Excitation of atoms by electrons in a strong optical field

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The excitation of atoms by electrons in a strong optical field is studied taking into account virtual photon absorption by the target. A method for calculating the excitation cross sections, based on averaging the instantaneous rate of the process over the period of the external field, is developed in the low-frequency approximation. A generalized Bethe–Born formula in the form of an expansion in powers of the strength of the external field is derived for weak fields whose frequency is comparable to the transition frequency of the atom.

1. INTRODUCTION

The excitation of atoms and ions in collisions with fast electrons in a laser field is being intensively studied in connection with different laser-plasma problems, including the study of laser-induced sparks[1] and of mechanisms for producing population inversion in gases. This problem has two significant aspects. First, the electron distribution function changes in a strong optical or microwave field.[2] For narrow pulses, whose duration is shorter than the collision time, the electron distribution function can be obtained simply by replacing in the field of the wave the momentum by an oscillatory momentum that oscillates in time.[3] Thus the function becomes nonstationary and averaging, for example, of single-particle observables should also include, beside the standard averaging over the coordinates and momenta, time averaging over a period of the oscillations. For observables which are not explicitly time-dependent, standard averaging over a period of the distribution function itself can be performed.

Another important circumstance is that in strong fields the elementary processes themselves can differ appreciably from those in an equilibrium medium.

In Refs. 4 and 5 it was shown that the cross sections for stimulated bremsstrahlung and absorption accompanying scattering of electrons by atoms and ions changes as a result of the influence of the electromagnetic wave on the free electron. In Refs. 6–11 these investigations were elaborated to take into account the internal dynamics of the target under the action of an external field.

The excitation of atoms in collisions with fast electrons in a strong optical field was apparently first studied in Ref. 7, where the motion of the electron was described in the approximation of a prescribed classical trajectory in an external electromagnetic field. A full quantum-mechanical analysis was performed in Ref. 10. In these papers, however, the direct effect of the field on the atomic electrons was actually assumed to be negligibly small, but no mention of this assumption was made. The computation results are presented in the form of Bessel-function series which, in the case of strong fields, converge slowly. An attempt to take into account the change in the properties of the target in an external electromagnetic field and the effect of this change on collisional-excitation processes was made in Ref. 8. However, the technique employed in Ref. 8 for reducing the problem to the stationary scattering theory seems to us unwieldy and unsuitable for practical calculations.

At the same time, all the required results can be obtained by nonstationary perturbation theory in a basis of quasienergy eigenfunctions. Recall that the quasienergy state u_μ is an eigenfunction of the equation

$$[H(t) - i\partial/\partial t]u_\mu = \varepsilon_\mu u_\mu \tag{1}$$

with a periodic Hamiltonian $H(t+T) = H(t)$.

The eigenvalues ε_μ are called quasienergies, and the set of solutions u_μ forms a complete orthogonal set of functions in the coordinate representation,[12] so that

$$\int u_\mu^*(\xi,t)u_\nu(\xi,t)d\xi = \delta_{\mu\nu}. \tag{2}$$

This makes it possible to separate the problems of interaction of systems with an external electromagnetic field and with one another. The first problem reduces to determining the quasienergy states, in this case the states of the atom (ion) and the incident electron. The second problem for a fast incident electron, whose velocity is much higher than that of the atomic electrons in the outer shell ($v \gg v_a$), can be solved by first-order perturbation theory, i.e., in Born's approximation.[13]

In the present paper we construct a formalism that makes it possible to calculate the excitation of atoms and ions colliding with electrons in a strong optical field, taking into account the direct action of the field on both the incident electron and the target. In so doing, excitation processes and the stimulated bremsstrahlung are described, taking into account the dynamics of the target, in a unified manner. Efficient methods are also developed for performing calculations in the low-frequency limit (on the basis of the sudden-impact approximation) and in the approximation of a nearly resonant weak field (generalized Bethe–Born approximation).

