The energy spectrum and hole relaxation in tunnel-coupled double quantum wells

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We calculate the energy spectrum and tunneling relaxation of hole population in double quantum wells. We use an analytical approximation that describes the mixing of states of heavy (h) and light (l) holes in the case of a large difference in their effective masses ($m_h \gg m_l$). Not only do the effective masses corresponding to longitudinal motion made smaller by such mixing; the tunneling matrix element acquires a dependence on the $2D$ momentum p (the case of zero tunnel coupling could manifest itself here as p increases). The well-to-well relaxation rate for holes in the event of scattering on acoustic phonons is examined as a function of temperature and the splitting energy Δ for a pair of tunnel-coupled levels (the latter is determined by the parameters of the structure or the strength of the transverse electric field). The Δ-dependence of this rate is found to agree with the experimental data.

1. INTRODUCTION

Tunneling of electrons and holes has been observed in various types of heterostructures (through a single or double barrier, from a quantum well to a continuous spectrum) and in double quantum wells. Tunneling of photoexcited electrons in tunnel-coupled quantum wells is studied by optical methods (see literature cited in Refs. 1–3). Tunneling relaxation of holes has also been observed in such experiments,[4–8] with this process considerably slower.

In describing hole tunneling, as in the case of passage of holes through a barrier,[9–11] one must allow for the mixing of states of the h- and l-subbands, which noticeably alters the results of a numerical calculation[12] that takes into account the process of well-to-well tunneling relaxation of holes in which impurities participate. This paper studies analytically, in the $m_l \ll m_h$ approximation, the process of well-to-well tunneling relaxation of holes in which acoustic phonons participate, a process predominant in unalloyed structures at moderate excitation levels. Here the drastic transformation of the spectrum, the nature of tunneling in double quantum wells, and the transition-matrix element (in view of the multicomponent nature of the hole wave functions) lead to a noticeable modification of the relaxation process.

Describing quantum-size holes is an extremely cumbersome procedure even in the case of a single quantum well,[13] owing to the mixing, at finite longitudinal $2D$ momenta p, of the wave-function components corresponding to the $\pm\frac{3}{2}$ and $\pm\frac{1}{2}$ angular momenta. The mixing dramatically changes the dispersion law of the ground state in the low-momentum range $p \leqslant \sqrt{m_l/m_h}(\hbar/d_{\mathrm{L,R}})$, where $d_{\mathrm{L,R}}$ are the widths of the left (L) and right (R) quantum wells. This fact, noted earlier for quantum-size films,[14] calls for numerical calculations to describe the energy spectrum, scattering, or optical transitions of $2D$-holes. Building an analytical theory for such processes proves possible in the case of markedly different effective masses of h- and l-holes in narrow-gap double quantum wells. Here the contribution of states with angular momenta $\pm\frac{1}{2}$ is taken into account by allowing for the low-energy asymptotic behavior of the Green function of l-holes. As a result the spin-degenerate wave function of the ground state of l-holes can be found explicitly from a one-component Schrödinger equation in which the l-contribution is described by an integral term. The energy spectrum ε_p in the low-momentum range is obtained from a simple transcendental equation describing the transition from the parabolic dispersion law (with a mass of the order of m_l) for $p \ll \sqrt{m_l/m_h}(\hbar/d_{\mathrm{L,R}})$ to the spectrum with a heavy mass for $p \geqslant \sqrt{m_l/m_h}(\hbar/d_{\mathrm{L,R}})$. Subbarrier penetration by the h-component of the wave function is determined by the characteristic length $\varkappa_h^{-1}$, where $\hbar\varkappa \simeq \sqrt{2m_h U_v}$, with U_v the valence-band gap at the heterobarrier, and the l-component contribution decreases under the barrier over a distance of the scale $\varkappa_l^{-1}$, where $\hbar\varkappa_l \simeq \sqrt{2m_l U_v}$.

Using these solutions as a basis in the L and R tunnel-coupled quantum wells makes it possible not only to find the energy spectrum and the wave functions in an approximation similar to the one used in Refs. 3 and 15 (here the tunneling matrix element proves to be p-dependent and can vanish) but also to write simple expressions for the matrix elements representing the interaction of holes with acoustic phonons (earlier in Ref. 16 only numerical results were obtained, and those pertained to a single quantum well). We write below concentration balance equations for photoexcited holes and obtain an expression for the time of well-to-well tunneling relaxation caused by DA interaction with acoustic phonons (for definiteness we assume that $d_{\mathrm{L}} < d_{\mathrm{R}}$, with the hole going from the left well to the right). We restrict our discussion to the case of low hole concentrations, so that Coulomb relaxation is insignificant owing to interparticle scattering or scattering on acceptors. Since in what follows we allow only for the coupling of a pair of the lowest hole states of the left and right quantum wells, the parabolic approximation can be used to describe the energy spectra of these states. We consider the situation

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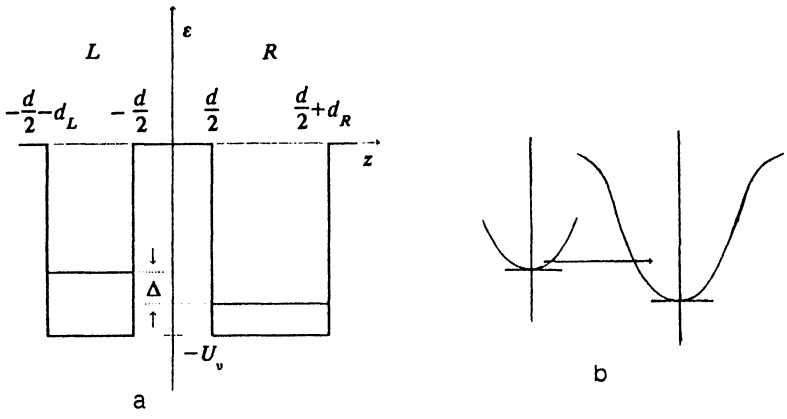

FIG. 1. (a) The energy structure of a double quantum well. (b) The spectrum of holes in the lowest subbands of the L and R quantum wells, and well-to-well relaxation.

in which processes involving optical phonons are forbidden (the conditions imposed on the level separation Δ and temperature T are similar to those discussed in Ref. 3 for the case of electron relaxation). Here the matrix elements representing the transition between hole states in double quantum wells differ from similar expressions for electrons because of the contribution of the l-components of the wave function. This leads to a complex dependence of the rate of tunneling relaxation on the average hole energy and on the level energy gap Δ, whose variation is possible in structures placed in a transverse electric field.

