Dynamic modulation of an ultrashort high-intensity laser pulse in matter

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The time-dependent spatially two-dimensional problem of the self-channeling of a high-power ultrashort laser pulse in matter is solved. The relativistic and ponderomotive nonlinearities are treated for free electrons, with the ions assumed to be at rest. The novelty of the model consists in including the longitudinal component of the ponderomotive force for the electrons along with the wave interaction between neighboring transverse segments of the pulse. The two-dimensional calculations are visualized as a sequence of "snapshots" of the pulse as a whole at different times. In the process of nonlinear propagation, strong dynamic modulation of the spatial shape of the pulse in the direction of propagation is observed, which causes the spectrum to broaden.

At the present time the interaction of ultrashort ($\tau < 1$ ps) high-power ($P > 10^{10}$ W) pulses from excimer[1] and neodymium[2] lasers with gaseous media is being studied actively both experimentally and theoretically. One of the important directions of this research is the study of the propagation of intense laser pulses in the nonlinear plasma medium formed by the pulses themselves. When a laser pulse is focused in light gases (H_2, He, N_2, CO_2,...), nearly complete ionization of the gas atoms occurs at the leading edge of the pulse. Under these conditions relativistic and ponderomotive nonlinearities are present and the Kerr nonlinearity is suppressed. In this paper we treat the time-dependent spatially two-dimensional picture of the propagation of an ultrashort high-power laser pulse in plasma under conditions such that the relativistic—ponderomotive nonlinearity dominates. We show that a pulse propagating in the self-channeling regime undergoes strong time-dependent spatial modulation, which changes its spectral makeup.

1. PHYSICAL MECHANISMS

The possibility of self-focusing of a light beam in a nonlinear medium was first pointed out by G. A. Askar'yan.[3] It was also he who predicted the thermal and ponderomotive mechanisms that give rise to self-focusing. The work of A. I. Akhiezer and R. V. Polovin[4] drew attention to the dependence of the mass of an electron oscillating in a strong field on the intensity of this field. The repulsive force acting on a free electron, causing it to move from the region of an intense light field to the region occupied by a weaker field, was established by V. A. Gaponov and M. A. Miller.[5] The fact that the increase of the electron mass in a strong optical field leads to relativistic self-focusing of the radiation was apparently first pointed out by Max *et al.*[6] The critical relativistic self-focusing power was determined by Schmidt and Horton.[7] The nonlinear Schrödinger equation describing the combined relativistic-ponderomotive nonlinearity effects on propagating circularly polarized radiation was first obtained in Ref. 8. The effects of self-modulation of the spectrum of scattered radiation when a picosecond laser pulse propagates in a nonlinear medium were reported in Refs. 9–11. A more complete bibliography of papers on nonlinear propagation of high-power laser radiation in matter and on related topics can be found in several review articles[12–14] and also in two papers by Borisov *et al.*[15,16]

The *relativistic-ponderomotive self-channeling of ultrashort high-power laser pulses in plasma* is a physical effect which is of interest for a number of applications. The effect was predicted theoretically by Borisov *et al.*,[17] and then observed experimentally.[18] The theory has been described in Refs. 15, 16, and 19, and later experimental results are given in Ref. 20.

A high-power ultrashort laser pulse propagating in matter causes rapid nonlinear ionization of the atoms. As a result, a plasma consisting of multicharged ions and free electrons forms. The presence of the plasma reduces the index of refraction of the gas, and the resulting plasma column has nonlinear defocusing properties. However, there exist a number of physical mechanisms that lead to the nonlinear increase in the index of refraction in the region occupied by the strong field. These include a) the Kerr effect, in which the high-power laser radiation deforms the electron shells of atoms and ions, thereby creating nonlinear dipole moments induced by the strong field; b) the relativistic increase in mass of the free electrons oscillating in the strong field with high velocities, close to the speed of light; c) electron repulsion by the ponderomotive force from the region occupied by the field, which gives rise to a cavitation channel filled with heavy ions. In the case of fully ionized material the Kerr effect is absent. These mechanisms increase the index of refraction of the medium, and when the critical power is exceeded they lead to a regime in which a self-focused ultrashort laser pulse propagates.

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This regime differs from the familiar laser pulses propagation regimes. Its characteristic feature is that a pulse of length τ and transverse dimension d propagates without diffractive spreading in the transverse direction for some distance $L \gg L_d$, where $L_d = d^2/\lambda$ is the Rayleigh distance associated with the transverse dimension of the channel and satisfying $L \gg L_r$, where $L_r = c\tau$ is the longitudinal dimension of the pulse. Inside the pulse a region develops with a reduced electron density, which for high levels of the trapped power can turn into a cavity completely free of electrons. This cavity or hollow has a length less than L_r and translates with the pulse.

Borisov *et al.*[18,20] observed the self-channeled propagation of a KrF* excimer laser pulse ($\lambda = 0.248\ \mu$m, $\tau \approx 600$ fs, $P \approx 3 \cdot 10^{11}$ W) over a distance of up to 2 mm, which is equal to $\approx (50\text{–}100) L_d$. The channel radius was less than 1 μm. The intensities in the channel were estimated to be $I \approx 10^{20}$ W/cm^2. The distance L over which self-channeled propagation occurred was determined by the rate of pulse energy dissipation in the material.

The present work extends the theoretical treatment of the relativistic-ponderomotive self-channeling of an ultrashort high-power laser pulse in plasma. We add to previous work[15,16,19] by presenting a more general theory of the effect. We also report for the first time the results of numerical simulation of the three-dimensional (r,z,t) problem, illustrating the time-dependent spatially two-dimensional propagation of an ultrashort laser pulse as a single entity in matter. Note that all previous work only treated the steady two-dimensional (r,z) problem of the propagation of a thin transverse slice of a pulse, without treating the wave interaction between neighboring layers. The results of the theory described below paint a physically clear picture of the propagation of the pulse and predict that it undergoes time-dependent spatial modulation parallel to the direction of propagation. The latter is exhibited in the broadening of the spectrum of the scattered radiation, which may be confirmed experimentally.

2. GENERAL CONSIDERATIONS

To describe the propagation of a short intense laser pulse in matter we use the following approach. Since atoms and ions undergo rapid nonlinear photoionization at the leading edge of the pulse, most of the pulse propagates through an already existing plasma. Hence in what follows we will treat the propagation of radiation in plasma. The electrons are probably not heated to temperatures much greater than a few keV over the time during which the ultrashort high-power laser pulse acts on the material; this is shown by experiments with subpicosecond pulses from excimer and neodymium lasers.[2,14,20] On the other hand, the average energies of the electron oscillations in the fields of these pulses reach hundreds of keV. This implies that for a short high-power laser pulse the plasma through which it propagates is essentially cold.

We require that the average kinetic energy of the electron oscillations be much greater than their thermal energy, $w_{\text{ocu}} = mc^2(\gamma - 1) \gg 3T/2$, where we have written $\gamma^2 = 1 + I/I_r$ and $I_r = m^2\omega^2 c^3/4\pi e^2$ is the relativistic intensity.[4] In terms of the radiation intensity this criterion assumes the form $I[\text{W/cm}^2] \gg 1.65 \cdot 10^{16} T[\text{keV}]/\lambda^2[\mu\text{m}]$.

