Frequency conversion of squeezed light in a medium with a quadratic nonlinearity

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The propagation of squeezed light through a crystal with a quadratic nonlinearity is analyzed. Squeezed light with suppressed noise can be doubled in frequency or converted into a subharmonic with no changes in statistical properties. The analysis is carried out by the Langevin approach.

INTRODUCTION

The development of macroscopic sources of squeezed light[1,2] opens the door to experiments on the interaction of nonclassical electromagnetic fields with media. For example, the capabilities of a spectroscopy with a sensitivity better than the standard quantum limit, based on a tunable parametric generator of squeezed light, have been demonstrated.[3] A light source with a suitable spectral range is of interest for spectroscopic measurements, among others. In the present paper we discuss some conventional parametric systems in the role of frequency converters for squeezed light. We analyze the interaction of nonclassical states of the field in the examples of second-harmonic generation or frequency doubling and subharmonic generation or frequency division.

Parametric systems based on second-harmonic generation and subharmonic generation have been discussed widely in the literature.[6] These systems are usually regarded as sources of nonclassical fields in layouts with optical resonators.[7,8] Some of these subharmonic-generation systems have been implemented experimentally.[9,10] Experiments on second-harmonic generation were carried out in Refs. 11 and 12. Pereira *et al.*[11] detected light with sub-Poisson photon statistics at the fundamental frequency. This light exhibited a noise suppression 13% below the level of the standard quantum limit. Sizmann *et al.*[12] have detected light with a 40% suppression of the amplitude noise at the frequency of the second harmonic. It was shown in Refs. 13 and 14 that parametric resonator systems can be efficient converters of squeezed light.

In the present paper we examine a resonator-free version of harmonic generation, as shown schematically in Fig. 1. This would appear to be the simplest optical system from the experimental standpoint. We are particularly interested in situations with a high conversion coefficient and a pronounced pump depletion. Analysis shows that just in such situations is squeezed light converted from one frequency to another without a disruption of the nonclassical statistics. Because of certain physical aspects of harmonic-generation processes, light which is squeezed in amplitude is converted efficiently during second-harmonic generation, while light which is squeezed in phase is converted efficiently in the course of subharmonic generation. There is the interesting possibility of converting amplitude-squeezed light in the course of subharmonic generation. A mechanism operates to smooth out phase fluctuations in this case, while amplitude fluctuations or intensity fluctuations simply grow. The subharmonic light at the exit from the system is nevertheless in an amplitude-squeezed state if this was the state of the pump wave at the entrance.

The characteristics of the fields in these situations cannot be calculated through a small number of iterations of the equations of motion, as is customary in efforts to analyze resonator-free schemes.[4,15] For our description we adopt the transport theory developed by Golubev[16] to solve the space-time problem of the interaction of a quantized field with a medium. In Sec. 1 of this paper we use that equation to derive a starting equation for the density matrix of the electromagnetic field. To calculate observables we take the Langevin approach, which was formulated in Ref. 5 for space-time problems. The Langevin equations are given in Sec. 2. In Sec. 3 we determine the harmonic-generation regimes for a long crystal, in which high conversion coefficients can be achieved. In Secs. 4 and 5 we calculate the correlation functions for the intensity and phase fluctuations. We also calculate the noise spectra in the approximation of small fluctuations. The conversion of squeezed light in a long crystal is analyzed in Sec. 6 for the second-harmonic-generation case and in Secs. 7 and 8 for the subharmonic-generation case.

1. INITIAL EQUATIONS

Figure 1 shows the optical layout for harmonic generation. Light with frequencies ω_1 and ω_2 and intensities I_{1_0} and I_{2_0} is incident on the transparent nonlinear crystal. After passing through a filter with a bandwidth $\Delta\omega$ at the frequencies ω_1 and ω_2, the light reaches a measurement system. Here the light is received by a homodyne or heterodyne method, and the the photocurrent spectrum or the noise spectrum $i^{(2)}_{1,2}(z,\Omega)$ is measured.

To describe the propagation of the light in the medium we use a transport theory in which the evolution of the statistical properties can be found from the space-time equation for the density matrix of the electromagnetic field:[16]

$$(\partial_t + V\partial_z)\rho = L(a^+,a)\rho. \qquad (1)$$

The right side of this equation is determined by the interaction with the medium. The operators a^+ and a are local operators which create and annihilate photons at the point

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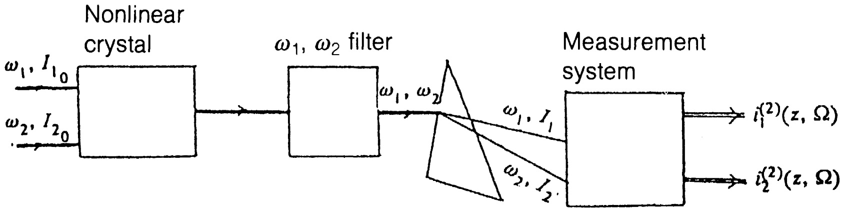

FIG. 1. Optical layout.

z of the coarse spatial scale, with a length scale l of the spatial cell. This localization arises if the operators a^+ and a are defined as packets of modes of the normalization volume with wave vectors in the interval $\Delta k = 2\pi l^{-1}$. Such packets have a spectral width $\Delta\omega = c2\pi l^{-1}$.