2. LOW-ENERGY HOLE STATES IN DOUBLE QUANTUM WELLS

Hole states in tunnel-coupled double quantum wells are described in the $(\mathbf{p},z)$-representation by a four-row wave function $\psi_{\mathbf{p}z}^{1-4}$. It has also been found expedient to isolate in the Schrödinger equation the components $\psi_{\mathbf{p}z}^{1,4}$ and $\psi_{\mathbf{p}z}^{2,3}$ describing, at $\mathbf{p}=0$, the spin-degenerate states of the h- and l-holes, respectively. For these components in the isotropic approximation we have the following system of equations:

$$\left[\frac{\hat{p}_z^2}{2m_h}+\frac{p^2}{8}\left(\frac{3}{m_l}+\frac{1}{m_h}\right)+U(z)-E\right]\begin{bmatrix}\psi_{\mathbf{p}z}^1\\ \psi_{\mathbf{p}z}^4\end{bmatrix}$$
$$=\frac{\sqrt{3}}{8}\left(\frac{1}{m_l}-\frac{1}{m_h}\right)\begin{bmatrix}2i\hat{p}_z p_- & -p_-^2\\ -p_+^2 & 2i\hat{p}_z p_+\end{bmatrix}\begin{bmatrix}\psi_{\mathbf{p}z}^2\\ \psi_{\mathbf{p}z}^3\end{bmatrix}, \quad (1)$$

$$\left[\frac{\hat{p}_z^2}{2m_l}+\frac{p^2}{8}\left(\frac{1}{m_l}+\frac{3}{m_h}\right)+U(z)-E\right]\begin{bmatrix}\psi_{\mathbf{p}z}^2\\ \psi_{\mathbf{p}z}^3\end{bmatrix}$$
$$=\frac{\sqrt{3}}{8}\left(\frac{1}{m_l}-\frac{1}{m_h}\right)$$
$$\times\begin{bmatrix}-2i\hat{p}_z p_+ & -p_-^2 & -p_+^2 & -2i\hat{p}_z p_-\end{bmatrix}\begin{bmatrix}\psi_{\mathbf{p}z}^1\\ \psi_{\mathbf{p}z}^4\end{bmatrix}, \quad (2)$$

in which $p_\pm=(p_x\pm ip_y)$ and $\hat{p}_z=-i\hbar(d/dz)$. The potential energy $U(z)=U_L(z)U_R(z)$ is determined by the position of the extrema in the v-band and describes a barrier of width d between the L and R quantum wells of width d_L and d_R, with the band gap at heterojunctions equal to U_v (see the energy diagram in Fig. 1).

We exclude the l-component of the wave function from this system of equations by employing a Green function $G_\zeta(z,z')$ defined by the equation

$$\left[\frac{\hat{p}_z^2}{2m_l}+U(z)+U_v-\zeta\right]G_\zeta(z,z')=\delta(z-z'),$$

$$\zeta=E-\frac{p^2}{8}\left(\frac{1}{m_l}+\frac{3}{m_h}\right)+U_v. \quad (3)$$

Considering the case of small $2D$ momenta, $p\ll\pi\hbar/d_{L,R}$, we discard in the right-hand sides of Eqs. (1) and (2) the off-diagonal matrix elements proportional to p^2 and arrive at identical equations for the h-components $\psi_{\mathbf{p}z}^1$ and $\psi_{\mathbf{p}z}^4$ satisfying Eq. (1), so that the spin-degenerate states are described by the following column matrices:

$$\begin{bmatrix}\varphi_{pz}^h\\ \frac{ip_+}{p}\varphi_{pz}^l\\ 0\\ 0\end{bmatrix},\quad \begin{bmatrix}0\\ 0\\ \frac{ip_-}{p}\varphi_{pz}^l\\ \varphi_{pz}^h\end{bmatrix}, \quad (4)$$

with the wave function φ_{pz}^h being the solution of the integro-differential equation

$$\left[\frac{\hat{p}_z^2}{2m_h}+\frac{p^2}{8}\left(\frac{3}{m_l}+\frac{1}{m_h}\right)+U(z)-E\right]\varphi_{pz}^h$$
$$=\frac{3p^2}{16}\left(\frac{1}{m_l}-\frac{1}{m_h}\right)^2$$
$$\times\int dz'\hat{p}_z G_{zeta}(z,z')\hat{p}_{z'}\varphi_{pz'}^h, \quad (5)$$

and φ_{pz}^l related to φ_{pz}^h by the formula

$$\varphi_{pz}^l=-\frac{\sqrt{3}}{4}\left(\frac{1}{m_l}-\frac{1}{m_h}\right)p\int dz' G_\zeta(z,z')\hat{p}_{z'}\varphi_{pz'}^h. \quad (6)$$

It has proved expedient to transform the right-hand side of Eq. (5) by isolating the integral term with the kernel $[\hat{p}_z\hat{p}_{z'}G(z,z')]$. In the process the contribution proportional to p^2 transforms into $p^2/2m_1$, in which $m_1\simeq\frac{4}{7}m_h$

when $m_h \gg m_l$. Bearing in mind the low-energy asymptotic value of the Green function, $G_0(z,z')$, we transform Eq. (5) to

$$\left[\frac{\hat{p}_z^2}{2m_h}+\frac{p^2}{2m_l}+U(z)-E\right]\varphi_{pz}^h+\frac{3p^2}{8m_l}\int dz'\Phi(z,z')\varphi_{pz'}^h=0, \tag{7}$$

where

$$\Phi(z,z')=\frac{1}{2m_l}[\hat{p}_z\hat{p}_{z'}G_0(z,z')]. \tag{8}$$

The fact that here we use $G_0(z,z')$ presupposes that $\zeta \ll \pi^2\hbar^2/2m_l d_{\mathrm{L,R}}^2$, which limits the energy interval considered.