Cold plasma in an electromagnetic field responds only to the electromagnetic forces, and the gasdynamic pressure and the processes giving rise to this pressure through electron heating can be neglected to lowest order. However, when charged particles move in superpowerful light fields their velocities are so large that it becomes necessary to take into account the relativistic increase in their masses. Thus, many properties of the nonlinear propagation of ultrashort pulses of high-power laser radiation in plasma can be understood if we make use of the electrodynamics of a cold relativistic plasma. This approach is relatively widespread, going back to the pioneering work of A. I. Akhiezer and R. V. Polovin.[4]

2.1. Equations in relativistically invariant form

Consider a plasma consisting of electrons with charge $q = -e$ and rest mass m and ions with charge $Q = Ze$ and rest mass M. The quantity $e = |q|$ is taken to be positive. In the absence of an external field we assume that the plasma is quasineutral. The electron density, velocity, momentum, and current are denoted by n, $\mathbf{u}$, $\mathbf{p}$, $\mathbf{j}$. The corresponding quantities for the ions are denoted by the corresponding upper case letters N, $\mathbf{U}$, $\mathbf{P}$, $\mathbf{J}$.

We introduce into the discussion the 4-velocities and 4-currents for electrons and ions, along with the 4-potential of the field, using the following formulas:

$$u^i = \left(\frac{c}{\sqrt{1 - u^2/c^2}}, \frac{\mathbf{u}}{\sqrt{1 - u^2/c^2}} \right), \tag{1}$$

$$U^i = \left(\frac{c}{\sqrt{1 - U^2/c^2}}, \frac{\mathbf{U}}{\sqrt{1 - U^2/c^2}} \right), \tag{2}$$

$$j^i = (c\rho_e, \mathbf{j}) = (cqn, qn\mathbf{u}), \tag{3}$$

$$J^i = (c\rho_i, \mathbf{J}) = (cQN, QN\mathbf{U}), \tag{4}$$

$$A^i = (\phi, \mathbf{A}). \tag{5}$$

We introduce the electromagnetic field tensor

$$F^{ik} = \frac{\partial A^k}{\partial x_i} - \frac{\partial A^i}{\partial x_k}. \tag{6}$$

The components of this tensor are equal to

$$F_{ik} = \begin{vmatrix} 0 & E_x & E_y & E_z \\ -E_x & 0 & -B_z & B_y \\ -E_y & B_z & 0 & -B_x \\ -E_z & -B_y & B_x & 0 \end{vmatrix}, \tag{7}$$

where

$$\mathbf{E} = \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} - \nabla \phi, \quad \mathbf{B} = [\nabla \times \mathbf{A}]. \tag{8}$$

The system of equations describing the combined motion of the field and plasma, including the electron and ion components, assumes the form

$$m\frac{\partial u_k}{\partial x^l}u^l=\frac{q}{c}F_{kl}u^l, \quad (9)$$

$$M\frac{\partial U_k}{\partial x^l}U^l=\frac{Q}{c}F_{kl}U^l, \quad (10)$$

$$\frac{\partial F_{ik}}{\partial x^l}+\frac{\partial F_{kl}}{\partial x^i}+\frac{\partial F_{li}}{\partial x^k}=0, \quad (11)$$

$$\frac{\partial F_{ik}}{\partial x_k}=-\frac{4\pi}{c}(j_i+J_i), \quad (12)$$

$$\frac{\partial j_i}{\partial x_i}=\frac{\partial J_i}{\partial x_i}=0. \quad (13)$$

Here Eqs. (9) and (10) describe the motion of the electron and ion components of the plasma in the field F_{ik}, and Eqs. (11) and (12) are the four-dimensional version of the Maxwell equations. Equations (13) are the conditions for conservation of the electron and ion 4-currents.

For the system (9)–(13) we can introduce the energy-momentum tensor

$$T^{ik}=\frac{m}{q}u^ij^k+\frac{M}{Q}U^iJ^k+\frac{1}{4\pi}\left(F^{il}F_l^k+\frac{1}{4}g^{ik}F_{lm}F^{lm}\right), \quad (14)$$

which for the solutions of this system of equations satisfies the conservation law

$$\frac{\partial T^{ik}}{\partial x^k}=0. \quad (15)$$

The system of equations (9)–(13), together with the requirement that the conservation law (15) hold for the energy-momentum tensor (14) of the material and field, constitutes the starting set of equations for the electrodynamics of a cold relativistic plasma and forms the basis for our subsequent treatment.

2.2. Basic equations in spatially three-dimensional form

Equations (9)–(13), written in three-dimensional form, become

$$\frac{\partial \mathbf{p}}{\partial t}+(\mathbf{u}\cdot\nabla)\mathbf{p}=q\left(-\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}-\nabla\phi+\frac{1}{c}[\mathbf{u}\times[\nabla\times\mathbf{A}]]\right), \quad (16)$$

$$\frac{\partial \mathbf{P}}{\partial t}+(\mathbf{U}\cdot\nabla)\mathbf{P}=Q\left(-\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t}-\nabla\phi+\frac{1}{c}[\mathbf{U}\times[\nabla\times\mathbf{A}]]\right), \quad (17)$$

$$\Box\mathbf{A}-\frac{1}{c}\frac{\partial}{\partial t}\nabla\phi=-\frac{4\pi}{c}(\mathbf{j}+\mathbf{J}), \quad (18)$$

$$\Delta\phi=-4\pi\rho, \quad (19)$$

$$(\nabla\cdot\mathbf{A})=0, \quad (20)$$

$$\mathbf{j}+\mathbf{J}=qn\mathbf{u}+QN\mathbf{U}, \quad (21)$$

$$\rho=qn+QN, \quad (22)$$

$$\mathbf{u}=\mathbf{p}/(m\gamma_e),\ \gamma_e^2=1+|\mathbf{p}|^2/(mc)^2, \quad (23)$$

$$\mathbf{U}=\mathbf{P}/(M\gamma_i),\ \gamma_i^2=1+|\mathbf{P}|^2/(Mc)^2. \quad (24)$$

Here the first two equations are the equations of relativistic hydrodynamics for the electron and ion components of a cold plasma in an electromagnetic field. The Maxwell equations (18) and (19) are written in terms of the field potentials. The fifth equation is the condition for the Coulomb gauge of the vector potential. The sixth and seventh equations determine the current and charge densities of the plasma. The eighth and ninth equations give the relativistic relation between the velocities and momenta of the electrons and ions.

Energy and Momentum Conservation Laws. The solution of Eqs. (16)–(24) must satisfy the energy and momentum conservation laws, which in four-dimensional notation take the form (15). The various components of the energy-momentum tensor (14) are determined by the following expressions:

$$T^{00}=\frac{nmc^2}{\sqrt{1-u^2/c^2}}+\frac{NMc^2}{\sqrt{1-U^2/c^2}}+\frac{1}{8\pi}(|\mathbf{E}|^2+|\mathbf{B}|^2), \quad (25)$$

$$T^{0\alpha}=\frac{nmcu_\alpha}{\sqrt{1-u^2/c^2}}+\frac{NMcU_\alpha}{\sqrt{1-U^2/c^2}}+\frac{1}{4\pi}[\mathbf{E}\times\mathbf{B}]_\alpha, \quad (26)$$

$$T^{\alpha\beta}=np_\alpha u_\beta+NP_\alpha U_\beta-\sigma_{\alpha\beta}, \quad (27)$$

$$\sigma_{\alpha\beta}=\frac{1}{4\pi}(E_\alpha E_\beta+B_\alpha B_\beta)-\frac{1}{8\pi}(|\mathbf{E}|^2+|\mathbf{B}|^2)\delta_{\alpha\beta}. \quad (28)$$

In three-dimensional notation the conservation laws assume the form

$$\frac{\partial}{\partial t}\left[mc^2\gamma_e n+Mc^2\gamma_i N+\frac{1}{8\pi}(|\mathbf{E}|^2+|\mathbf{B}|^2)\right]+\nabla\cdot\left(mc^2\gamma_e n\mathbf{u}+Mc^2\gamma_i N\mathbf{U}+\frac{1}{4\pi}[\mathbf{E}\times\mathbf{B}]\right)=0, \quad (29)$$

$$\frac{\partial}{\partial t}\left(np_\alpha+NP_\alpha+\frac{1}{4\pi c}[\mathbf{E}\times\mathbf{B}]_\alpha\right)+\frac{\partial}{\partial x_\beta}\times\left[(np_\alpha u_\beta+NP_\alpha U_\beta)+\frac{1}{8\pi}(|\mathbf{E}|^2+|\mathbf{B}|^2)\times\delta_{\alpha\beta}-\frac{1}{4\pi}(E_\alpha E_\beta+B_\alpha B_\beta)\right]=0. \quad (30)$$

Here $(|\mathbf{E}|^2+|\mathbf{B}|^2)/8\pi$ is the energy density of the electromagnetic field, $[\mathbf{E}\times\mathbf{B}]/4\pi$ is the energy flux of the field (the Poynting vector), and $\sigma_{\alpha\beta}$ is the momentum flux of the field (the stress tensor). The electric and magnetic field strengths are expressed in terms of the field potentials by means of Eqs. (8).

