Quasitransition surface wave radiation

M. E. Ryazanov and A. N. Safronov

Engineering-Physics Institute, 115409 Moscow, Russia
(Submitted 7 May 1993; resubmitted 7 June 1993)
Zh. Eksp. Teor. Fiz. **104**, 3512–3520 (October 1993)

The spectral and angular distribution of surface-wave radiation due to the elastic reflection of particles from the interface between two nonabsorptive media is obtained. It is suggested that the number of charged particles reflected from the surface may be estimated from the surface waves emitted by the particles

1. The excitation of electromagnetic surface waves by a charge in uniform rectilinear motion was discussed in Ref. 1, which predicts a marked increase in intensity for small glancing angles between the velocity of the particle and the surface of the substance. It is known, however, that for glancing angles less than the Lindhard angle, a charged particle does not penetrate a single crystal but is reflected specularly from its surface.[2] A particle incident at small glancing angles on the surface of an *amorphous* substance will also suffer specular reflection;[3] an upper bound on the value of the glancing angle for the specular reflection case has been obtained in Ref. 4. Thus, at small glancing angles a fast particle does not pass through the surface of the substance but rather is reflected from it. Therefore at small glancing angles the result of Ref. 1 on the surface-wave excitation intensity has nothing to do with the real physical situation, and so it is of interest to investigate surface-wave formation for specular reflection of particles from the surface. In the following we call this quasitransition radiation. It should be noted that the problem of surface-wave excitation due to a charge moving parallel to the surface has been examined in Refs. 5 and 6; the correspondence between the classical and quantum mechanical treatments of surface-wave formation due to the scattering of charged particles from a metal surface has been discussed in Ref. 7.

2. Suppose a particle of charge e travels in a medium of permittivity ϵ_2 and is specularly reflected from the plane interface with another medium, of permittivity ϵ_1 (see Fig. 1). In elastic scattering of a charged particle, the tangential velocity component v remains unchanged whereas the normal component u changes sign. The current density then has the form

$$\vec{j}(r,t)=e(\vec{v}+\vec{u})\delta(x-vt)\delta(y)\delta(z-ut)\Theta(-t)+e(\vec{v}-\vec{u})\delta(x-vt)\delta(y)\delta(z+ut)\Theta(t), \quad (1)$$

where $\Theta(t)=1$ for $t>0$ and $\Theta(t)=0$ for $t<0$. It is convenient to write down the Fourier transform of the field of a surface wave decaying into the interior of the medium as follows:

$$\vec{E}_{1,2}(r,t)=\int d\omega\int d^2q\vec{E}_{1,2}(q,\omega)\exp(iqR-i\omega t\pm\gamma_{1,2}z), \quad (2)$$

where $\gamma_{1,2}=\sqrt{q^2-(\omega/c)^2\epsilon_{1,2}}$, $\gamma_{1,2}>0$.

By solving Maxwell's equation for standard boundary conditions and the current (1), the Fourier components of the fields of interest can be obtained in straightforward fashion. For the surface *TE*-wave we have

$$\begin{aligned}\vec{E}_{1,2}&=(\pm i\gamma_{1,2}(\vec{q}/q^2)+\vec{l}_z)E_{1,2z}\\ \vec{H}_{1,2}&=(q_y\vec{l}_x-q_x\vec{l}_y)(\omega/c)(\epsilon_{1,2}/q^2)E_{1,2}z,\end{aligned} \quad (3)$$

where $\vec{l}_x$, $\vec{l}_y$, $\vec{l}_z$ are the unit vectors of the respective axes and $E_{2z}=(\epsilon_1/\epsilon_2)E_{1z}$ with

$$E_{1z}=-\frac{(e/\pi^2)u(q^2-(\omega/c^2)\epsilon_2qv)}{[(\epsilon_1\gamma_2+\epsilon_2\gamma_1)(\gamma_2^2u^2+(\omega-qv)^2)]}. \quad (4)$$