2. BASIC THEORETICAL RELATIONS

We employ nonstationary perturbation theory in a basis of quasienergy eigenfunctions. Let the complete Hamiltonian $H(t)$ of the atom+electron system have the form

$$H(t) = H_e(t) + H_a(t) + W,$$ (3)

where $H(t)_e(t)$ is the Hamiltonian of an electron in an external field with vector potential $\mathbf{A}(t)$:

$$H_e(t) = \frac{1}{2}\left(-i\mathbf{\nabla} - \frac{1}{c}\mathbf{A}(t)\right)^2,$$ (4)

where c is the speed of light in vacuum and $H_a(t)$ is the Hamiltonian of the atom in an external field. The operator W describes the interaction of the incident electron with the atom:

$$W = \sum_{j=1}^{N} \frac{1}{|\mathbf{r} - \mathbf{r}_j|} - \frac{Z}{r},$$ (5)

where $\mathbf{r}$ is the coordinate of the incident electron, $\mathbf{r}_j$ the coordinate of the jth electron of an N-electron atom, and Z the charge of the atomic nucleus.

Using the completeness of the basis of quasienergy functions, we seek the solution of the scattering problem as an expansion of the complete wave function ψ of the atom +electron system in terms of the wave functions of a free electron with momentum $\mathbf{p}$ in the field of a wave

$$\psi_{\mathbf{p}}(\mathbf{r},t) = \frac{1}{(2\pi)^{3/2}} \exp\left\{i\left[\mathbf{pr} - \frac{1}{2}\int_0^t \mathbf{P}(\tau)^2 d\tau\right]\right\},$$ (6)

$$\mathbf{P}(t) = \mathbf{p} - \frac{1}{c}\mathbf{A}(t),$$

and the quasienergy states $u_k(\mathbf{R},t)$ of the target atom in the form

$$\psi = \sum a_{k\mathbf{p}}(t)\psi_{\mathbf{p}}(\mathbf{r},t)u_k(\mathbf{R},t)\exp(-i\varepsilon_k t).$$ (7)

Substituting the wave function (7) into the Schrödinger equation with the Hamiltonian (3) for the complete system, we obtain the following equation for the amplitudes:

$$\frac{i\partial a_{k\mathbf{p}}}{\partial t} = \sum_{k_1\mathbf{p}_1} \langle \psi_{\mathbf{p}} u_k | W | \psi_{\mathbf{p}_1} u_{k_1}\rangle \exp[i(\varepsilon_k - \varepsilon_{k_1})t]a_{k_1\mathbf{p}_1}.$$ (8)

If the initial state of the system before the collision was the quasienergy state u_i and the state of an electron with quasimomentum $\mathbf{p}_0$ in the field of the wave, then in first-order perturbation theory in the interaction W the amplitude of the transition into the final state u_f of the target and the state $\psi_{\mathbf{p}}$ of the electron after the time t is

$$a_{f\mathbf{p}}^{(1)} = -i\int \langle \varphi_{\mathbf{p}} u_f | W | \varphi_{\mathbf{p}_0} u_i\rangle \exp\left|i(\varepsilon_f - \varepsilon_i)t\right.$$

$$\left. + i\int_0^t [\varepsilon_{\mathbf{p}}(\tau) - \varepsilon_{\mathbf{p}_0}(\tau)]d\tau\right|dt,$$ (9)

where $\varepsilon_{\mathbf{p}}(t) = \mathbf{P}^2(t)/2$ is the energy of the electron in the field of the wave at time t, and

$$\varphi_{\mathbf{p}} = (2\pi)^{-3/2}\exp(i\mathbf{pr}).$$

We note that if a different gauge is used and the electron-field interaction is chosen in the form $\mathbf{rE}(t)$, the function $\varphi_{\mathbf{p}}$ becomes correspondingly

$$\varphi_{\mathbf{p}} = (2\pi)^{-3/2}\exp[i\mathbf{P}(t)\mathbf{r}].$$

The expression for the transition amplitude (9) contains the product of complex conjugate functions $\varphi_{\mathbf{p}}$ and $\varphi_{\mathbf{p}_0}$. As a result, the additional identical phase factors will cancel one another and the results will not depend on the choice of gauge.