When describing a pair of adjacent tunnel-coupled levels corresponding to the ground states of holes in the L and R wells, we expand the solution to (7) in the basis of L- and R-orbitals:

$$\psi_{\mathrm{L}p}\varphi_{pz}^{(\mathrm{L})}+\psi_{\mathrm{R}p}\varphi_{pz}^{(\mathrm{R})}. \tag{9}$$

The functions $\varphi_{pz}^{(i)}$ $(i=\mathrm{L,R})$ satisfy the equations

$$\left[\frac{\hat{p}_z^2}{2m_h}+\frac{p^2}{2m_1}+U_i(z)-E_{ip}\right]\varphi_{pz}^{(i)}+\frac{3p^2}{8m_l}\int dz'\Phi(z,z')\varphi_{pz'}^{(i)}=0, \tag{10}$$

where $U_i(z)$ is the potential of the ith single well. Note that the orbitals introduced in this manner do not coincide at $p\neq 0$ with the wave functions of single quantum wells, since the kernel $\Phi(z,z')$ is linked [see Eq. (3)] to the Green function of a double-well structure. Substituting expansion (9) into Eq. (7), we arrive at an equation for the column of coefficients $\psi_{\mathrm{L}p}$ and $\psi_{\mathrm{R}p}$:

$$\begin{bmatrix}\frac{\Delta}{2}+\varepsilon_{\mathrm{L}p} & T_p\\ T_p & -\frac{\Delta}{2}+\varepsilon_{\mathrm{R}p}\end{bmatrix}\begin{bmatrix}\psi_{\mathrm{L}p}\\ \psi_{\mathrm{R}p}\end{bmatrix}=E\begin{bmatrix}\psi_{\mathrm{L}p}\\ \psi_{\mathrm{R}p}\end{bmatrix}, \tag{11}$$

where the energy E is measured (this convention is also employed below) from the point in the middle of the gap between the L and R levels, whose separation $\Delta = E_{\mathrm{L}p=0} - E_{\mathrm{R}p=0}$ is calculated using the values $E_{\mathrm{L}p}$ and $E_{\mathrm{R}p}$ obtained from the dispersion laws for the L and R wells by solving Eq. (10). In (11) we also used the dispersion laws for the L and R wells evaluated from the extrema of these states, $\varepsilon_{ip}=E_{ip}-E_{ip=0}$. The matrix element T_p describing the tunnel mixing of the L- and R-orbitals is defined as

$$T_p=\tfrac{1}{2}\int dz\varphi_{pz}^{(\mathrm{L})}[U_{\mathrm{L}}(z)+U_{\mathrm{R}}(z)]\varphi_{pz}^{(\mathrm{R})} \tag{12}$$

and is found to depend on the $2D$ momentum.

Equation (11) has been obtained on the basis of the usual assumptions of the smallness of Δ (in comparison to the distance to the next levels) and the tunneling exponential $\exp(-\varkappa_h d)$. Its solutions, which describe the hybridized states $|+\rangle$ and $|-\rangle$, are

$$\begin{aligned}|+\rangle&=N\begin{bmatrix}1\\ \frac{2T_p}{\Delta_p+\tilde{\Delta}_p}\end{bmatrix},\\ |-\rangle&=N\begin{bmatrix}-\frac{2T_p}{\Delta_p+\tilde{\Delta}_p}\\ 1\end{bmatrix},\quad N^2=\frac{\tilde{\Delta}_p+\Delta_p}{2\tilde{\Delta}_p},\\ E_{\pm p}&=\frac{\varepsilon_{\mathrm{R}p}+\varepsilon_{\mathrm{L}p}}{2}\pm\frac{\tilde{\Delta}_p}{2},\\ \Delta_p&=\Delta-(\varepsilon_{\mathrm{R}p}-\varepsilon_{\mathrm{L}p}),\quad \frac{\tilde{\Delta}_p}{2}=\sqrt{\left(\frac{\Delta_p}{2}\right)^2+T_p^2}.\end{aligned} \tag{13}$$

Here Δ_p and $\tilde{\Delta}_p$ are the initial and renormalized (due to tunneling) separations between the levels. The fact that the dispersion laws are not parabolic is due not only to the kinetic energies ε_{ip} but also to the variation of $\tilde{\Delta}_p$ with the $2D$ momentum. Note that when $\Delta \gg T_p$, the $|+\rangle$ states are localized mainly in the left well and the $|-\rangle$ states in the right.

Thus, to describe tunnel-coupled hole states in double quantum wells, we must find the kernel (8) $\Phi(z,z')$ defined in by solving Eq. (3), solve the eigenvalue problem (10), and calculate the tunneling matrix elements T_p according to (12).