3. Let us evaluate the spectral and angular distribution of the surface-wave power flow in the medium ϵ_2 through the plane perpendicular to the motion of the particle (i.e., the plane $x=x_0$):

$$\Delta E_2=(c/4\pi)\int_{-\infty}^{\infty}dt\int_{-\infty}^{\infty}dy\int_0^{\infty}dz[\vec{E}_2(r,t),\vec{H}_2(r,t)]_x. \quad (5)$$

Substituting (2) into (5) and integrating over t, y, and z we find

$$\Delta E_2=\pi c\int d\omega\int d^2q\int d^2q'\frac{[\vec{E}_2(q,\omega),\vec{H}_2(q',-\omega)]_x}{\gamma_2(q,\omega)+\gamma_2(q',-\omega)}\times\exp(i(q_x+q_x')x_0)\delta(q_y+q_y'). \quad (6)$$

Using (3) and (4) in integrating (6), and changing from integration over d^2q to integration over dq and $d\varphi$ ($d^2q=qdqd\varphi$), one finds

$$\Delta E_2=\frac{e^2}{2\pi^3}\int d\omega\int d\varphi\int dq\int dq_x'\times\frac{\epsilon_1^2u^2\omega\cos(\varphi)(q^2-(\omega/c^2)\epsilon_2qv\cos(\varphi))^2}{\epsilon_2\gamma_2(\gamma_2^2u^2+(\omega-qv\cos(\varphi))^2)^2}\times\frac{\exp(i(q_x+q_x')x_0)}{G(q,\omega)G(q^*,-\omega)}, \quad (7)$$

where φ is the polar angle, in the XY plane, between the wave vector $q=(q_x,q_y)$ and the X axis. In Eq. (7) we have

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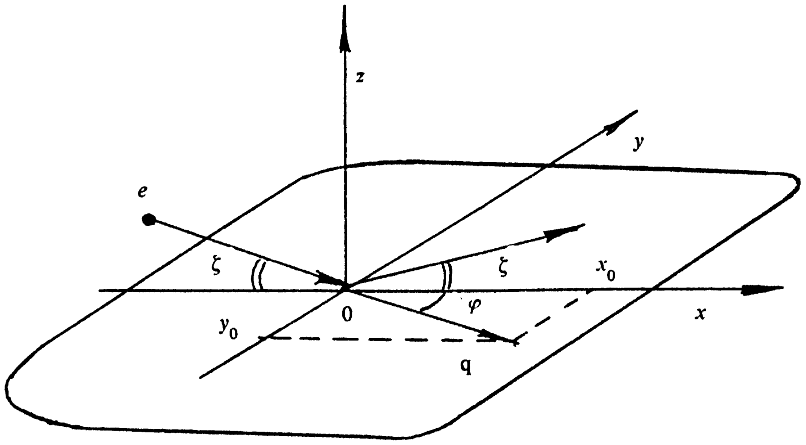

FIG. 1. Geometry of a particle reflecting from an interface. The interface coincides with the XY plane. A charged particle moves along the X axis in the medium ϵ_2 ($z>0$) and is reflected elastically from a medium with ϵ_1 ($z<0$). The point of observation has the coordinates (x_0,y_0); q is the wave vector.

introduced the notation $q^* = (q'_z, -q_y)$, $q_x=q\cos\varphi$, $G(q,\omega)=\epsilon_1\gamma_2+\epsilon_2\gamma_1$. As is well known, surface waves obey the dispersion relation $G(q,\omega)=\epsilon_1\gamma_2+\epsilon_2\gamma_1=0$, and the condition for the existence of surface waves is $\epsilon_1\epsilon_2<0$, $\epsilon_1+\epsilon_2<0$. Since the integrand in (7) oscillates as a function of q'_x, it follows that the integral is nonzero in the region $q'_x = -q_x$. Except in the resonant denominator $G(q^*,-\omega)$, which vanishes for a surface wave, we may take $q'_x = -q_x$ everywhere in (7). Near this value, we perform a series expansion of $G(q^*,-\omega)$ in q'_x and retain the first nonvanishing term to obtain

$$G(q^*,-\omega)\approx -q_x(q'_x+q_x)(\epsilon_1\gamma_1+\epsilon_2\gamma_2)/\gamma_1\gamma_2. \qquad (8)$$

