Generation of high-energy photons by ultrarelativistic electrons in a nonequilibrium plasma

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It is shown that the scattering of high-energy electrons in a nonequilibrium dense plasma can be used effectively for generating hard polarized radiation. Calculations performed for a dense lithium plasma showed that the yield of polarized photons with energy $0.68E$, where $E = 150$ GeV, is 10^6–$10^7\,s^{-1}$ per electron.

1. INTRODUCTION

Theoretists and experimentalists alike are becoming increasingly interested in the physics of γ–γ reactions. In this connection, the development of experimental methods for producing intense fluxes of high-energy (> 100 GeV photons would greatly expand the range of research on electroweak and strong interactions, supersymmetry, etc. For this reason, the solution of the problem of a source of high-energy phonons is crucial for further progress in this field of research.

Existing methods, based on bremsstrahlung from relativistic particles in heavy targets, do not produce intense photon fluxes in the very hard region in the tail of the spectral distribution. In spite of the many advantages of the method of inverse Compton scattering, there are two problems with it: first, the momenta of the accelerated particles must be matched with the momenta of the laser photons and, second, high laser power (> 1000 MW) is needed in order to transform efficiently the particle beam into scattered photons.

In the analysis of hard radiation processes accompanying the passage of a relativistic particle through condensed media, the medium is usually treated as a source of an external stationary field in which the particle, being scattered and using momentum, emits bremsstrahlung photons. Strictly speaking, in order to give a complete description of radiation phenomena accompanying the interaction of a fast particle with a many-body quantum system it is necessary to take into account the exchange with the system not only of momentum but also energy, the energy exchange process being accompanied by quantum transitions between the states of the system itself. The case when a high density of coherently excited states is created in the system is of special interest. A relativistic particle transversing such a system is scattered in the electromagnetic field generated by both the charges and transitions currents in the medium. For this reason, in addition to the usual bremsstrahlung caused by Coulomb collisions, Compton scattering of the electromagnetic field of the medium by the relativistic particle also makes a contribution.

The emission of coherent radiation accompanying the uniform motion of a charged particle in excited matter was first investigated in Ref. 1. The coherent emission processes is studied in Ref. 1 are unrelated to the acceleration of the charged particle and are an extension of Cherenkov radiation to the case of excited matter. Nonetheless, in many cases, when, for example, the excited states of the particles of the system are not degenerate or are specially prepared, the value of the transverse acceleration of a relativistic particle (averaged over the positions of the particles in the system) is found to be nonzero, and there arises a coherent contribution to the radiation (coherent conversions of excitations[2,3]), and the spectral density of the contribution has a maximum in the hard part of the spectrum and is proportional to the squared density of particles in the system. Due to the latter feature the yield of polarized radiation in the hard part of the spectrum is significant and under some conditions can be comparable to the incoherent bremsstrahlung background.

It can be shown that the value of the electric field intensity in a medium consisting of single-electron atoms averaged over the ensemble and over the positions of the particles in the system is [see also the derivation of Eq. (4.8) below]:

$$\langle\langle \mathscr{E} \rangle\rangle = 4\pi e N \sum_{mn} \langle n | \rho^{(S)}(t) | m \rangle \langle m | \mathbf{r} | n \rangle , \tag{1.1}$$

where N is the number of atoms per unit volume, $\langle n|\rho^{(S)}(t)|m\rangle$ are the elements of the single-atom density matrix in the Schrödinger representation, and $e\langle m|\mathbf{r}|n\rangle$ is a matrix element of the dipole moment between the states n and m.

It is obvious that the field $\langle\langle \mathscr{E} \rangle\rangle$ is determined by the evolution of the off-diagonal elements of the density matrix, since the diagonal matrix elements of the dipole moment are zero. Under the usual conditions, when the atomic system is an incoherent mixture, the off-diagonal elements of the density matrix are zero and, therefore, the average field $\langle\langle \mathscr{E} \rangle\rangle$ is also zero. On the other hand, under nonequilibrium conditions the off-diagonal elements of the density matrix decay rapidly due to relaxation mechanisms with characteristic times on the order of the lifetime of the excited states. For this reason, the field $\langle\langle \mathscr{E} \rangle\rangle$ should vanish quite rapidly after the source responsible for the departure from equilibrium no longer acts on the system.

It should also be noted that when the states n are degenerate the average field $\langle\langle \mathscr{E} \rangle\rangle$ can be zero because the dipole moment vectors corresponding to transitions between degenerate states are oppositely oriented. If, however, a state of the system is specially prepared with nonzero average dipole moment and there exists a mechanism for "replenishing" the off-diagonal elements of the density matrix, then the average field $\langle\langle \mathscr{E} \rangle\rangle$ will be nonzero and the motion of a relativistic

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particle in such a field is similar to the motion in an electric undulator with "optical" frequency equal to the frequency of the corresponding transition $n \to m$.

These conditions can be realized if, first, the nonequilibrium system is placed in an external magnetic or electric field (which lifts the degeneracy) and, second, such a system is subjected to steady resonant pumping by an external alternating field (for example, laser radiation) whose frequency is close to that of some transition $n \to m$. Whether the external source of excitation is a beam of charged particles injected into the medium or external electromagnetic irradiation is essentially irrelevant to the basic questions concerning electronic relaxation and the theoretical analysis of the required properties of nonequilibrium plasma steadily maintained in this manner. In any case, since this is accompanied by excitation of quantum states, in order to solve the problem posed it is first necessary to calculate the parameters of nonequilibrium and to analyze the population kinetics of the excited states.

In this paper we study the possibility of generation of hard photons by an ultrarelativistic electron beam in the process of scattering in a nonequilibrium medium, in which a high density of nondegenerate coherent quantum states is created. A model of a steady source of hard photons is proposed and analyzed on the basis of coherent conversion of atomic excitations. The system of units with $\hbar = c = 1$ is employed throughout.