For a linearly polarized field

$$\mathbf{A}(t) = -(c\mathbf{E}/\omega)\cos(\omega t),$$ (10)

$$\int_0^t [\varepsilon_{\mathbf{p}}(\tau) - \varepsilon_{\mathbf{p}_0}(\tau)]d\tau = \tfrac{1}{2}(p^2 - p_0^2)t + \mathbf{aq}\sin(\omega t),$$ (11)

where $\mathbf{a} = \mathbf{E}/\omega^2$ is the amplitude of oscillations of the electron in the optical field and $\mathbf{q} = \mathbf{p} - \mathbf{p}_0$ is the transferred momentum.

In accordance with Floquet's theorem, an arbitrary quasienergy state u_μ can be represented as an expansion over quasienergy harmonics

$$u_k = \sum u_k^n \exp(in\omega t).$$ (12)

Substituting Eqs. (11) and (12) into Eqs. (9) and using the generating function for Bessel functions

$$\exp[i\lambda\sin(\omega t)] = \sum_{\nu=-\infty}^{\infty} J_\nu(\lambda)\exp(i\nu\omega t),$$

we obtain the squared probability amplitude for scattering of an electron with excitation of the atom from state i into state f in the following form:

$$|a_{f\mathbf{p}}^{(1)}|^2 = 4t\sum_M \left|\sum_{\nu+n-m=M} \left\langle u_f^n\right| \sum_{j=1}^{N}\exp(-i\mathbf{q}_M\mathbf{r}_j)\right.$$

$$\left. - Z\left|u_i^m\right\rangle J_\nu(\mathbf{q}_M\mathbf{a})/q_M^2\right|^2 \delta[(\varepsilon_f - \varepsilon_i)$$

$$+ (p^2 - p_0^2)/2 - \omega M].$$ (13)

The corresponding differential excitation cross section of the atom is

$$d\sigma_{fi} = 4\sum_M \frac{p_M}{p_0}\int\frac{1}{q_M^4}\left|\sum_{\nu+n-m=M}[F_{fi}^{nm}(\mathbf{q}_M)\right.$$

$$\left. - Z\delta_{fi}]J_\nu(\mathbf{q}_M\mathbf{a})\right|^2 d\Omega_{\mathbf{p}_M},$$ (14)

where $F_{fi}^{nm}(\mathbf{q}_M) = \langle u_f^n | \sum\exp(-i\mathbf{q}_M\mathbf{r}_j)|u_i^m\rangle$ is the dynamical form factor of the transition from the quasienergy harmonic m of the initial state i into the quasienergy harmonic n of the final state f. The quasienergy and quasimomentum in the initial and final states satisfy the relation

$$(\varepsilon_f - \varepsilon_i) + (p^2 - p_0^2)/2 - \omega M = 0$$

for each channel M in the inner sum of the expression (14).

The result above has a simple physical interpretation. When scattering of an electron excites an atom, the energies of both the electron and the atom in the final state can change. Some energy can be transferred from the external field to both the scattered electron and, by direct virtual absorption of the field, to the atom. We consider below only the nonresonant case, when the detuning from multiphoton resonance is greater than the total width of the levels.

The situation of exact resonance must be studied separately. This situation is in some sense simpler, since in this case the density-matrix formalism in the two-level approximation can be employed. The resonance problem, however, falls outside the scope of the present paper, where we study frequencies at which the gaseous target is transparent. In the case of single-photon resonance this problem was solved in Ref. 14.