Substituting (8) into (7) and using the limiting relation

$$\frac{\sin(x_0(q_x+q'_x))}{(q_x+q'_x)}\approx\pi\delta(q_x+q'_x),$$

valid for large values of x_0, we can perform the integral over dq'_x and we obtain

$$\Delta E_2=\frac{-ie^2}{2\pi^2}\int d\omega\int d\varphi\int dq \times\frac{\epsilon_1^2u^2\omega\gamma_1(q^2-(\omega/c^2)\epsilon_2 qv\cos(\varphi))^2}{\epsilon_2 q(\gamma_2^2u^2+(\omega-qv\cos(\varphi))^2)^2} \times\frac{1}{G(q,\omega)(\epsilon_1\gamma_1+\epsilon_2\gamma_2)}. \qquad (9)$$

To integrate (9) over dq, we use the relation

$$\mathrm{Re}\frac{i}{G+i0}=\pi\delta(G)=\pi\frac{\delta(q-q_0)}{\left|\frac{\delta G}{\delta q}\right|}\Bigg|_{G(q_0)=0} \qquad (10)$$

to obtain the spectral-angular distribution of the surface-wave power flow in the ϵ_2 medium:

$$\frac{d^2E_2}{d\omega d\varphi}=\frac{e^2}{2\pi c^3}\frac{u^2\epsilon_2|\epsilon_1|^5\left[1-(v/c)\cos(\varphi)\epsilon_2\sqrt{\frac{\epsilon_1+\epsilon_2}{\epsilon_1\epsilon_2}}\right]^2}{\left[\left(1-(v/c)\cos(\varphi)\sqrt{\frac{\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2}}\right)^2+\frac{u^2\epsilon_2^2}{c^2|\epsilon_1+\epsilon_2|}\right]^2|\epsilon_1+\epsilon_2|^{3/2}}\frac{1}{(\epsilon_1|\epsilon_1|+\epsilon_2|\epsilon_2|)^2}. \qquad (11)$$

A similar argument yields the spectral-angular distribution of the power flow due to the quasitransition surface-wave radiation in the ϵ_1 medium:

$$\frac{d^2E_1}{d\omega d\varphi}=\frac{e^2}{2\pi c^3}\frac{u^2\epsilon_1|\epsilon_1^2\epsilon_2^3|\left[1-(v/c)\cos(\varphi)\epsilon_2\sqrt{\frac{\epsilon_1+\epsilon_2}{\epsilon_1\epsilon_2}}\right]^2}{\left[\left(1-(v/c)\cos(\varphi)\sqrt{\frac{\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2}}\right)^2+\frac{u^2\epsilon_2^2}{c^2|\epsilon_1+\epsilon_2|}\right]^2|\epsilon_1+\epsilon_2|^{3/2}}\frac{1}{(\epsilon_1|\epsilon_1|+\epsilon_2|\epsilon_2|)^2}. \qquad (12)$$

The power flows in the two media are in opposite directions, the resultant flow being directed away from the particle reflection point. In a transparent medium the energy losses of the particle are limited to radiative losses, so that the spectral-angular distribution of the particle energy losses is given by

$$\frac{d^2E}{d\omega d\varphi}=\frac{2e^2}{\pi c^3}\frac{u^2|\epsilon_1^3\epsilon_2|\left[1-(v/c)\cos(\varphi)\epsilon_2\sqrt{\frac{\epsilon_1+\epsilon_2}{\epsilon_1\epsilon_2}}\right]^2}{\left[\left(1-(v/c)\cos(\varphi)\sqrt{\frac{\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2}}\right)^2+\frac{u^2\epsilon_2^2}{c^2|\epsilon_1+\epsilon_2|}\right]^2}\frac{1}{|\epsilon_1|\epsilon_1|+\epsilon_2|\epsilon_2||}. \tag{13}$$