If a parametric interaction of light with a medium is modeled by a standard effective Hamiltonian, the right side of (1) takes the usual form

$$(\partial_t + V\partial_z)\rho = i\hbar^{-1}[H,\rho]$$

$$H = i\hbar\frac{\chi}{2}a_1^2 a_2^+ + \text{H.a.} \qquad (2)$$

Here χ is the nonlinearity of the medium, and a_1 and a_2 are boson operators of modes [of wave packets with frequencies ω_1 and ω_2] and wave vectors $k(\omega_1)$ and $k(\omega_2)$. In Eq. (2) we are assuming that the frequencies and wave vectors are related by

$$\omega_1 + \omega_1 - \omega_2 = 0,$$

$$k(\omega_1) + k(\omega_1) - k(\omega_2) = \Delta k = 0.$$

The quantity V in Eq. (2) is the mode propagation velocity.

When there is a wave detuning, the characteristics of the fields at the exit acquire a factor[17,18]

$$\frac{\sin(\Delta k l_a/2)}{\Delta k l_a/2},$$

where l_a is the length of the crystal. For large detunings ($\Delta k l_a \gg 1$) the generation process is thus inefficient. By setting $\Delta k = 0$ we are implicitly assuming $\Delta k l_a \ll 1$.

Our starting equation, (2), describes the interaction of only two modes. It is not intuitively obvious that these two modes will be singled out in this resonator-free layout. Furthermore, in the case of subharmonic generation a continuum of modes arises at the frequencies $\omega_1 + \varepsilon$, $\omega_1 - \varepsilon$, and $\omega_2 = (\omega_1 + \varepsilon) + (\omega_1 - \varepsilon)$. These modes interact in the nonlinear medium. These modes may have a substantial effect if their wave detunings, which can be written in the form

$$k(\omega_1 + \varepsilon) + k(\omega_1 - \varepsilon) - k(\omega_2) \approx \Delta k + \frac{\partial^2 k}{\partial \varepsilon^2}\varepsilon^2,$$

are small. In this case the relation

$$\frac{\partial^2 k}{\partial \varepsilon^2}\varepsilon^2 l_a \ll 1$$

determines the frequency band $\delta\varepsilon$ in which all modes with frequencies $\omega_1 \pm \varepsilon$, $\varepsilon \lesssim \delta\varepsilon$, must be taken into consideration in the description of the interaction. We thus need a multimode model.

However, our modes at ω_1 and ω_2 are wave packets of width $\Delta\omega = 2\pi c/L$. We set $\Delta\omega = \delta\varepsilon$. The description of harmonic generation by Eq. (2) then becomes valid again.

Using a diagonal representation of the intensity matrix, we write Eq. (2) for the Glauber quasiprobability:

$$(\partial_t + V\partial_z)P(\alpha_1,\alpha_2;z,t)$$

$$= \left\{\frac{\partial}{\partial\alpha_1}\Lambda_1 + \frac{\partial^2}{\partial\alpha_1^2}\Lambda_2 + \frac{\partial}{\partial\alpha_2}\Lambda_3\right\}P(\alpha_1,\alpha_2;z,t) + \text{c.c.} \qquad (3)$$

$$\Lambda_1 = \chi\alpha_1^*\alpha_2, \quad \Lambda_2 = \tfrac{1}{2}\chi\alpha_2, \quad \Lambda_3 = -\tfrac{1}{2}\chi\alpha_1^2.$$

Specifying the statistics of the light at the entrance to the medium by means of (3), we can determine all the field characteristics at the exit, in particular, the noise spectrum $i_{1,2}^{(2)}(z,\Omega)$. The basic approximation which we will use is our condition that the fluctuations are small. Introducing the polar coordinates $\alpha_s = \sqrt{u_s}\exp\{i\varphi_s\}$, $s = 1,2$, we thus assume that the fluctuations of the intensity ε_s and of the difference phase μ around their semiclassical values I_s and Ψ are small:

$$u_s = I_s(z,t) + \varepsilon_s, \quad \varepsilon_s \ll I_s(z,t),$$
$$2\varphi_1 - \varphi_2 = \Psi(z,t) + \mu, \quad \mu \ll \Psi(z,t). \qquad (4)$$

Condition (4) makes it possible to simplify Eq. (3) by linearizing it.

