Accretion onto a neutron star in the presence of intense radiation

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We derive the complete set of quasihydrodynamic equations describing the interaction of plasma with a strong radiation field, and use it to study spherically symmetric Eddington accretion onto a neutron star. We obtain expressions that describe the spatial distribution of density, temperature, and the photon-gas and plasma flux. The overall flow, including all of the derived numerical values, depends on a single external parameter—the relative proximity to the Eddington limit.

Rapid accretion onto a compact object such as a neutron star or black hole is a common astrophysical phenomenon, and is thought to occur in x-ray sources, active galactic nuclei, and quasars. Accretion results in the generation of radiation at power levels that approach the Eddington limit, giving rise to a strong interaction between the radiation and the accreting plasma, and leading to radiative slowing of the plasma flow and to energy exchange between plasma and radiation via Compton processes. Ultimately, a self-consistent distribution of the parameters of the infalling plasma and the counterstreaming radiation is produced over the entire interaction region.

The basic features of this process can be studied in a natural way under the very simplest conditions by assuming spherical accretion; in the present paper, we solve for spherically symmetric accretion onto a neutron star. In Sec. 1, we make use of our previous results[1] to derive the complete set of hydrodynamic equations describing the dynamics of the various fluxes and the interaction between radiation and plasma (we present the equations for the general nonspherical case in the Appendix). Although the radiation usually does not conform to an equilibrium distribution under these conditions, the hydrodynamic system still has a closed solution, inasmuch as the radiation and plasma temperatures are small compared to mc^2. It is important here that the theory make it possible to determine these temperatures accurately. In Sec. 2, we study the equations thus derived in detail, and obtain a general solution. This enables us to construct a complete picture of the flow, to find the density, temperature, and flux distribution of both the plasma and radiation, and to fully elucidate the physics of all interaction processes.

We emphasize that the overall structure of the flow, including the spatial distribution and the numerical values of all of the fundamental macroscopic quantities (flow rates, fluxes, densities and temperatures of radiation and plasma), is dictated by a single free parameter—the relative proximity to the Eddington limit. We will show, in particular, that radiation and plasma temperatures are highest rather far from the stellar surface. As the Eddington limit is approached, the maximum temperature can rise to $T\simeq 20$ keV or more.

It should also be pointed out that spherical accretion has been considered before (see, e.g., Refs. 2–5), but the dynamical equations used were not based on the complete kinetic theory, so that in many instances they are not entirely correct. The most thoroughgoing investigation was the one by Miller,[5] who nonetheless did not obtain a complete solution—for example, the plasma and radiation temperatures went entirely unexamined.

1. THE COMPLETE SET OF HYDRODYNAMIC EQUATIONS FOR RADIATION AND PLASMA IN SPHERICALLY SYMMETRIC ACCRETION[1)]

We consider accretion onto a neutron star. It will be shown below that the dynamics of that process can be considered in the weakly relativistic approximation, which, when we describe the radiation, enables us to neglect—among other things—photon emission and absorption, pair annihilation, and nonlinear photon-photon interactions.

In the spherically symmetric case, the kinetic equation for the photon occupation numbers $n(r;\lambda,k)$ can then be written in the form

$$\frac{\partial n}{\partial t}+\frac{c}{r^2}\frac{\partial}{\partial r}(r^2 n\lambda)+\frac{c}{r}\frac{\partial}{\partial\lambda}[(1-\lambda^2)n]=\left(\frac{\delta n}{\delta t}\right)_{\rm ph}. \qquad (1)$$

Here r is the radius measured from the center of the neutron star and $\lambda=\cos\vartheta$, where ϑ is the angle between the radius vector $\mathbf{r}$ and the photon momentum $\mathbf{k}$. The photon distribution F is simply related to the occupation numbers n:

$$F=\frac{2n}{h^3},$$

where $h=2\pi\hbar$ is Planck's constant. The quantity $(\delta n/\delta t)_{\rm ph}$ on the right-hand side of (1) accounts for Compton collisions between photons and electrons.

For most photons, we normally have

$$k\ll\sqrt{T_{\rm ph}m}\ll mc,$$

where $T_{\rm ph}$ is the effective temperature of the photons and m is the electron mass; for the most part, electrons here undergo scattering through some angle in momentum space. In Ref. 1, we obtained for this case an expression for the integral over photon collisions with electrons. While the latter was an integral expression in λ, it was nevertheless a differential expression in k.

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Expanding Eq. (1) in Legendre polynomials,

$$n(r;\lambda,k)=\sum_i P_i(\lambda)n_i(r;k)$$

we obtain a set of recursion relations for the coefficients $n_i(r;k)$:

$$\frac{\partial n_0}{\partial t}+\frac{c}{3r^2}\frac{\partial}{\partial r}(r^2n_1)=\left(\frac{\delta n}{\delta t}\right)^{(0)}_{\rm ph},$$

$$\frac{\partial n_1}{\partial t}+c\frac{\partial n_0}{\partial r}+\frac{2}{5}\frac{c}{r^3}\frac{\partial}{\partial r}(r^3n_2)=\left(\frac{\delta n}{\delta t}\right)^{(1)}_{\rm ph},$$

$$\text{.....................} \tag{2}$$

$$\frac{\partial n_i}{\partial t}+c\left[\frac{ir^{i-1}}{2i-1}\frac{\partial}{\partial r}\left(\frac{n_{i-1}}{r^{i-1}}\right)+\frac{i+1}{(2i+3)r^{i+2}}\frac{\partial}{\partial r}(r^{i+2}n_{i+1})\right]=\left(\frac{\delta n}{\delta t}\right)^{(1)}_{\rm ph}.$$