2.3. Separation of the charged-particle momentum into rotational and potential components

Let us consider Eq. (16). In what follows we will regard the ions as fixed and leave Eq. (17) out of the discussion. The term $(\mathbf{u}\cdot\nabla)\mathbf{p}$ can be transformed as follows:

$$(\mathbf{u}\cdot\nabla)\mathbf{p}=(m\gamma_e)^{-1}(\mathbf{p}\cdot\nabla)\mathbf{p}$$
$$=(m\gamma_e)^{-1}(\tfrac{1}{2}\nabla|\mathbf{p}|^2-[\mathbf{p}\times[\nabla\times\mathbf{p}]])$$
$$=mc^2\nabla\gamma_e-[\mathbf{u}\times[\nabla\times\mathbf{p}]]. \quad (31)$$

Substituting in Eq. (16), we rewrite the equation in the equivalent form

$$\frac{\partial}{\partial_t}\left(\mathbf{p}+\frac{q}{c}\mathbf{A}\right)-\left[\mathbf{u}\times\left[\nabla\times\left(\mathbf{p}+\frac{q}{c}\mathbf{A}\right)\right]\right]=-q\nabla\phi-mc^2\nabla\gamma_e. \quad (32)$$

We look for a solution of this equation as a sum of three components

$$\mathbf{p}=-\frac{q}{c}\mathbf{A}+\mathbf{p}_0+\mathbf{p}_1, \quad (33)$$

where $\mathbf{p}_0$ is a vector derivable from a potential and $\mathbf{p}_1$ is a rotational vector:

$$[\nabla\times\mathbf{p}_0]=0,\quad (\nabla\cdot\mathbf{p}_0)\neq 0, \quad (34)$$

$$[\nabla\times\mathbf{p}_1]\neq 0,\quad (\nabla\cdot\mathbf{p}_1)=0. \quad (35)$$

Substituting (33) in (32) and taking the curl of both sides of the equation, we find an equation for the momentum vorticity $\mathbf{M}=[\nabla\times\mathbf{p}_1]$:

$$\frac{\partial\mathbf{M}}{\partial t}-[\nabla\times[\mathbf{u}\times\mathbf{M}]]=0. \quad (36)$$

Taking the divergence of the same equation we find for $D=(\nabla\times\mathbf{p}_0)$

$$\frac{\partial D}{\partial t}D-(\nabla\cdot[\mathbf{u}\times\mathbf{M}])=-q\Delta\phi-mc^2\Delta\gamma_e. \quad (37)$$

The system of equations consisting of (36) and (37) is nonlinear, since $\mathbf{u}$ is given in terms of $\mathbf{p}$ by the relativistic relation, which also depends on $\mathbf{p}_0$ and $\mathbf{p}_1$.

There is a simplifying factor that permits us to treat the problem without taking into account the vorticity $\mathbf{M}$. Specifically, if the laser pulse is incident on a plasma in which there is no rotational motion in the initial state, then at subsequent times no vorticity will develop in the plasma. This is implied by Eq. (36). In fact, if at time $t=0$ we have $\mathbf{M}=0$ and $\partial\mathbf{M}/\partial\mathbf{x}_\alpha=0$ in the plasma, then $\partial\mathbf{M}/\partial t=0$ holds everywhere, and consequently no vorticity can be created. In the remainder of this work we will consider the equations of motion for the charged components of the plasma under conditions such that no vorticity is present.

Note that under actual experimental conditions rotational fluctuations can evidently be present in the initial state. Under these conditions, although the quantities $\mathbf{M}$ can be close to zero, the derivatives are nonzero: $\partial\mathbf{M}/\partial\mathbf{x}_\alpha\neq 0$. The question as to whether these fluctuations can develop into full-scale electron vortices affecting the dynamics of the laser radiation during the time the laser pulse is acting is an interesting one, but it lies outside the scope of the present work.

Thus in what follows we set $\mathbf{p}_1\equiv 0$. Under these conditions the equation of motion (16) for the electrons reduces to

$$\mathbf{p}=-\frac{q}{c}\mathbf{A}+\mathbf{p}_0, \quad (38)$$

$$\partial_t\mathbf{p}_0=-q\nabla\phi-mc^2\nabla\gamma_e. \quad (39)$$

The electron current in this approximation can be written

$$\mathbf{j}=qn\mathbf{u}=\frac{qn}{m\gamma_e}\mathbf{p}=-\left(\frac{q^2n}{mc\gamma_e}\right)\mathbf{A}+\frac{qn}{m\gamma_e}\mathbf{p}_0. \quad (40)$$

The representation (38)–(40) becomes most perspicuous when the Maxwell equations are written in the Coulomb gauge, which implies that the vector potential is a solenoidal vector. In this case Eqs. (38)–(40) constitute an expansion of the momentum of the electron component of the plasma into rotational $-(q/c)\mathbf{A}$ and potential $\mathbf{p}_0$ components. This expansion facilitates solution of the problem.

3. SPATIALLY TWO-DIMENSIONAL TIME-DEPENDENT PROBLEM

In this section we derive simpler equations approximately describing the time-dependent spatially two-dimensional nonlinear propagation of ultrashort high-power circularly and linearly polarized laser pulses in a cold plasma. In the most simplified case these equations are versions of the nonlinear Schrödinger equation (NLSE) with a special form of nonlinearity.

3.1. Circularly Polarized Waves

We treat a circularly polarized wave by assuming that the potential part of the electron motion is time-independent:

$$\mathbf{p}_0=0,\quad \partial\mathbf{p}_0/\partial t=0. \quad (41)$$

This approximation is valid if the laser pulse length is greater than the time required to expel an electron from the strong-field region, $\tau>r/c\approx\lambda/c=1-3$ fs. In the field of this wave the electron moves in a circular orbit in the xy plane, perpendicular to the direction of wave propagation. In this limit it is unnecessary to take the p_z component into account. In the field of a circularly polarized wave with no transverse variation a z component of the vector potential nevertheless develops. The condition $(\nabla\cdot\mathbf{A})=0$ implies $|a_z|\approx k^{-1}\partial|a|/\partial r$. The A_z component is small under the condition that the field varies slowly in the transverse direction over distances of order $\lambda/6$. The presence of A_z gives rise to a component $p_z\ll p_\perp$.

Here we consider the case of a circularly polarized wave, keeping a small longitudinal component of the vector potential in the theory. The potential motion of the electron gas is treated as steady. The presence of a rapidly oscillating longitudinal component of the vector potential, and hence of the electron momentum, causes harmonics to be generated. Because the longitudinal components are small it is reasonable to take into account only the production of the third harmonic. Below we derive a system of coupled equations describing the propagation of radiation at the fundamental frequency and that of the third harmonic. Neglecting third-harmonic generation, we find a nonlinear wave equation describing the propagation of radiation at the fundamental frequency.