2. LANGEVIN EQUATIONS

For the dimensionless intensities I_s and difference phase Ψ we find from (3) a familiar systems of equations:

$$(\partial_t + V\partial_z)I_1 = 2\chi I_1\sqrt{I_2}\cos\Psi,$$

$$(\partial_t + V\partial_z)I_2 = -\chi I_1\sqrt{I_2}\cos\Psi, \qquad (5)$$

$$(\partial_t + V\partial_z)\Psi = \frac{\chi}{2}\left(\frac{I_1}{\sqrt{I_2}} - 4\sqrt{I_2}\right)\sin\Psi.$$

These equations have the two integrals

$$I_1+2I_2=I_{1_0}+2I_{2_0},$$
$$I_1\sqrt{I_2}\sin\Psi=I_{1_0}\sqrt{I_{2_0}}\sin\Psi_0. \tag{6}$$

For simplicity we are assuming that the intensities at the entrance, I_{s_0}, $s=1, 2$, and the difference phase Ψ_0 are independent of t.

We restrict the analysis to the case

$$C_0=\cos\Psi_0=\pm 1.$$

It can be seen from (6) that this phase difference does not change as the light propagates through a medium if the intensities are nonzero. The amplitude and phase fluctuations become independent and can be treated separately.

Introducing the variables $z'=z$, $\theta=t-z/V$, and switching to the new dimensionless coordinate

$$y=-\frac{\chi}{2V}\int_0^z dz' I_1(z')\sqrt{I_2(z')},$$

we can put the linearized equation for the quasiprobability in the form

$$\partial_y P=\left\{2C_0\left(2\frac{\partial}{\partial\varepsilon_1}-\frac{\partial}{\partial\varepsilon_2}\right)\left(\frac{\varepsilon_1}{I_1}+\frac{1}{2}\frac{\varepsilon_2}{I_2}\right)-2C_0\frac{\partial^2}{\partial\varepsilon_1^2}\right.$$
$$-C_0\left[\left(\frac{4}{I_1}-\frac{1}{I_2}\right)\frac{\partial}{\partial\mu}+\left(\frac{2B}{I_1}+\frac{A}{I_2}\right)\frac{\partial}{\partial\varphi}\right]\mu$$
$$\left.+\frac{1}{2}C_0\left(4\frac{\partial^2}{\partial\mu^2}+B^2\frac{\partial^2}{\partial\varphi^2}+4B\frac{\partial^2}{\partial\varphi\partial\mu}\right)\right\}P. \tag{7}$$

Here the phase variable $\varphi=A\varphi_2+B\varphi_1$ is identical to φ_1 if $A=0$ and $B=1$, while it is identical to φ_2 if $A=1$ and $B=0$. The small-fluctuation condition is not necessary for the phases φ_1 and φ_2 separately.

Diffusion matrix (7) is not positive definite. We can nevertheless write corresponding Langevin equations, as was shown in Ref. 19. In the case at hand, these equations are

$$\partial_y\begin{pmatrix}\varepsilon_1\\ \varepsilon_2\end{pmatrix}=A_I\begin{pmatrix}\varepsilon_1\\ \varepsilon_2\end{pmatrix}+D_I^{1/2}\eta(y,\theta),$$
$$\partial_y\begin{pmatrix}\mu\\ \varphi\end{pmatrix}=A_\varphi\begin{pmatrix}\mu\\ \varphi\end{pmatrix}+D_\varphi^{1/2}\eta(y,\theta), \tag{8}$$

where the matrices A_I and A_φ are defined by

$$A_I=\begin{bmatrix}-4\dfrac{C_0}{I_1}; & -2\dfrac{C_0}{I_2}\\ 2\dfrac{C_0}{I_1}; & \dfrac{C_0}{I_2}\end{bmatrix},$$

$$A_\varphi=\begin{bmatrix}C_0\left(\dfrac{4}{I_1}-1\right); & 0\\ C_0\left(\dfrac{2A}{I_1}+\dfrac{B}{I_2}\right); & 0\end{bmatrix},$$

and the nonvanishing elements of diffusion matrices D_I and D_φ are

$$D_{\varepsilon_1\varepsilon_1}=-4C_0,\qquad D_{\mu\mu}=4C_0/I_1^2,$$
$$D_{\varphi\varphi}=B^2C_0/I_1^2,\qquad D_{\mu\varphi}=D_{\varphi\mu}=2BC_0/I_1^2.$$

Since we are examining the evolution of light as it propagates, there are some features in the formulation of the Langevin approach which stem from the specification of the random sources $\eta(y,\theta)$. As was shown in Ref. 5, the correlation functions of the random sources should be taken in the form

$$\langle\eta(y,\theta)\eta(y',\theta')\rangle=\delta(y-y')l\delta_l(\theta-\theta'),$$
$$\langle\eta(0,\theta)\eta(y,\theta)\rangle=0,\quad y\neq 0. \tag{9}$$

The quantity $l\delta_l(\tau)$ in (9) represents a large-scale dimensionless δ-function; it is introduced in the transport theory by the relation[16] $l\delta_l(\tau)=\sin x/x$, $x=2\tau c/l$.