We shall see below that when the rate of accretion onto the neutron star is close to the Eddington limit we have

$$N_e\sigma_{\rm Th}L_{\rm ph}\gg 1 \tag{3}$$

over a large spatial region of the flow, where N_e is the electron number density, $L_{\rm ph}\sim r$ is the characteristic spatial scale length over which the electron distribution function varies, and

$$\sigma_{\rm Th}=\frac{8\pi}{3}\left(\frac{e^2}{mc^2}\right)^2$$

is the Thomson cross section for photon scattering by electrons. When (3) holds, photons are strongly scattered at all angles, and their distribution function is virtually isotropic. In the usual way, this then enables us to truncate the sequence of equations (2) in the small parameter (3). Retaining only the two leading terms, we have

$$n(r;\lambda,k)=n_0(r;k)+\lambda n_1(r;k),$$

$$\frac{c}{3r^2}\frac{\partial}{\partial r}(r^2n_1)=\frac{N_e\sigma_{\rm Th}}{k^2}\frac{\partial}{\partial k}\left\{\frac{k^4}{m}\left[n_0+n_0^2+\frac{1}{c}\left(T_e+\frac{mV_e^2}{3}\right)\frac{\partial n_0}{\partial k}\right]-\frac{k^3}{3}V_en_1\right\}, \tag{4}$$

$$c\frac{\partial n_0}{\partial r}=-N_e\sigma_{\rm Th}\left(cn_1-V_ek\frac{\partial n_0}{\partial k}\right), \tag{5}$$

where $V_e=-\langle v_r\rangle$ is the radial velocity of the electron flux directed toward the center of attraction, averaged over the ensemble of particles,

$$T_e=\frac{2}{3N_e}\int f_e\frac{m}{2}\left(\mathbf{v}-\langle v_r\rangle\frac{\mathbf{r}}{r}\right)^2d^3p$$

is the effective temperature of the electrons, and $\mathbf{p}\approx m\mathbf{v}$ is the momentum of the electrons.

The first group of terms on the right-hand side of (4) are the same as the corresponding terms in the Kompaneets equation,[6] while the other terms proportional to V_e and V_e^2 account for modifications to that equation relating to motion in the electron flow.

We introduce next a set of parameters that characterize the radiation as a continuous medium:

$$N_{\rm ph}=\frac{2}{h^3}\int nd^3k\approx\frac{8\pi}{h^3}\int_0^\infty n_0k^2dk,$$

$$J_{\rm ph}=\frac{2c}{h^3}\int n\lambda d^3k\approx\frac{8\pi c}{3h^3}\int_0^\infty n_1k^2dk,$$

$$U_{\rm ph}=\frac{2c}{h^3}\int nkd^3k\approx\frac{8\pi c}{h^3}\int_0^\infty n_0k^3dk, \tag{6}$$

$$S_{\rm ph}=\frac{2c^2}{h^3}\int n\lambda kd^3k\approx\frac{8\pi c^2}{3h^3}\int_0^\infty n_1k^3dk,$$

$$T_{\rm ph}=\frac{c}{4}\left[\int_0^\infty(n_0+n_0^2)k^4dk\right]\Big/\left[\int_0^\infty n_0k^3dk\right].$$

In these equations, $N_{\rm ph}$ and $J_{\rm ph}$ are the mean photon number density and radial flux, $U_{\rm ph}$ and $S_{\rm ph}$ are the mean photon energy density and flux, and $T_{\rm ph}$ is the effective photon temperature, which is the same as the true temperature for equilibrium photons with a Planck distribution. Integrating (4) and (5) with the appropriate weighting factors from (6), we obtain the quasihydrodynamic equations

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2J_{\rm ph})=0,$$

$$\frac{\partial N_{\rm ph}}{\partial r}=-\frac{3N_e\sigma_{\rm Th}}{c}(J_{\rm ph}+VN_{\rm ph}),$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2S_{\rm ph})=\frac{N_e\sigma_{\rm Th}}{c}\left[\frac{4U_{\rm ph}}{m}(T_e-T_{\rm ph})+V\left(S_{\rm ph}+\frac{4}{3}VU_{\rm ph}\right)\right], \tag{7}$$

$$\frac{\partial U_{\rm ph}}{\partial r}=-\frac{3N_e\sigma_{\rm Th}}{c}\left(S_{\rm ph}+\frac{4}{3}VU_{\rm ph}\right).$$

This set of equations must be supplemented by the analogous hydrodynamic equations for the plasma flow. Owing to quasineutrality, the number densities of electrons and ions are approximately equal:

$$N_i=N_e=N, \tag{8a}$$

and noting that div $J=0$ implies the impossibility of a continuous current flowing into the star ($J=0$) in the spherically symmetric case, we have an analogous equation for the mean electron and ion velocities,

$$V_i=V_e=V. \tag{8b}$$

Taking advantage of the specific form of the electron-photon interaction operator,[1] we find that the steady-state hydrodynamic equations for the plasma flow take the form[2)]

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2NV)=0, \tag{9a}$$

$$m_i V \frac{\partial}{\partial r} V + \frac{1}{N} \frac{\partial}{\partial r} [N(T_e+T_i)] = -\frac{\tilde{\gamma}}{r^2} + \frac{\sigma_{\rm Th}}{c} \times \left(S_{\rm ph} + \frac{4}{3} V U_{\rm ph} \right), \tag{9b}$$

$$\frac{3}{2} \frac{\partial}{\partial r} (T_e+T_i) - (T_e+T_i) \frac{1}{N} \frac{\partial N}{\partial r} = \frac{4\sigma_{\rm Th} U_{\rm ph}}{mcV} (T_e - T_{\rm ph}), \tag{9c}$$

$$\frac{3}{2} \frac{\partial T_i}{\partial r} - \frac{T_i}{N} \frac{\partial N}{\partial r} = \frac{3m\nu_{ei}}{m_i V} (T_i - T_e), \quad \nu_{ei} = \frac{4\sqrt{2\pi} e^4 \tilde{Z} N \ln \Lambda}{3\sqrt{m} T_e^{3/2}}. \tag{9d}$$

Here T_e and T_i are the electron and ion temperatures, m_i is the mean ion mass in the plasma, $\tilde{\gamma} = G m_i M$, where G is the gravitational constant and M is the mass of the star, ν_{ei} is the effective electron-ion collision rate, $\ln \Lambda \approx 10$–20 is the Coulomb logarithm, and $\tilde{Z} = \Sigma \tilde{Z}_i^2 N_i / N$ is the mean charge on a plasma ion. The left-hand sides of Eqs. (9a)–(9d) are in standard form. On the right-hand side of Eq. (9b), which describes the variation of plasma momentum, we have included both the gravitational field and photon-electron interactions. The energy balance equation (9c) has accordingly a term for the Compton energy exchange between electrons and photons. We neglect thermal conductivity and any additional heat sources or sinks. Recall that throughout this paper, we neglect any process that violates photon number conservation (such as bremsstrahlung, resonant photon absorption by ions, etc.).

Equations (7)–(9) comprise a complete set of hydrodynamic equations for radiation and plasma in steady-state spherically symmetric accretion. To study this system, we must set down the boundary conditions (which in the present case are specified on the surface of the neutron star, $r = r_0$, and at $r \to \infty$).