Next, we use the normalized physical variables $\widetilde{\mathbf{A}}=e\mathbf{A}/mc^2$, $\widetilde{\phi}=e\phi/mc^2$, $\widetilde{\mathbf{u}}=\mathbf{u}/c$, $\widetilde{\mathbf{p}}=\mathbf{p}/mc$, $\widetilde{n}=n/n_0$. In what follows we omit the tilde. In the approximation (41) we have the following system of equations:

$$\Box\mathbf{A}_\perp=\frac{1}{c}\frac{\partial}{\partial t}\nabla_\perp\gamma+k_p^2\cdot\frac{n}{\gamma}\mathbf{A}_\perp, \tag{42}$$

$$(\nabla\cdot\mathbf{A})=0, \tag{43}$$

$$n=1+k_p^{-2}\Delta\gamma, \tag{44}$$

$$\gamma^2=1+|\mathbf{A}|^2. \tag{45}$$

Here we have written $k_p=\omega_{p,0}/c$, where $\omega_{p,0}=(4\pi e^2n_0/m)^{1/2}$ is the plasma frequency in the unperturbed plasma. Note that because of the approximations we have made the system (42)–(45) is not completely equivalent to the original equations.

We look for a solution of Eqs. (42)–(45) in the following form:

$$\mathbf{A}=\mathbf{A}_\perp+\mathbf{A}_\parallel$$
$$=\sum_n\tfrac{1}{2}(\mathbf{e}_x+i\mathbf{e}_y)a_\perp^{(n)}(\mathbf{r},t)\exp[i(n\omega t-k_nz)]$$
$$+\sum_n\tfrac{1}{2}e_za_\parallel^{(n)}(\mathbf{r},t)\exp[i(n\omega t-k_nz)]+\text{c.c.} \tag{46}$$

We write the square of the absolute value of the vector potential as follows:

$$|\mathbf{A}|^2=A_x^2+A_y^2+A_z^2$$
$$=|a_\perp|^2+\tfrac{1}{4}(a_\parallel e^{i\phi}+a_\parallel^*e^{-i\phi})^2$$
$$=|a_\perp|^2+\tfrac{1}{2}|a_\parallel|^2+\tfrac{1}{4}(a_\parallel^2e^{i2\phi}+a_\parallel^{*2}e^{-i2\phi}). \tag{47}$$

Here $\phi=\omega t-k_1z$.

We introduce the following notation (here the symbol a without a superscript refers to the primary wave):

$$B_0=1+|a_\perp|^2+\tfrac{1}{2}|a_\parallel|^2, \tag{48}$$

$$C_0=\tfrac{1}{4}a_\parallel^2(1+|a_\perp|^2+\tfrac{1}{2}|a_\parallel|^2)^{-1}, \tag{49}$$

$$\gamma_0=B_0^{1/2}. \tag{50}$$

We write the relativistic γ factor in the form

$$\gamma=\gamma_0(1+C_0e^{-2i\phi}+C_0^*e^{-i2\varphi})^{1/2}. \tag{51}$$

Next, we carry out an expansion and retain only the terms which are first order in C_0. The complex quantity C_0 enters in the problem as a small parameter. The term $c^{-1}(\partial/\partial t)_t\nabla_{\perp\gamma}$ does not contribute to generating the third harmonic. The result of the calculations for the case in which only the two frequencies ω and 3ω are retained in the summation (46) takes the form

$$\Box[a_\perp^{(1)}\exp[i(\omega t-k_1z)]]$$
$$=k_p^2\gamma_0^{-1}(1+k_p^{-2}\Delta\gamma_0)a_\perp^{(1)}\exp[i(\omega t-k_1z)]$$
$$+G^*a_\perp^{(3)}\exp[i(\omega t-k_3z+2k_1z)], \tag{52}$$

$$\Box[a_\perp^{(3)}\exp[i(3\omega t-k_3z)]]$$
$$=k_p^2\gamma_0^{-1}(1+k_p^{-2}\Delta\gamma_0)a_\perp^{(3)}\exp[i(3\omega t-k_3z)]$$
$$+Ga_\perp^{(1)}\exp[i(3\omega t-3k_1z)], \tag{53}$$

$$G=\gamma_0^{-1}\{-\tfrac{1}{2}C_0+\tfrac{1}{2}k_p^{-2}[\Delta(\gamma_0C_0)-C_0\Delta\gamma_0-4k_1^2\gamma_0C_0]\}. \tag{54}$$

Applying the operation

$$\Box=\Delta_\perp+\frac{\partial^2}{\partial z^2}-C^{-2}\frac{\partial^2}{\partial t^2}$$

to the left sides of (52) and (53) and using the dispersion relation

$$k_1^2=k_0^2-k_p^2,\quad k_3^2=9k_0^2-k_p^2, \tag{55}$$

we obtain the following equations:

$$\left(\frac{1}{v_g^1}\frac{\partial}{\partial t}a_\perp^{(1)}+\frac{\partial}{\partial z}a_\perp^{(1)}\right)+\frac{i}{2k_1}\Box a_\perp^{(1)}+\frac{i}{2k_1}k_p^2$$
$$\times\left[1-\frac{(1+k_p^{-2}\Delta\gamma_0)}{\gamma_0}\right]a_\perp^{(1)}-\frac{i}{2k_1}G^*a_\perp^{(3)}$$
$$\times\exp[-i(k_3-3k_1)z], \tag{56}$$

$$\left(\frac{1}{v_g^3}\frac{\partial}{\partial t}a_\perp^{(3)}+\frac{\partial}{\partial z}a_\perp^{(3)}\right)+\frac{i}{2k_3}\Box a_\perp^{(3)}+\frac{i}{2k_3}k_p^2$$
$$\times\left(1-\frac{1+k_p^{-2}\Delta\gamma_0}{\gamma_0}\right)a_\perp^{(3)}-\frac{i}{2k_3}Ga_\perp^{(1)}$$
$$\times\exp[i(k_3-3k_1)z]. \tag{57}$$

These equations can be still further simplified if we use the approximation in which the complex amplitudes are slowly varying. This means that the field amplitudes vary slowly over distances on the order of a wavelength in the direction of propagation and over times on the order of the period of the high-frequency field oscillations:

$$\frac{\partial a}{\partial z},\ \frac{1}{c}\frac{\partial a}{\partial t}\ll k_1a. \tag{58}$$

In this approximation the Dalembertian and the Laplacian appearing in the nonlinearity in Eqs. (56) and (57) must be replaced with transverse Laplacians $\Box\Rightarrow\Delta_\perp$, $\Delta\Rightarrow\Delta_\perp$. As a result we find equations which are classified as nonlinear Schrödinger equations (NLSE). These equations are widely employed to describe the propagation of electromagnetic gains in nonlinear media. In the solutions of Eqs. (56) and (57), in which the Dalembertian is reduced to the transverse Laplacian $\Box\Rightarrow\Delta_\perp$, the following conservation law holds:

$$\left(\frac{1}{v_g^1}\frac{\partial}{\partial t}+\frac{\partial}{z}\right)k_1\int|a_\perp^{(1)}|^2dxdy$$

$$+\left(\frac{1}{v_g^3}\frac{\partial}{\partial t}+\frac{\partial}{\partial z}\right)k_3\int|a_\perp^{(3)}|^2dxdy=0. \tag{59}$$

This equation is well known in nonlinear optics; it describes the conservation of photon number in processes in which the frequencies are transformed parametrically.