3. CALCULATING THE PARAMETERS OF THE TUNNELING HAMILTONIAN

We calculate the dispersion laws for ε_{ip} and the tunneling matrix element T_p, which enter into the two-level problem (11), for the low-momentum region

$$p \ll \sqrt{\frac{m_l}{m_h}}\frac{\hbar}{d_{\mathrm{L,R}}}. \tag{14}$$

In this approximation the solutions to Eqs. (10) assume the form

$$\varphi_{pz}^{(i)}=\sqrt{\frac{2}{d_i}}\begin{cases}\frac{\pi}{\varkappa_h d_i}\exp\left[\varkappa_h\left(z_i+\frac{d_i}{2}\right)\right]-\frac{3p^2}{8m_l U_v}\int_{-d_i/2}^{d_i/2}dz_i'\Phi(z,z')\cos\frac{\pi z_i'}{d_i} & \text{if } z_i<-\frac{d_i}{2},\\ \cos k_i z_i \quad \text{if } -\frac{d_i}{2}<z_i<\frac{d_i}{2},\\ \frac{\pi}{\varkappa_h d_i}\exp\left[-\varkappa_h\left(z_i-\frac{d_i}{2}\right)\right]-\frac{3p^2}{8m_l U_v}\int_{-d_i/2}^{d_i/2}dz_i'\Phi(z,z')\cos\frac{\pi z_i'}{d_i} & \text{if } z_i>\frac{d_i}{2},\end{cases} \tag{15}$$

where $z_L=z_L(z)=z+(d+d_L)/2$, and $z_R=z_R(z)=z-(d+d_R)/2$. In the subbarrier regions we have retained the contributions that are proportional to p^2 and contain slowly decreasing exponentials. Such contributions determine the subbarrier penetration by the l-components of the wave function, which is described by the integral term in (10). The transverse wave number k_i is determined from the transcendental dispersion equation $\tan(k_i d_i/2)=\varkappa_h/k_i$ (here the fact that $\varkappa_h d_i \gg 1$ implies that $k_i \simeq \pi/d_i$). The dispersion laws can be found from (10) in the first order in the integral contribution (in this approximation they are found to be parabolic),

$$\varepsilon_{ip}=E_{ip}-E_{ip=0}$$
$$=\frac{p^2}{2m_i}+\frac{3}{8}\frac{p^2}{m_l}\int dz\int dz'\varphi_{pz}^{(i)}\Phi(z,z')\varphi_{pz'}^{(i)}$$
$$\simeq\frac{3}{8}\frac{p^2}{m_l}\frac{2}{d_i}\int_{-d_i/2}^{d_i/2}dz_i\int_{-d_i/2}^{d_i/2}dz_i'\cos\frac{\pi z_i}{d_i}\Phi(z,z')$$
$$\times\cos\frac{\pi z_i'}{d_i}\equiv\frac{p^2}{2m_i^*}, \qquad (16)$$

and for the tunneling matrix element we obtain, by substituting the orbitals (15) into (12), the following:

$$T_p=-\frac{2U_v}{\sqrt{d_L d_R}}\int_{-d_L/2}^{d_L/2}dz_L\cos\frac{\pi z_L}{d_L}$$
$$\times\left\{\frac{\pi}{\varkappa_h d_R}\exp\left[\varkappa_h\left(z_L-\frac{d_L}{2}-d\right)\right]\right.$$
$$\left.-\frac{3p^2}{8m_l U_v}\int_{-d_R/2}^{d_R/2}dz_R'dz_R'\Phi(z,z')\cos\frac{\pi z_R'}{d_R}\right\}. \qquad (17)$$

Now we employ the fact that the kernel $\Phi(z,z')$ required to calculate (16) and (17) proves to be independent of z and z' both for values of z and z' in the same well (case I) and for values in different wells (case II):

$$\Phi(z,z')=\begin{cases}\dfrac{c_i}{d_i} & \text{(I)},\\ \dfrac{2\tilde{c}\exp(-\varkappa_l d)}{\varkappa_l d_L d_R} & \text{(II)},\end{cases} \qquad (18)$$

where the constants c_L, c_R, and $\tilde{c}$ are given in the Appendix. These constants become equal to unity when $\varkappa_l d_{L,R}\gg 1$. The integrals in (16) and (17) can easily be evaluated. The effective mass m_i^* is defined by the expression $m_i^*/m_l=\pi^2/6c_i$, and for the tunneling matrix element we have

$$T_p=-\frac{4\pi^2 U_v}{(\varkappa_h\bar{d})^3}\left[e^{-\varkappa_h d}-\frac{3p^2\bar{d}^2}{\pi^4\hbar^2}\left(\frac{m_h}{m_l}\right)^{3/2}\tilde{c}e^{-\varkappa_l d}\right],$$
$$\bar{d}=\sqrt{d_L d_R}. \qquad (19)$$

The heavy-hole contribution defined by the first term on the right-hand side of (19) differs from the electron contribution[3,15] in the substitution of m_h for the electron effective mass.[1)] The second term can dominate because in addition to a small factor $(p\bar{d}/\hbar)^2$ it contains a large mass ratio and an exponential factor that decreases less rapidly as d grows. The difference in sign between these two contributions is determined by the fact that in the subbarrier regions the orbitals (15) consist of two terms with opposite signs (the kernel $\Phi(z,z')$ proves to be positive everywhere). Hence, T_p vanishes at a certain value of the longitudinal momentum.

Note that both the emergence of nodes at the tails of the ground-state wave functions and the dependence of the coefficients c_L and c_R on the widths of, respectively, the right and left wells (this dependence becomes essential when $\varkappa_l\sim 1$) are due to the contribution of the mixing of l-states to the formation of the ground state described by the integral term in Eq. (10).