Note that Eqs. (7) and (9) are conservation laws. Thus, the total flux of accreting matter should be given in the present context by

$$I_0 = 4\pi r^2 N V = \frac{1}{m_i} \frac{dM_{\rm accr}}{dt}, \tag{10}$$

where $dM_{\rm accr}/dt$ is the total flux of matter incident upon the star per unit time ($dM_{\rm accr}/dt = \text{const}$ in the steady state). The total photon flux emerging from the neutron star and counterpropagating with respect to the plasma flux is also constant,

$$I^{\rm ph} = 4\pi r^2 J_{\rm ph} = I_0^{\rm ph}. \tag{11}$$

In contrast to (10), however, it cannot be specified *a priori*, but must instead be determined by solving the problem in a self-consistent manner.

From (7) and (9), one can also easily obtain the conservation law for the combined energy of radiation and plasma moving in a gravitational field,

$$\frac{\partial}{\partial r} \mathscr{L}_{\rm tot} = 0,$$
$$\mathscr{L}_{\rm tot} = \mathscr{L} - \left[\frac{m_i V^2}{2} + \frac{5}{2} (T_e+T_i) - \frac{\tilde{\gamma}}{r} \right] I_0, \tag{12}$$

where $\mathscr{L} = 4\pi r^2 S_{\rm ph}(r)$ is the total energy flux through a sphere of radius r. The constant $\mathscr{L}_{\rm tot}$ can also be determined by solving the complete problem. At the surface of the star ($r = r_0$), however, $\mathscr{L}$ is related to I_0. In fact, if we assume that in the steady state at $r = r_0$, all of the kinetic energy of plasma incident upon the neutron star surface is transformed into radiation, then

$$\mathscr{L} \Big|_{r=r_0} = I_0 \left[\frac{m_i V^2}{2} + \frac{5}{2} (T_e+T_i) \right] \Big|_{r=r_0}. \tag{13}$$

For (7) and (9) to form a completely self-contained set of equations, they must also be supplemented with a boundary condition at $r \to \infty$, which applies, for example, to the plasma velocity V. Since the plasma density remains finite as $r \to \infty$, when the flux I_0 is constant we would naturally expect

$$V|_{r=\infty} = 0. \tag{14}$$

Equations (10)–(14) constitute a complete set of boundary conditions for Eqs. (7)–(9), which we can now proceed to solve.

2. STRUCTURE OF THE FLOW IN THE PLASMA-RADIATION INTERACTION REGION

To solve (7)–(9) for the combined dynamics of plasma and radiation, it is convenient to change to dimensionless quantities

$$y = \frac{r_0}{r}, \quad g = \frac{V}{V_s}, \quad Z = \frac{\mathscr{L}}{\mathscr{L}_s},$$
$$\mathscr{P} = \frac{4\pi r_0^2 U_{\rm ph} V_s}{\mathscr{L}_s}, \quad \mathscr{R} = \frac{4\pi r_0^2 N_{\rm ph} V_s}{I_0^{\rm ph}}, \quad \chi_{e,i} = \frac{T_{e,i} r_0}{\tilde{\gamma}}, \tag{15a}$$

where

$$\mathscr{L}_s = I_0 \frac{\tilde{\gamma}}{r_0} \equiv q \mathscr{L}_{\rm Edd}, \quad V_s = \left(\frac{2\tilde{\gamma}}{m_i r_0} \right)^{1/2}, \quad q = \frac{I_0 \sigma_{\rm Th}}{4\pi c r_0}. \tag{15b}$$

Here $\mathscr{L}_{\rm Edd} = 4\pi \tilde{\gamma} c / \sigma_{\rm Th} \approx 1.26 \cdot 10^{38}$ $(M/M_\odot)$ erg/s is the Eddington luminosity. The constancy of the energy flux (12), together with the boundary condition (13), yield

$$z = g^2 + 1 - y + \frac{5}{2} (\chi_e + \chi_i). \tag{16}$$

The boundary condition (14) then means that

$$g = 0 \quad \text{at } y = 0.$$

We therefore obtain from (13)

$$z_\infty \equiv z|_{y=0} = 1 + \tfrac{5}{2} (\chi_e + \chi_i).$$

In the dimensionless variables (15), Eqs. (7)–(9), together with (10)–(13), can be rewritten as

$$\frac{d\mathscr{R}}{dy}=\frac{3q}{g}(y^2+g\mathscr{R}),$$

$$\frac{d\mathscr{P}}{dy}=\frac{q}{g}(3y^2Z+4g\mathscr{P}),\tag{17}$$

$$\frac{d}{dy}(g^2+\chi_e+\chi_i)+(\chi_e+\chi_i)\frac{d}{dy}\ln\left(\frac{y^2}{g}\right)$$

$$=1-q-q\left[g^2-y+\frac{5}{2}(\chi_e+\chi_i)+\frac{4}{3}\frac{g}{y^2}\mathscr{P}\right].$$

Here we have plugged the expression for z from (16) into the third of Eqs. (17). As we shall see below, that is just the equation which governs the structure of the plasma-radiation interaction region.

It is clear from (17) that the parameter q in (15b), which is determined by the magnitude of the accretion flow, is always less than unity, and the accretion regime depends critically on the relation between q and 1. Thus, for $q\ll 1$, accretion takes place far below the Eddington rate, and matter can flow unimpeded onto the star since the radiation is too weak to exert any kind of decelerating influence on the accretion flow.

In the present study we have dealt principally with the most interesting case, for which $q\simeq 1$; the radiation has then a substantial effect on accretion dynamics. The basic parameter governing the scale size of the interaction region is

$$\varepsilon=1-q>0.\tag{18}$$

We consider the case in which the parameters that we have determined, $\chi_{e,i}$ in (15a) and ε in (18), satisfy the condition

$$\chi_{e,i}\ll\varepsilon y<1.\tag{19}$$

The left-most inequality here means that the plasma is cold, so that its hydrodynamic pressure has little effect on the plasma flow. From (17) we then obtain

$$z=1-g^2-y,\tag{20a}$$

$$\frac{d\mathscr{P}}{dy}=\frac{1-\varepsilon}{g}\left[3y^2(1+g^2-y)+4g\mathscr{P}\right],\tag{20b}$$

$$\frac{dg}{dy}=\frac{1-\varepsilon}{2g}\left[\frac{\varepsilon}{1-\varepsilon}-g^2+y-\frac{4}{3}\frac{g}{y^2}\mathscr{P}\right].\tag{20c}$$

If the functions $\mathscr{P}(y)$ and $g(y)$ here are known, the first of Eqs. (17) can be easily integrated:

$$\mathscr{R}(y)=\exp[3(1-\varepsilon)y]\left[\mathscr{R}(y_0)\exp[-3(1-\varepsilon)y_0]+3(1-\varepsilon)\int_{y_0}^{y}\frac{x^2}{g(x)}\exp[-3(1-\varepsilon)x]dz\right].\tag{21}$$

