Can an atom "be set in rotation"?

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We discuss the effect of light-polarization drag by a rotating medium (the rotational analog of the Fizeau effect) for radiation whose frequency is close to that of a resonant transition. We show that the two terms contributing to the effect, the dispersion term and the term caused by the Coriolis force, exactly balance each other in a rotating rarefied gas. The shift of the m-component energy levels caused by the Coriolis force can also redistribute the equilibrium populations, that is, cause a contribution of the type of a C-term by analogy with term classification scheme similar used to that in the Faraday-effect theory. In this sense only an atom whose ground state is degenerate in the m-components can be made to rotate.

1. INTRODUCTION

In a recent paper[1] the question was posed of whether an atom could be made to rotate. This, at first glance paradoxical, question emerged in connection with the effect of the luminiferous ether "being dragged" by a rotating medium, predicted by Fermi[2] in 1923 and measured in experiments by Jones[8] in 1976. The effect consists in the polarization plane being dragged when light propagates through a rotating medium.

From the viewpoint of theory, several terms contribute to the effect. One is purely electrodynamic. It was calculated by Fermi[2] and its origin is similar to the mechanism of the translational Fizeau effect. Suppose that linearly polarized light travels through a layer of thickness L of a rotating medium parallel to the rotation axis. The angle of rotation of the polarization plane is

$$\varphi_{\text{ed}} = \Omega t \alpha, \quad t = Ln/c, \quad \alpha = 1 - 1/n^2, \tag{1}$$

where t is time it takes light to travel through the layer of thickness L with a velocity c/n, α the Fresnel ether-drag coefficient, and Ω the angular velocity of rotation of the medium. The second term was calculated by Player[4] and allows for dispersion in the drag coefficient α:

$$\delta\alpha_{\text{disp}} = \frac{\omega}{n}\frac{dn}{d\omega}. \tag{2}$$

This correction appears because of the Doppler frequency shift: the impinging light with a circular polarization $\sigma = \pm 1$ and an initial frequency ω_0 in the reference frame in which the medium is at rest will have a frequency $\omega = \omega_0 + \sigma\Omega$.

The third term appears because the rotating reference frame is not inertial and has no analog for the translational Fizeau effect. In the first order in Ω this manifests itself in a Coriolis force. If nuclear motion and electron spin are ignored, the gyromagnetic ratio for the orbital motion of electrons is constant, $g = e/2m_ec$. In view of this, under

such (fairly stringent) constraints, the Coriolis force $\mathbf{F} = 2m\mathbf{v}\times\mathbf{\Omega}$ is equivalent to the Lorentz force $\mathbf{F} = e\mathbf{v}\times\mathbf{H}/c$, with $\mathbf{H} = 2mc\mathbf{\Omega}/e$ (the Larmor theorem). Hence, as shown in Ref. 5, in this approximation

$$\delta\alpha_{\text{Cor}} = \frac{2mc}{e}V\frac{c}{n}, \tag{3}$$

where V is the Verdet constant entering into the Faraday effect and characterizing the polarization rotation rate, which is proportional to the strength H of the external field: $\varphi_{\text{Farad}} = VHL$.

Both the dispersion and the Coriolis-Faraday contributions exceed near a strong resonance ($|\omega - \omega_{21}| \ll \omega$) the electrodynamic contribution by a factor equal to $\omega/|\omega - \omega_{21}|$, where ω_{21} is the frequency of an atomic or molecular transition. Hence, a straightforward idea is to use resonant gases. But it is exactly in this case that we cannot ignore spin-orbit coupling and the effect of nuclear motion on the electron shell.

The aim of the present paper is to study the rotational Fizeau effect for a special medium: a rarefied gas near a resonant transition. This takes into account all effects associated with the Coriolis force. The final result proves somewhat unexpected: the dispersion and Coriolis terms balance each other exactly. The only possibility of making an atom rotate is to have the Coriolis force redistribute the equilibrium populations of the m-components of the ground state. This is an analog of the "paramagnetic C-term" in the language used in classifying various contributions in the standard theory of the Faraday effect.[6,7]

2. THE CORIOLIS HAMILTONIAN

It is well known from the basic principles of mechanics[8] that the rotation of the reference frame can be allowed for in the first order by introducing the following correction to the Hamiltonian:

$$\delta H = -\mathbf{M}\cdot\mathbf{\Omega}, \tag{4}$$

where $M = M(r_i, p_i)$ is the total angular momentum vector of the system. Since we use the Hamiltonian form of equations (in contrast to the Lagrange form), M must be expressed in terms of the coordinates r_i and the generalized momenta p_i (instead of velocities).