3. SUDDEN-IMPACT APPROXIMATION

We consider the low-frequency limit. In this case the excitation of the corresponding high-quasienergy harmonics in the wave function of the target is exponentially weak,[15] and we neglect it. The entire temporal factor in the transition amplitude (9) then has the form

$$S = \int_0^t \exp\left[i(\varepsilon_f - \varepsilon_i)t + i\int_0^t [\varepsilon_{\mathbf{p}}(\tau) - \varepsilon_{\mathbf{p}_0}(\tau)]d\tau\right]dt.$$

If the integral is calculated at times t, such that

$$\min(|\varepsilon_i|^{-1}, |\varepsilon_f|^{-1}, p^{-2}, p_0^{-2}) \ll t \ll 1/\omega,$$

then the energy of the electron in the field of the wave does not change much over this time, and we can write approximately

$$\int [\varepsilon_{\mathbf{p}}(\tau) - \varepsilon_{\mathbf{p}_0}(\tau)]d\tau \approx t[\varepsilon_{\mathbf{p}}(0) - \varepsilon_{\mathbf{p}_0}(0)].$$

In this approximation it is actually assumed that in a single collision an electron strikes the atom with a definite momentum determined both by the initial momentum of the electron and the component acquired from the field. Since the time of the collision is not fixed, the averaging in the final expressions for the transition probabilities per unit time must be over the phase of the field or, equivalently, over the period of the field.

This is the so-called sudden-impact approximation, which was considered in Refs. 16 and 17 in the general case on the basis of an operator expansion. For the particular case of a collision in a linearly polarized field, results similar to ours were obtained in Ref. 18. There, however, in contrast to our work, the result is obtained by direct summation of the cross section over the number of photons absorbed by the electron. We propose a more direct and general method. Moreover, in Ref. 18 in the final result the cross section is averaged over the phase of the field, whereas the correct averaging procedure must be performed for observables, i.e., for the probability. This dif-

ference is fundamental when the vibrational momentum acquired from the field becomes comparable to the initial momentum of the electron in the absence of a field.

The excitation cross section, being a function of time, is expressed in terms of the generalized momentum $\mathbf{P}(t)$ in the standard manner[13]

$$d\sigma_{fi} = \frac{4P(t)}{P_0(t)q^4}\left|\left\langle f\left|\sum_{j=1}^N \exp(-i\mathbf{q}\mathbf{r}_j)\right|i\right\rangle\right|^2 d\Omega, \quad (15)$$

where the transferred momentum $\mathbf{q} = \mathbf{P}(t) - \mathbf{P}_0(t) = \mathbf{p} - \mathbf{p}_0$ is independent of the both the field and time, and the summation extends over all N electrons in the target. The process rate $\langle v\sigma_{fi}\rangle$ averaged over the phase of the field makes it possible to introduce the average cross section $\langle \sigma_{fi}\rangle$ by dividing the rate of the process by the velocity v_0 of the incident electrons in the absence of the field. In specific calculations the Stark shift of the initial and final levels of the atom must also be taken into account.

It is also easy to verify that in the Bethe–Born approximation

$$\langle \sigma_{fi}\rangle = \langle p_0/P_0(t)\rangle\sigma_{fi}, \quad (16)$$

where σ_{fi} is the cross section for excitation of the target by an electron without a field.

Our procedure makes it possible to find easily the cross section for the excitation of atoms and ions by electron impact in a laser field from data on cross sections in the absence of a field. The change in the cross section can be very notable. Thus, the cross section for excitation of atoms by 100-eV electrons increases by 10% in a 10.6-μm laser field of intensity $2 \cdot 10^{11}$ W/cm^2 for an angle $\phi = 0°$ between the polarization vector of the laser radiation and the direction of motion of the electrons incident on the atom.