Expression for the γ Factor. From the condition $\nabla\cdot\mathbf{A}=0$ it follows that

$$a_\parallel\approx-\frac{i}{k_1}\left(\frac{x+iy}{r}\right)\frac{\partial}{\partial r}a_\perp=-\frac{i}{k}e^{i\psi}\frac{\partial}{\partial r}a_\perp\,. \tag{60}$$

Here ψ is the azimuthal angle. We see that

$$|a_\parallel|^2=\frac{1}{k^2}\left|\frac{\partial}{\partial r}a_\perp\right|^2. \tag{61}$$

If we write $a_\perp=|a_\perp|\exp(i\psi)$, then

$$\gamma_0^2=1+|a_\perp|^2+\frac{1}{2k_1^2}\left[\left(\frac{\partial}{\partial r}|a_\perp|\right)^2+|a_\perp|^2\left(\frac{\partial}{\partial r}\psi\right)^2\right]. \tag{62}$$

Charge Conservation. Note that in the problem defined by Eqs. (42)–(45) the total charge is conserved. Specifically,

$$\int_V(n-1)dV=k_p^{-2}\int_V\Delta\gamma dV=0 \tag{63}$$

since $\nabla\gamma$ vanishes at infinity. This fact is an additional argument in favor of the present model.

Conditions for Phase Synchronization. When the third harmonic is generated in the plasma in the direction in which the primary wave is propagating, the condition for phase synchronization is only satisfied for a short distance. Using the dispersion relation we estimate this distance from the condition

$$|(k_3-3k_1)|z_c=\pi, \tag{64}$$

whence

$$z_c\approx\frac{3}{8}\lambda_0(\omega/\omega_p)^2,(\omega_p/\omega)^2\ll 1. \tag{65}$$

In experiments with gases $(\omega/\omega_p)^2\approx 0.1-0.01$ holds. This means that $z_c\approx(10\text{–}40)\lambda_0$. For a KrF* excimer laser at $\lambda_0=0.248\cdot10^{-4}$ cm the phase synchronization distance is less than 10 μm. Hence it makes no sense to talk about any significant pumping of energy from the primary wave into the third harmonic. In what follows we will refrain from questions involving third-harmonic generation and will analyze Eq. (56) for the primary wave, setting $G^*=0$.

3.2. Linearly Polarized Waves

To study the propagation of a linearly polarized wave in the plasma we neglect the longitudinal component of the vector potential and the transverse component of the potential part of the electron momentum,

$$A_\parallel=0,\quad \mathbf{p}_{0,\perp}=0. \tag{66}$$

We obtain the following equations:

$$\square\mathbf{A}_\perp=c^{-1}\frac{\partial}{\partial t}\nabla_\perp\phi+k_p^2n\mathbf{u}_\perp\,, \tag{67}$$

$$c^{-1}\frac{\partial^2}{\partial t\partial z}\phi+k_p^2nu_\parallel=0, \tag{68}$$

$$\Delta\phi=k_p^2(n-1), \tag{69}$$

$$\mathbf{p}=\mathbf{A}_\perp+\mathbf{p}_{0\parallel}\,, \tag{70}$$

$$\nabla_\perp(\phi-\gamma)=0, \tag{71}$$

$$c^{-1}\frac{\partial p_{0,\parallel}}{\partial t}=\frac{\partial(\phi-\gamma)}{\partial z}, \tag{72}$$

$$\mathbf{u}=\mathbf{p}/\gamma,\quad \gamma^2=1+|\mathbf{A}_\perp|^2+p_{0,\parallel}^2\,. \tag{73}$$

From (72) it follows that

$$\frac{\partial^2\phi}{\partial t\partial z}=\frac{1}{c}\frac{\partial^2p_{0,\parallel}}{\partial t^2}+\frac{\partial^2\gamma}{\partial t\partial z}. \tag{74}$$

Replacing $\partial^2\phi/\partial t\partial z$ in Eq. (68) by means of (74) and replacing $\nabla_\perp\phi$ in Eq. (67) with $\nabla_\perp\gamma$ from (71), and also expressing n from (69) in the following fashion:

$$n=1+k_p^{-2}\Delta\phi=1+k_p^{-2}\left(\Delta_\perp\phi+\frac{\partial^2\phi}{\partial z^2}\right)$$

$$=1+k_p^{-2}\left(\Delta_\perp\gamma+\frac{\partial^2\gamma}{\partial z^2}+c^{-1}\frac{\partial^2p_{0,\parallel}}{\partial t\partial z}\right)$$

$$=1+k_p^{-2}\left(\Delta\gamma+c^{-1}\frac{\partial^2p_{0,\parallel}}{\partial t\partial z}\right), \tag{75}$$

we arrive at a system of equations

$$\square\mathbf{A}_\perp=c^{-1}\frac{\partial}{\partial t}\nabla_\perp\gamma+k_p^2n\gamma^{-1}\mathbf{A}_\perp\,, \tag{76}$$

$$c^{-2}\frac{\partial^2p_{0,\parallel}}{\partial t^2}+c^{-1}\frac{\partial^2\gamma}{\partial t\partial z}+k_p^2n\gamma^{-1}p_{0,\parallel}=0, \tag{77}$$

$$n=1+k_p^{-2}\left(\Delta\gamma+c^{-1}\frac{\partial^2p_{0,\parallel}}{\partial t\partial z}\right), \tag{78}$$

$$\gamma^2=1+|\mathbf{A}_\perp|^2+p_{0,\parallel}^2\,. \tag{79}$$

The physical meaning of Eq. (77) is as follows. The equation

$$c^{-2}\frac{\partial^2p_{0,\parallel}}{\partial t^2}+c^{-1}\frac{\partial^2\gamma}{\partial t\partial z}=0 \tag{80}$$

in a uniform monochromatic field

$$\mathbf{A}_\perp=\tfrac{1}{2}\mathbf{e}_xa_\perp\exp[i(\omega t-kz)]+\text{c.c.} \tag{81}$$

has the solution

$$p_{0,\parallel}=p_{0,\parallel}^{\text{os}}=\frac{|a_\perp|^2}{2(1+|a_\perp|^2/2)^{1/2}}\left[\cos^2(\omega t-kz)-\frac{1}{2}\right], \tag{82}$$

which after being integrated twice describes the familiar relativistic figure-eight for the electron trajectory. In a nonuniform field this is superposed on the directed electron drift[21] $p_{0,\parallel}=p_{0,\parallel}^{dr}+p_{0,\parallel}^{os}$.

The equation

$$c^{-2}\frac{\partial^2 p_{0,\parallel}}{\partial t^2}+k_p^2 n\gamma^{-1}p_{0,\parallel}=0, \tag{83}$$

describes oscillations of the longitudinal electron momentum with the plasma frequency, altered by relativistic effects.

In the set of equations Eq. (77) describes the combined effect of the electromagnetic field and plasma oscillations on the longitudinal electron momentum.

With due regard to the complexity of the nonlinear equations (76)–(79), we can suggest the following approximate approach for describing the propagation of linearly polarized laser radiation in a plasma. We look for a solution in the form of an expansion in harmonics of the frequency:

$$\mathbf{A}_\perp=\sum_n \tfrac{1}{2}\mathbf{e}_x a_\perp^{(n)}(\mathbf{r},t)\exp[i(n\omega t-k_n z)]+\text{c.c.} \tag{84}$$

and at the same time describe the longitudinal electron momentum using (82) with $a_\perp=a_\perp^{(1)}$. Then, just as in the case of a circularly polarized wave, we find the oscillatory γ factor. Then, expanding the γ factor we obtain a system of coupled equations for the harmonic amplitudes. Since there is no phase matching in the plasma between the fundamental and the harmonics, we can disregard the process of harmonic generation. Under these conditions the equation describing propagation of the fundamental takes the form (56) with $G^*=0$, and the expression for the nonoscillatory γ factor (disregarding the drift component of the longitudinal momentum, which is usually smaller than the oscillatory part) is the following:

$$\gamma_0^2=1+\frac{1}{2}|a_\perp|^2\left[1+\frac{|a_\perp|^2}{16(1+|a_\perp|^2/2)}\right]. \tag{85}$$

We can also use the approximation of slowly varying complex amplitudes. In this case the Dalembertian and Laplacian in the nonlinear term must be replaced with the transverse Laplacian.