The matrices A and D in the Langevin equations [and, correspondingly, the coefficients in (7)] depend on the coordinates through dimensionless intensities. In an analysis of time evolution these quantities are constants. Explicit expressions for $I_s(z)$ can be found from system (5), which takes the following simple form in the case at hand:

$$\partial_y I_1=-4C_0,$$
$$\partial_y I_2=2C_0.$$

3. GENERATION REGIMES

If the conditions

$$I_{1_0}\gg I_{2_0},\quad C_0=1, \tag{10}$$

hold for the light intensities and the difference phase at the entrance, then the intensity of the wave at the frequency ω_2 will grow in the medium. This is second-harmonic generation. Under the conditions

$$I_{1_0}\ll I_{2_0},\quad C_0=-1, \tag{11}$$

subharmonic generation arises, and the intensity of the wave at ω_1 increases. To describe these situations we introduce the parameters

$$p_0=(1+I_{1_0}/2I_{2_0})^{-1},$$
$$p=(1+I_1/2I_2)^{-1}, \tag{12}$$
$$p=p_0I_2/I_{2_0},$$

where the intensities $I_{1,2}=I_{1,2}(z)$ are determined by the solution of (5). The quantities I_s in (12) can be expressed in terms of the dimensional intensities or power levels of the fields. The parameters p_0 and p then take on a simple meaning and can easily be found from experimental data. In the case of second-harmonic generation, for example, the quantity p is the ordinary coefficient of conversion into the second harmonic in terms of the intensity or the power.

We are interested primarily in situations in which the conversion coefficients are large, and the depletion of the pump wave must be taken into account. We denote by z_∞ the length of a medium which is such that the conversion coefficients are large. The length z_∞ depends primarily on the geometric length; it also depends on the nonlinearity of the crystal and the entrance intensities. This length can be found from the condition

$$I_2(z_\infty) \gg I_1(z_\infty) \quad (13)$$

in the case of second-harmonic generation or from the condition

$$I_1(z_\infty) \gg I_2(z_\infty) \quad (14)$$

in the case of subharmonic generation. We say that a crystal of length z_∞ is "long." Let us formulate conditions (10) and (11) for a long crystal. In the case of second-harmonic generation, for example, we have

$$p_0 \ll 1, \quad p \lesssim 1, \quad C_0 = 1. \quad (15)$$

In the case of subharmonic generation we have

$$p_0 \lesssim 1, \quad p \ll 1, \quad C_0 = -1. \quad (16)$$

The situations described by (15) and (16) are known[18] to be stable, by virtue of the boundary conditions which have been selected. These situations are moreover easy to realize experimentally. For the case of second-harmonic generation, for example, the crystal is illuminated by only the light at the frequency ω_2. Light at the frequency of the second harmonic arises in the medium, since the difference phase with $C_0 = 1$ is stable.

4. CALCULATION OF NOISE SPECTRA

We introduce normally ordered correlation functions of the intensity fluctuations,

$$E_s(z,\tau) = I_s^{-1}(z) \langle \varepsilon_s(z,t) \varepsilon_s(z,t+\tau) \rangle, \quad (17)$$

and of the phase fluctuations,

$$\Phi_s(z,\tau) = 4 I_s(z) \langle \varphi_s(z,t) \varphi_s(z,t+\tau) \rangle, \quad (18)$$

where $s = 1, 2$ for the waves at ω_1, ω_2. If the fluctuations are small, the measured photocurrent spectra or noise spectra $i^{(2)}(z,\Omega)$ in the case of direct (homodyne) detection are [see (17)]

$$i_s^{(2)}(z,\Omega) = 1 + q l^{-1} \int_{-\infty}^{\infty} d\tau \exp\{i\Omega\tau\} E_s(z,\tau). \quad (19)$$

In the case of heterodyne detection we have

$$i_s^{(2)}(z,\Omega) = 1 + \tilde{q} l^{-1} \int_{-\infty}^{\infty} d\tau \exp\{i\Omega\tau\} \Phi_s(z,\tau). \quad (20)$$

Here q and $\tilde{q}$ are the quantum efficiencies of the photodetectors; the normalization for $i^{(2)}(z,\Omega)$ has been chosen in such a way that the level of the shot noise or the quantum limit corresponds to one.

To calculate the correlation functions we can use the Langevin equations. Here we need to specify the statistics of the light at a boundary, i.e., the quantities $E_s(0,\tau)$ and $\Phi_s(0,\tau)$, which are determined by the sources. In order to analyze the physical states of the field, we restrict the discussion to resonator sources which emit light in a phase- or amplitude-squeezed state. There are some well-known theoretical models[20,21] for such sources; several of these models have been realized experimentally.[1,2] For such sources we have

$$E(0,\tau) = \xi(0) \exp(-\Gamma |\tau|),$$

where the parameter $\xi(0)$ of the statistics at the boundary of the medium, is related by $\xi(0) = Cl\xi$ to the intraresonator parameter ξ, which is usually involved in theoretical descriptions.[6] Here C is the resonator width, and Γ is the width of the spectrum of amplitude fluctuations. Here we have $\xi \geqslant -1/2$. In the case $\xi < 0$, the light from such a source is squeezed in amplitude and has a sub-Poissonian photon statistics. Its noise spectrum is

$$i^{(2)}(0,\Omega) = 1 + 2q\xi \frac{C}{\Gamma} \frac{\Gamma^2}{\Gamma^2 + \Omega^2}. \quad (21)$$

In the case $\xi = -1/2$ and $C = \Gamma$, there is almost no noise at all in the low-frequency region ($\Omega = 0$).