2. RELAXATION EQUATIONS FOR THE DENSITY MATRIX

Below we write out the system of equations describing the relaxation of a homogeneous atomic plasma with a simple chemical composition. We assume that a Maxwellian distribution with temperatures $T_e = T_a = T$ is established beforehand over the translational degrees of freedom of the electrons and heavy particles. In addition, we assume that the system is subjected to an alternating field $V(t)$, whose frequency Ω is close to the frequency of some atomic transition $2 \to 1$. We represent the interaction of the atoms with the applied field as follows:

$$V(t) = \frac{1}{2} V[\exp(i\Omega t) + \exp(-i\Omega t)] . \tag{2.1}$$

For electrical dipole transitions, for example, the interaction operator is

$$V(t) = -e\langle 2|\mathbf{r}|1\rangle \mathbf{E}(t) , \tag{2.2}$$

where $\mathbf{E}(t) = \mathbf{E}\cos(\Omega t)$ is the intensity of the external electric field.

The complete equation for the elements of the density matrix in the Schrödinger representation, taking into account the interaction with the field (2.1) and the relaxational processes accompanying the interaction of the atoms with free electrons and with one another, has the form

$$\dot{\rho}^{(S)}_{km} = -iE_{km}\rho^{(S)}_{km} - i\langle k|[V(t), \rho^{(S)}(t)]|m\rangle + \sum_{ln} R_{kmln}\rho^{(S)}_{ln}, \tag{2.3}$$

where $E_{km} = E_k - E_m$, E_m is a bound-state energy, and R_{kmln} is a relaxational matrix, whose elements depend on the character of the relaxational processes which are taken into account. The most general expression for the relaxational matrix is given, for example, in Ref. 4.

It is convenient to analyze Eqs. (2.3) for the off-diagonal elements of the density matrix ($k \neq m$) in the interaction representation. Taking into account the explicit form of the interaction operator (2.1) and the properties of the relaxational matrix (see Ref. 4), it can be shown that the off-diagonal elements of the density matrix satisfy the equations

$$\dot{\rho}_{km} = -\gamma_{km}\rho_{km} - \frac{i}{2}[\exp(i\Omega t) + \exp(-i\Omega t)]\sum_l [V_{kl}\rho_{lm}\exp(-iE_{lk}t) - V_{lm}\rho_{kl}\exp(-iE_{ml}t)] . \tag{2.4}$$

The coefficients γ_{km} in Eq. (2.4) are, generally speaking, complex:

$$\gamma_{km} = \gamma'_{km} + i\gamma''_{km} ,$$

and they have the following physical meaning: γ''_{km} characterize the magnitude of the shift of the line on the transition $k \to m$ due to relaxational mechanisms and γ'_{km} is the relaxational broadening of the corresponding line.

Taking into account the fact that the frequency Ω is close to the frequency of the atomic transition $2 \to 1$ substantially simplifies Eqs. (2.4). In this case the nonzero off-diagonal elements of the density matrix will be ρ_{12} and ρ_{21}, which satisfy the equation

$$\dot{\rho}_{21} = \dot{\rho}^{*}_{12} = -\gamma_{21}\rho_{21} - \frac{i}{2}V_{21}(\rho_{11} - \rho_{22})\exp(iE_{21}t) \times [\exp(i\Omega t) + \exp(-i\Omega t)] . \tag{2.5}$$

In the resonance region $\Omega \approx E_{21}$ and the low-frequency term $\exp[i(\Omega - E_{21})t]$ makes the main contribution to the right-hand side of Eq. (2.5). For this reason, in a first approximation the rapidly oscillating terms $\exp[i(\Omega + E_{21})t] \approx \exp(i2\Omega t)$ can be neglected (the so-called "rotating wave approximation"). Then, instead of Eq. (2.5), we have the equation

$$\dot{\rho}_{21} = -\gamma_{21}\rho_{21} - \frac{i}{2}V_{21}(\rho_{11} - \rho_{22})\exp[i(E_{21} - \Omega)t] . \tag{2.6}$$

The general solution of Eq. (2.6) corresponding to the stationary case $\dot{\rho}_{11} = \dot{\rho}_{22} = 0$ is

$$\rho_{21} = -\frac{i}{2}V_{21}(\rho_{11} - \rho_{22})\exp(-\gamma_{21}t) \times \left\{ \int_{t_0}^{t} dt \exp[\gamma_{21}t + i(E_{21} - \Omega)t] + \text{const} \right\} . \tag{2.7}$$

If, initially, at time $t_0 \to -\infty$, before the interaction $V(t)$ is switched on, the atomic system is an incoherent mixture, i.e., $\rho_{21} = 0$ at $t = t_0$, then the constant is zero. Thus it follows from Eq. (2.7) that

$$\rho_{21} = \rho^{*}_{12} = -\frac{1}{2}V_{21}(\rho_{11} - \rho_{22})\frac{\exp[i(E_{21} - \Omega)t]}{E_{21} - \Omega - i\gamma_{21}} . \tag{2.8}$$

In the Schrödinger representation the solutions (2.8), are, correspondingly,

$$\rho_{21}^{(S)}(t) = \rho_{12}^{*(S)}(t) = -\frac{1}{2} V_{21}(\rho_{11} - \rho_{22}) \frac{\exp(-i\Omega t)}{E_{21} - \Omega - i\gamma_{21}} . \tag{2.9}$$

The equations (2.4) [or (2.6)] must be supplemented by a system of equations for the diagonal elements of the density matrix, which are the relative populations of the atomic states N_m ($\rho_{mm} = N_m/N$). Such a system includes the equations of population balance together with the additional condition of particle number conservation:

$$N = N_+ + \sum_m N_m = \text{const} \tag{2.10}$$

(for a quasineutral plasma, we have $N_+ = N_e$, where N_+ and N_e are the ion and electron densities, respectively).

The population balance equations can be written in the form

$$\frac{dN_n}{dt} = \sum_{m=1}^{m_{max}} K_{mn} N_m + D_n . \tag{2.11}$$

In what follows we consider the case of steady recombination, i.e., the right-hand side of Eq. (2.11) is identically zero. The solutions of Eqs. (2.4), as already mentioned above, must be found from the condition that the diagonal elements of the density matrix are stationary $\dot{\rho}_{mm} = 0$.

The elements of the truncated relaxation matrix K_{mn} (in contrast to the full matrix R_{mnlk}) give the average number of transitions of an atom per unit time from the state m into the state n. The diagonal element K_{nn} determines the total flow of particles out of the state n per unit time. The quantity D_n characterizes inflow from the continuum.

