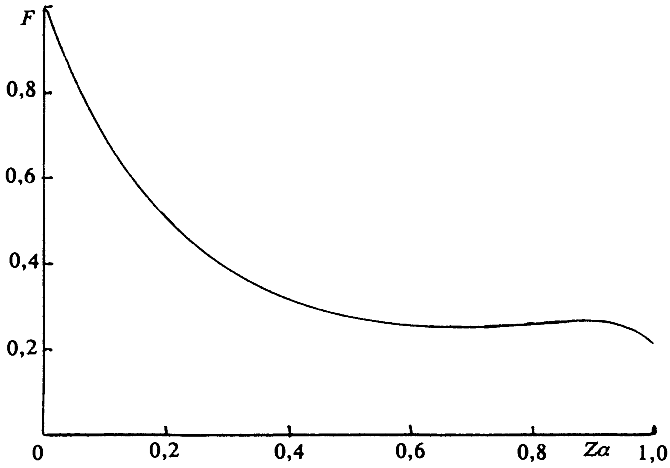

FIG. 1. The function $F(Z\alpha)$ determining the dependence of the cross section σ_γ for $n\gg 1$ [see formula (15)].

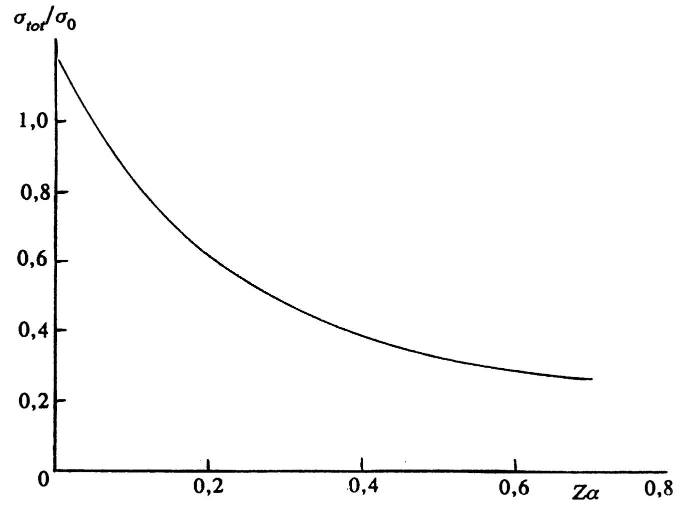

FIG. 2. Total cross section of the process in units of σ_0.

$$\sigma=\sigma_0\frac{F(Z\alpha)}{(\gamma_1+n)^3}, \tag{14}$$

where $\sigma_0=4\pi\alpha(Z\alpha)^5/m\omega$, and

$$\begin{aligned}F(Z\alpha)&=e^{-\pi Z\alpha}\\&\times\int_{-1}^{1}du\left(\frac{1-u}{1+u}\right)^{iZ\alpha}e^{2iZ\alpha u}\\&\times\sum_{M=0}^{\infty}\frac{(Z\alpha)^{2(d-2)}}{M!\Gamma(\gamma+d+1)}\\&\times\int_0^1 dy(y^2-u^2)^{d-1}\{R(2d+1)\\&\times[R(1-u^2)+y^2-1)]\\&+(Z\alpha)^2(1-u^2)[(1-y^2)(2R+1)+u^2-y^2]\},\end{aligned} \tag{15}$$

with

$$d=M+\gamma_1,\quad \gamma_1=\sqrt{1-(Z\alpha)^2},\quad R=d+iZ\alpha u.$$

As $Z\alpha\to 0$, the function $F(Z\alpha)$ tends to unity, which agrees with the results of calculations in the Born approximation.[13] Note that, as formula (13) implies, for $Z\alpha\ll 1$ the pair-production cross section with capture into a state with $l\geqslant 2$ contains an additional (in comparison with $l=1$) suppression factor proportional to $(Z\alpha)^{2(l-1)}$ (see the discussion in Ref. 6).

The result of summation in (15) can be expressed in terms of the Bessel function $J_{2\gamma_1}(2Z\alpha\sqrt{u^2-y^2})$ and its derivative. This expression can be obtained directly from Eq. (4) if we note that for $n\gg 1$ the small $\tau\sim k/E_n=Z\alpha(n+\gamma)^{-1}$ contribute to (5). Following this path in deriving the asymptotic behavior, we can easily see that in calculating the corrections to formula (14) the quantity $(k/E_n)^2$ is the expansion parameter. If we allow

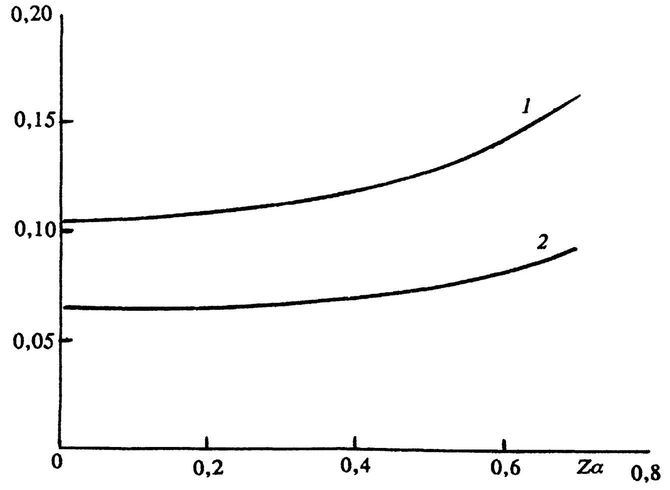

FIG. 3. The dependence on $Z\alpha$ of the contributions $\sigma_L/\sigma_{\text{tot}}$ (curve *1*) and $\sigma_{M'}/\sigma_{\text{tot}}$ (curve *2*) to the total cross section.