We take the correlation function $\Phi(0,\tau)$ in the form

$$\Phi(0,\tau) = \zeta(0) \exp(-\gamma |\tau|), \qquad \zeta(0) = Cl\zeta. \quad (22)$$

This approach corresponds to a source with small phase fluctuations. The meaning of the quantities $\zeta(0)$ and ζ is analogous to that of $\xi(0)$ and ξ. For our model we have $\zeta \geqslant -1/2$. The case $\zeta < 0$ corresponds to a phase-squeezed state with a noise spectrum as in (21):

$$i^{(2)}(0,\Omega) = 1 + 2\tilde{q}\zeta \frac{C}{\gamma} \frac{\gamma^2}{\gamma^2 + \Omega^2}.$$

Here γ is the spectral width of the phase fluctuations.

Solving the Langevin equations, we find that the correlation functions at the exit from the medium can be written in the form

$$\begin{aligned} E_s(z,\tau) &= D_s(z) l \delta_l(\tau) + W_{s_1}(z) E_1(0,\tau) + W_{s_2}(z) E_2(0,\tau), \\ \Phi_s(z,\tau) &= F_s(z) l \delta_l(\tau) + U_{s_1}(z) \Phi_1(0,\tau) + U_{s_2}(z) \Phi_2(0,\tau), \end{aligned} \quad (23)$$

where all the entrance cross-correlation functions have been set equal to zero. The propagators W and U in (23) describe the conversion of the incoherent component of the light. For coherent light we have $\Phi(0,\tau) = E(0,\tau) = 0$. For the light at the frequency ω_1 we have

$$W_{11} = (1-p_0)(1-p)[1 + \tfrac{1}{2}\sqrt{p} \ln K + \sqrt{pp_0}(1-p_0)^{-1}]^2,$$

$$W_{12} = 2(1-p)[\sqrt{p_0} - \sqrt{p} + \tfrac{1}{2}\sqrt{pp_0} \ln K]^2,$$

$$U_{11} = (1-p)(1-p_0)[1 - \tfrac{1}{2}\sqrt{p_0} \ln K + \sqrt{pp_0}(1-p)^{-1}]^2,$$

$$U_{12} = \tfrac{1}{2}(1-p)[\tfrac{1}{2}(1-p_0) \ln K + \sqrt{p_0}$$

$$-(1-p_0)\sqrt{p}(1-p)^{-1}]^2,$$

where p_0 and p are defined in (12), and

$$K=(1-\sqrt{p})(1-\sqrt{p_0})[(1+\sqrt{p})(1+\sqrt{p_0})]^{-1}.$$

At the entrance to the crystal, i.e., at $z=0$, we have $p=p_0$, $W_{12}=U_{12}=0$, and $W_{11}=U_{11}=1$. For the light at the frequency ω_2 we have

$$W_{21}=\tfrac{1}{8}(1-p_0)(1-p)^2[\ln K-2\sqrt{p}(1-p)^{-1}$$
$$+2\sqrt{p_0}(1-p_0)^{-1}]^2,$$
$$W_{22}=[1-p+\sqrt{pp_0}-\tfrac{1}{2}(1-p)\sqrt{p_0}\ln K]^2,$$
$$U_{21}=\tfrac{1}{2}p_0(1-p_0)[\sqrt{pp_0}\ln K+2\sqrt{p_0}-2\sqrt{p}]^2,$$
$$U_{22}=pp_0^{-1}\left[1+\tfrac{1}{2}\sqrt{p_0}(1-p_0)\left(\ln K+\frac{2}{\sqrt{p}}-\frac{2}{\sqrt{p_0}}\right)\right]^2.$$

Note that these equations were derived from the Langevin equations in which the random sources were set equal to zero. In other words, only a semiclassical approach is required for calculating them.

The propagators D and F in (23) represent the noise due to the medium itself; it is δ-correlated in time. To calculate this noise, we must use quantum theory. For the light at ω_1, for example, we find

$$D_1=-p+(1-p)[(\sqrt{p}-\sqrt{p_0})^2+p(1-p_0)^{-1}$$
$$+p(p_0+1)\tfrac{1}{4}\ln^2 K+\sqrt{p}(1+p_0-\sqrt{pp_0})\ln K],$$
$$F_1=-1+\tfrac{1}{8}(1-p)(1-p_0)^2\ln K-\tfrac{1}{2}(1-p_0)$$
$$\times[(1+p_0)\sqrt{p}+(1-p)\sqrt{p_0}]\ln K+(1-p_0)$$
$$\times(1-p+\sqrt{pp_0})+\tfrac{1}{2}p_0(1-p-p_0)+\frac{1}{2}\frac{p-p_0^2}{1-p}.$$