The sudden-impact and Born approximations are independent. For this reason, in the low-frequency limit more accurate experimental or theoretical data on the excitation cross sections can be used for averaging the process rate over the period of the field. As an example, the cross section for excitation of the metastable state 2^3S in the He atom was calculated for two cases: 1) neodymium-laser radiation ($\hbar\omega = 1.17$ eV) under the experimental conditions of Ref. 19) CO$_2$-laser radiation ($\hbar\omega = 0.117$ eV) under the experimental conditions of Ref. 20.

In the first case (neodymium laser) the radiation intensity was assumed to be $4 \cdot 10^{10}$ W/cm^2. The angle ϕ between the polarization vector of the laser radiation and the direction of motion of the electrons incident on the atoms is 45°. The calculated difference of the excitation cross section in the presence and absence of the field is displayed in Fig. 1 (dashed line). The figure also displays the experimental data of Ref. 19 as well as the error. The solid curve represents Chichkov's computational results[21] multiplied by 4. As we can see, our calculation is closer to the experimental results.

We note that the computation results are very sensitive to the behavior of the cross section (which we took from the data of Ref. 22) near threshold in the absence of a field. This apparently can explain why our result differs appre-

σ, 10⁻¹⁸ a.u.
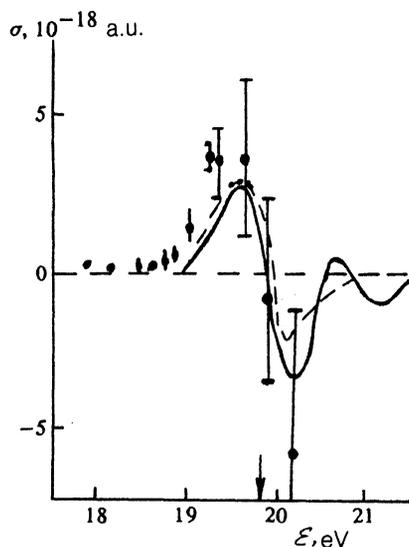
FIG. 1. Excitation cross section of the metastable state 2^3S in the He atom as a function of the energy of the incident electron near the reaction threshold under conditions corresponding to the experiment of Ref. 19.

ciably from the results of Ref. 21, in spite of the closeness of the approximations employed. In addition, in accordance with the remark made in Ref. 19, we increased somewhat the intensity used in the calculations.

Figure 2 displays the computation results for the excitation of He(2^3S) by electrons in CO_2-laser radiation. The energies are measured from the reaction threshold in photon energy units. The solid curve 1 corresponds to the experimental number of excited states in the absence of a field. The other dashed curves (2–6) correspond to the calculated difference of the number of metastables with and without a field. The figure also displays the experimental data together with an indication of the errors. The theoretical curves were constructed taking into account the fact that the low-frequency polarizability of He(2^3S) is quite high and is equal to 46.8 atomic units.[23] This leads to a Stark shift of the state and increases the excitation threshold of the 2^3S state by 0.05 eV.

It is evident that the experimental and theoretical results agree satisfactorily with one another.