We see that the propagation of circularly and linearly polarized radiation is described approximately by the same wave equation or the NLSE. However, the γ factors are different. It is noteworthy that the γ factors do not in fact differ much from the model expression

$$\gamma_0^2=1+|a_\perp|^2. \tag{86}$$

(For a linearly polarized wave the factor 1/2 goes away because the amplitude is renormalized.) In this connection we can model both cases qualitatively by using the wave equation (56) with $G^*=0$ or the NLSE and the model γ factor (86). Note that the wave equation (56) with $G^*=0$ and the model γ factor has a Hamiltonian.[1)]

3.3. Transformation to the Comoving Reference Frame

The wave equation describing the nonlinear propagation of ultrashort high-power laser pulses in a plasma was obtained above, and takes the following form:

$$\left(\frac{1}{v_g}\frac{\partial}{\partial t}+\frac{\partial}{\partial z}\right)a+\frac{i}{2k}\left[\Box+k_p^2\left(1-\frac{(1+k_p^{-2}\Delta\gamma)}{\gamma}\right)\right]a=0. \tag{87}$$

In solving Eq. (87) in practice we use two alternate forms of the variables, depending on the type of initial conditions. Both transformations are based on the intrinsic properties of Eq. (87).

If we are treating a semiinfinite medium $z>0$ and the pulse enters this medium moving in the positive z direction, it is convenient to perform the following change of variables:

$$\xi=z,\quad \tau=t-z/V_g. \tag{88}$$

Equation (87) then assumes the form

$$\frac{\partial a}{\partial\xi}+\frac{i}{2k}\left\{\Delta_\perp+\frac{\partial^2}{\partial\xi^2}-\frac{2}{v_g}\frac{\partial^2}{\partial\xi\partial\tau}+\frac{1}{v_g^2}\left(1-\frac{v_g^2}{c^2}\right)\frac{\partial^2}{\partial\tau^2}+k_p^2\left[1-\gamma^{-1}\right.\right.$$
$$\left.\left.\times\left[1+k_p^{-2}\left(\Delta_\perp\gamma+\frac{\partial^2\gamma}{\partial\xi^2}-\frac{2}{v_g}\frac{\partial^2\gamma}{\partial\xi\partial\tau}+\frac{1}{v_g^2}\frac{\partial^2\gamma}{\partial\tau^2}\right)\right]\right]\right\}a=0. \tag{89}$$

If the problem is posed on the infinite domain $-\infty<z<\infty$ and at time $t=0$ the pulse is prescribed with a known form, then it is convenient to make another change of variables:

$$\xi=v_g t-z,\quad \tau=t. \tag{90}$$

Equation (87) then assumes the form

$$\frac{1}{v_g}\frac{\partial a}{\partial\tau}+\frac{i}{2k}\left\{\Delta_\perp+\left(1-\frac{v_g^2}{c^2}\right)\frac{\partial^2}{\partial\xi^2}-\frac{2v_g}{c^2}\frac{\partial^2}{\partial\xi\partial\tau}-\frac{1}{c^2}\frac{\partial^2}{\partial\tau^2}+k_p^2\right.$$
$$\left.\times\left[1-\gamma^{-1}\left[1+k_p^{-2}\left(\Delta_\perp\gamma+\frac{\partial^2\gamma}{\partial\xi^2}\right)\right]\right]\right\}a=0. \tag{91}$$

The transformation (88) corresponds to a rotation of the slanted region of propagation of a pulse in the t, z plane parallel to the z axis. The transformation (90) rotates the propagation region parallel to the time axis. Convenience in solving the problem and the initial conditions determine which transformation to choose.

In the approximation of slowly varying complex amplitudes, Eq. (89) takes the form

$$\frac{\partial a}{\partial\xi}+\frac{i}{2k}\left[\Delta_\perp+k_p^2\left(1-\frac{1+k_p^{-2}\Delta_\perp\gamma}{\gamma}\right)\right]a=0. \tag{92}$$

It was precisely this equation which was treated in previous work.[8,15,16,19]

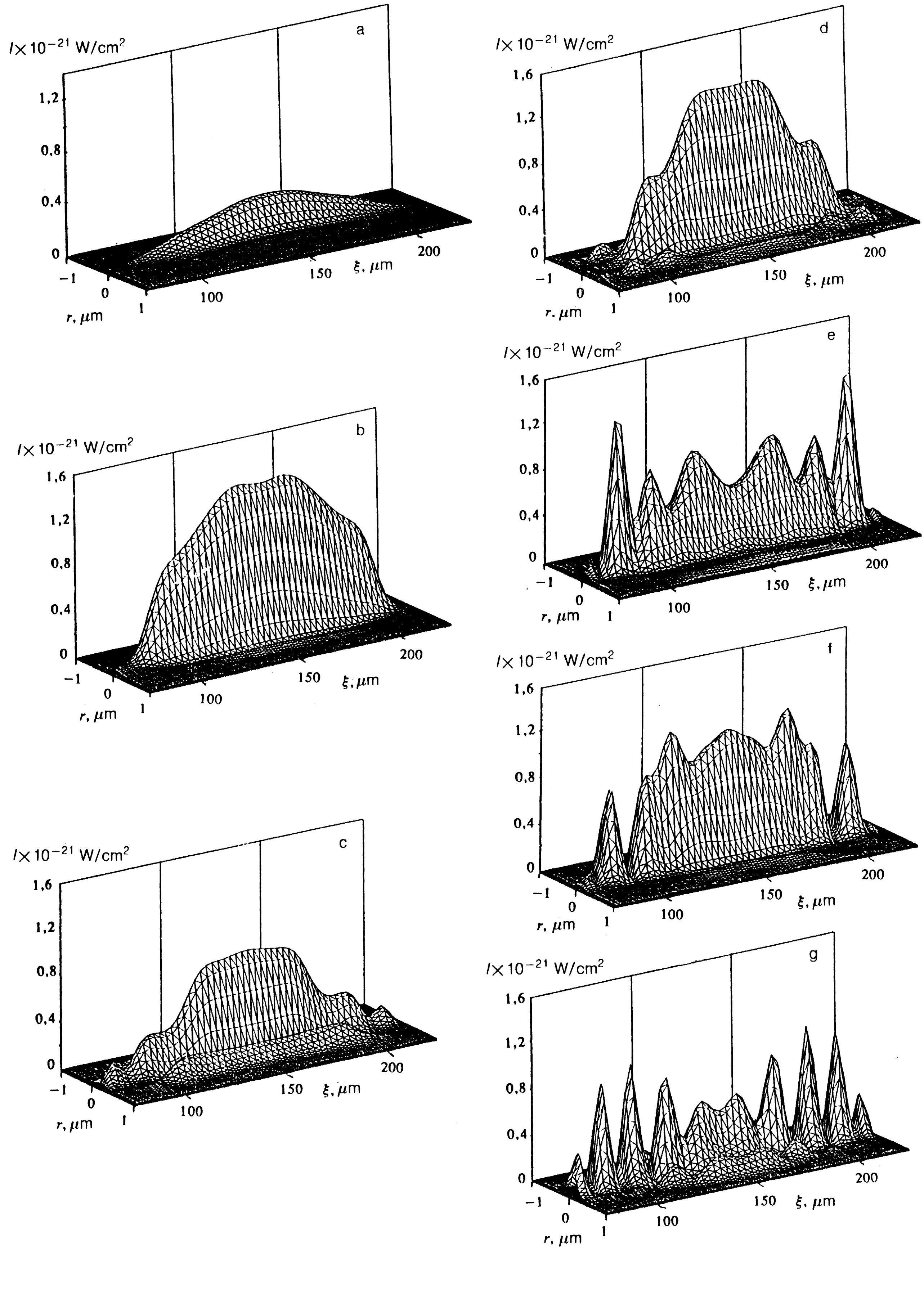

FIG. 1. A succession of "snapshots," illustrating the self-channeling of an ultrashort high-power laser pulse in plasma (τ, fs): a) 112.5; b) 150; c) 262.5; d) 300; e) 337.5; f) 375; g) 487.5.