When the degree of ionization is high the relaxation processes are determined completely by collisions with electrons and by radiative transitions. According to Refs. 5 and 6, for $N_e/N \approx 10^{-6}$ "quenching" by electrons and by atoms become equally efficient. Thus for $N_e/N > 10^{-6}$ electronic collisions dominate, and heavy particles can influence the population only in the presence of resonant processes[7] (charge exchange, resonant transfer of excitation, or the Penning effect).

Thus we write, taking into account only radiative transitions and collisions with electrons,

$$K_{mn} = V_{mn} N_e , \quad m < n , \tag{2.12a}$$

$$K_{mn} = \tilde{V}_{mn} N_e + A_{mn} , \quad m > n , \tag{2.12b}$$

where A_{mn} is the probability of a spontaneous radiative transition, V_{mn} is the rate of inelastic collisions with excitation of an atom, and $\tilde{V}_{mn}$ is the rate of superelastic collisions with deexcitation of an atom. The two collisional transition rates are related by the principle of detailed balance:

$$\tilde{V}_{mn} = \frac{g_n}{g_m} V_{nm} \exp\left(\frac{|E_{mn}|}{T_e}\right) , \quad m > n \tag{2.13}$$

where g_n is the statistical weight of the state n.

The elements of the relaxation matrix (2.12) must also be supplemented by terms which describe transitions induced between atomic states by the external field (2.1). The corresponding probabilities per unit time for nondegenerate states have the form

$$W_{nm} = W_{mn} = |\langle m|V|n\rangle|^2 \frac{\gamma'_{mn}}{(\Omega - E_{mn} - \gamma''_{mn})^2 + \gamma'^2_{mn}} . \tag{2.14}$$

On the basis of the physical meaning of the elements of the matrices γ_{mn} and K_{mn} considered above we can write the useful relation

$$\text{Re}\,\gamma_{mn} \approx 0,5|K_{mm} + K_{nn}| . \tag{2.15}$$

In a dense low-temperature plasma radiative recombination can be neglected compared with three-particle recombination, and it can be assumed that

$$D_m = V_{em} N_e^3 , \tag{2.16}$$

where V_{em} is the rate of three-body recombination in the state m. A diagonal element of the relaxation matrix can then be written as

$$K_{nn} = -\sum_m K_{nm} - V_{ne} N_e , \tag{2.17}$$

where V_{ne} is the rate of ionization of the level n by free electrons. This rate is related to the inverse process, three-body recombination, by the principle of detailed balance:

$$V_{em} = V_{me} \frac{g_m}{g_e g_+} \left(\frac{2\pi}{mT_e}\right)^{3/2} \exp\left(\frac{|E_m|}{T_e}\right) , \tag{2.18}$$

where g_e and g_+ are the statistical weights of the electron and ion, respectively.

Semiempirical formulas, which agree well with existing experimental data, are usually employed to calculate the collisional transition rates V_{nm} and V_{me}. Detailed reviews and monographs on this topic, which give more complete information about the character of collisional transitions in different atoms, are available (see, for example, Refs. 7 and 8). In the next section more useful formulas are presented and the relaxation matrix is calculated for a dense lithium plasma.

The formulas (2.8) and (2.9), derived above, together with the numerical solution of the stationary system of equations (2.11) make it possible to estimate the average field $\langle\langle\mathscr{E}\rangle\rangle$ in accordance with Eq. (1.1). If, in particular, it is assumed that $|V_{21}| \approx 10^{-19}$ ergs, which corresponds to an external excitation field of approximately 10 V/cm, then the average field generated in the medium, according to Eq. (2.9) with $N > 10^{20}$ cm^{-3} and $\langle 2|\mathbf{r}|1\rangle \approx 10^{-8}$ cm will be, in order of magnitude, $> 10^6$ V/cm. This high value of the average field $\langle\langle\mathscr{E}\rangle\rangle$ is due to the resonant character of the interaction $V(t)$ of the external source with the plasma.

As pointed out in the Introduction, if the states are degenerate, then the field $\langle\langle\mathscr{E}\rangle\rangle$ can vanish completely because the dipole moment vectors between states with the same energy are oppositely oriented. This situation can be eliminated, for example, by applying a magnetic field that lifts the degeneracy.

Consider a two-level system in which the state 2 is doubly degenerate. Let the state vectors be $|2'\rangle$ and $|2''\rangle$, respectively. According to Eq. (1.1), the average field $\langle\langle\mathscr{E}\rangle\rangle$ is

$$\langle\langle\mathscr{E}\rangle\rangle = 8\pi eN\,\text{Re}(\langle 2'|\rho^{(S)}(t)|1\rangle\langle 1|\mathbf{r}|2'\rangle + \langle 2''|\rho^{(S)}(t)|1\rangle\langle 1|\mathbf{r}|2''\rangle) . \tag{2.19}$$

If the degenerate states correspond to different projections of the orbital angular momentum (for example, P-state, $l = \pm 1$), then

$$\langle 1|\mathbf{r}|2'\rangle = -\langle 1|\mathbf{r}|2''\rangle$$

and the field (2.19) is identically zero. Now let the degeneracy be lifted by an external magnetic field with intensity H and frequency such that

$$\Omega = E_2 - E_1 + \Delta ,$$

where Δ is the shift of the eigenvalue E_2 due to the external magnetic field:

$$\Delta = g\mu_B H ,$$

μ_B is the Bohr magneton and g is the gyromagnetic ratio. It is easy to see from Eqs. (2.19) and (2.9) that the ratio of the two terms of different sign in Eq. (2.19) in this case is $1 + \Delta/|K_{11} + K_{22}|$ in order of magnitude. Consider as an example the $2P$ and $2S$ atomic states of the lithium atom. The energy of the degenerate transition $2P$–$2S$ is 1.844 eV ($\lambda = 6707.8$ Å). In an external magnetic field of intensity $H = 1.54\cdot 10^5$ Oe the energy of one of the $2P$–$2S$ transitions which are split apart ($l = -1$) is equal to the energy of argon laser photons[9] ($\Omega = 1.838$ eV, $\lambda = 6730$ Å), and the shift is $\Delta = 6.345\cdot 10^{-3}$ eV. It is easy to show that

$$1 + \Delta/|K_{11} + K_{22}| \approx 10^3$$

and the contribution of states to Eq. (2.19) which do not satisfy the resonance condition (in this case the $2P$ state with projection of the orbital angular momentum $l = 1$) can be neglected. Thus the above estimates show that it is indeed possible to obtain a polarized ensemble of atoms by applying an external magnetic field and a resonant alternating electromagnetic field.