A complete solution of the entire problem thus requires only that (20b) and (20c) be solved with the boundary condition–

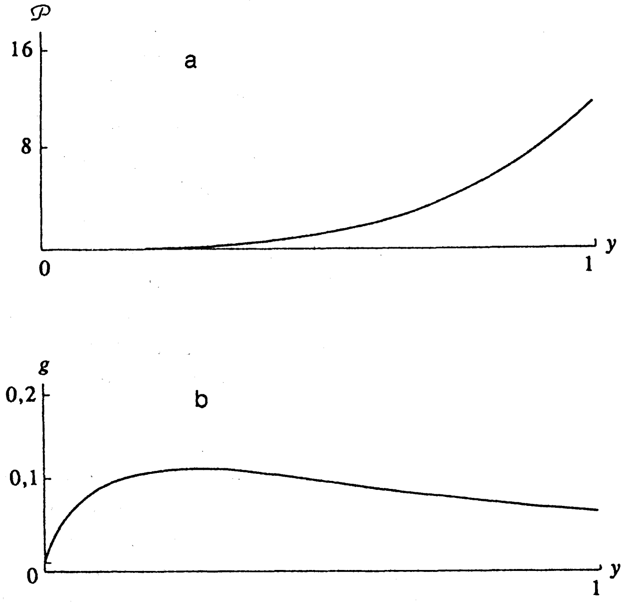

FIG. 1. Spatial distribution for $\varepsilon=0.1$ of the photon energy density (a) and plasma flow velocity (b), normalized as in (15).

$$g\to 0\quad\text{as } y\to 0.\tag{22}$$

Far from the star, where

$$y\ll\varepsilon$$

(i.e., $r\gg r_0/\varepsilon$), the sole consistent solution of (20) that satisfies (22) takes the form

$$g=\sqrt{\varepsilon y},$$

$$\mathscr{P}\approx\mathscr{R}\approx\frac{6}{5}y^{5/2}\frac{1-\varepsilon}{\sqrt{\varepsilon}},\tag{23}$$

$$z\approx 1.$$

In this domain, the plasma dynamics are determined by the difference between the gravitational force and the pressure due to the directed radiative flux. Since the intensity of the latter here is essentially constant $[\mathscr{L}\approx(1-\varepsilon)\mathscr{L}_{\text{Edd}}]$, these competing forces will have the same spatial dependence proportional to $1/r^2$, and respectively a velocity dependence proportional to $1/r^{-1/2}$ (23).

We solved Eqs. (20a)–(20c) numerically, with boundary conditions (22). The desired solution, which tends to the asymptotic values (23), has the following basic properties. When $\varepsilon>\varepsilon_{\text{cr}}\approx 0.28$, the function $g(y)$ increases monotonically up to $y=1$; it becomes nonmonotonic when $\varepsilon<\varepsilon_{\text{cr}}$ (see Fig. 1). Near some critical value $y=y_{\text{cr}}\equiv r_0/r_{\text{cr}}$ (where r_{cr} is the critical radius) there is a maximum in $g(y)$, and furthermore, as the flow approaches the star, the directional plasma flow falls off. Here $\mathscr{P}(y)$ and $\mathscr{R}(y)$ always increase monotonically with y.

Let us clarify the physical nature of the nonmonotonic spatial dependence of the plasma flow velocity. In an optically dense medium, the decelerating force on the plasma flow consists of two parts [see Eq. (9b)]: the pressure force due to beaming of the photons and proportional to S_{ph}, and the viscous damping force due to the anisotropic flow of the plasma itself, which is proportional to $VU_{\rm ph}$ (and which vanishes in a frame of reference comoving with the plasma). When the rate of accretion is near the Eddington limit (i.e., when ε is small enough), the total radiative flux $\mathscr{L}$ near the star, which is proportional to $r^2S_{\rm ph}$, starts to decrease, gravitation is no longer balanced by radiation pressure, and the plasma flow begins to accelerate. Furthermore, close to the star, the photon energy density rises dramatically, and consequently so does the viscous damping force. This leads to a gradual reestablishment of balance between the gravitational force and radiation pressure, and after passing the critical radius $r_{\rm cr}$, the plasma flow starts to decelerate.

The rise in luminosity and decrease in radiative energy density with distance from the star have to do with energy and momentum conservation. As the plasma flow slows, some fraction of its kinetic energy is converted into photons, which in the steady state yields a rise in the radiative flux (i.e., the luminosity) with distance from the star. At the same time, the momentum transferred to the photons increases the inward radial component of the radiative momentum flux, and in the presence of strong photon scattering, the corresponding component of the momentum flux is proportional to the radiative energy density $U_{\rm ph}$. The fact that the radiative flux increases and the energy density decreases with distance from the star inevitably leads to more and more narrowly beamed radiation, which in turn ultimately leads to the free expansion of photons sufficiently far from the star (see below).

We now consider the case

$$\varepsilon \ll 1 \tag{24}$$

in more detail. We first study the region $y \sim \varepsilon$ (i.e., $r \sim r_0/\varepsilon$) far from the star. Renormalizing variables with

$$y=\varepsilon\tau, \quad g=\varepsilon\zeta, \quad \mathscr{P}=\varepsilon^2\xi, \quad \mathscr{R}=\varepsilon^2\eta, \tag{25}$$

substituting (25) into (20), and retaining leading terms of the expansion in the small parameter ε, we obtain

$$z=1$$

for this region, and the pair of universal equations

$$\frac{d\zeta}{d\tau}=\frac{1}{2\zeta}\left[1+\tau-\frac{4}{3}\frac{\zeta\xi}{\tau^2}\right], \qquad \frac{d\eta}{d\tau}=\frac{d\zeta}{d\tau}=\frac{3\tau^2}{\zeta}, \tag{26}$$

with asymptotic boundary conditions at $\tau \ll 1$ [see (23)]

$$\zeta=\sqrt{\tau}, \quad \xi=\frac{6}{5}\tau^{5/2}. \tag{27}$$

The asymptotic behavior is different for $\tau \gg 1$:

$$\tau \approx \frac{4}{3}\frac{\zeta\xi}{\tau^2}, \qquad \eta \approx \xi \approx C\tau^2, \quad \zeta \approx \frac{3}{4C\tau}, \quad C \approx 0.11. \tag{28}$$

The constant in Eq. (28) has been obtained numerically by matching to the exact solution. The solution for ζ and ξ given by (28) also holds when $y \gg \varepsilon$ out to $y=1$. The various asymptotic expansions verify this statement.