4. GENERALIZED BETHE–BORN FORMULA

We now consider the limiting case of relatively low intensity and more photons. The atom can then be virtually excited efficiently by several photons, while the laser
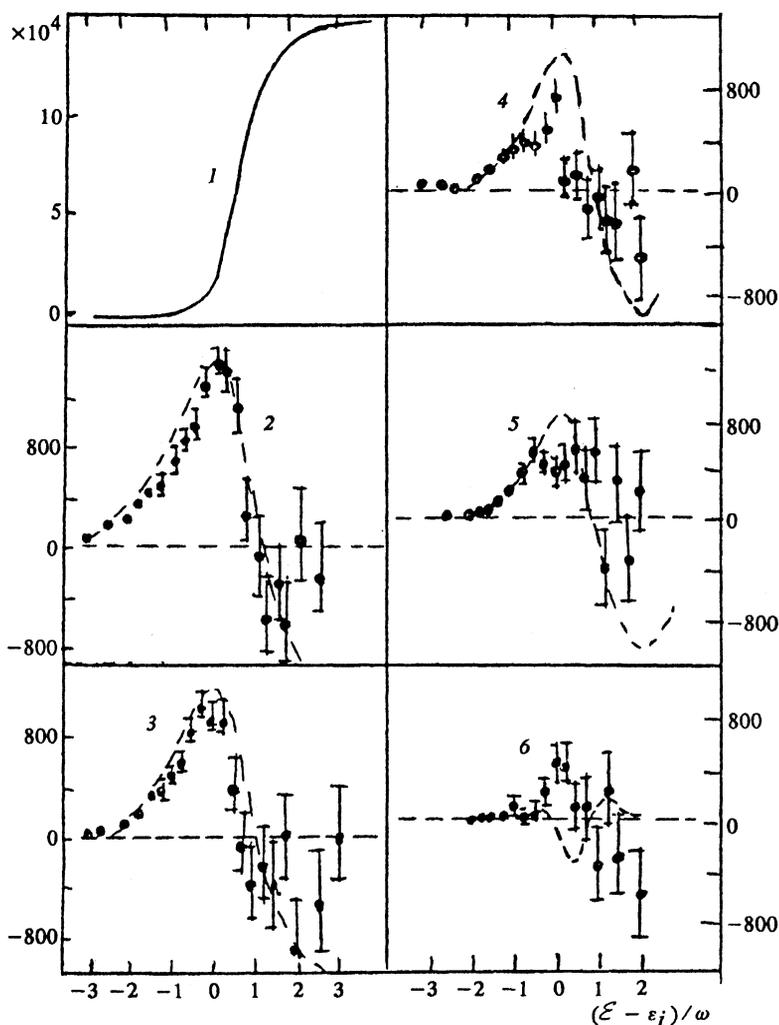
FIG. 2. Number of metastable He atoms as a function of the energy of the incident electron near the reaction threshold under conditions corresponding to the experiment of Ref. 20 over 5000 laser pulses: 1—signal in zero field, 2—$\phi=0$, 3—$\phi=30°$, 4—$\phi=45°$, 5—$\phi=60°$, 6—$\phi=90°$. The energy of the incident electron is measured from the threshold in units of the photon energy.

$(\mathcal{E} - \varepsilon_i)/\omega$

radiation distorts only slightly the motion of the incident electron. In other words, we assume that the amplitude $a=E/\omega^2$ of the oscillations of the incident electron in an external field is much smaller than atomic dimensions and $qa \ll 1$. Then we can set $J_0=1$ and $J_n=0$ for $n>1$. Correspondingly,

$$d\sigma_{fi}=4\sum_M \frac{p_M}{p_0}\int \frac{1}{q_M^4}\Big| \sum_{n-m=M} [F_{fi}^{nm}(\mathbf{q}_M)$$

$$-Z\delta_{fi}]\Big|^2 d\Omega_{\mathbf{p}_M}. \tag{17}$$

It is well known[13] that the transitions with small transferred momentum, less than the characteristic atomic momentum $(q \ll q_a)$, make the main contribution to the total cross section for excitation of an atom by electron impact. For $qr \ll 1$ the dipole approximation, i.e., the expansion $\exp(-i\mathbf{q}\mathbf{r}_j)\approx 1-i\mathbf{q}\mathbf{r}_j$, can be used. Integrating over the angles of emergence of the electron scattered with momentum $\mathbf{p}$, we obtain the following generalization of the well-known Bethe–Born approximation:

$$\sigma_{fi}=\sigma_{fi}^B+\frac{4}{p_0^2}\cos^2\beta \sum_{M=1} \ln|\varkappa p/(\varepsilon_f-\varepsilon_i-M\omega)|$$

$$\times (E/2)^{2M}|\chi^M(M\omega;\ n\{f\},\ m\{i\}|^2, \tag{18}$$

where $\varkappa=\min\{|2\varepsilon_f|^{1/2},\ |2\varepsilon_i|^{1/2}\}$,

$$\chi^M=\sum \langle f|\mathbf{de}G_{\varepsilon_f-\omega}\mathbf{de}G_{\varepsilon_f-2\omega}...\mathbf{de}G_{\varepsilon_f-n\omega}$$

$$\times \mathbf{de}G_{\varepsilon_i+m\omega}\mathbf{de}...G_{\varepsilon_i+\omega}\mathbf{de}|i\rangle, \tag{19}$$