3. RELAXATION MATRIX OF A LITHIUM PLASMA

In order to determine the transition rates and to calculate the elements of the relaxation matrix we employ the empirical data presented in Ref. 10. The rate of a collisional transition from a level characterized by the quantum numbers n_1, l_1 to the level n_2, l_2 is determined by the formula

$$V_{n_1l_1,n_2l_2} = 10^{-8}\left(\frac{\mathrm{Ry}}{\Delta E}\frac{E_{n_1l_1}}{E_{n_2l_2}}\right)^{3/2}\frac{e^{-x}G(x)}{2l_2+1}\,,\mathrm{cm^3/s}, \qquad (3.1)$$

where

$$G(x) = A\frac{[x(x+3)]^{1/2}}{x+\chi}f(x)\,,$$

$$f(x) = \ln(16 + 1/x)\,, \qquad |\Delta l\,| = 1\,, \qquad (3.2)$$

$$f(x) = 1\,, \qquad |\Delta l\,| \neq 1\,,$$

F_{nl} is the energy of the state n, l (measured from the continuum), ΔE is the energy of the corresponding transition, $x = |\Delta E|/T_e$, and Ry is the Rydberg constant. The values of the parameters A and χ depend on the transition and are tabulated in Ref. 10.

The collisional ionization rates V_{me} can be approximated quite accurately by the Beigman–Vainshtein formula[11]

$$V_{me} = 1{,}744\cdot 10^{-8}\left(\frac{\mathrm{Ry}}{\Delta \mathrm{E}}\right) e^{-x}G_1(x)\,,\ \mathrm{cm^3/s}, \qquad (3.3)$$

where the function $G_1(x)$ is tabulated for a wide range of values of the parameter x. For values of x outside the interval given in the tables of Ref. 11 the following approximations can be used:

$$x < 0{,}02\,, \qquad G_1(x) = 3{,}85x^{1/2}\ln(1{,}42x)\,, \qquad (3.4)$$

$$x > 163{,}84\,, \qquad G_1(x) = 0{,}859x^{-1/2}\,.$$

For further calculations we consider the example of a dense ideal plasma with atomic density $N = 10^{22}$ cm^{-3} at a temperature of 0.23 eV (or 2668 K). This choice of values is dictated by the fact that the vaporization temperature of lithium is 0.14 eV (or 1590 K) and the density of the solid phase is $N = 4.6\cdot 10^{22}$ cm^{-3}. A dense gas with this density and temperature exerts a pressure of $2.6\cdot 10^5$ Pa (or 2.6 atm) on the vessel walls. Obviously, there is no difficulty in producing and maintaining a medium without such parameters.

The equilibrium free-electron density is determined from the well-known Saha distribution (see, for example, Ref. 12):

$$\frac{N\alpha^2}{1-\alpha} = \frac{g_e g_+}{g_m}\left(\frac{mT}{2\pi}\right)^{3/2}\exp\left(-\frac{I}{T}\right),$$

where I is the ionization potential and $\alpha = N_e/N$ is the degree of ionization, which even for $T < I$ is quite high and becomes greater than 10^{-6} at temperatures $T > 0.2$ eV. In the latter case, as indicated above, the main mechanism of electronic relaxation is collisions with free electrons and the relaxation kinetics is determined by the equations presented in Sec. 2.

The values of the elements of the relaxation matrix and the collisional transition rates for a low-temperature ($T < 1.0$ eV) dense lithium plasma are given in Table I.

4. EQUATIONS OF MOTION AND RETARDED POTENTIALS

The classical equations of motion of a relativistic particle in an electromagnetic field produced by the medium have the form

$$\frac{d\mathbf{p}}{dt} = -e\frac{\partial\langle \mathbf{A}\rangle}{\partial t} - e\nabla\langle\varphi\rangle + e[\mathbf{v}, \mathrm{rot}\langle \mathbf{A}\rangle]\,, \qquad (4.1)$$

where $\mathbf{p}$ is the momentum of the relativistic particle and $\langle \mathbf{A}\rangle$ and $\langle\varphi\rangle$ are the ensemble-averaged potentials of the field generated by the particles of a nonrelativistic quantum system.

In order to solve this problem we employ a modified semiclassical method. A rigorous justification of this method is given, for example, in Ref. 13. The incident electron is considered to be a classical spinless particle, and the retarded potentials of the external field are replaced by the matrix elements of Heisenberg operators, determined by the transition currents between states of a nonrelativistic quantum system, for example, an ensemble of single-electron atoms which scatter the incident electrons:

$$\mathbf{A}_{mn}(\mathbf{r}, t) = \sum_\alpha \int d\mathbf{r}'\mathbf{j}_{mn}(\mathbf{r}')$$

$$\times\frac{\exp(-iE_{mn}t + iE_{mn}|\mathbf{r} - \mathbf{r}' - \mathbf{r}_\alpha|)}{|\mathbf{r} - \mathbf{r}' - \mathbf{r}_\alpha|}\,, \qquad (4.2)$$

TABLE I. Relaxational matrix and collisional transition rates in a lithium plasma with $N = 10^{22}$ cm^{-3} and $T = 0.23$ eV.