The solution is given to high accuracy for all y by

$$\zeta(\tau) \approx \frac{3\sqrt{\tau}}{3+4C\tau^{3/2}}, \quad \xi \approx \frac{6}{5}\tau^{5/2}+C\tau^4. \tag{29}$$

On the other hand, the expressions $z \approx 1$ and $\mathscr{R} \approx \mathscr{P}$ as given by (28) only hold when $y \lesssim \varepsilon$. When $y \sim 1$, z is given by the general equation (20a), and by that time $\mathscr{R}$ and $\mathscr{P}$ will have begun to diverge appreciably. When $q=1-\varepsilon \approx 1$, we have according to (28) and (21)

$$\mathscr{P} \approx C\frac{y^4}{\varepsilon^2}, \quad g \approx \frac{3}{4C}\frac{\varepsilon^2}{y}, \quad \frac{d\mathscr{R}}{dy} \approx \frac{4\mathscr{R}}{y}+3\mathscr{R}, \qquad \mathscr{R} \approx \frac{8}{27}\frac{C}{\varepsilon^2}\left[e^{3y}-\left(1+3y+\frac{9}{2}y^2+\frac{9}{2}y^3\right)\right]. \tag{30}$$

Under the Eddington accretion conditions (24), therefore, Eqs. (20a), (29), and (30), supplemented by (15) and (25), account for the spatial distribution of luminosity, energy density, photon number density, and the mean rate of plasma mass flow. It is clear from these expressions that the small quantity ε given by (18) is the dominant control parameter. The characteristic spatial scale length $r_{\rm cr}$ of the interaction region between plasma and radiation, at which the plasma flow velocity is a maximum, is thus inversely proportional to ε:

$$r_{\rm cr} \approx 0.5\frac{r_0}{\varepsilon}. \tag{31}$$

As a result, when the total flux I_0 of accreting plasma approaches the critical value

$$I_{\rm Edd}=\frac{4\pi c r_0}{\sigma_{\rm Th}} \equiv \frac{r_0}{\gamma}\mathscr{L}_{\rm Edd}, \tag{32}$$

the spatial structure of the interaction region extends to infinity.[3)]

It is also clear from (29) that the flow attains its maximum velocity at $r=r_{\rm cr}$ (31). Under the conditions given by (24), it will then always be much less than the characteristic velocity V_s of (15), the free-fall velocity of matter in the star's gravitational field. For $r \ll r_{\rm cr}$ according to (28), the flow velocity V will fall linearly with radius r, and will be exceedingly small near the star:

$$V|_{r=r_0} \approx 7\varepsilon^2 V_s. \tag{33}$$

The luminosity $\mathscr{L}$ drops off rather abruptly near the stellar surface, possibly owing to the influence of small terms neglected above. Instead of Eq. (20a), we therefore make use of the more general expression (16) and obtain

$$\mathscr{L}(r) \approx \mathscr{L}_{\rm Edd}\left[1-\frac{r_0}{r}+g^2+\frac{5}{2}(\chi_e+\chi_i)\right]. \tag{34}$$

According to (10), (15), and (28), we have

$$N \approx \frac{0.15}{4\pi\varepsilon^2 r_0^2} \frac{I_0}{V_s} \left(\frac{r_0}{r}\right)^3. \tag{35}$$

At the same time, the photon energy density rises faster than this:

$$U_{\text{ph}} \approx \frac{0.11}{\varepsilon^2} \frac{\mathscr{L}_s}{4\pi r_0^2 V_s} \left(\frac{r_0}{r}\right)^4. \tag{36}$$

This is also approximately how the photon number density varies as well [see (30)]. Right at the surface of the star we have

$$N_{\text{ph}}\Big|_{r=r_0} \approx 2.1 \frac{I_0^{\text{ph}}}{\mathscr{L}_s} U_{\text{ph}}\Big|_{r=r_0}. \tag{37}$$

We will determine the constant I_0^{ph} in (37) below.

On the whole, these features of the solution for the plasma flow velocity, luminosity, and photon energy density are consistent with the results obtained by Miller.[5] The spatial distributions of the photon number density and of the plasma and radiation temperature, however, were previously not studied, and we now look into this problem in somewhat more detail.

From Eqs. (9c) and (9d), we have for the dimensionless variables $\chi_{e,i}$

$$\frac{d}{dy}\left[\frac{(\chi_e+\chi_i)g^{2/3}}{y^{4/3}}\right] = \frac{4}{3} q \frac{\mathscr{P}}{gy^2} \frac{m_i}{m} \frac{(\chi_{\text{ph}}-\chi_e)g^{2/3}}{y^{4/3}},$$

$$\frac{d}{dy}\left[\frac{\chi_i g^{2/3}}{y^{4/3}}\right] = \left(\frac{2}{\pi}\right)^{1/2} q \frac{y^2}{g^2} \tilde{Z} \ln \Lambda \frac{m}{m_i} \left(\frac{mc^2}{T_e}\right)^{3/2} \times \frac{c^2}{V_s^2} \frac{(\chi_e-\chi_i)g^{2/3}}{\chi_e^{3/2} y^{4/3}}, \tag{38b}$$

where $\chi_{\text{ph}} = T_{\text{ph}} r_0 / \tilde{\gamma}$. To evaluate the parameters in Eq. (38a), we begin by noting that near the Eddington limit ($q \approx 1$), the ratio $gy/\mathscr{P} \simeq 3\varepsilon^4/4c^2y^4$ will be much less than unity in the near zone, where $y \gg \varepsilon$, and of order unity in the far zone, i.e., at $y \sim \varepsilon$. With the foregoing in mind, we obtain from (38a)

$$\frac{|\chi_{\text{ph}}-\chi_e|}{(\chi_e+\chi_i)} \sim \frac{m}{m_i} \frac{gy}{\mathscr{P} q} \ll 1.$$

Hence, by virtue of the smallness of $mgy/m_i \mathscr{P} q$, we conclude that the effective photon temperature is essentially equal to the electron temperature. Similarly, we find from Eq. (38b) that

$$\frac{|\chi_e-\chi_i|}{\chi_i} \sim \frac{1}{q\tilde{Z} \ln \Lambda} \frac{m_i V_s^2}{mc^2} \left(\frac{T_e}{mc^2}\right)^{3/2} \frac{g^2}{y}.$$

In the far zone, $y \lesssim \varepsilon$, (23) yields $g^2/y = \varepsilon$. We will show below [see (42)] that $T_e \sim 1$ keV$/\sqrt{\varepsilon}$, and despite the presence of the large parameter $m_i/m \sim 10^3$, we always have

$$|T_e - T_i| \ll T_i.$$

This inequality is much more easily satisfied in the near zone $y \sim 1$, and thus all particle bunches wind up at virtually the same temperature:

$$T_i \approx T_e \approx T_{\text{ph}}.$$

The reason is that the photon number density is several orders of magnitude higher that that of the electrons (see below), so the photon gas is effectively a heat bath for the electrons. Moreover, the electrons and ions in the plasma also exchange a significant amount of heat, leading to rapid redistribution of internal energy between these groups of particles.