σ_{fi}^B is the Born cross section for the excitation of an atom in the absence of a field and χ^M is, evidently, the atomic nonlinear susceptibility tensor convoluted with the polarization vectors of the laser radiation. The quantity β is the angle between the momentum $\mathbf{p}_0$ of the incident electron and the polarization vector $\mathbf{e}$ of the laser radiation.

This expression has quite a simple physical meaning. An atom colliding with an electron can be excited via different channels. One channel is the channel without absorption (emission) of photons by the atom. This channel $(M=0)$ corresponds to the standard Bethe–Born approximation. The terms in the formula (18) with $M\neq 0$ take into account the possibility of absorption of M photons from the field in the process of the collision with excitation. In the process m photons are absorbed before the collision and n photons are absorbed after the collision. The amplitudes of processes with a constant value of the sum $M=n-m$ absorbed (emitted) photons interfere with one another.

It is obvious that the contribution of virtual excitation of the atom by the laser radiation to the total cross section for the excitation of an atom by electron impact is greatest near an intermediate resonance. Then the weight of the corresponding quasienergy harmonic becomes large and excitation seemingly proceeds not from the ground state but rather from an excited state.
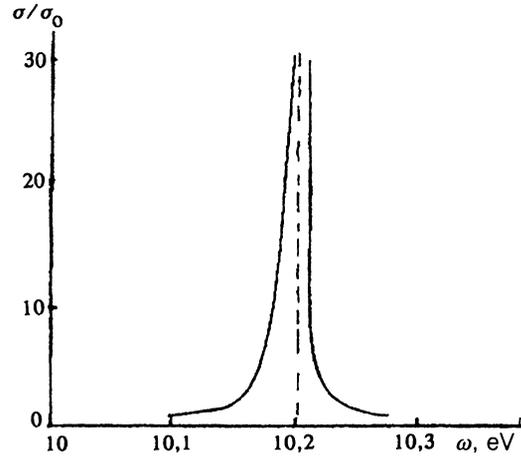
FIG. 3. Frequency dependence of the ratio of the excitation cross sections of the $3s$ states of the hydrogen atom with and without allowance for the infuence of a field with intensity $5 \cdot 10^{-7}$ V/cm. The energy of the incident electron is 500 eV.

As an example we consider the excitation of the $1s$–$3s$ transition in the hydrogen atom. Approximating the hydrogen excitation cross sections calculated in the Born approximation,[24] we obtain a formula for the total excitation cross section of $1s$–ns transitions taking into account the virtual excitation of the atom by the field:

$$\sigma_{fi}=\frac{\pi n C}{(n^2-1)^2}\left(\frac{u}{u+1}\right)^{1/2}\frac{1}{u+\varphi}+\tfrac{1}{2}\cos^2(\beta)$$

$$\times \ln|\varkappa p_0/(\varepsilon_f-\varepsilon_i-\omega)||\chi(\omega)E^2/\mathscr{E}^2, \tag{20}$$

where $\mathscr{E}$ is the energy of the incident electron, $\chi(\omega)$ is the nonlinear susceptibility taking into account virtual excitation of the atom, and $u=(\mathscr{E}-\varepsilon_f-\varepsilon_i)/(\varepsilon_f-\varepsilon_i)$. The values of the coefficients C and φ depend on the principal quantum number n of the final state and are tabulated in Ref. 24.