For the light at ω_2 we find

$$D_2=-\tfrac{1}{2}p(1-p_0)+(1-p)\{\sqrt{pp_0}-\tfrac{1}{2}(1-p)p_0$$
$$\times(1-p_0)^{-1}-(p-p_0)(1-p_0)^{-1}+\tfrac{1}{8}(1-p)$$
$$\times(1+p_0)\ln^2 K-\tfrac{1}{2}[\sqrt{p_0}(1-p)$$
$$+\sqrt{p}(1+p_0)]\ln K\},$$
$$F_2=\tfrac{1}{4}p(1-p_0^2)\ln^2 K+\sqrt{p}(1-p_0)[1+p_0-\sqrt{pp_0}]\ln K$$
$$+p_0(p-p_0)-2\sqrt{p}(1-p_0)(\sqrt{p_0}-\sqrt{p}).$$

These expressions are valid for any medium, in particular, a long crystal. The basic approximation here is the requirement that the fluctuations be small.

5. FLUCTUATIONS OF THE DIFFERENCE PHASE

At the exit from the medium, the correlation function of the difference phase,

$$M(z,\tau)=\langle\mu(z,t)\mu(z,t+\tau)\rangle,$$

is given by an expression like (23):

$$M(z,\tau)=R(z)l\delta_l(\tau)+R_0(z)M(0,\tau),$$

where

$$R_0=p_0(1-p_0)^2[p(1-p)^2]^{-1}$$

is found from the solution of the semiclassical problem, while the propagator

$$R=\tfrac{1}{2}(p^2-p_0^2)[p(1-p)^2(I_{1_0}+2I_{2_0})]^{-1}$$

is found from a quantum analysis.

If the harmonic generation is to be a stable process, the fluctuations in the medium must remain small: $R_0(z)\ll 1$, $|R(z)|\ll 1$. The situation with $p=0$ or $p=1$ is a critical one. This case cannot be dealt with under the approximations which we have been using. Analysis shows that the inequalities which are necessary can be satisfied for the cases of generation in a long crystal described by (17) and (18) in Sec. 3 of this paper. A necessary condition here is that the dimensionless intensities at the entrance be large:

$$I_{1_0}+2I_{2_0}\gg 1. \tag{24}$$

Analysis of the correlation function $M(z,\tau)$ with $M(0,\tau)=0$ shows that it may be negative in the case $p<p_0$. The meaning here is that the difference phase and the phases of the fields are squeezed separately in the case of subharmonic generation.

6. CONVERSION OF AMPLITUDE-SQUEEZED LIGHT TO THE DOUBLED FREQUENCY

Let us examine second-harmonic generation in a long crystal. Setting $p_0\approx 0$ and $p\approx 1$ (in the sense that we have $p_0\ll 1$ and $p_0\ll p<1$), we find from (23)

$$\begin{aligned}E_1(z_\infty,\tau)&=-l\delta_l(\tau),\\ E_2(z_\infty,\tau)&=-\tfrac{1}{2}l\delta_l(\tau)+\tfrac{1}{2}E_1(0,\tau).\end{aligned} \tag{25}$$

The fluctuations at the exit will be small if

$$|E_1(z_\infty,\tau)|\ll I_1(z_\infty)$$

or

$$1\ll I_1(z_\infty)\approx I_{1_0}(1-p).$$

This condition is satisfied if the dimensionless intensity at the entrance is large, in accordance with (24): $I_{1_0}\gg(1-p)^{-1}$. The noise spectra corresponding to (25) for the waves at ω_1 and ω_2 are

$$\begin{aligned}i_1^{(2)}(z_\infty,\Omega)&=1-q,\\ i_2^{(2)}(z_\infty,\Omega)&=1+q\left(-\frac{1}{2}+\xi_1\frac{C_1}{\Gamma_1}\frac{\Gamma_1^2}{\Gamma_1^2+\Omega^2}\right),\end{aligned} \tag{26}$$

where the noise spectrum of the fundamental light (that at the frequency ω_1) at the entrance to the medium has been taken in the form in (21). If we assume that the fundamental light at the entrance was in an amplitude-squeezed state with suppressed noise, i.e., if we assume $\xi_1=-1/2$ and $C_1=\Gamma_1$, then the expression for $i_2^{(2)}(z_\infty,\Omega)$ becomes

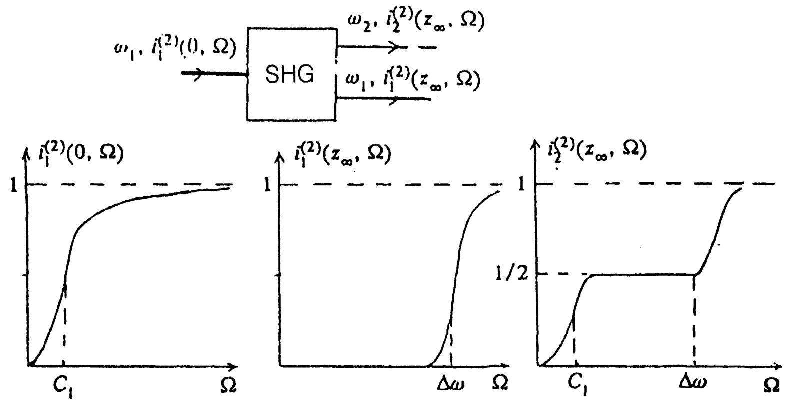

FIG. 2. Conversion of amplitude-squeezed light in the course of second-harmonic generation (SHG).

$$i_2^{(2)}(z_\infty,\Omega)=1-q. \tag{27}$$

Figure 2 shows the noise spectra for this case.