4. WELL-TO-WELL TRANSITION PROBABILITIES

When considering transitions between the $|\pm\rangle$ states defined in (13), we must know the transition matrix element reflecting the interaction of holes with acoustic phonons. In the deformation-potential approximation such an interaction is described by the product of the 4-by-4 deformation-potential matrix D_{ij} and the deformation tensor. Expanding this product in phonon modes yields[17]

$$\sum_{\mathbf{Q}\lambda}\sqrt{\frac{\hbar Q}{2\rho s_\lambda V}}\{\mathscr{D}_{\mathbf{Q}\lambda}\hat{b}_{\mathbf{Q}\lambda}\exp[i(\mathbf{q}\cdot\mathbf{r}_\perp+q_z z)]+\text{H.c.}\}, \qquad (20)$$

$$\mathscr{D}_{\mathbf{Q}\lambda}=\frac{i}{2Q}\sum_{ij}D_{ij}(e_{\mathbf{Q}\lambda}^i Q_j+e_{\mathbf{Q}\lambda}^j Q_i), \qquad (21)$$

where $\mathbf{Q}=(\mathbf{q},q_z)$ is the phonon wave vector, λ numbers the phonon modes, ρ is the density of the crystal, V the normalization volume, s_λ the speed of sound of the λth mode, $\hat{b}_{\mathbf{Q}\lambda}$ the annihilation operator, and $e_{\mathbf{Q}\lambda}^i$ the λth mode polarization vector. The expression for matrix D_{ij} can be found in Ref. 17.

The probability of the transition between states $|+\mathbf{p}\rangle$ and $|-\mathbf{p}'\rangle$ due to the contribution of the λth phonon mode to (20) is given by the following expression:

$$W_\lambda(+\mathbf{p}|-\mathbf{p})=\frac{\pi}{\rho s_\lambda^2 V}\sum_{q_z\sigma\sigma'}|\langle+\sigma\mathbf{p}|\mathscr{D}_{\mathbf{Q}\lambda}\exp(iq_z z)$$
$$\times|-\sigma'\mathbf{p}'\rangle|^2\omega_{\mathbf{Q}\lambda}\coth\frac{\hbar\omega_{\mathbf{Q}\lambda}}{2T_{\text{ph}}}$$
$$\times\delta(E_{+p}-E'_{-p}), \qquad (22)$$

where we have summed over the spin variables, $\omega_{\mathbf{Q}\lambda}=s_\lambda Q$ is the phonon frequency, T_{ph} the phonon temperature, and $\mathbf{q}=(\mathbf{p}-\mathbf{p}')/\hbar$ due to conservation of the $2D$ momentum. In Eq. (22) we used the elastic approximation in describing scattering, which is justified if the characteristic phonon energy $\hbar s_\lambda/d_i$ is much lower than $\tilde{\Delta}_p$. Here the characteristic wave number q_z proves to be of the order of $\bar{d}^{-1}$. Since in absolute value the longitudinal wave vector

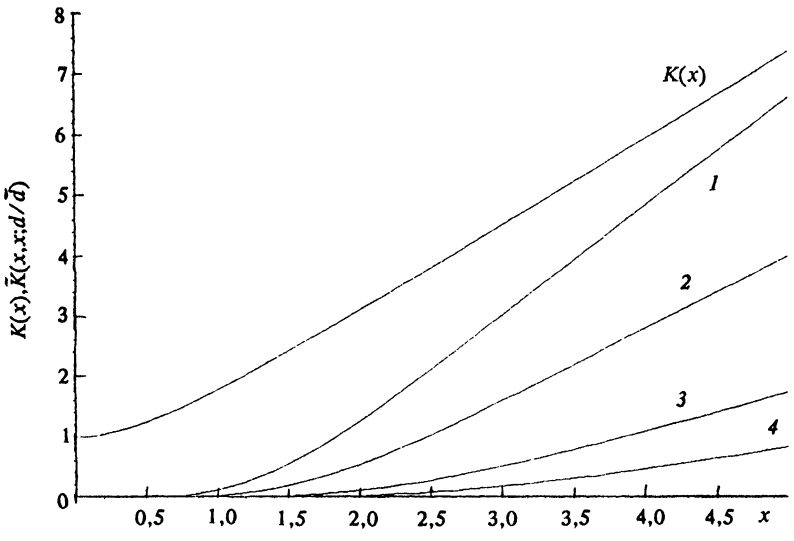

FIG. 2. Graphs of $K(x)$ and $\tilde{K}(x,x;d/\bar{d})$. Curves 1, 2, 3, and 4 correspond to the function $\tilde{K}(x,x;d/\bar{d})$ (magnified tenfold) at $d/\bar{d}=0.25$, 0.5, 1.0, and 1.5, respectively.

$\mathbf{q}=(\mathbf{p}-\mathbf{p}')/\hbar$ is much smaller than $\bar{d}^{-1}$, calculation of the transition probability can be simplified if we assume that $q \ll q_z$. Here the matrices (21) for one longitudinal mode (LA) and two transverse modes ($TA_{1,2}$) are

$$\mathscr{D}_{\mathbf{Q}\,\mathrm{LA}} \simeq i \begin{pmatrix} a-b & 0 & 0 & 0 \\ 0 & a+b & 0 & 0 \\ 0 & 0 & a+b & 0 \\ 0 & 0 & 0 & a-b \end{pmatrix},$$

$$\mathscr{D}_{\mathbf{Q}\,\mathrm{TA}_1} \simeq \frac{id}{2q} \begin{pmatrix} 0 & -q_- & 0 & 0 \\ -q_+ & 0 & 0 & 0 \\ 0 & 0 & 0 & q_- \\ 0 & 0 & q_+ & 0 \end{pmatrix}, \tag{23}$$

$$\mathscr{D}_{\mathbf{Q}\,\mathrm{TA}_2} \simeq \frac{d}{2q} \begin{pmatrix} 0 & -q_- & 0 & 0 \\ q_+ & 0 & 0 & 0 \\ 0 & 0 & 0 & q_- \\ 0 & 0 & -q_+ & 0 \end{pmatrix},$$