We consider now how to determine the radiation temperature T_{ph}. It can be seen from (6) that it is determined by a higher moment of the photon distribution function than are U_{ph} and S_{ph}, so from the standpoint of determining the radiation temperature, the hydrodynamic equations (7)–(9) do not form a closed set. This has not been an important issue thus far, since by virtue of (19) thermal effects have been assumed to make only a minor contribution to the overall balance of forces. Strictly speaking, finding the actual value of T_{ph} requires that one solve the full set of kinetic equations (4) and (5). But there is one favorable circumstance that enables us to find the temperature accurately without recourse to the solution of the full kinetic equation.

We first assume that the photons have a Wien distribution,

$$n_0 = \exp[-ck/T_{\text{ph}}^*]. \tag{39}$$

We then have

$$T_{\text{ph}}^* = \frac{c}{3}\left[\int_0^\infty n_0 k^3 dk\right] \Big/ \left[\int_0^\infty n_0 k^2 dk\right] \equiv \frac{U_{\text{ph}}}{3N_{\text{ph}}}. \tag{39a}$$

If on the other hand the photons have a Planck distribution

$$n_0 = \left[\exp\left(\frac{ck}{T_{\text{ph}}}\right) - 1\right]^{-1},$$

then

$$T_{\text{ph}}^* \simeq 0.9 T_{\text{ph}}.$$

It can easily be shown that the approximate value of T_{ph}^* given by (39a) will deviate only slightly from the true temperature T_{ph} for any reasonable distribution of the isotropic part of the photon distribution function n_0 over energy.[4)] For example, even in the limiting case in which n_0 is a step function in energy (momentum),

$$n_0 = \begin{cases} 1 & \text{for } ck \leqslant T_{\text{ph}}, \\ 0 & \text{for } ck > T_{\text{ph}}, \end{cases}$$

surely a worst case, the error is at most 25%:

$$T_{\text{ph}}^* = 1.25 T_{\text{ph}}.$$

Finally, kinetic considerations (which lie outside the scope of the present paper) show that at the dominant energies the distribution most closely resembles (39), so that we can assume throughout that $T_{\text{ph}} \approx T_{\text{ph}}^*$. This makes it possible to study the radiation temperature distribution (and thus the plasma temperature distribution) using the solutions obtained above, namely (29) and (30).

It follows from (30) and (39a), in particular, that when $\varepsilon \ll 1$, the common temperature at $r \sim r_0$ of all interacting particles increases monotonically with r. At $r \gg r_0$, $\mathscr{P}(y) \approx \mathscr{R}(y)$, and the temperature tends to the constant value T_∞

$$T_\infty = T_e \approx T_i \approx T_{\rm ph} \approx T_0 \frac{\mathscr{P}(y)\mathscr{R}(1)}{\mathscr{R}(y)\mathscr{P}(1)} \approx 2.1 T_0 \frac{\mathscr{P}(y)}{\mathscr{R}(y)}, \tag{40}$$

approximately twice as high as the temperature $T_0 \approx T^*_{\rm ph}(r_0)$ at the neutron star surface. This temperature increase is related to the fact that when photons are strongly scattered by the moving plasma flow, a portion of the kinetic energy of directed plasma motion is transformed into internal energy of the photon gas.

Kinetic considerations [see Eq. (5)] show that the monotonic increase in radiation temperature with height r plays a decisive role in the overall energy balance of the interacting radiative and plasma fluxes. As a result of this increase, the distribution function for photons of a given energy, beginning at some threshold value, rises exponentially with r (the threshold energy depends on r, however, monotonically tending to infinity as $r \to \infty$). The advent of a reversed gradient in the distribution function results in photons of sufficiently high energy being directed toward the neutron star. This effect is enhanced by momentum exchange with the infalling plasma flow. It is precisely this reverse flux of high-energy photons that reduces the radiative energy flux $\mathscr{L}$ near the star's surface [see (34)] at the same time that the total photon flux $I^{\rm ph}$ naturally remains constant (11).

To finally find the temperature profile, we see from (40) that it is necessary to determine the temperature T_0 at the surface of the star. Accreting matter heats the surface, and since the matter at the stellar surface is quite dense, the radiation is rapidly thermalized. Note also that radiation near the surface is extremely close to being isotropic; the photon flux from the surface is strongly scattered by the accreting plasma into a thin surface layer, and essentially all of it is returned to the star's surface, where it is then reradiated, and so on. This also facilitates the establishment of thermal equilibrium in the surface layer. To high accuracy, it can therefore be assumed that the surface of the star radiates as a black body. Substituting the Planck distribution into (6), we have

$$U_{\rm ph}(r_0) = \frac{T^4_{\rm ph}(r_0)}{(\hbar c)^3 \pi^2} \int_0^\infty \frac{x^3 dx}{e^x - 1} = \frac{\pi^2 T^4_{\rm ph}(r_0)}{15(\hbar c)^3},$$

$$N_{\rm ph}(r_0) = \frac{T^3_{\rm ph}(r_0)}{(\hbar c)^3 \pi^2} \int_0^\infty \frac{x^2 dx}{e^x - 1} \approx 0.244 \frac{T^3_{\rm ph}(r_0)}{(\hbar c)^3}, \tag{41}$$

$$T_0 \approx T^*_{\rm ph}(r_0) \approx T_{\rm ph}(r_0) \approx 0.37 \frac{U_{\rm ph}(r_0)}{N_{\rm ph}(r_0)}.$$

Equations (41) and (36) then yield the final expression for the photon temperature at the neutron-star surface,

$$T_0 \approx \frac{0.34}{\sqrt{\varepsilon}} \left(\frac{\hbar^3 c^3 \mathscr{L}_{\rm Edd}}{r_0^2 V_s} \right)^{1/4}$$

$$\approx \frac{0.84}{\sqrt{\varepsilon}} \left(\frac{M}{M_\odot} \right)^{1/8} \left(\frac{10 \text{ km}}{r_0} \right)^{3/8} \text{ (keV)}, \tag{42}$$

where $M_\odot$ is the mass of the sun.