Two circumstances associated with these results deserve comment. First, the fundamental light at the exit is squeezed regardless of its statistics at the entrance. That this is true becomes obvious when we note that this light undergoes an absorption as it propagates through the medium. For ideal photodetection ($q=1$), we conclude from (26) and (27) that the noise of the fundamental light and that of the second harmonic are completely suppressed over the broad frequency band $\Delta\omega$. The band $\Delta\omega$ may be considerably broader than the width of the noise spectrum at the entrance, which is determined by Γ_1. Second, squeezing in the second harmonic arises not only because of the nonlinear medium but also because of the squeezing of the fundamental light. If follows from (27) that squeezed light with suppressed noise in a long crystal is converted to the doubled frequency without a disruption of the statistics. At the exit from a long crystal the fundamental light is of low intensity and is squeezed. This state is called an "amplitude-squeezed vacuum." A squeezed vacuum is of interest from the practical standpoint, since its use in a system of heterodyne detection increases the sensitivity of the photodetection.

7. CONVERSION OF PHASE-SQUEEZED LIGHT TO THE HALVED FREQUENCY

Let us examine subharmonic generation. For a long crystal, under the conditions $p_0\approx 1$ and $p\approx 0$, i.e., under the assumptions $p\ll 1$ and $p\ll p_0<1$, we find from (23)

$$\Phi_1(z_\infty,\tau)=-\tfrac{1}{2}\,l\delta_l(\tau)+\tfrac{1}{2}\,\Phi_2(0,\tau),$$
$$\Phi_2(z_\infty,\tau)=-l\delta_l(\tau). \tag{28}$$

The condition that the fluctuations be small, which is required in this case so that the noise spectrum can be written as in (20), reduces to the requirement

$$|\Phi_1(z_\infty,\tau)|\ll I_1(z_\infty)\approx 2I_{2_0}(1-p),$$

$$|\Phi_2(z_\infty,\tau)|\ll I_2(z_\infty)\approx I_{2_0}p.$$

These inequalities hold if the dimensionless intensities at the entrance are again assumed to be large in accordance with (24). For the light at ω_2, with the entrance correlation function as in (22), the noise spectra in the case of heterodyne detection at the exit from a long crystal are

$$i_2^{(2)}(z_\infty,\Omega)=1-\tilde{q},$$
$$i_1^{(2)}(z_\infty,\Omega)=1+\tilde{q}\left(-\frac{1}{2}+\zeta_2\frac{C_2}{\gamma_2}\frac{\gamma_2^2}{\gamma_2^2+\Omega^2}\right). \tag{29}$$

Expressions (29) have the same structure as the noise spectra in the course of second-harmonic generation according to (26). The conversion of phase squeezing therefore proceeds in a manner analogous to the conversion of amplitude squeezing in the case of second-harmonic generation. In a long crystal, phase-squeezed light with suppressed noise is thus converted to the subharmonic frequency without a disruption of the statistics. At the frequency ω_1, a state with a small dimensionless intensity arises at the exit from a long crystal. This is a phase-squeezed state: a squeezed vacuum.

8. CONVERSION OF AMPLITUDE-SQUEEZED LIGHT TO A SUBHARMONIC FREQUENCY

Under conditions corresponding to subharmonic generation, a physical mechanism operates to smooth out phase fluctuations. This mechanism stems from nonlinear phase locking. In this case, intensity fluctuations should grow. For subharmonic generation in a long crystal we find the following expressions for the correlation functions of the intensity fluctuations from (23):

$$E_1(z_\infty,\tau)=l\delta_l(\tau)+2E_2(0,\tau),$$
$$E_2(z_\infty,\tau)=\tfrac{1}{2}\,(1-p_0)^{-1}l\delta_l(\tau)+\tfrac{1}{4}\,E_2(0,\tau)\ln^2K. \tag{30}$$

The requirement that the fluctuations be small, which in the case at hand means

$$|E_2(0,\tau)|\ll I_2(z_\infty)\approx I_{2_0}p,$$

can be satisfied if we require, in accordance with (24), that the dimensionless intensity at the entrance, I_{2_0}, be large: $I_{2_0}\gg[p(1-p_0)]^{-1}$. It follows from (25) and (26) that, for example, in the case $E_2(0,\tau)=0$, with coherent light incident at the entrance, the crystal simply increases the intensity fluctuations. However, the fluctuations at the exit depend on the statistics at the entrance. Specifying the entrance statistics as in (21), we find, for the light at ω_1,

$$i_1^{(2)}(z_\infty,\Omega)=1+q\left(1+4\xi_2\frac{C_2}{\Gamma_2}\frac{\Gamma_2^2}{\Gamma_2^2+\Omega^2}\right). \tag{31}$$