$n-m$	K_{nm}, 1/s	K_{mn}, 1/s	V_{nm}, cm^3/s	V_{mn}, cm^3/s
1—1	$-1{,}8\cdot10^{7}$	$-1{,}8\cdot10^{7}$	0	0
2—1	$1{,}9\cdot10^{10}$	$1{,}8\cdot10^{7}$	$8{,}3\cdot10^{-7}$	$7{,}9\cdot10^{-10}$
3—1	$1{,}4\cdot10^{9}$	$5{,}9\cdot10^{2}$	$6{,}3\cdot10^{-8}$	$2{,}6\cdot10^{-14}$
4—1	$1{,}2\cdot10^{9}$	$2{,}2\cdot10^{2}$	$5{,}4\cdot10^{-8}$	$9{,}5\cdot10^{-15}$
5—1	$4{,}8\cdot10^{7}$	$1{,}1\cdot10^{1}$	$2{,}1\cdot10^{-9}$	$4{,}9\cdot10^{-16}$
6—1	$1{,}1\cdot10^{8}$	$7{,}0\cdot10^{-1}$	$5{,}0\cdot10^{-9}$	$3{,}1\cdot10^{-17}$
7—1	$7{,}6\cdot10^{7}$	$6{,}3\cdot10^{-1}$	$3{,}3\cdot10^{-9}$	$2{,}7\cdot10^{-17}$
2—2	$-3{,}8\cdot10^{10}$	$-3{,}8\cdot10^{10}$	0	0
3—2	$2{,}0\cdot10^{10}$	$8{,}6\cdot10^{6}$	$8{,}7\cdot10^{-7}$	$3{,}8\cdot10^{-10}$
4—2	$2{,}5\cdot10^{9}$	$4{,}6\cdot10^{5}$	$1{,}1\cdot10^{-7}$	$2{,}0\cdot10^{-11}$
5—2	$7{,}8\cdot10^{8}$	$1{,}7\cdot10^{5}$	$3{,}1\cdot10^{-8}$	$7{,}5\cdot10^{-12}$
6—2	$1{,}5\cdot10^{9}$	$9{,}1\cdot10^{3}$	$6{,}3\cdot10^{-8}$	$4{,}0\cdot10^{-13}$
7—2	$1{,}2\cdot10^{8}$	$1{,}0\cdot10^{3}$	$5{,}1\cdot10^{-9}$	$4{,}4\cdot10^{-14}$
3—3	$-9{,}2\cdot10^{10}$	$-9{,}2\cdot10^{10}$	0	0
4—3	$1{,}1\cdot10^{11}$	$4{,}5\cdot10^{10}$	$4{,}7\cdot10^{-6}$	$2{,}0\cdot10^{-6}$
5—3	$6{,}5\cdot10^{9}$	$3{,}7\cdot10^{9}$	$2{,}8\cdot10^{-7}$	$1{,}6\cdot10^{-7}$
6—3	$5{,}8\cdot10^{9}$	$8{,}6\cdot10^{7}$	$2{,}5\cdot10^{-7}$	$3{,}7\cdot10^{-9}$
7—3	$3{,}0\cdot10^{9}$	$6{,}1\cdot10^{7}$	$1{,}3\cdot10^{-7}$	$2{,}7\cdot10^{-9}$
4—4	$-3{,}3\cdot10^{12}$	$-3{,}3\cdot10^{12}$	0	0
5—4	$2{,}3\cdot10^{12}$	$3{,}0\cdot10^{12}$	$9{,}9\cdot10^{-5}$	$1{,}3\cdot10^{-4}$
6—4	$7{,}1\cdot10^{10}$	$2{,}5\cdot10^{9}$	$3{,}1\cdot10^{-6}$	$1{,}1\cdot10^{-7}$
7—4	$3{,}7\cdot10^{9}$	$1{,}8\cdot10^{8}$	$1{,}6\cdot10^{-7}$	$7{,}8\cdot10^{-9}$
5—5	$-4{,}5\cdot10^{12}$	$-4{,}5\cdot10^{12}$	0	0
6—5	$3{,}2\cdot10^{10}$	$8{,}2\cdot10^{8}$	$1{,}4\cdot10^{-6}$	$3{,}6\cdot10^{-8}$
7—5	$1{,}3\cdot10^{10}$	$4{,}4\cdot10^{8}$	$5{,}5\cdot10^{-7}$	$1{,}9\cdot10^{-8}$
6—6	$-6{,}2\cdot10^{11}$	$-6{,}2\cdot10^{11}$	0	0
7—6	$2{,}3\cdot10^{11}$	$3{,}2\cdot10^{11}$	$1{,}0\cdot10^{-5}$	$1{,}4\cdot10^{-5}$
7—7	$-2{,}4\cdot10^{13}$	$-2{,}4\cdot10^{13}$	0	0

$$\varphi_{mn}(\mathbf{r}, t) = \sum_{\alpha} \int d\mathbf{r}' \rho_{mn}(\mathbf{r}') \times \frac{\exp(-iE_{mn}t + iE_{mn}|\mathbf{r} - \mathbf{r}' - \mathbf{r}_\alpha|)}{|\mathbf{r} - \mathbf{r}' - \mathbf{r}_\alpha|}, \tag{4.3}$$

where

$$\mathbf{j}_{mn}(\mathbf{r}') = \frac{e}{m} \psi^*_m(\mathbf{r})\mathbf{p}\psi_n(\mathbf{r}),$$

$$\rho_{mn}(\mathbf{r}') = e\psi^*_m(\mathbf{r})\psi_n(\mathbf{r}).$$

We write the statistical ensemble-averaged potentials of the field as follows:

$$\langle \mathbf{A} \rangle = \sum_{mn} \rho^{(H)}_{mn} \mathbf{A}_{mn}(\mathbf{r}, t), \tag{4.4}$$

$$\langle \varphi \rangle = \sum_{mn} \rho^{(H)}_{mn} \varphi_{mn}(\mathbf{r}, t), \tag{4.5}$$

where $\rho^{(H)}_{mm}$ are the elements of the reduced single-atom density matrix in the Heisenberg representation, which are calculated in the basis of functions $\psi_n(\mathbf{r})$.

If the system under consideration consists of an ensemble of single-electron atoms, then the potential of the nuclei or ionic cores Φ, which are assumed to be stationary, must also be added to the average potential $\langle \varphi \rangle$ in Eq. (4.1). In this case the basis functions $\psi_n(\mathbf{r})$ are the single-electron wave functions of an isolated atom.

In order to investigate the electromagnetic radiation arising with scattering by a many-body quantum system the spectral density must be averaged over the coordinates of the particles of the system. In so doing, the coherent and incoherent contributions to the spectral density can be separated. The coherent radiation in the dipole approximation is determined by the average (over the positions of the particles of the system) value of the transverse acceleration of the relativistic particle. The incoherent radiation is determined by fluctuations of the transverse acceleration caused by uncorrelated collisions of the relativisitic particle with atoms of the system.