Making use of (42) and the definition (15) of $\mathscr{P}$ and $\mathscr{R}$, we obtain the last unknown $I_0^{\rm ph}$ in (37):

$$I_0^{\rm ph} \approx 0.176 \frac{\mathscr{L}_{\rm Edd}}{T_{\rm ph}(r_0)}. \tag{43}$$

Equations (40), (42), and (43), in conjunction with (29) and (30), yield the complete solution of this problem. Note that according to (42), the maximum temperature of the plasma flow (40) is

$$T_{\max} \approx \frac{1.76}{\sqrt{\varepsilon}} \left(\frac{M}{M_\odot} \right)^{1/8} \left(\frac{10 \text{ km}}{r_0} \right)^{3/8} \text{ (keV)}. \tag{44}$$

Recall that the present theory holds when $\varepsilon y > \chi_{e,i} = \chi$ [see (19)]. Bearing that constraint in mind, and also the fact that the far zone $y \sim \varepsilon$ plays an important role in the solution obtained above, we assume $\varepsilon = \varepsilon_{\min} \sim \sqrt{\chi}$ to estimate the maximum temperature. Taking $\bar{\gamma}/r_0 \approx 140$ MeV $\cdot (M/M_\odot)(10 \text{ km}/r_0)$, we obtain from (44)

$$\varepsilon_{\min} \simeq 0.01 \left(\frac{M_\odot}{M} \right)^{7/20} \left(\frac{r_0}{10 \text{ km}} \right)^{1/4},$$
$$T_\infty^{\max} \simeq 20 \text{ keV} \cdot \left(\frac{M}{M_\odot} \right)^{3/10} \left(\frac{10 \text{ km}}{r_0} \right)^{1/2}. \tag{45}$$

Thus, the characteristic temperatures of x-ray sources undergoing Eddington accretion of plasma onto a neutron star can be as high as 20 keV; they can probably be even higher if ε is less than $\varepsilon_{\min}$ (45). In the latter case, however, the contribution made by hydrodynamic pressure must be taken into account in the overall balance of forces. Note that Comptonized x-ray and gamma-ray spectra consistent with such temperatures have been observed on more than one occasion[7]—in particular, they have recently been observed coming from the source at the center of the Milky Way.[8]

To complete the picture presented by Eddington flow, we conclude by addressing the beaming of the radiative flux. For qualitative analysis, it is convenient to define two characteristic velocities:

$$u = \frac{J_{\rm ph}(r)}{N_{\rm ph}(r)} \equiv V_s \frac{y^2}{\mathscr{R}(y)}, \tag{46}$$

the effective photon transport velocity, and

$$w = \frac{S_{\rm ph}(r)}{U_{\rm ph}(r)} \equiv V_s \frac{y^2}{\mathscr{P}(y)} z(y), \tag{47}$$

the effective photon energy transport velocity.

In wide-angle scattering, i.e., when the distribution function exhibits only weak beaming, we have

$$w, u \ll c. \tag{48}$$

Equations (4) and (5) are no longer applicable when (48) breaks down.[5)]

Consider first the region with radius

$r \ll r_{cr} \sim 0.5 r_0/\varepsilon.$

Equations (30) and (16) then yield

$$w \simeq 9V_s\left(\frac{r\varepsilon}{r_0}\right)^2\left[1-\frac{r_0}{r}+\left(\frac{V}{V_s}\right)^2+5\frac{T_{ph}r_0}{\tilde{\gamma}}\right] \ll c,$$

$$u \simeq 2V_s\left(\frac{r\varepsilon}{r_0}\right)^2\frac{y^4}{e^{3y}-(1+3y+(9/2)y^2+(9/2)y^3)} \ll c,$$

i.e., the conditions for strong scattering are satisfied by a wide margin. Note in addition that near the surface of the star, the stricter inequality $w \ll u \ll c$ holds, implying an abrupt drop in luminosity while the total number of photons remains constant (see above). The ratio of the number density of photons to plasma particles is

$$\frac{N_{ph}}{N_e}=\frac{\mathscr{R}_g}{y^2}\frac{I_0^{ph}}{I_0}=\frac{\mathscr{P}g}{3y^2\chi} \gg 1. \qquad (49)$$

Equation (49) makes it clear that in the present setting of the problem there are always many more photons than electrons.

Now consider the region with

$r \gtrsim r_{cr},$

in which $w \approx u$; it is quite difficult then to satisfy (48), since V_s/c is not a terribly small number for a neutron star (it is of order 0.4–0.5). The requirement (48) breaks down when $r/r_{cr} > (c/V_s)^2$. Here this means that an isotropic photon distribution becomes a beamed one, i.e., the radiation is gradually transformed into a narrow flux of photons freely traversing an optically thin medium.

CONCLUSION

We have used kinetic theory to derive a closed set of quasihydrodynamic equations describing the interaction between an optically thick plasma flow and a strong radiation field during rapid accretion onto a neutron star. We obtained the steady-state solution for the spherically symmetric equations under conditions in which the energy of the accreting plasma flow is entirely converted into gamma rays, which then exert a significant back influence on the overall dynamics of the plasma flow.

We have treated in detail the case in which the accretion rate comes close to its maximum possible value, whereupon the luminosity far from the star approaches the Eddington limit. We have calculated the magnitude and spatial distribution of the fundamental parameters of the plasma flow and photon gas (plasma number density, flow velocity, photon number and energy density, gamma-ray flux, luminosity, and the temperature of all interacting particles). It is important to emphasize that all of the results, including all numerical values, have been obtained in an entirely self-consistent manner, using a single predetermined parameter ε, which characterizes the relative proximity to the Eddington limit.

As a result, we have shown that the characteristic scale of the plasma-radiation interaction region is inversely proportional to ε. The photon flux exerts a strong decelerating and thermally stabilizing influence on the plasma. All groups of interacting particles ultimately wind up at essentially the same temperature, which rises with distance from the star. The temperatures attained far from the star are approximately twice as high as those at its surface. The temperature also rises with decreasing ε, and can reach 20 keV or more.