Figure 3 displays the computed ratios of the cross sections calculated with zero field and with a field of intensity $5 \cdot 10^{-7}$ V/cm and $\beta=0$. The energy of the incident electron is 500 eV. The excitation cross section of the $3s$ state of the hydrogen atom in the absence of the laser radiation is $8.013 \cdot 10^{-2}$ (πa_0^2), where a_0 is the Bohr radius. One can see from the figure that as the frequency of the external field approaches an intermediate resonance frequency the total excitation cross section increases rapidly, owing to the virtual excitation of the atom by the field.

5. CONCLUSION

We studied above the excitation of atoms by electrons in a strong optical field, taking into account virtual absorption of photons by the target. Our results extend those of Refs. 4, 10, 25, and 26, which are incorporated in our results as special cases.

A simple and efficient method was developed for calculating excitation cross sections in the low-frequency approximation. The method is based on averaging the instantaneous rate of the process over the period of the external

field. This method makes it possible to determine the cross section in the presence of a field from the theoretical or experimental values of the cross sections in the absence of a field.

Our approximation is an extension of the low-frequency approximation employed in Ref. 5 for elastic stimulated bremsstrahlung to include the inelastic process. In Ref. 27 it is asserted that the low-frequency approximation is incorrect, except at high energies, where the first Born approximation is applicable[4] and where it agrees with results of Ref. 5. This assertion is based on the fact that in the limit $\omega \to 0$ Bersons' quasiclassical result[28] does not pass into the results of Ref. 5. However, a more careful analysis shows that this conclusion is premature.

In Ref. 5 it is proved, by analysis of the properties of the solution of the scattering equation to all orders of perturbation theory in the interaction of the electron with the scattering potential, that the low-frequency approximation is applicable. This result is thus well founded. We note that the low-frequency approximation refers to a rather strong stimulated bremsstrahlung effect with emission (absorption) of a large number of photons, $n \sim pE/\omega^2 \gg 1$.

The proof in Ref. 5 pertains precisely to this case, though the large dimensionless parameter n is not singled out in Ref. 5 as a criterion for applicability of the results. For small n the agreement with the first Born approximation is only formal and the low-frequency approximation itself breaks down. For this reason, there is indeed an error in Ref. 5 in the section where the approximation is applied to single-photon processes.

We note that the low-frequency approximation is also more related to quantum mechanics, since the question of measurability is closely connected with it. In this case, the measurement times and the rate of change of the parameters of the quantum-mechanical system during the measurements are restricted by the uncertainty principle.

As for the quasiclassical approximation,[27] for bremsstrahlung on a hydrogen-atom nucleus this approximation is valid for $1/p \gg 1$ and, in addition, it implies that the trajectory is distorted very little by the external field. This imposes automatically the restriction $n\omega/\mathscr{E} \ll 1$ and simultaneously $p \gg E/\omega$. Since the quasiclassical theory is applicable, we obtain $E/\omega \ll 1$. This condition obviously breaks down in the limit $\omega \to 0$. Thus the quasiclassical approximation and the low-frequency approximation (or the sudden-impact approximation) correspond to the first terms of two different asymptotic expansions and do not convert into one another. The fact that these two asymptotic expansions are identical can be shown by re-expanding one of them with respect to the parameter of the other. This comparison was not made because the corrections to the first term of both the low-frequency and quasiclassical approximations are unavailable at the present time.

For fields whose frequency is comparable to that of atomic transitions and under conditions when the motion of the incident electron is distorted very little by the external field, a generalized Bethe–Born formula was obtained in the form of an expansion in powers of the strength of the external field. Besides the dipole matrix elements, in terms of which the cross section in the absence of a field is expressed, higher-order nonlinear susceptibilities also appear in the generalized Bethe–Born formula. Near resonance virtual excitation of the atom by the field can significantly increase the total cross section for excitation by electron impact.

The results of this work can also be extended to ionization of atoms by electron impact in the presence of a strong optical field.

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Translated by M. E. Alferieff