In order to analyze the coherent radiation the equation (4.1) averaged over the coordinates of the particles in the system must be solved. As a result of such averaging we obtain

$$\frac{d\mathbf{p}}{dt} = -e\frac{\partial}{\partial t}\langle\langle \mathbf{A} \rangle\rangle, \tag{4.6}$$

where the double brackets indicate averaging over the coordinates of the particles and statistical averaging over the ensemble. It is easy to show that

$$\langle\langle \mathbf{A} \rangle\rangle = -4\pi N \sum_{mn} \rho^{(H)}_{mn} \langle m | \frac{e}{m}\mathbf{p} | n \rangle \frac{\exp(-iE_{mn}t)}{E^2_{mn}} = 4i\pi e N \sum_{mn} \rho^{(H)}_{mn} \langle m | \mathbf{r} | n \rangle \frac{\exp(-iE_{mn}t)}{E_{mn}}. \tag{4.7}$$

The intensity of the electric field, averaged in this manner, is [see also Eq. (1.1)]

$$\langle\langle \mathscr{E}\rangle\rangle = \frac{\partial}{\partial t}\langle\langle A\rangle\rangle = 4\pi N\sum_{mn}\rho^{(H)}_{mn}\langle m|\mathbf{r}|n\rangle\exp(-iE_{mn}t)$$
$$= 4\pi eN\sum_{mn}\rho^{(S)}_{mn}\langle m|\mathbf{r}|n\rangle . \qquad (4.8)$$

Thus Eq. (4.6) together with the relation (4.8) will make it possible to determine the average value of the transverse acceleration and to calculate the coherent contribution to the electromagnetic radiation of the relativistic particle.

An equation describing the fluctuations of the transverse acceleration can easily be derived from Eqs. (4.1) and (4.6):

$$\frac{d}{dt}\delta\mathbf{p} = -e\frac{\partial}{\partial t}(\langle A\rangle - \langle\langle A\rangle\rangle)$$
$$- e\nabla(\langle\varphi\rangle + \Phi) + e[\mathbf{v}, \mathrm{rot}\langle A\rangle] , \qquad (4.9)$$

where $\delta\mathbf{p} = \mathbf{p} - \langle\mathbf{p}\rangle$, and we have used the fact that averaging the last two terms on the right-hand side of Eq. (4.1) over the positions of the particles in the system gives zero. If only the Coulomb interaction of the relativistic electron with the atoms of the system is taken into account in Eq. (4.9), then

$$\frac{d}{dt}\delta\mathbf{p} = e\sum_{\alpha}\nabla\Phi(\mathbf{r}-\mathbf{r}_{\alpha})$$
$$+ e^2\sum_{\alpha}\sum_{n}\rho^{(S)}_{nn}(t)\int d\mathbf{r}'\psi^*_n(\mathbf{r}')\frac{\mathbf{r}-\mathbf{r}'-\mathbf{r}_{\alpha}}{|\mathbf{r}-\mathbf{r}'-\mathbf{r}_{\alpha}|^3}\psi_n(\mathbf{r}') , \qquad (4.10)$$

where $\Phi(\mathbf{r}-\mathbf{r}_{\alpha})$ is the potential of the nuclei or ion cores. The derivation of Eq. (4.10) took into account the fact that the off-diagonal matrix elements (4.3) are zero.

The solution of the equations of motion (4.1), (4.6), and (4.10) can be represented as a power series in the small parameter $1/E$, where E is the energy of the incident particle.[14] Then we write the transverse component of the acceleration of the particle as follows:

$$\dot{\mathbf{v}}_{\perp}(t) = \langle\dot{\mathbf{v}}_{\perp}(t)\rangle + \delta\dot{\mathbf{v}}_{\perp}(t) , \qquad (4.11)$$

where

$$\langle\dot{\mathbf{v}}_{\perp}(t)\rangle = -\frac{e}{E}\langle\langle\mathscr{E}_{\perp}\rangle\rangle + O(E^{-2}) , \qquad (4.12)$$

$$\delta\dot{\mathbf{v}}_{\perp}(t) = -\frac{e}{E}\sum_{\alpha}\nabla_{\perp}\Phi(\mathbf{r}-\mathbf{r}_{\alpha})$$
$$+ \frac{e^2}{E}\sum_{\alpha}\sum_{n}\rho^{(S)}_{nn}(t)\int d\mathbf{r}'\psi^*_n(\mathbf{r}')\frac{(\mathbf{r}-\mathbf{r}'-\mathbf{r}_{\alpha})_{\perp}}{|\mathbf{r}-\mathbf{r}'-\mathbf{r}_{\alpha}|^3}\psi_n(\mathbf{r}')$$
$$+ O(E^{-2}) . \qquad (4.13)$$

The value of the radius vector of the unperturbed rectilinear motion $\mathbf{r} = \mathbf{r}_0 + \mathbf{v}_0 t$, where $\mathbf{r}_0$ and $\mathbf{v}_0$ are the initial coordinate and velocity of the particle, must be substituted on the right-hand side of the relation (4.13).

5. SPECTRAL DENSITY OF THE RADIATION

If over the coherence length the scattering angle of the particle is small compared with the characteristic emission angle m/E, then the formula for the spectral density of the radiation in the frequency interval $(\omega, \omega + d\omega)$, taking recoil into account in the quasiclassical approximation, can be written in the form[15]

$$\frac{dE}{d\omega} = \frac{e^2}{4\pi}\frac{E^2+E'^2}{EE'}\omega$$
$$\times\int_{p'}^{\infty}\frac{dp}{p^2}\left[1 - \frac{2p'}{p}\left(1-\frac{p'}{p}\right)\left(1-\frac{\omega^2}{E^2+E'^2}\right)\right]|\mathbf{W}(p)|^2, \qquad (5.1)$$

where $E' = E - \omega$, $p' = m^2\omega/2EE'$, and $\mathbf{W}(p)$ are the Fourier components of the transverse acceleration:

$$\mathbf{W}(p) = \int dt\frac{d\mathbf{v}_{\perp}}{dt}e^{ipt} . \qquad (5.2)$$

It is easily shown from Eq. (4.12) that the maximum angle of deflection in the case of motion in the field (4.8) is approximately $\theta_{\max} \approx 4\pi e^2 N a_0 \Lambda_{mn}/E$, where a_0 is the Bohr radius and $\Lambda_{mn} = 1/E_{mn}$. From the condition for the applicability of the formula (5.1) $\theta_{\max} \ll m/E$ it follows that the dipole approximation is valid if

$$\frac{4\pi}{137}Na_0\Lambda_{mn} \ll \frac{1}{\lambda_e},$$

where λ_e is the Compton wavelength of the electron. It is easy to see that for $E_{mn} \approx 1$ eV the last condition always holds with a large margin. In accordance with Eq. (4.11) we can write

$$\mathbf{W}(p) = \langle\mathbf{W}(p)\rangle + \delta\mathbf{W}(p) . \qquad (5.3)$$

