Vector properties of the radiation-pressure force for an atom in a magnetic field

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A general expression is derived for the radiation-pressure force for an atom which is moving perpendicular to a static magnetic field **H**. This field is collinear with the propagation direction of two counterpropagating light waves with a linear, circular, or elliptical polarization. The derivation is carried out by perturbation theory in the resonance approximation. The behavior of the radiation-pressure force as a function of the frequency and polarization of the counterpropagating waves is analyzed in the case of slight saturation and for an arbitrary value of **H**. In a strong magnetic field, the vector properties and quantitative characteristics of the radiation-pressure force change substantially upon a switch from the resonance at the central frequency of the transition to resonances at frequencies of transitions between Zeeman sublevels.

The radiation-pressure force (RPF) has been the subject of tens of experimental and theoretical studies, which are cited in some monographs.[1–3] The theoretical papers have essentially been restricted to a scalar model of a two-level atom, with the degeneracy of the quantum states ignored.[4–6] This circumstance has made it possible to study the behavior of the RPF for an arbitrary intensity of the resonant light waves. As was shown in Refs. 7 and 8, attempts to incorporate a degeneracy of levels in a calculation of the RPF in studies of the radiative cooling of atoms and the trapping of atoms in an optical field run into serious mathematical difficulties in the case of arbitrary angular momenta and intense light waves. These difficulties worsen considerably when there is a magnetic field **H**.

In some recent experiments[9,10] the vector properties of the RPF have been studied along with other topics. Grimm *et al.*[11] observed a new magnetooptic force with some novel vector properties. They gave a clear description of the phenomenon they observed without going into detail on the degeneracy of the resonance levels of the atom at $\mathbf{H}=0$ or their splitting into Zeeman sublevels in the presence of a magnetic field. In this connection, the intensities of the light waves should be kept low in order to find a clear physical interpretation of the vector properties of the RPF. This approach makes it possible to incorporate an arbitrary degeneracy of the resonance levels with respect to angular-momentum projections in a study of the RPF, regardless of whether there is a magnetic field.

In the present paper we examine the vector properties and quantitative characteristics of the RPF for an atom in a weak electric field of two counterpropagating light waves. The waves have an identical frequency ω but arbitrary amplitudes and polarizations. The waves are propagating collinear with a static magnetic field **H**. The atom of interest is moving along a direction perpendicular to **H** and can be in resonance states with arbitrary angular momenta allowed by the selection rules for a dipole interaction with the counterpropagating waves.

In this problem, the symmetry for the atom in the given external fields can be taken into account fully in solving the equation for the density matrix incorporating the degeneracy of the levels with respect to angular-momentum projections. As a result we obtain a general formula which gives a unified description of the RPF for an arbitrary frequency ω and for all possible amplitudes and polarizations of the waves.

We find that in a magnetic field, near the resonance at the central frequency of the transition, $\omega=\omega_{ba}$, the Zeeman sublevels with positive and negative angular-momentum projections participate on an equal basis in the interaction of the atom with the counterpropagating waves. This is a fundamental point, determining the vector properties of the RPF. In this case the RPF breaks up into terms which are even and odd with respect to **H** or with respect to the detuning from resonance, $\omega-\omega_{ba}$. The direction of the terms of the RPF which are even with respect to **H** is determined by the given wave vectors; in some cases it is also determined by the sign of the detuning from resonance, $\omega-\omega_{ba}$. The direction of the terms which are odd with respect to **H** depends on the magnetic field. If we interchange the waves, the terms which are even in **H** are conserved, while those which are odd change sign. If counterpropagating waves with $\omega=\omega_{ba}$ and equal amplitudes $R_1=R_2$ have identical polarizations, the RPF vanishes. If the directions of the wave vectors of the counterpropagating waves are held constant, and the right- and left-hand circular polarizations are interchanged with $\omega=\omega_{ba}$ and $R_1=R_2$, there is no change in the RPF.