It follows from (31) that if the pump light at the entrance is amplitude-squeezed with suppressed noise in the low-

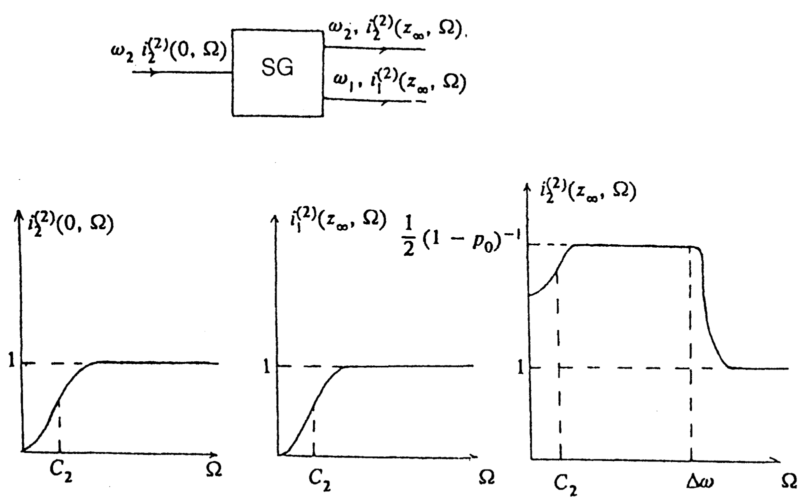

FIG. 3. Conversion of amplitude-squeezed light during subharmonic generation (SG).

frequency region, i.e., if $\xi_2=-1/2$ and $C_2=\Gamma_2$, then the subharmonic light has the same properties in the frequency band $\Omega \ll C_2$:

$$i_1^{(2)}(z_\infty,\Omega)=1+q\left(1-\frac{C_2^2}{C_2^2+\Omega^2}\right).$$

Outside this band, $\Omega \gg C_2$, however, the noise is nearly twice as high as the shot noise. The reason is that the crystal generates a high intensity noise, which is "white" in the $\Delta\omega$ observation band according to (30). The nature of the spectra in this case is illustrated in Fig. 3.

In the case of subharmonic generation, light which is amplitude-squeezed with suppressed noise is converted to the halved frequency, with no changes in the properties of the noise in the low-frequency region. This result follows from simply an analysis of the process by which the light propagates through the medium.

Let us consider the uncertainty relations for the case of the conversion of amplitude-squeezed light under subharmonic-generation conditions. We introduce the usual quadrature operators

$$X_1=a^+ +a,$$

$$X_2=i(a^+ -a),$$

for which the uncertainty relations

$$\langle(\Delta X_1)^2\rangle\langle(\Delta X_2)^2\rangle \geqslant 1 \quad (32)$$

hold. Here

$$\langle(\Delta X)^2\rangle=\langle X^2\rangle-\langle X\rangle^2=\langle(\Delta X)^2\rangle_N+1,$$

and the index N corresponds to a normally ordered operator. Under the assumption that the fluctuations are small, the normal variances of the quadratures can easily be related to the correlation functions of the intensity and the phase which were introduced in Sec. 4:

$$\langle(\Delta X_1)^2\rangle_N=E(\tau=0),$$

$$\langle(\Delta X_2)^2\rangle_N=\Phi(\tau=0).$$

As a result, (32) becomes

$$(1+E_1(z,0))(1+\Phi_1(z,0))\geqslant 1, \quad (33)$$

where the subscript 1 means that the correlation functions correspond to the light at ω_1. From (30) and (28) we have

$$E_1(z_\infty,0)=1+2E_2(0,0),$$

$$\Phi_1(z_\infty,0)=-\tfrac{1}{2}+\tfrac{1}{2}\Phi_2(0,0).$$

We then find

$$(1+E_1(z_\infty,0))(1+\Phi_1(z_\infty,0))$$

$$=(1+E_2(0,0))(1+\Phi_2(0,0)).$$

The product of uncertainties at the entrance for the light at ω_2 is thus reproduced precisely at the exit from a long crystal for the light at ω_1.

CONCLUSION

Transport theory has been used to analyze resonator-free parametric systems in the role of frequency converters of nonclassical states of light. We have focused on second-harmonic generation and subharmonic generation with high conversion coefficients. These cases cannot be analyzed by the customary iterative calculation methods. The results derived here demonstrate the promising outlook for the use of optical systems of this sort for frequency conversion of squeezed light.

Amplitude-squeezed light with suppressed noise in the low-frequency region can be converted to the doubled frequency under conditions corresponding to second-harmonic generation, or to the halved frequency under conditions corresponding to subharmonic generation, with no changes in properties.

Phase-squeezed light can be converted in the same way, but only to the halved frequency in subharmonic generation.

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