Since averaging of $\delta\mathbf{W}(p)$ over the positions of the particles gives zero, it is easy to show that after averaging the spectral density (5.1) will have the form of a sum of two terms:

$$\frac{dE}{d\omega} = \frac{dE^{coh}}{d\omega} + \frac{dE^{incoh}}{d\omega}, \qquad (5.4)$$

where the first term on the right-hand side of Eq. (5.4) describes coherent radiation and is determined by the Fourier components $|\langle\mathbf{W}(p)\rangle|^2$ and the second term describes incoherent radiation and is determined by the Fourier components $\langle|\delta\mathbf{W}(p)|^2\rangle$. The theory of incoherent radiation in a medium is quite well developed (see, for example, Refs. 13–15), and for this reason in what follows we consider in detail only the coherent radiation.

Using the previously obtained solution of Eq. (2.4) for a two-level system (2.9) in the formulas (4.8), (4.12), and (5.2) we can find the average value of the Fourier components of the transverse acceleration of a relativistic electron moving in the field (4.8):

$$\langle\mathbf{W}(p)\rangle = \frac{4\pi^2 e^2 N}{E}(\rho_{11}-\rho_{22})\left[V_{21}\langle 1|\mathbf{r}_{\perp}|2\rangle\frac{\delta(p-\Omega)}{E_{21}-\Omega-i\gamma_{21}}\right.$$
$$\left. + V_{12}\langle 2|\mathbf{r}_{\perp}|1\rangle\frac{\delta(p+\Omega)}{E_{21}-\Omega-i\gamma^*_{21}}\right] . \qquad (5.5)$$

The spectral density of coherent radiation per unit time, according to Eqs. (5.1) and (5.5), is

$$\frac{dI^{coh}}{d\omega} = (2\pi)^2 |\langle 1|\mathbf{r}_\perp|2\rangle|^2$$

$$\frac{e^6 N^2 |B_{21}|^2}{m^2 E_{21}} \frac{E^2 + E'^2}{EE'} \frac{E'}{E} \frac{\omega}{\omega'} F(\omega), \tag{5.6}$$

or

$$\frac{dI^{coh}}{d\omega} = 2\pi^2 \Lambda_{21}^4 A_{21} r_e^2 N^2 |B_{21}|^2 \frac{E^2 + E'^2}{EE'} \frac{E'}{E} \frac{\omega}{\omega'} F(\omega),$$

where $\omega' = 2EE'E_{21}/m^2$, $A_{21} = 2e^2 E_{21}^3 |\langle 1|\mathbf{r}_\perp|2\rangle|^2$ is the probability of a spontaneous radiative transition,

$$F(\omega) = \left[1 - \frac{2\omega}{\omega'}\left(1 - \frac{\omega}{\omega'}\right)\left(1 - \frac{\omega^2}{E^2 + E'^2}\right)\right]\theta(\omega' - \omega),$$

and $\theta(x)$ is the Heaviside unit function. The coefficient $|B_{21}|$ is defined as follows:

$$B_{21} = \frac{V_{21}}{\mathrm{Re}\,\gamma_{21}}(\rho_{11} - \rho_{22}),$$

$$\text{or} \quad |B_{21}|^2 = \frac{2W}{|K_{11} + K_{22}|}(\rho_{11} - \rho_{22})^2. \tag{5.7}$$

The upper limit of the spectrum, according to Eq. (5.6), is determined by the relation

$$\omega \le \omega_{max} = \frac{2E^2}{m^2} E_{21} \frac{m^2}{m^2 + 2EE_{21}}, \tag{5.8}$$

and the maximum value of the spectral density (5.6) is reached at $\omega = \omega_{\max}$. The dependence of $|B_{21}|^2$ on the probability W of resonant pumping, calculated in an eight-level model with the previously chosen parameters of the plasma ($N = 10^{22}$ cm^{-3} and $T = 0.23$ eV) is presented in Fig. 1. For $W = 0$ the states 2 and 1 are incoherent and the off-diagonal matrix elements are identically zero. Therefore $|B_{21}|^2$ and together with it the spectral density of the coherent radiation (5.6) are zero. As $W \to \infty$ the populations of the states 2 and 1 become equal to one another (see Fig. 1) and, according to Eqs. (2.9) and (5.7), $|B_{21}|^2$ approaches zero. The maximum value of $|B_{21}|^2$ is reached for $W = 10^9$

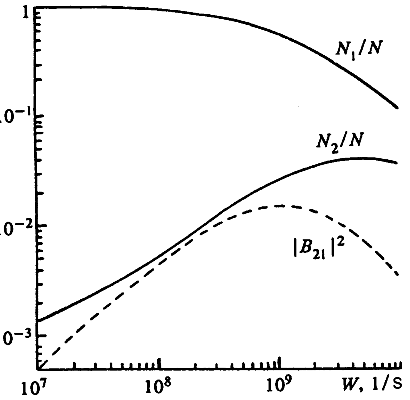

FIG. 1. Some elements of the density matrix of a dense nonequilibrium lithium plasma as a function of the probability of excitation of the $2P$–$2S$ transition by external resonance radiation ($N = 10^{22}$ cm^{-3}, $T = 0.23$ eV).