Near Zeeman resonances, in contrast, this equivalence of the Zeeman levels is disrupted, since a leading role is played by exclusively the resonance Zeeman sublevels. As a result, there are substantial changes in the properties of the RPF. In a strong magnetic field, for example, this separation of the RPF into terms even and odd in **H** or $\omega-\omega_{ba}$ no longer occurs, regardless of the polarizations of the counterpropagating waves. There is also a change in the way in which the RPF depends on the angle between the polarization planes of the waves. If the waves have identical amplitudes and polarizations (linear, right- or left-hand circular, or elliptical), the corresponding RPF is nonzero,

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in contrast with the resonance $\omega=\omega_{ba}$. After an interchange of right- and left-hand circular polarizations of the counterpropagating waves, with no change in the amplitudes, in the phase difference, or in the directions of the wave vectors, we find that the RPFs for individual pairs of Zeeman resonances trade places with respect to the new counterpropagating waves.

1. FORMULATION OF THE PROBLEM; CALCULATION METHOD

Let us consider an atom which is moving perpendicular to a static magnetic field **H** and also perpendicular to the propagation direction of two traveling light waves with identical frequencies $\omega_1=\omega_2=\omega$ and oppositely directed wave vectors $\mathbf{k}_1=-\mathbf{k}_2$, as has been the case in several experiments (see, for example, Refs. 10–14).

We direct the Cartesian z axis along the direction collinear with $\mathbf{k}_1$ and $\mathbf{k}_2$. The resultant electric field of these counterpropagating waves is then

$$\mathbf{E}=[\mathbf{l}_1R_1\exp(i\Phi_1)+\mathbf{l}_2R_2\exp(i\Phi_2)]\exp(-i\omega t)+\text{c.c.},\tag{1}$$

where

$$\Phi_1=\mathbf{k}_1\mathbf{r}-\alpha_1,\quad \Phi_2=\mathbf{k}_2\mathbf{r}-\alpha_2,$$

$\mathbf{l}_1$ and $\mathbf{l}_2$ are unit polarization unit vectors, R_1 and R_2 are real amplitudes, and α_1 and α_2 are constant phase shifts.

For linearly polarized waves, the polarization wave vectors are

$$\mathbf{l}_n=\mathbf{l}_x\cos\phi_n+\sigma_1\mathbf{l}_y\sin\phi_n,\quad n=1,2,\tag{2}$$

where

$$\sigma_n=\mathbf{l}_z\mathbf{k}_n/k,\quad n=1,2,$$

$\mathbf{l}_x$, $\mathbf{l}_y$, and $\mathbf{l}_z$ are unit vectors along the Cartesian coordinates x, y, and z; and the positive direction for the angle ϕ_n is the clockwise direction if we are looking along the unit vector $\mathbf{l}_z$, regardless of the direction of $\mathbf{k}_n$.

In the case of elliptically polarized waves whose polarization ellipses have parallel axes, with a right-hand polarization, we have

$$\mathbf{l}_n=\mathbf{l}_x\cos\psi_n+i\sigma_n\mathbf{l}_y\sin\psi_n,\quad n=1,2,\tag{3}$$

while for a left-hand polarization we replace (3) by

$$\mathbf{l}_n=-\mathbf{l}_x\sin\psi_n+i\sigma_n\mathbf{l}_y\cos\psi_n,\quad n=1,2.\tag{4}$$

The argument ψ_n in (3) and (4) takes on values on the interval $0\leqslant\psi_n\leqslant\pi/2$. This argument characterizes the semiaxes of the polarization ellipses, which are equal to $\cos\psi_n$ and $\sin\psi_n$ in (3) and $\sin\psi_n$ and $\cos\psi_n$ in (4). In particular, at the value $\psi_n=\pi/4$ the polarization ellipses become circles of unit radius, and Eqs. (3) and (4) describe right- and left-hand circular polarizations. In the cases $\psi_n=0$ and $\psi_n=\pi/2$, the polarization ellipses become deformed into orthogonal line segments. These segments are on the x and y axes in the case of (3) and on the y and x axes in the case of (4). In this case, Eqs. (3) and (4), with (1), describe linearly polarized waves with polarization planes which are parallel or orthogonal. The variable-sign factor $\sigma_n=\pm1$ appears in (2)–(4) because of the common rule for describing the polarization vectors of the two counterpropagating waves, (1).

The frequency ω is close to the frequency $\omega_{ba}=(E_b-E_a)/\hbar$, which is the frequency of a transition between two states of the atom with a zero nuclear spin. These states are characterized by, along with the energies E_a and E_b ($E_a<E_b$), the quantum numbers J_a, J_b, M_a, and M_b, where J_a and J_b corresponds to the angular-momentum operators $\mathbf{J}_a$ and $\mathbf{J}_b$ in the lower and upper levels, and M_a and M_b describe the projections $\mathbf{l}_z\mathbf{J}_a$ and $\mathbf{l}_z\mathbf{J}_b$ of these angular momenta onto the quantization axis, which is directed along the unit vector $\mathbf{l}_z$.