for the first correction term in this parameter, the cross section of the process for $n \gg 1$ has the form

$$\sigma=\sigma_0\frac{F(Z\alpha)}{(Y_1+n)^3}\left[1-\left(\frac{Z\alpha}{\gamma_1+n}\right)^2 f(Z\alpha)\right]. \tag{16}$$

The explicit expression for $f(Z\alpha)$ is too cumbersome to be given here. Estimates show that for $0.1 < Z\alpha < 0.9$ the function $f(Z\alpha)$ varies in the interval from 0.9 to 2, which enables estimating the accuracy of formula (14). Interestingly, if in (16) we put $f(Z\alpha)=4/3$ this formula approximates at $Z\alpha \leqslant 0.7$ the results of the numerical calculations for $\sigma(l=1,n=1)(\sigma_{2s_{1/2}}+\sigma_{2p_{1/2}})$ carried out in Ref. 6, with an accuracy higher than 1.6%.

The function $F(Z\alpha)$ specified by (15) is depicted in Fig. 1. In calculating it we first evaluate the integral with respect to y via (10) and then the integral with respect to u, so that the result is expressed in terms of a double sum containing hypergeometric functions. Note that as $Z\alpha$ grows the function $F(Z\alpha)$ falls of quite rapidly at first owing to the factor $\exp(-\pi Z\alpha)$ in (15), but remains practically constant in the interval $0.55 < Z\alpha < 0.95$.

The approximate expressions (13) and (16) obtained for the partial (with given values for l and n) cross sections $\sigma_\gamma(l,n)$ make it possible to find, with an accuracy higher than 1%, the total cross section of the process:

$$\sigma_{\text{tot}}=\sum_{n=0}^{\infty}\sum_{l=1}^{\infty}\sigma_\gamma(l,n).$$

For the contribution to this sum by the K-shell [$1s_{1/2}$ states; $\sigma_K=\sigma_\gamma(1,0)$] and the L-shell [$2s_{1/2}$, $2p_{1/2}$, and $2p_{3/2}$ states; $\sigma_L=\sigma_\gamma(1,1)+\sigma_\gamma(2,0)$] we use Pratt's results,[6] and for the sum $\sigma_{M'}=\sigma_{\text{tot}}-\sigma_K-\sigma_L$, of all other contributions, we use formulas (13) and (16). The result for the ratio $\sigma_{\text{tot}}/\sigma_0$ is depicted in Fig. 2 in the interval $0 < Z\alpha < 0.7$. The reader can see that exact allowance for the Coulomb field radically changes the result in comparison to the Born approximation.

Figure 3 depicts the dependence on $Z\alpha$ of the relative contributions $\sigma_L/\sigma_{\text{tot}}$ and $\sigma_{M'}/\sigma_{\text{tot}}$ to the total cross section. Clearly, for all values of $Z\alpha$ the cross section with production of an electron in the ground state is predominant, but as $Z\alpha$ increases the relative contribution of other state increases.

[1] K. Rumrich, K. Momberger, G. Soff *et al.*, Phys. Rev. Lett. **66**, 2613 (1991).
[2] J. C. Wells, V. E. Oberacker, A. S. Umar *et al.*, Phys. Rev. A **45**, 6296 (1992).
[3] C. A. Bertulani and G. Baur, Phys. Rep. **163**, 299 (1988).
[4] J. Eichler, Phys. Rep. **193**, 167 (1990).
[5] R. H. Pratt, Phys. Rep. **117**, 1017 (1960).
[6] R. H. Pratt, Phys. Rep. **119**, 1619 (1960).
[7] A. Sommerfeld and A. W. Maue, Ann. Physik **22**, 629 (1935); W. H. Furry, Phys. Rev. **46**, 391 (1934).
[8] A. I. Milstein and V. M. Strakhovenko, Phys. Lett. **90A**, 447 (1982).
[9] A. I. Milstein and V. M. Strakhovenko, Phys. Lett. **92A**, 381 (1982).
[10] A. I. Mil'shteĭn and V. M. Strakhovenko, Zh. Eksp. Teor. Fiz. **85**, 14 (1983) [Sov. Phys. JETP **58**, 8 (1983)].
[11] V. B. Berestetskiĭ, E. M. Lifshitz, and L. P. Pitaevskiĭ, *Quantum Electrodynamics*, 2nd ed., Pergamon Press, Oxford (1982).
[12] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series and Products*, 4th ed., Academic Press, New York (1966).
[13] F. Sauter, Ann. Physik **11**, 454 (1931).

Translated by Eugene Yankovsky