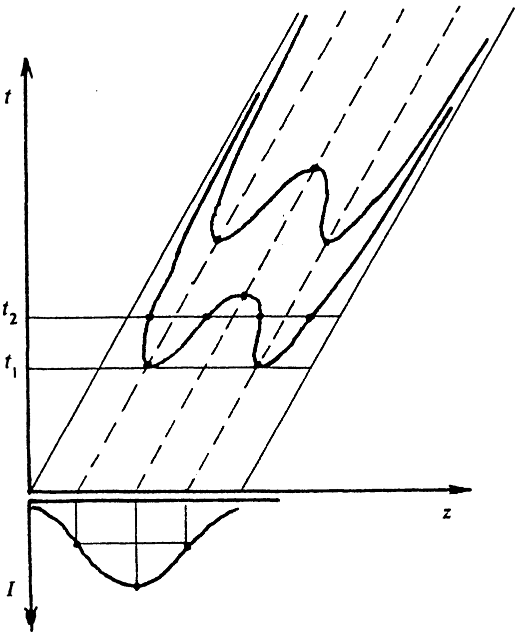

FIG. 2. Trajectories of the foci in the z,t plane, obtained in Ref. 19 by solving the NLSE with the relativistic-ponderomotive nonlinearity, dropping the second derivatives parallel to the direction of pulse propagation.

4. TIME-DEPENDENT SPATIALLY TWO-DIMENSIONAL PROPAGATION OF AN ULTRASHORT HIGH-POWER LASER PULSE IN PLASMA

Here we present the results of a numerical simulation of the following equation:

$$\frac{1}{v_g}\frac{\partial a}{\partial \tau}+\frac{i}{2k}\left\{\Delta_\perp+\left(1-\frac{v_g^2}{c^2}\right)\frac{\partial^2}{\partial \xi^2}+k_p^2\right.$$
$$\left.\times\left[1-\gamma^{-1}\left[1+k_p^{-2}\left(\Delta_\perp\gamma+\frac{\partial^2\gamma}{\partial\xi^2}\right)\right]\right]\right\}a=0. \quad (93)$$

Below in the two-dimensional calculations we use a model expression for the γ factor (86).

At the initial time $\tau=0$ we assume that a laser pulse is given with intensity profile ($\xi=v_g t-z$)

$$I(r,\xi,\tau=0)=I_0\exp\left[-\left(\frac{r}{r_0}\right)^{N_1}-\left(\frac{\xi}{L}\right)^{N_2}\right]. \quad (94)$$

We assume that the initial phase is planar. The field amplitudes are taken to vanish at infinity:

$$a(r=\infty,\xi,\tau)=a(r,\xi=\pm\infty,\tau)=0. \quad (95)$$

On the axis we impose the condition

$$\frac{\partial a}{\partial r}(r=0,\xi,\tau)=0. \quad (96)$$

We have solved Eq. (93) with the initial and boundary conditions (94)–(96). The values of the radiation and plasma parameters were taken as follows: $\lambda=0.248$ μm (KrF* excimer laser[1]), initial focal spot radius $r_0=3$ μm, which corresponds to the focusing system used in Ref. 20, electron density $n_e=7.5\cdot10^{20}$ cm^{-3} (N_2 gas at pressure ≈ 1 atm), and laser pulse length $2\tau=0.8$ ps, corresponding to a length $L=120$ μm. The ratio of the squares of the plasma and laser frequencies in this case is $\omega_p^2/\omega^2=0.043$. The initial peak intensity was taken to be $I_0=2.98\cdot10^{19}$ W/cm^2. (The radiation and plasma parameters given above correspond to the constants $a_1=248.6$ and $a_2=2/3$ used in Refs. 15, 16, and 19.) Initially the transverse and longitudinal intensity distribution was taken to be Gaussian, i.e., we took $N_1=2$, $N_2=2$ in Eq. (94). In this case the laser pulse contains an energy $E=6.0$ J, and the initial power in the transverse cross section corresponding to the peak intensity is equal to $P_0=8.4\cdot10^{12}$ W. From Refs. 8 and 19 the critical power of the relativistic-ponderomotive self-focusing is equal to $P_{cr}=1.62\cdot10^{10}(\omega/\omega_p)^2$

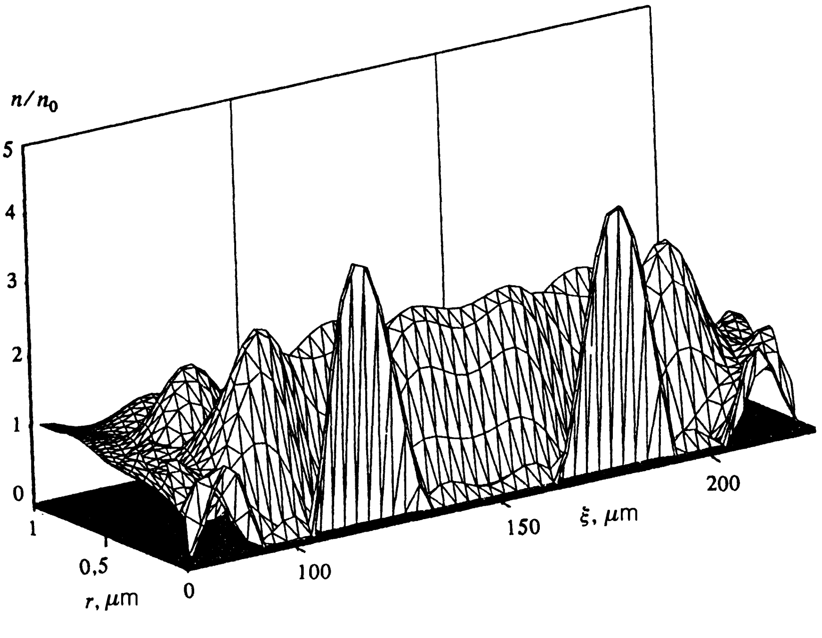

FIG. 3. Electron density profile at time $\tau=300$ fs.

$=3.77 \cdot 10^{11}$ W. Thus, we are treating a case in which the initial power is substantially greater than the critical value for the central regions of the laser pulse.

Equation (93) has a conservation law

$$\frac{\partial}{\partial t}\int_0^\infty rdr \int_{-\infty}^\infty d\xi |a|^2=0. \tag{97}$$

In the numerical integration we check the calculations by evaluating this integral.

Whenever this problem is solved numerically the high-frequency region of the spatial perturbations is cut off. For this problem we could not use a PC (running at 33 MHz) if spatial perturbations with $k_{\parallel} > 4800 \text{ cm}^{-1}$ were allowed to develop. However, as our calculations show, high-frequency longitudinal perturbations occur at substantial distances from the start of the calculations (in fact, beyond the points at which the first and second foci form).