APPENDIX

We present for the record equations describing the interaction of radiation with an optically thick plasma and generalizing Eqs. (4), (5), (7), and (9) to the nonstationary case with an arbitrary spatial distribution:

$$n(\mathbf{r};k)=n_0(\mathbf{r};k)+\mathbf{n}_1(\mathbf{r};k)\mathbf{k}/k,$$

$$\frac{\partial n_0}{\partial t}+\frac{c}{3}\operatorname{div}\mathbf{n}_1=\frac{N_e\sigma_{Th}}{k^2}\frac{\partial}{\partial k}\left\{\frac{k^4}{m}\left[n_0+n_0^2+\frac{1}{c}\left(T_e+\frac{mV_e^2}{3}\right)\frac{\partial n_0}{\partial k}\right]-\frac{k^3}{3}\mathbf{V}_e\mathbf{n}_1\right\}, \qquad (A1)$$

$$c\,\nabla n_0=-N_e\sigma_{Th}\left(c\mathbf{n}_1-\mathbf{V}_e k\frac{\partial n_0}{\partial k}\right), \qquad (A2)$$

$$\frac{\partial N_{ph}}{\partial t}+\operatorname{div}\mathbf{J}_{ph}=0, \qquad (A3)$$

$$\nabla N_{ph}=-\frac{3N_e\sigma_{Th}}{c}(\mathbf{J}_{ph}-\mathbf{V}_e N_{ph}), \qquad (A4)$$

$$\frac{\partial U_{ph}}{\partial t}+\operatorname{div}\mathbf{S}_{ph}=\frac{N_e\sigma_{Th}}{c}\left[\frac{4U_{ph}}{m}(T_e-T_{ph})-\mathbf{V}_e\left(\mathbf{S}_{ph}-\frac{4}{3}\mathbf{V}_e U_{ph}\right)\right], \qquad (A5)$$

$$\nabla U_{ph}=-\frac{3N_e\sigma_{Th}}{c}\left(\mathbf{S}_{ph}-\frac{4}{3}\mathbf{V}_e U_{ph}\right), \qquad (A6)$$

$$\frac{\partial N_{e,i}}{\partial t}+\operatorname{div}(N_{e,i}\mathbf{V}_{e,i})=0, \qquad (A7)$$

$$m_e\frac{d_e\mathbf{V}_e}{dt}+\frac{1}{N_e}\nabla(N_eT_i)=\frac{\sigma_{Th}}{c}\left(\mathbf{S}_{ph}-\frac{4}{3}\mathbf{V}_eU_{ph}\right)-e\mathbf{E}-\frac{e}{c}[\mathbf{V}_e\mathbf{B}]-\mathbf{R}, \qquad (A8)$$

$$m_i\frac{d_i\mathbf{V}_i}{dt}+\frac{1}{N_i}\nabla(N_iT_i)=-\nabla\Psi+e\mathbf{E}+\frac{e}{c}[\mathbf{V}_i\mathbf{B}]+\mathbf{R}, \qquad (A9)$$

$$\frac{3}{2}N_e\frac{d_eT_e}{dt}-T_e\frac{d_eN_e}{dt}=\frac{4\sigma_{\rm Th}U_{\rm ph}}{mc}N_e(T_{\rm ph}-T_e)+\frac{3m\nu_{ei}}{m_i}N_e(T_i-T_e)+\text{div}(\hat{\varkappa}_e\nabla T_e)+Q_e, \quad (A10)$$

$$\frac{3}{2}N_i\frac{d_iT_i}{dt}-T_i\frac{d_iN_i}{dt}=\frac{3m\nu_{ei}}{m_i}N_i(T_e-T_i)+\text{div}(\hat{\varkappa}_i\nabla T_i)+Q_i, \quad (A11)$$

where to extend the generality, we have included a number of terms in these equations that were neglected in (9). Here **B** and **E** are the magnetic and electric field strengths, Ψ is the gravitational potential, **R** is the frictional force between ions and electrons, $\hat{\varkappa}_{e,i}$ are the electron and ion thermal conductivities (tensors in general), $Q_{e,i}$ are additional heat sources or sinks (ohmic dissipation, etc.), and

$$\frac{d_{e,i}}{dt}\equiv\frac{\partial}{\partial t}+\mathbf{V}_{e,i}\nabla.$$

We have assumed here that even when (8a) holds, if we are dealing with a general case lacking spherical symmetry [see (8b)], the mean electron and ion velocity vectors $\mathbf{V}_e$ and $\mathbf{V}_i$ will frequently fail to coincide. Note that we have neglected derivatives with respect to time in the kinetic equation for the beamed part of the photon distribution function (A2), and likewise in the resultant Eqs. (A4) and (A6) for the momenta, since otherwise we would have exceeded the assumed accuracy dictated by (3).

We also draw attention to the fact that according to (A6) and (A8) the total force due to radiation pressure on electrons, apart from any dependence on the direction of the mean electron velocity $\mathbf{V}_e$ or the photon energy flux $\mathbf{S}_{\rm ph}$, always points along the gradient of the photon energy density $U_{\rm ph}$.

In the general case, the energy conservation law that replaces (12) takes the form

$$\frac{\partial}{\partial t}\left[N_i\left(\frac{m_iV_i^2}{2}+\frac{3}{2}T_i\right)+N_e\left(\frac{m_eV_e^2}{2}+\frac{3}{2}T_e\right)+U_{\rm ph}\right]+\text{div}\left[\mathbf{S}_{\rm ph}-\hat{\varkappa}_e\nabla T_e-\hat{\varkappa}_i\nabla T_i+\left(\frac{m_iV_i^2}{2}+\Psi+\frac{5}{2}T_i\right)N_i\mathbf{V}_i+\left(\frac{m_eV_e^2}{2}+\frac{5}{2}T_e\right)N_e\mathbf{V}_e\right]=Q_e+Q_i. \quad (A12)$$

[1)]The equations appropriate to the nonstationary case with arbitrary spatial dependence in presented in the Appendix.

[2)]Here we assume the hydrodynamic conditions $l_{e,i}/r\ll 1$, where $l_{e,i}$ is the mean free path of electrons or ions. Under these conditions, the distribution function of the bulk of the thermal ions and electrons is close to Maxwellian, with corresponding temperatures $T_{e,i}$.

[3)]Recall that the foregoing solution holds only for $\varepsilon y\gg\chi_{e,i}$ (19). When $\varepsilon y\lesssim\chi_{e,i}$, the hydrodynamic pressure of the plasma must be included in the overall balance of forces.

[4)]It must be borne in mind that the concept of the true photon temperature is defined, strictly speaking, only for a Planck distribution. For an arbitrary distribution function, the temperature makes sense only as a ratio of the appropriate moments [see (6)].

[5)]Using Eq. (35) for the plasma number density, it can easily be shown that the strong scattering condition (3) and the weak beaming condition (48) are equivalent.

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