We note that the minimum radiation, from Eqs. (11)–(13), is observed at the angle $\varphi_{\min}$:

$$\varphi_{\min}=\arccos\left((c/v)\frac{1}{\epsilon_2}\sqrt{\frac{\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2}}\right). \tag{14}$$

In the nonrelativistic limit the spectrum simplifies to

$$\frac{d^2E_1}{d\omega d\varphi}=\frac{e^2}{2\pi c^3}\frac{u^2\epsilon_1|\epsilon_1^2\epsilon_2^3|}{|\epsilon_1+\epsilon_2|^{3/2}(\epsilon_1|\epsilon_1|+\epsilon_2|\epsilon_2|)^2}, \tag{15a}$$

$$\left|\frac{d^2E_2}{d^2E_1}\right|=\left|\frac{\epsilon_1}{\epsilon_2}\right|^2. \tag{15b}$$

Note that Eqs. (11), (12), and (13) are obtained under the assumption that no absorption takes place in the media ϵ_1 and ϵ_2 for the wave zone

$$|r_0-r_e(t)|\left(\frac{\omega}{c}\right)\sqrt{\frac{\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2}}\gg 1 \tag{16}$$

[$r_e=(x_e,0,z)$ is the position of the particle at time t] for distances from the trajectory which are large compared to the radiation formation length l_f (see Ref. 8):

$$|r_0-r_e(t)|\gg l_f$$

$$l_f=2\pi v/\omega\left|1-\frac{v}{c}\cos(\varphi)\sqrt{\frac{\epsilon_1\epsilon_2}{\epsilon_1+\epsilon_2}}\right|. \tag{17}$$

In the case where the radiation formation length is large [e.g., for a grazing particle when the observation angle is $\varphi=\arccos((c/v)\sqrt{\epsilon_1\epsilon_2/(\epsilon_1+\epsilon_2)})$], the relations (11), (12), and (13) become invalid. It follows that, by formally taking the limit $\beta_z=u/c\to 0$ in Eqs. (11) and (12), it is not possible to obtain the surface-wave radiation relations for a particle moving parallel to the interface. This requires a separate surface-wave generation analysis for the wave zone, for distances less than the radiation formation length.

4. In order to obtain the spectral and angular distribution of surface-wave radiation at distances less than the radiation formation length from the particle trajectory, it is convenient to utilize a method employed in Ref. 1 in the transition radiation problem. For simplicity, we will analyze the surface-wave generation process for the case of a particle being reflected from a vacuum-medium interface, i.e., we set $\epsilon_1=\epsilon<-1$ and $\epsilon_2=1$. For a particle approaching the interface, the normal component of the strength vector in the vacuum, E_{2z}, is

$$E'_{2z}(r,\omega)=-ie\frac{\omega\sqrt{\omega c}\sqrt{-2i\epsilon}}{c^2(\epsilon^2-1)\sqrt{\pi}}\left(\frac{\epsilon}{\epsilon-1}\right)^{5/4}\int_{-\infty}^{0}dt$$
$$\times\frac{\beta_x\cos(\alpha(t))+i\beta_z\sqrt{\epsilon}}{\sqrt{\rho(t)}}\exp\left(-\frac{\omega z}{c\sqrt{\epsilon-1}}\right)$$
$$\times\exp\left(i\omega t+\frac{i\omega}{c}\sqrt{\frac{\epsilon}{\epsilon-1}}\rho(t)+\omega\frac{\beta_z t}{\sqrt{\epsilon-1}}\right), \tag{18}$$

where $\beta=v/c$, $\alpha(t)=\arctan(y/x-x_e)$, and $\rho(t)=\sqrt{(x-x_e)^2+y^2}$. For a particle reflected from the interface, the normal component of the field strength in the vacuum, E''_{2z}, may be obtained from E'_{2z} by replacing β_z by β_z and changing the time integration limits to $(0,\infty)$. The remaining field components are given by

$$\begin{aligned}E'_{2\perp}&=\frac{-i}{\sqrt{\epsilon}}(l_x\cos(\alpha)+l_y\sin(\alpha))E'_{2z},\\ H'_{2\perp}&=\sqrt{\frac{\epsilon-1}{\epsilon}}(l_x\sin(\alpha)-l_y\cos(\alpha))E'_{2z}.\end{aligned} \tag{19}$$