where the deformation constants describe the displacement of the valence-band extremum due to hydrostatic deformation (a), and its splitting under uniaxial compression along a cubic axis (b) or along the cube's diagonal (d). The unit polarization vectors in (23) are selected as follows: $\mathbf{e}_{\mathrm{LA}} \| \mathbf{Q}$, $\mathbf{e}_{\mathrm{TA}_1} \perp \mathbf{e}_{\mathrm{LA}}, z$, and $\mathbf{e}_{\mathrm{TA}_2} \perp \mathbf{e}_{\mathrm{LA}}, \mathbf{e}_{\mathrm{TA}_1}$. In the approximation adopted here, as (23) clearly shows, the LA-mode does not mix the l- and h-components of the hole states, and the TA-mode mixes the components φ^h_{pz} and $(ip_\pm/)\varphi^l_{pz}$ of the column matrices (4). In this process the transition probabilities acquire additional small factors $(p\bar{d}/\hbar)^2$, and in the situation considered the transverse modes do not contribute much. (These modes can make a sizable contribution at $p\bar{d}/\hbar \sim 1$, as they do in the case of a single quantum well.[18])

The probability $W_{\mathrm{LA}}(+\mathbf{p}|-\mathbf{p}')$ defined in (22) is expressed in terms of the wave functions of the hybridized h-states obtained from (9) in the following manner:

$$W_{\mathrm{LA}}(+\mathbf{p}|-\mathbf{p}') \simeq \frac{T_{\mathrm{ph}}(a-b)^2}{\hbar\rho s^2_{\mathrm{LA}} S} \int dz \int dz' \times \phi(z-z')\varphi^{h,+*}_{pz}\varphi^{h,+}_{pz'}\varphi^{h,-}_{p'z}\varphi^{h,-*}_{p'z'} \times G(E_{+p}-E_{-p'}), \tag{24}$$

$$\phi(z-z') = \int dq_z \frac{\hbar\omega_{q_z}}{2T_{\mathrm{ph}}} \coth \frac{\hbar\omega_{q_z}}{2T_{\mathrm{ph}}} \exp[iq_z(z-z')],$$

with S the normalization area. Substituting the wave functions defined in (9), (10), and (13) into (24) and restricting our discussion to the approximation $(T_p/\Delta)^2 \ll 1$, we arrive at an expression for W_{LA} in the form

$$W_{\mathrm{LA}}(+\mathbf{p}|-\mathbf{p}') \simeq \frac{3\pi T_{\mathrm{ph}}(a-b)^2}{\hbar\rho s^2_{\mathrm{LA}} S\Delta^2} \left\{ \frac{T^2_{p'}}{d_{\mathrm{L}}} K\left(\frac{u_{\mathrm{L}}}{T_{\mathrm{ph}}}\right) + \frac{T^2_p}{d_{\mathrm{R}}} K\left(\frac{u_{\mathrm{R}}}{T_{\mathrm{ph}}}\right) + \frac{T_{p'}T_p}{\sqrt{d_{\mathrm{L}}d_{\mathrm{R}}}} \times \tilde{K}\left(\frac{u_{\mathrm{L}}}{T_{\mathrm{ph}}}, \frac{u_{\mathrm{R}}}{T_{\mathrm{ph}}}; \frac{d}{\bar{d}}\right) \right\} \delta(E_{+p}-E_{-p'}), \tag{25}$$

where $u_i = \hbar s_{\mathrm{LA}}/d_i$, and the functions $K(x)$ and $\tilde{K}(x_1,x_2;d/\bar{d})$ after integration over z, z', and q_z by the residue method can be represented in the form of rapidly converging series. For $x, x_{1,2} \ll 1$ (high temperatures, when the phonons are uniformly distributed), $K(x) \simeq 1$ and $\tilde{K}(x_1,x_2;d/\bar{d}) \simeq 0$. For $x, x_{1,2} \gg 1$ (zero-point oscillations) these functions exhibit a linear asymptotic behavior, with $K(x) \simeq 1.47x$. The graphs of $K(x)$ and $\tilde{K}(x_1,x_2;d/\bar{d})$ at $x_1=x_2=x$ (the approximation of quantum wells with nearly equal widths) are depicted in Fig. 2. The function $\tilde{K}(x,x;d/\bar{d})$ decreases as the ratio $d/\bar{d}$ increases.

Note that since the final-state momentum p' is of the order of $\sqrt{2m_l\Delta}$ (as the law of energy conservation

$E_{+p}=E_{-p'}$ implies), the second term in the expression for T_p [see Eqs. (19) and (A8)] proves to be of the order of $\Delta\exp(-\varkappa_l d)$. Hence, in conditions of elastic scattering the strong inequality $(T_p/\Delta)^2\ll 1$, which is used here and below and means that the overlap of the wave functions of the states $|+\rangle$ and $|-\rangle$ is small, can be satisfied only if $\exp(-\varkappa_l d)\ll 1$, that is, when the tunnel coupling is weak not only with respect to heavy holes but also with respect to light holes.

5. TUNNELING RELAXATION TIMES

When the overlap of the states in the left and right quantum wells is small, quasiequilibrium intrawell distributions $f_{\pm p}(t)$ rapidly set in the wells. These distributions depend on the hole temperature T_h (which we assume to be the same for both wells) and the Fermi quasilevels of the wells determined by the concentrations $n_\pm(t)$. The slow relaxation of the populations $n_\pm(t)$ is determined by the well-to-well transition rate

$$\left(\frac{\partial\Delta_n}{\partial t}\right)_T=\frac{2}{S}\sum_{pp'}W_{LA}(+\mathbf{p}|-\mathbf{p}')[f_{-p}(t)-f_{+p}(t)], \tag{26}$$

in terms of which we can write the concentration balance equation

$$\frac{dn_+(t)}{dt}=-\frac{dn_-(t)}{dt}=-\left(\frac{\partial\Delta n}{\partial t}\right)_T, \tag{27}$$