In addition to the electric field in (1), the atom interacts with the static magnetic field **H**, which is collinear with $\mathbf{k}_1$ and $\mathbf{k}_2$, and which splits the lower and upper levels into Zeeman sublevels:

$$E_{M_a}=E_a+\hbar\Omega_aM_a,\quad E_{M_b}=E_b+\hbar\Omega_bM_b,\tag{5}$$

where

$$\Omega_a=g_a\mu_B(\mathbf{l}_z\mathbf{H})/\hbar,\quad \Omega_b=g_b\mu_B(\mathbf{l}_z\mathbf{H})/\hbar,\quad \mu_B=|e|\hbar/2mc,$$

g_a and g_b are gyromagnetic factors, μ_B is the Bohr magneton, and e and m are the charge and mass of an electron.

The state of the atom in electric field (1) and magnetic field **H** is described by a quantum-mechanical equation for the density matrix ρ in the JM representation:

$$\left[\frac{\partial}{\partial t}+\mathbf{v}\nabla+i(\omega_{ba}+\Omega_bM_b-\Omega_aM_a)+\gamma_{ba}\right]\rho_{M_bM_a}=\frac{i}{\hbar}(\mathbf{E}\mathbf{d}_{M_bM'_a}\rho_{M'_aM_a}-\rho_{M_bM'_b}\mathbf{E}\mathbf{d}_{M'_bM_a}),\tag{6}$$

$$\left[\frac{\partial}{\partial t}+\mathbf{v}\nabla+i\Omega_b(M_b-M'_b)+\gamma_b\right]\rho_{M_bM'_b}=\frac{i}{\hbar}(\mathbf{E}\mathbf{d}_{M_bM_a}\rho_{M_aM'_b}-\rho_{M_bM_a}\mathbf{E}\mathbf{d}_{M_aM'_b}),\tag{7}$$

$$\left[\frac{\partial}{\partial t}+\mathbf{v}\nabla+i\Omega_a(M_a-M'_a)+\gamma_a\right]\rho_{M_aM'_a}=\frac{\gamma_a}{2J_a+1}\delta_{M_aM'_a}+\frac{\gamma(2J_b+1)}{|d_{ba}|^2}\mathbf{d}_{M_aM_b}\rho_{M_bM'_b}\mathbf{d}_{M'_bM'_a}+\frac{i}{\hbar}(\mathbf{E}\mathbf{d}_{M_aM_b}\rho_{M_bM'_a}-\rho_{M_aM_b}\mathbf{E}\mathbf{d}_{M_bM'_a}),\tag{8}$$

where

$$\mathbf{k}_1\mathbf{v}=0,\quad \gamma_{ba}=(\gamma_a+\gamma_b)/2,$$

$$\gamma=4|d_{ba}|^2\omega_{ba}^3/3\hbar c^3(2J_b+1),$$

$\mathbf{v}$ is the velocity of the atom; $\mathbf{d}_{M_bM_a}$ is a matrix element of the operator $\mathbf{d}$, which represents the electric dipole moment of the atom; d_{ba} is the reduced dipole moment; γ_{ba} is the half-width of the spectral line of the resonance transition; $\hbar\gamma_a$ and $\hbar\gamma_b$ are the homogeneous widths of the lower level E_a and the upper one E_b; γ is the probability for the spontaneous emission of a photon $\hbar\omega_{ba}$ by the isolated atom; and a repeated matrix index implies a summation.

The term containing γ_a on the right side of Eq. (8) reflects the circumstance that, before it enters field (1), the atom is in a steady state in the lower level, which is described by a steady-state solution of Eqs. (6)–(8) with $\mathbf{E}=0$ and $\mathbf{H}\neq 0$ in the form

$$\rho^{(0)}_{M_aM'_a}=\delta_{M_aM'_a}/(2J_a+1),\quad \rho^{(0)}_{M_bM'_b}=\rho^{(0)}_{M_bM_a}=0,$$

where we have assumed an equiprobable distribution between Zeeman sublevels in the magnetic field, since $\hbar\Omega_a(M_a-M'_a)$ is small in comparison with the energy per degree of freedom in the case of a statistical energy distribution.