It is convenient to denote by $F(t)$ the argument of the exponential in (18):

$$F(t)=i\omega t+i\frac{\omega}{c}\sqrt{\frac{\epsilon}{\epsilon-1}}\rho(t)+\frac{\omega\beta_z t}{\sqrt{\epsilon-1}},$$

$$F_1=\mathrm{Re}\,F;F_2=\mathrm{Im}\,F.$$

In the near-field region $|r_0-r_e|\sim l_f$, the integrand in (18) oscillates and one can perform the integration using the stationary phase method. The stationary phase point in question there corresponds to the value t_s for which $F'_2(t_s)=0$. This implies

$$\beta_x\cos\alpha_s=\sqrt{\frac{\epsilon-1}{\epsilon}}, \tag{20}$$

where α_s is the value of the angle α at time t_s. The time interval Δt during which the radiation is observable is determined from the relation $|F_2(t)-F_2(t_s)|\sim 1$. Hence, by $F'_2=0$, we have

$$\Delta t_s\approx\frac{1}{\beta_x}\sqrt{\frac{\rho(t_s)}{\omega c\sin^2(\alpha_s)}}\left|\frac{\epsilon-1}{\epsilon}\right|^{1/4}. \tag{21}$$

In order that the integrand change little in the time Δt_s we need

$$\frac{\beta_z\omega\Delta t_s}{\sqrt{\epsilon-1}}\ll 1. \tag{22}$$

The condition (22) may be written in the form

$$\rho(t_s) \ll \sqrt{\frac{\epsilon-1}{\epsilon}} \frac{c}{\beta_z^2\omega} (1-\epsilon+\beta_x^2\epsilon). \tag{23}$$

For $\beta_z \to 0$, the restriction (23) has little importance for the use of the stationary phase method. Integrating Eq. (18) we find

$$E'_{2z}(r,\omega) = -\frac{2ie\epsilon^2\omega\sqrt{-2i}}{c^2(\epsilon^2-1)(\epsilon-1)} \frac{\beta_x \cos\alpha_s + i\beta_z\sqrt{\epsilon}}{\beta_x|\sin(\alpha_s)|} \times \exp\left(\frac{-\omega z}{c\sqrt{(\epsilon-1)}}\right) \times \exp\left(i\omega t_s + i(\omega/c)\sqrt{\frac{\epsilon}{\epsilon-1}}\rho(t_s) + \frac{\omega\beta_z t_s}{\sqrt{\epsilon-1}}\right) \tag{24}$$

For a particle which has been reflected from the interface, the vacuum field $E''_{2z}(r,\omega)$ is obtained using the replacement

$$E''_{2z} = E'_{2z}(\beta_z \to -\beta_z) \tag{25}$$

Equations (24) and (25) hold for different regions in space: Eq. (24) is valid for those observation points for which the coordinates x_0, y_0 lie outside the angle $-\alpha_s \leqslant \varphi \leqslant \alpha_s$, where $\varphi = \arctan(y_0/x_0)$ (region I, Fig. 2); Eq. (25) is valid for observation points inside the angle $-\alpha_s \leqslant \varphi \leqslant \alpha_s$ (region II, Fig. 2), these being the only points accessible to the radiation emitted at an angle to the x axis. By calculating the power flow through the $y = y_0$ plane for a surface wave due to a particle approaching the interface, one obtains the spectral distribution of the radiation energy per unit path of the particle for observation points outside the angle $-\alpha_s \leqslant \varphi \leqslant \alpha_s$:

$$\frac{d^2E_2}{d\omega dx} = \frac{2e^2\epsilon^3\omega}{\beta_x c^2(\epsilon-1)(\epsilon^2-1)^2} \frac{\epsilon-1+\epsilon^2\beta_z^2}{\sqrt{1-\epsilon+\epsilon\beta_x^2}} \times \exp\left(\frac{-2\omega\beta_z}{c\beta_x\sqrt{\epsilon-1}}\left|x - y_0\cot(\alpha_s)\right|\right), \tag{26}$$

where $\cot(\alpha_s) = \sqrt{\epsilon-1}/\sqrt{\beta_x^2\epsilon-\epsilon+1}$. For observation points inside the angle $-\alpha_s \leqslant \varphi \leqslant \alpha_s$, the energy distribution will also satisfy Eq. (26). Analogously to Eq. (26), one obtains a relation for the spectral distribution of the radiation per unit path of the particle in the medium:

$$\frac{d^2E_1}{d\omega dx} = \frac{2e^2\epsilon\omega}{\beta_x c^2(\epsilon-1)(\epsilon^2-1)^2} \frac{\epsilon-1+\epsilon^2\beta_z^2}{\sqrt{1-\epsilon+\epsilon\beta_x^2}} \times \exp\left(\frac{-2\omega\beta_z}{c\beta_x\sqrt{\epsilon-1}}\left|x - y_0\cot(\alpha_s)\right|\right). \tag{27}$$

Note that in the limiting case $\omega\beta_z t_s/\sqrt{\epsilon-1} \sim 1$, we have from Eq. (26)

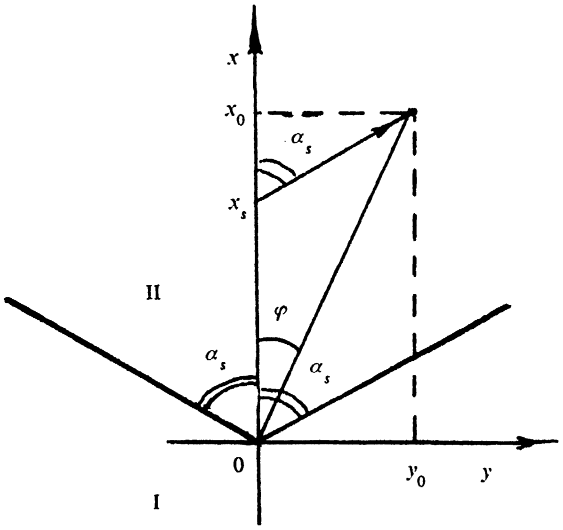

FIG. 2. Integration domains as used in the stationary phase method. Region I: outside the angle range $-\alpha_s \leqslant \varphi \leqslant \alpha_s$. Region II: inside the range $-\alpha_s \leqslant \varphi \leqslant \alpha_s$. The diagram corresponds to the case in which surface-wave radiation comes to the observation point (x_0, y_0) following the reflection of the particle at the point (0,0).

$$\frac{d^2E_2}{d\omega} = \frac{2e^2\epsilon^3}{\beta_x c\sqrt{\epsilon-1}(\epsilon^2-1)^2} \frac{\epsilon-1+\epsilon^2\beta_z^2}{\sqrt{1-\epsilon+\epsilon\beta_x^2}} \times \exp\left(\frac{-2\omega\beta_z}{c\beta_x\sqrt{\epsilon-1}}\left|x - y_0\cot(\alpha_s)\right|\right). \tag{28}$$

In the other limiting case, $\Delta t_s \ll t_s \ll \sqrt{\epsilon-1}/\beta_z\omega$, a coherent particle-surface interaction takes place, analogous to that for a particle moving parallel to the surface at a small distance above.[5,6] We note also that the relations (11) and (12) may be derived from the general expressions (18) and (19) in the limit of large distances from the particle reflection point.