with the total hole concentration $n=n_+(t)+n_-(t)$ constant. Assuming that the hole gas is nondegenerate, we introduce instead of (26) the well-to-well tunneling relaxation rate ν according to the relation

$$\left(\frac{\partial\Delta_n}{\partial t}\right)_T=-\nu n_+(t)\left(1-\exp\frac{\mu_{-t}-\mu_{+t}-\Delta}{T_h}\right), \tag{28}$$

where μ_{-t} and μ_{+t} are the Fermi quasilevels of the wells in the right and left quantum wells, and obtain an explicit expression for ν. Here we bear in mind that the hole spectra for $(T_p/\Delta)^2\ll 1$ and $\exp(-\varkappa_l d)\ll 1$ (see above) are

$$E_{+p}=\frac{\Delta}{2}+\frac{p^2}{2m_L^*},\quad E_{-p'}=-\frac{\Delta}{2}+\frac{p'^2}{2m_R^*},$$

$$m_i^*=m_l\frac{\pi^2(\varkappa_l d_i+2)}{6\varkappa_l d_i},\quad i=L,R. \tag{29}$$

Summing over $\mathbf{p}$ and $\mathbf{p}'$ yields

$$\nu=\frac{\pi^2 T_{ph}(a-b)^2}{2\hbar\rho s_{LA}^2 U_v\, d_R d_L^2}\left(\frac{\varkappa_l d_R+2}{\varkappa_l d_R}\right)\times\left(\frac{\varkappa_l d_L}{\varkappa_l d_L+2}\right)^2\left\{\left(\frac{t_0}{\Delta}\right)^2\eta_h\exp(-2\varkappa_h d)-\frac{2t_0}{\Delta}\eta_{hl}\exp[-(\varkappa_h+\varkappa_l)d]+\eta_l\exp(-2\varkappa_l d)\right\}, \tag{30}$$

where t_0 appeared because of the dimensional factor in the tunneling matrix element (19), and the factors η_h, η_l, and η_{hl} describe the heavy (h), light (l), and "mixed" (hl) contributions to tunneling and are given by the following expressions:

$$\eta_h=K\left(\frac{u_L}{T_{ph}}\right)+\frac{d_L}{d_R}K\left(\frac{u_R}{T_{ph}}\right)+\left(\frac{d_L}{d_R}\right)^{1/2}\tilde{K}\left(\frac{u_L}{T_{ph}},\frac{u_R}{T_{ph}};\frac{d}{\bar{d}}\right),$$

$$\eta_l=K\left(\frac{u_L}{T_{ph}}\right)+\frac{2T_h}{\Delta}+\frac{2T_h^2}{\Delta^2}\left(1+\frac{d_R(\varkappa_l d_L+2)^2}{d_L(\varkappa_l d_R+2)^2}\right),$$

$$\eta_{hl}=K\left(\frac{u_L}{T_{ph}}\right)+\left(\frac{d_L}{d_R}\right)^{1/2}\tilde{K}\left(\frac{u_L}{T_{ph}},\frac{u_R}{T_{ph}};\frac{d}{\bar{d}}\right)+\frac{T_h}{\Delta}\left(1+\frac{\varkappa_l d_L+2}{\varkappa_l d_R+2}\right),$$

$$t_0=\left(\frac{m_l}{m_h}\right)^{1/2}\frac{\hbar^2\pi^2(\varkappa_l d_L+2)}{2m_h d_R d_L\varkappa_l d_L}. \tag{31}$$

For small values of Δ the main contribution to the right-hand side of (30) is provided by the first term inside the braces. In this case the tunneling relaxation rate is determined by the tunneling exponential $\exp(-2\varkappa_h d)$ and has a Δ^{-2}-dependence. For

$$\Delta>\Delta_0,\quad \Delta_0=\sqrt{2}t_0\exp[-(\varkappa_h-\varkappa_l)d] \tag{32}$$

the predominant term is the last, which results from the mixing of the heavy and light components of the hole wave function. The tunneling relaxation rate is determined by the exponential $\exp(-2\varkappa_l d)$ and for $\Delta\gg T_h$ is independent of Δ.

6. CONCLUSION

Generally, the problem of the nature of hole relaxation in double quantum wells is extremely cumbersome due to the necessity of allowing for the tunnel mixing of the states of the left and right quantum wells, the mixing of heavy and light components of a four-row wave function, and the interaction of holes with acoustic phonons (and in the general case with other scatterers). Solving it requires numerical calculations for specific parameters of the structure (which is done in Refs. 4 and 12). Using an analytical procedure in which for $m_l/m_h\ll 1$ the l-components $\psi_{pz}^{2,3}$ are excluded makes it possible not only to obtain simple analytical expressions suitable for making order-of-magnitude estimates but also to demonstrate the features of tunneling related to the large subbarrier contribution due to l-states. These features distinguish the current situation from the electron case.

Let us recall the approximations used in this paper. Proceeding from the Luttinger equations (1) and (2) for

the isotropic spectrum model to the approximate equations (6) (with $\zeta=0$) and (7) requires the validity of the strong inequalities $m_l/m_h \ll 1$ and $pd_{L,R}/\hbar \ll 1$. In proceeding to Eq. (11), which describes the tunneling hybridization of only the two lowest states of the left and right quantum wells, we needed even stronger conditions,[14] and $\Delta \ll \varepsilon_R^{(21)}$, where $\varepsilon_R^{(21)}$ is the energy difference between the second and first levels of the dimensional quantization of heavy holes in the right quantum well. In principle, calculation is possible without the last two conditions, but the formulas describing tunneling become much more cumbersome. Finally, in calculating the well-to-well tunneling rate (Secs. 4 and 5) we assumed that the levels in the right and left wells are far from being in tunneling resonance: $\Delta \gg T_p, T_h$. If this is not so, the well-to-well relaxation rate proves comparable to the intrawell relaxation rate, quasiequilibrium intrawell distributions have no time to set in, and introducing ν loses all meaning.