Figure 1 shows the results of solving the problem. Figures 1a, b, corresponding to the times $\tau=112.5$ and 150 fs, respectively, illustrate the formation of the first focus by the central part of the pulse. Figure 1c ($\tau=262.5$ fs) shows how the field intensity is distributed after passing through the first focus. Two peaks can be seen, moving out toward the edges of the pulse, along with a ring which develops around the central part. The first focus disperses on account of energy transfer to the ring structure. Figure 1d ($\tau=300$ fs) illustrates the formation of the second focus. Shown is the complicated structure of the central part of the pulse, consisting of four symmetrically distributed peaks, of which the two outermost are moving toward the ends and the two inner ones are moving toward the center, producing a powerful central focus. Figures 1e, f, and g illustrate the subsequent transformation of the pulse at successive times $\tau=337.5$ fs, 375 fs, and 487.5 fs. Figure 1d shows the decay of the second pulse. The subsequent evolution of the pulse is accompanied by a strong spatial self-modulation of its shape, i.e., it breaks up into a large number of peaks parallel to the direction of propagation.

We can understand the main reason for the occurrence of self-modulation of the pulse if we use the model of the motion of the foci in the (z,t) plane developed in Ref. 19. These foci arise in the process of self-focusing of different transverse layers of the ultrashort pulse (Fig. 2). (Note that in Ref. 19 the second derivatives $\partial^2/\partial\xi^2$ were not treated, so the results of that work can only be applied in a qualitative fashion. Nevertheless, this is still possible, since in the initial stages of pulse propagation the derivatives are small because the pulse is stretched out longitudinally.) Different initial transverse sections of the pulse are focused at different distances, since they are transporting different amounts of power. If the initial peak power is greater than or of order the relativistic value, $I_0 > I_r = 4.46 \cdot 10^{19}$ W/cm^{-2}, which serves as a universal parameter of the medium and of the radiation wavelength, then the trajectory of the first focus in the (z,t) plane has three turning points (Fig. 2). The pulse therefore begins to focus at its leading and trailing edges, where two foci develop symmetrically about the center of the pulse (the time t_1 indicated in Fig. 2). Then each of these foci breaks up into two more (time t_2). Of these the two outer ones move away toward the edges of the pulse and the two inner ones proceed toward the center, merge, and then disappear (undergoing conversion to the ring structure). For the conditions considered in the present work the depth of the dip on the trajectory of the first focus is 6 fs (Ref. 19). This process is therefore actually somewhat smeared out. The two inner peaks are not resolved, and the overall picture resembles the formation of a single broad focus in z which then decays into three foci, two of which propagate to the ends while one remains at the center and then merges into the ring structure. Thus, the main reason for the self-modulation of an ultrashort laser pulse is that the different transverse sections are focused at different distances, since they have different amounts of power.

As a result of the self-focusing process the pulse breaks up into several peaks. After this the principal reason for the enhancement of the spatial self-modulation of the pulse comes into play. This is the longitudinal ponderomotive effect, i.e., the second derivatives $\partial^2\gamma/\partial\xi^2$ in the ponderomotive term. In order for this derivative to play a role, it is necessary that the pulse envelope have several peaks as a function of z. This means that a considerable number of concavities and convexities develop for the function γ. Near the peaks of the foci we have $\partial^2\gamma/\partial\xi^2<0$, and this term only enhances the transverse ponderomotive effect $\Delta_\perp \gamma<0$. The electrons are expelled from the focal regions more effectively. However, in the intervals between foci of the z axis the γ surface is concave downward and $\partial^2\gamma/\partial\xi^2>0$ holds. Consequently, this term has the opposite sign and partly cancels the transverse ponderomotive effect $\Delta_\perp \gamma<0$. The electrons are less effectively repelled from the interfocal regions, and the repulsion may shut off entirely. Figure 3 illustrates this situation. Two peaks in the electron density are shown, situated in the regions between the foci. On these electron density bumps the radiation undergoes enhanced refraction in the transverse direction. As a result, the field intensity in the interfocal region decreases. The spatial self-modulation of the pulse is enhanced.

In Eq. (93) there is one second derivative of ξ left in addition to the transverse diffraction operator $\Delta_\perp a$. The role of this ponderomotive effect is small, since it is multiplied by a small coefficient.

Reference 22 is of considerable interest. There an equation of the form (93) with a Kerr nonlinearity was solved for a wide beam (in slab geometry), under conditions such that the numerical coefficient of the derivative $\partial^2 a/\partial\xi^2$ present in addition to the diffraction operator was relatively large. [The equation of Ref. 22 is the same as (93) with the change in notation $\tau\leftrightarrow\xi$.] It was shown that the second derivative given above can go over to self-modulation of the pulse before self-focusing occurs.

Our treatment differs from that of Ref. 22 through the use of a different type of nonlinearity, and also of a small number of coefficients in the derivative noted above. The pulse undergoes self-focusing more rapidly, and the resulting self-modulation is due to the second derivative $\partial^2\gamma/\partial\xi^2$

in the nonlinear term, not to that in the diffraction operator.

5. CONCLUSION

In this work we have proposed a theoretical model for the propagation of an ultrashort ($10<\tau<1000$ fs) high-power ($P>10^{10}$ W) laser pulse in plasma, based on the use of a model in which the electrostatic and ponderomotive forces are instantaneously in balance under relativistic conditions for the electron component and the assumption that the plasma ions are stationary. We have presented a systematic derivation of the equation describing propagation of circularly and linearly polarized laser radiation, including the previously disregarded corrections to the expressions for the relativistic γ factors, taking into account the increase in the electron mass due to oscillations in the intense light field. We have shown that both cases are described by the same equation for the complex field amplitude, where the γ factors have different forms but are not very different from the model expression $\gamma^2=1+|a|^2$.

We have reported the first solutions of the time-dependent spatially two-dimensional problem involving the evolution of an ultrashort high-power laser pulse in the plasma with the relativistic-ponderomotive nonlinearity. In contrast to previous investigations,[8,15,16,19] the second derivatives $\partial^2/\partial\xi^2$ in the diffraction operator and in the ponderomotive term have been retained in Eq. (93).

We have presented visualizations of the three-dimensional solutions in the form of a sequence of "snapshots," illustrating the time evolution of an ultrashort pulse as a single entity. Thus, we have numerically simulated the dynamics of a self-channeled ultrashort high-intensity laser pulse in matter.

The calculations reveal that the pulse undergoes strong longitudinal spatial self-modulation as it propagates in the fully ionized plasma under conditions such that the relativistic-ponderomotive nonlinearity applies. The effect is spatially two-dimensional in nature. Different transverse sections of the pulse are focused at different distances, since they carry different powers. Self-focusing of the pulse causes it to become peaked primarily in the direction of propagation z. In the regions between the peaks the longitudinal ponderomotive effect balances the transverse effect, and there the bunched electrons are retained. The radiation undergoes refraction in these clumps in the transverse directions, enhancing the dips in the intensity between adjoining peaks. The self-modulation of the pulse is manifested in broadening of the spectrum of the scattered laser radiation, which can be detected experimentally.

Borisov *et al.*[15,16] established that quite arbitrary two-dimensional solutions of the NLSE (92) asymptotically approach the zeroth eigenmode of this equation when the power threshold condition[19] is satisfied. The present work shows that the real nature of the self-channeling of an ultrashort laser pulse in matter is complicated by the strong self-modulation of its shape parallel to the direction of propagation.

Note that the question regarding the asymptotic properties of the solutions for systems with dissipation is meaningful only when energy is supplied externally.[23]

The investigation carried out in the present work does not answer all possible questions. In order to develop a more complete description of the nonlinear propagation of an ultrashort laser pulse in plasma it is necessary to solve Eq. (91), including in addition to (93) the cross-derivative $\partial^2/\partial\xi\partial\tau$ and the second derivative term $\partial^2/\partial\tau^2$ in the wave operator. Moreover, refinements in the description of the behavior of the electron component of the plasma in a strong light field, introduction of dissipation of the field energy to the model, self-consistent ionization, and other effects are possible.

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[1)] We are grateful to O. B. Shiryaev for this information (private communication).

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