5. In summary, the physics of surface-wave generation, for a charged particle reflected from an interface, depends sensitively on the relation between the distance from the particle (to the observation point) and the radiation formation length. We may regard the radiation formation zone as a surface-wave source. Generally speaking, such a source is extended parallel to the projection of the particle velocity on the interface. If the distance to the detector is very large compared to the source size (or the coherence length), then the source in question may be regarded as a point-source ["small" formation zone, Eqs. (11)–12)]. If the distance to the detector is less than, or comparable to, the source size ["large" formation zone, Eqs. (26)–(27)], then the surface wave is mainly emitted normal to the axis of the source. Naturally, this gives rise to different angular distributions of the surface waves being radiated, and one can speak of two different kinds of radiated surface waves. Also, the dependence of the radiation on the incident-particle glancing angle is different: For large radiation formation lengths, the intensity varies inversely as the particle

glancing angle $\zeta=\arctan(u/v)$, whereas for a small radiation formation length the intensity is proportional to the square of the glancing angle.

It is interesting to compare the surface-wave formation intensity for a charge passing through the surface (transition radiation, intensity $d^2_{2,tr}$) with that for the specular reflection case (quasitransition radiation, $d^2_{2,qtr}$). If the incident particle characteristics are the same, the ratio of these intensities has the form

$$f(\omega,\varphi)=\left(\frac{d^2E_{2,tr}}{d^2E_{2,qtr}}\right)=\frac{2(\epsilon+1)^2((\sqrt{\epsilon-1}-\beta_x\cos(\varphi)\sqrt{\epsilon})^2+\beta_z^2)(1+\epsilon\beta_z^2-\beta_z^2)}{\epsilon(\sqrt{\epsilon}-\beta_x\cos(\varphi)\sqrt{\epsilon-1})^2((\sqrt{\epsilon-1}-\beta_x\cos(\varphi)\sqrt{\epsilon})^2+\epsilon^2\beta_z^2)}, \tag{29a}$$

$$\left(\frac{d^2E_{1,tr}}{d^2E_{1,qtr}}\right)=\epsilon\left(\frac{d^2E_{2,tr}}{d^2E_{2,qtr}}\right). \tag{29b}$$

In the case when the incidence angle of the beam is close to the Lindhard angle, the particles are partly reflected from the interface and partly penetrate through it. If the radiation is incoherent, then a knowledge of the total (transition-plus-quasitransition) radiation intensity enables one to estimate both the number of surface-penetrating particles, N_p, and of those reflected, N_r, by measuring the radiation intensity $W_1(\omega,\varphi)$ in the medium and that in the vacuum, $W_2(\omega,\varphi)$.

$$W_{1,2}(\omega,\varphi)=N_p\left(\frac{d^2E_{tr}}{d\varphi d\omega}\right)_{1,2}+N_r\left(\frac{d^2E_{qtr}}{d\varphi d\omega}\right)_{1,2}, \tag{30}$$

where $(d^2E_{qtr}/d\varphi d\omega)_{1,2}$ is given by the relations (11)–(12), and $(d^2E_{tr}/d\varphi d\omega)_{1,2}$ is the spectral-angular distribution of the transition radiation.[8] From Eq. (30), it follows that the number of surface-reflected particles is

$$N_r=\frac{\left(\epsilon\dfrac{W_2(\omega,\varphi)}{\left(\dfrac{d^2E_{qtr}}{d\varphi d\omega}\right)_2}-\dfrac{W_1(\omega,\varphi)}{\left(\dfrac{d^2E_{qtr}}{d\varphi d\omega}\right)_1}\right)}{\epsilon-1}. \tag{31}$$

The number of the interface-penetrating particles can be estimated from the experimentally measured quantities W_1 and W_2 using the relation

$$N_p=\frac{\left(\dfrac{\epsilon W_1(\omega,\varphi)}{\left(\dfrac{d^2E_{tr}}{d\varphi d\omega}\right)_1}-\dfrac{W_2(\omega,\varphi)}{\left(\dfrac{d^2E_{tr}}{d\varphi d\omega}\right)_2}\right)}{(\epsilon-1)}. \tag{32}$$

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Translated by E. Strelchenko

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