To check the numerical factor and the sign in Eq. (4), we take the Hamiltonian

$$H(r,p) = U(r) + \frac{p^2}{2m} - M \cdot \Omega, \quad M = r \times p, \tag{5}$$

and show that the standard Hamilton equations

$$\dot{r} = \frac{\partial H}{\partial p}, \quad \dot{p} = -\frac{\partial H}{\partial r} \tag{6}$$

lead to the equations of Newton's second law in the presence of a Coriolis force. Indeed, combining Eqs. (5) and (6) yields

$$\frac{dr}{dt} = \frac{p}{m} + r \times \Omega, \quad \frac{dp}{dt} = -\frac{\partial U}{\partial r} + p \times \Omega. \tag{7}$$

We see that the generalized momentum $p = mv - mr \times \Omega$ differs from mv. Substituting the first equation of (7) into the second, we get

$$m \frac{d^2 r}{dt^2} = -\frac{\partial U}{\partial r} + 2mv \times \Omega, \tag{8}$$

which corresponds exactly to the case where the Coriolis force is taken into account and the centrifugal force, proportional to ω^2, is ignored.

Since the correction (4) to the Hamiltonian was obtained in Ref. 8 for the most general case, vector M must include on an equal basis all the following terms; namely, the orbital and spin angular momenta of the electrons, the orbital and spin angular momenta of the nuclei, etc. In what follows we employ the fact that for a rarefied gas the angular momentum M of the particle considered (an atom or molecule) remains constant between collisions.

3. ETHER DRAG BY A ROTATING MEDIUM

The contribution of the specified transition $\hbar\omega_{21}$ between two states of a particle to the refractive index of a rarefied gas, $n_\pm(\omega)$ for clockwise polarization $(\sigma = +1)$ or counterclockwise polarization $(\sigma = -1)$, is given by the formula

$$n_\pm(\omega) = 1 + 2\pi N \sum_{m_1} \frac{1}{g_1} \frac{|\langle 2, m_1 + \sigma | \hat{d} | 1, m_1 \rangle|^2}{\hbar(\omega_{21} - \omega - i\Gamma_{21})}. \tag{9}$$

Here $N(\text{cm}^{-3})$ is the density of the medium, Γ_{21} the homogeneous linewidth, $g_1 = 2J_1 + 1$ the degree of degeneracy of the ground state with total angular momentum J_1, and $m_1 = J_{1z}$. The notation $|\langle a | \hat{d} | b \rangle|^2$ stands for the scalar product $\langle a | \hat{d} | b \rangle \cdot \langle b | \hat{d} | a \rangle$, with $\hat{d}$ the Hermitian operator of the dipole-moment vector. The selection rules $m_2 = m_1 + \sigma = m_1 \pm 1$ automatically allow for the resonance contribution to n of circularly polarized light with a definite circular polarization, $\sigma = +1$ or $\sigma = -1$.

Equation (9) has been written for the case of a medium that is not rotating. Rotation can be taken into account perturbatively by introducing a first-order correction to the Hamiltonian.

$$\delta\hat{H} = -\hbar\Omega\hat{J}_z, \tag{10}$$

where for simplicity we have assumed that $\Omega = \Omega e_z$, with the light propagating in the direction specified by vector e_z. From (10) follow the first-order corrections to the energy levels:

$$\delta E_{1,m_1} = -\hbar\Omega m_1, \quad \delta E_{2,m_2} = -\hbar\Omega m_2. \tag{11}$$

The Coriolis shift of the energy levels given by (11) has two consequences. First, Zeeman splitting of the transition frequencies occurs:

$$\omega_{2,m_2,1,m_1} = \omega_{21}^{(0)} + \Omega(m_1 - m_2), \tag{12}$$

which for the case of a resonance with circularly polarized light, $m_2 = m_1 + \sigma$, yields

$$\omega_{21}^{(\sigma)} = \omega_{21}^{(0)} - \sigma\Omega. \tag{13}$$

The effect of Zeeman splitting on the denominator in (9) is similar to the A-term in the theory of the Faraday effect. Owing to the additional term (10) in the Hamiltonian, the wave functions $\langle 2, m_2 |$ and $| 1, m_1 \rangle$ in Eq. (9) with a definite value of J_z do not mix. Hence, the corrections to the refractive index similar to the B-term in the Faraday-effect theory are absent in our case of freely orienting particles.