In this paper we have shown that when inequality (32) holds true in double quantum wells, the tunneling relaxation rate is determined by an exponential function containing the light-hole mass and is independent on the level separation Δ. This property is the main qualitative result of our study and enables explaining the results of experiments[6,7] in which the method of measuring the photoluminescence decay time was used to establish the well-to-well tunneling relaxation rates for holes in GaAs/AlGaAs/GaAs in double quantum wells as functions of the transverse electric field (which fixes the value of Δ). These rates proved practically independent of Δ within a broad range of values Δ (up to the point where higher hole levels begin to participate in tunneling relaxation or where processes involving optical phonon emission manifest themselves). However, the estimates done via Eqs. (30) and (31) using the parameters established in the experiments have yielded results smaller than those obtained in the experiments by a factor greater than ten. This suggests that scattering on impurities and other effects ignored here played a more prominent role in the experiments.[6,7] Nevertheless, it is clear from the above that the well-to-well relaxation rate for holes interacting elastically via any short-range potential $V(\mathbf{r}_\perp, z)$ whose Fourier transform $V(\mathbf{q},q_z)$ can be assumed independent of $\mathbf{q}=(\mathbf{p}-\mathbf{p}')/\hbar$ in conditions of the given problem (this refers to, say, point defects and the heteroboundary roughness described by a correlation length $l_{cor} < \bar{d}\sqrt{m_h/m_l}$) and also independent of Δ in conditions (32). Note that these conditions do not depend explicitly on the characteristics of the short-range scattering mechanism because a transition to $\nu(\Delta)=\text{const}$ is provoked by the dependence (19) on the square of the final momentum $2m_R^*\Delta$. The experimental data[7] can be used to obtain the value of Δ corresponding to such a transition, with the value (of the order of 8 meV) agreeing well with the value Δ_0 estimated according to (32) for the parameters of the experiment.[7]

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APPENDIX

The kernel of the integro-differential equations (7) and (1) is related to the low-energy asymptotic Green function $G_0(z,z')$ through Eq. (8). The function $G_0(z,z')$ is specified by the equations

$$-\frac{d^2}{dz^2}G_0(z,z')=\frac{2m_l}{\hbar^2}\delta(z-z')$$

$$\text{if } z\in\left[-\frac{d}{2}-d_L,-\frac{d}{2}\right]\cup\left[\frac{d}{2},d_R+\frac{d}{2}\right], \tag{A1}$$

$$\left(-\frac{d^2}{dz^2}+\varkappa_l^2\right)G_0(z,z')=\frac{2m_l}{\hbar^2}\delta(z-z')$$

$$\text{if } z\in\left[-\infty,-\frac{d}{2}-d_L\right]\cup\left[-\frac{d}{2},\frac{d}{2}\right]\cup\left[d_R+\frac{d}{2},\infty\right] \tag{A2}$$

and satisfies the following boundary conditions:

$$G_0(z,z') \quad \text{and} \quad \frac{d}{dz}G_0(z,z) \tag{A3}$$

are continuous at the heteroboundaries,

$$G_0(z,z')=0 \quad \text{at } z=\pm\infty. \tag{A4}$$

We write the function $\Phi(z,z')$ required in calculating (16) and (17) explicitly for intervals of variation of z and z' lying inside the left and right quantum wells:

$$(1)\quad \Phi(z,z')=\frac{c_L}{d_L} \quad \text{if } z,z'\in\left[-\frac{d}{2}-d_L-\frac{d}{2}\right], \tag{A5}$$

$$(2)\quad \Phi(z,z')=\frac{c_R}{d_R} \quad \text{if } z,z'\in\left[\frac{d}{2},d_R+\frac{d}{2}\right], \tag{A6}$$

with

$$c_i=\left(\frac{\varkappa_i d_i}{\varkappa_i d_i+2}-\frac{\varkappa_l^2 d_L d_R\exp(-2\varkappa_i d)}{(\varkappa_i d_L+2)(\varkappa_i d_R+2)}\right)\times\left(1-\frac{\varkappa_l^2 d_L d_R\exp(-2\varkappa_i d)}{(\varkappa_i d_L+2)(\varkappa_i d_R+2)}\right)^{-1}, \tag{A7}$$

and

$$(3)\quad \Phi(z,z')=\frac{2\tilde{c}\exp(-\varkappa_i d)}{\varkappa_i d_L d_R}$$

$$\text{if } z\in\left[-\frac{d}{2}-d_L,-\frac{d}{2}\right],$$

$$z'\in\left[\frac{d}{2},d_R+\frac{d}{2}\right], \tag{A8}$$

with

$$\tilde{c}=\frac{\varkappa_l^2 d_L d_R}{(\varkappa_i d_L+2)(\varkappa_i d_R+2)-\varkappa_l^2 d_L d_R\exp(-2\varkappa_i d)}.$$

We also give the expression for $G_0(z,z')$ in this region:

$$G_0(z,z') = \Phi(z,z') \frac{2m_l}{\hbar^2} \left(\frac{1}{\varkappa_l} + \frac{d}{2} + d_L + z \right)$$

$$\times \left(\frac{1}{\varkappa_l} + \frac{d}{2} + d_R + d_R - z' \right). \quad \text{(A9)}$$

For

$$z \in \left[\frac{d}{2}, d_R + \frac{d}{2} \right], \quad z' \in \left[-\frac{d}{2} - d_L, -\frac{d}{2} \right]$$

the functions $\Phi(z,z')$ and $G_0(z,z')$ are expressed by (A8) and (A9) with z replaced by z' and vice versa.

[1)]Note that in Ref. 3 the expressions for the pre-exponential factor of the tunneling matrix element and the columns $|\pm\rangle$ are inaccurate.

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