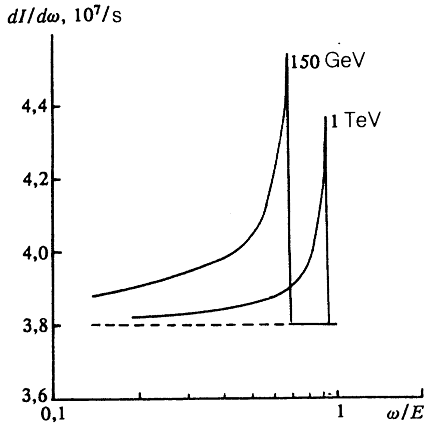

FIG. 2. Spectral density of coherent radiation together with the incoherent bremsstrahlung background with conversion of the $2P$–$2S$ transition in a dense lithium plasma.

s^{-1}, which corresponds to an external pump field of intensity $\sim 10^2$ V/cm (we note that the intensity of the field in state-of-the-art high-power lasers reaches values of $\sim 10^8$ V/cm).

The spectral density of the coherent radiation is presented in Fig. 2 together with the incoherent bremsstrahlung background. The integral photon yield in a narrow frequency interval near the maximum ($\omega_{\max} \approx 0.63E$) is

$$N_\gamma = \int_{0,9\omega_{max}}^{\omega_{max}} \frac{d\omega}{\omega} \frac{dI^{coh}}{d\omega} = 4{,}6 \cdot 10^6 \text{ (1/s, per electron)}.$$

For comparison we note that a somewhat lower photon yield (10^4–10^5 s^{-1}) was obtained in experiments on Compton backscattering[16] using a 10^3 MW argon laser.

6. POLARIZATION OF THE COHERENT RADIATION

We now estimate the polarization properties of this coherent radiation using the classical formula for the intensity of the electromagnetic field of radiation neglecting recoil:[17]

$$\mathbf{E}(p) = \frac{e}{R_0}\exp(ikR_0)\left(\frac{p}{q}\right)^2 [[\mathbf{u}(\mathbf{u} - \beta\mathbf{i})]\langle \mathbf{W}(q)\rangle],$$

where $q = p(1 - \beta\,\mathbf{u}\cdot\mathbf{i})$, $\mathbf{u}$ is the direction of propagation of the electromagnetic wave, $\mathbf{i}$ is the direction of motion of the relativistic particle, β is the relative velocity of the particle, and $\mathbf{k}$ is the wave vector of the emitted electromagnetic wave.

We choose the polarization vectors in the following form:

$$\mathbf{e}_1 = \frac{[\mathbf{ui}]}{\sqrt{1 - (\mathbf{ni})^2}}, \quad \mathbf{e}_2 = \frac{\mathbf{u}(\mathbf{ui}) - \mathbf{i}}{\sqrt{1 - (\mathbf{ui})^2}}.$$

Next we write the intensity of the electric field in the form

$$\mathbf{E}(p) = \mathbf{e}_1 E_1(p) + \mathbf{e}_2 E_2(p), \quad E_i(p) = \mathbf{e}_i \mathbf{E}(p).$$

It is obvious that in the case $\mathbf{u}\|\mathbf{i}$ the coherent radiation under consideration in the direction of motion of the particle is linearly polarized and the polarization vector lies in the

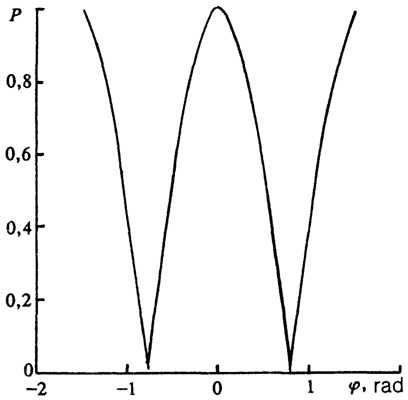

FIG. 3. Degree of linear polarization as a function of the azimuthal angle.

plane of the vectors $\langle \mathbf{r}_\perp \rangle$ and $\mathbf{i}$ (recall that the photons are emitted in a narrow cone of angles $\sim 1/\gamma$ around the direction of motion of the particle).

It is convenient to characterize the degree of linear polarization in the plane of the vectors $\langle \mathbf{r}_\perp \rangle$ and $\mathbf{i}$ by the following formula

$$P = \left| \frac{|E_1(p)|^2 - |E_2(p)|^2}{|E_1(p)|^2 + |E_2(p)|^2} \right| .$$

The dependence of P on the azimuthal angle φ can be calculated using the expression for $E_i(p)$ (see Fig. 3). Thus linearly polarized radiation with degree of polarization $P \approx 1$ can be obtained by collimating in the measurements the photon flux in the direction $\langle \mathbf{r}_\perp \rangle$ (or a direction orthogonal to this direction); it should be kept in mind, however, that the formula employed (in which recoil is neglected) could somewhat overestimate the degree of linear polarization of the radiation.

7. CONCLUSIONS

The analysis performed above shows that scattering of high-energy electrons in a nonequilibrium dense plasma can be effectively used to generate hard polarized radiation. Calculations performed for lithium plasma with density 10^{22} cm^{-3} at a temperature of 0.23 eV and subjected to steady resonant pumping with external laser radiation with field intensity 10^2 V/cm and frequency 1.838 eV reveal that the yield of polarized photons with energy $0.68E$ where $E = 150$ GeV, is 10^6–10^7 1/sec per electron. Further elaboration of the theory of bremsstrahlung of ultrarelativistic particles in nonequilibrium plasma and development of methods of optimal pumping of the plasma should make it possible to develop a new source of high-energy photons for investigating the structure of matter. A possible schematic diagram of such a source is presented in Fig. 4.

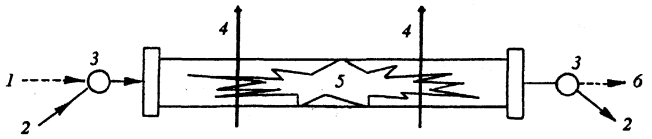

FIG. 4. Schematic diagram of a source of high-energy photons: *1*) flux of laser photons ($\lambda = 6730$ Å, $E = 10^2$ V/cm); *2*) high-energy electron beam (150 GeV); *3*) rotating magnet; *4*) magnetic field ($H = 1.56 \cdot 10^5$ Oe); *5*) dense lithium plasma ($N = 10^{22}$ cm^{-3}, $T = 0.23$ eV); *6*) flux of high-energy photons ($\omega = 0.68\,E$, $N_\gamma = 4.6 \cdot 10^6$ 1/sec).

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