Second, the effect of the splitting (11) of the energy levels on the refractive index (9) is due to the redistribution of the equilibrium populations over the m_1-sublevels of the ground state:

$$\frac{1}{g_1} \rightarrow \exp\left(-\frac{m_1\hbar\Omega}{k_BT}\right)\left[\sum_{m_1'} \exp\left(-\frac{m_1'\hbar\Omega}{k_BT}\right)\right]^{-1}, \tag{14}$$

with k_BT the temperature in energy units. To within terms of the first order in Ω, Eq. (14) assumes the form

$$\frac{1}{g_1} \rightarrow \frac{1}{g_1}\left(1 - \frac{m_1\hbar\Omega}{k_BT}\right). \tag{15}$$

The effect of redistribution over the sublevels is similar to the "paramagnetic C-term" in the Faraday-effect theory.[6,7] In the first order in Ω, the contributions described by Eqs. (13) and (15), respectively, can be studied independently. Together with these, we discuss the dispersion contribution in the same first order in Ω.

Let us consider the sum of two contributions, the Coriolis A-term, which is due to the Zeeman splitting (13), and the dispersion term. The unit vectors e_x' and e_y' of the rotating reference frame are related to the unit vectors e_x and e_y of the laboratory reference frame by the following formulas:

$$e_x = e_c\cos\Omega t - e_y'\sin\Omega t, \quad e_x = e_c\sin\Omega t + e_y'\cos\Omega t. \tag{16}$$

For clockwise-polarized light $[\sigma = +1$ and $e_+ = (e_x + ie_y) \times e^{-i\omega t}/\sqrt{2}]$, we can write in the rotating reference frame

$$E_+ = \frac{e'_x + ie'_y}{\sqrt{2}} e^{-i(\omega - \Omega)t}. \tag{17}$$

Similarly, for counterclockwise-polarized light [$e_- = (e_x - ie_y)e^{-i\omega t}/\sqrt{2}$], by going over to the rotating reference frame we get the factor $e^{-i(\omega + \Omega)t}$. This means that when measured in the reference frame rotating together with the medium the frequency of the light field acting on the medium is

$$\omega_{act} = \omega - \sigma\Omega. \tag{18}$$

Hence, simultaneous allowance for the rotational "Doppler effect" [Eqs. (17) and (18)] and the Coriolis force [Eq. (13)] yields new resonance denominators coinciding (!) with the initial denominators:

$$\omega_{21}^{\sigma} - \omega_{act} - i\Gamma_{21} = (\omega_{21}^{(0)} - \sigma\Omega) - (\omega - \sigma\Omega) - i\Gamma_{12}$$

$$\equiv \omega_{21}^{(0)} - \omega - i\Gamma_{12}. \tag{19}$$

Thus, we have discovered a remarkable feature: the two resonance contributions, the dispersion contribution and the Coriolis A-term, exactly balance each other. Such balance was predicted earlier by starting from other assumptions,[5] namely, from the Becquerel formula[9,10] linking the Verdet constant with dispersion. In itself the Becquerel formula is valid only in the spherically symmetric potential of an atom for particles with a fixed gyromagnetic ratio $e/2mc$, that is, for spinless electrons. The reasoning in the present paper shows that the balance between the Coriolis and dispersion terms also holds true for the contribution of the resonant transition in a rarefied gas, that is, for freely oriented atoms or molecules when such factors as spin, spin-orbit coupling, nuclear motion in a molecule, and others are taken into account.

The contribution of the C-term, that is, the variation of the equilibrium populations of the m-components, is, in order of magnitude,

$$\delta n \sim (n-1)\frac{\hbar\Omega}{k_B T}. \tag{20}$$

The ω-dependence of this term is the same as that of the electrodynamic term. In order of magnitude,

$$\frac{\varphi C}{\varphi_{ed}} \sim \frac{\omega}{k_B T/\hbar}. \tag{21}$$

The case of alkali atoms, where the ground state is spin-degenerate, $s = \frac{1}{2}$, requires special consideration. As

the Kramers theorem implies (see, e.g., Ref. 11), electrostatic Coulomb interaction cannot change the spin orientation in collisions. This means that the time it takes the populations to relax over the m_1-sublevels may prove to be much longer than the time of ordinary gas-kinetic relaxation. Hence, the onset of equilibrium populations may require considerable time after the vessel begins to rotate.

4. CONCLUSION

Generally speaking, the balance between strong resonant terms of order $\omega(\partial n/\partial\omega)$ in a rarefied gas is almost obvious. Indeed, in the time intervals between collisions an atom or molecule moves freely, that is, by inertia. In the course of these intervals the particle "feels" no rotation. In a rotating reference frame we must allow for two terms, the rotational (Doppler) shift of the light frequency [Eq. (18)] and the Coriolis precession of the angular momentum [Eqs. (11) and (13)]. On the other hand, in the laboratory reference frame the balance between these terms is almost obvious.

Thus, the answer to the question posed in Ref. 1 of whether an atom can be made to rotate is in the affirmative in the sense of the populations being redistributed over the m-components of the degenerate ground state. However, the redistribution results in a rather weak rotation of the plane of polarization of the transmitted light, even near resonance.

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