As the initial time for Eqs. (6)–(8) we use the instant $t=0$, at which the atom enters electric field (1), at the point $x=y=0$, $z\neq 0$.

By virtue of the resonance interaction with counterpropagating waves (1), the atom experiences an RPF.[1–8] When the degeneracy of the levels with respect to angular-momentum projections in the $\mathbf{H}=0$ case is taken into account, and when the presence of a Zeeman splitting of the levels, (5), in the case $\mathbf{H}\neq 0$ is taken into account, this RPF is conveniently written in the form

$$\mathbf{F}=\langle \mathrm{Tr}\,\rho\nabla(\mathbf{dE})\rangle, \tag{9}$$

where $\langle\cdots\rangle$ means an average over the time t between t and $t+2\pi/\omega$.

Expression (9) was derived with the help of a quantum-mechanical equation of motion. However, the RPF could also be derived from the conservation of electromagnetic energy for an atomic gas:

$$\int_V \dot{\mathbf{P}}\mathbf{E}dV=\oint_f \mathbf{S}d\mathbf{f},$$

where $\mathbf{S}$ is the Poynting vector, $\mathbf{P}=N\,\mathrm{Tr}\,\rho\mathbf{d}$ is the vector dielectric polarization of the atomic gas, and N is the density of resonance atoms in an arbitrary volume V bounded by the surface f. The flux of the Poynting vector across the closed surface f on the left side of this equality is proportional to the force applied to the active atoms in the volume V. We thus find the RPF which is exerted on an individual resonant atom; it is expressed in a different way:

$$\mathbf{F}=\frac{\mathbf{k}_1}{\omega}\left\langle \mathbf{E}_1\frac{d}{dt}\mathrm{Tr}\,\rho\mathbf{d}\right\rangle+\frac{\mathbf{k}_2}{\omega}\left\langle \mathbf{E}_2\frac{d}{dt}\mathrm{Tr}\,\rho\mathbf{d}\right\rangle,$$

where $\mathbf{E}_1+\mathbf{E}_2=\mathbf{E}$, and $\mathbf{E}_1$ and $\mathbf{E}_2$ are the electric fields of the waves in (1).

According to (6)–(8), the density matrix which is nondiagonal in the indices a and b can be written in the resonance approximation as follows:

$$\rho_{M_bM_a}=r_{M_bM_a}\exp(-i\omega t),$$

where $r_{M_bM_a}$ is a slow function of time t in comparison with $\exp(-i\omega t)$. The two expressions written above for the RPF thus take the same form after an average is taken over the time t between t and $t+2\pi/\omega$:

$$\mathbf{F}=-i\mathbf{k}_1 r_{M_bM_a}\{\mathbf{d}_{M_aM_b}[\mathbf{l}_1^*R_1\exp(-i\Phi_1)-\mathbf{l}_2^*R_2 \times\exp(-i\Phi_2)]\}+\text{c.c.}$$

To calculate (9), we solved Eqs. (6)–(8) in the resonance approximation by perturbation theory for a weak field $\mathbf{E}$ in the absence of saturation. We allowed for the circumstance that in a study of the vector properties of the RPF it is sufficient to find the first nonvanishing result of the calculation of (9), which is quadratic in the field $\mathbf{E}$. In this approximation the term containing γ in Eq. (8), which describes the arrival of the atom in the lower level because of spontaneous emission in the upper level, makes no contribution.

2. RADIATION-PRESSURE FORCE NEAR RESONANCE, $\omega=\omega_{ba}$

To simplify the equations we assume that the amplitudes R_1 and R_2 of counterpropagating waves (1) are constant. In a perturbation theory quadratic in the field $\mathbf{E}$, the RPF in (9) thus breaks up into two quite different parts:

$$\mathbf{F}=\mathbf{F}_1+\mathbf{F}_2, \tag{10}$$

which are given in the case of a linear polarization (2) by

$$\mathbf{F}_1=\mathbf{k}_1(\hbar\gamma_{ba})^{-1}|d_{ba}|^2\{(R_1^2-R_2^2)[S'(1)+S'(-1)] +2R_1R_2[S(1)+S(-1)]\cos\phi\sin(\Phi_1-\Phi_2)\}, \tag{11}$$

$$\mathbf{F}_2=-2\mathbf{k}_1(\hbar\gamma_{ba})^{-1}|d_{ba}|^2R_1R_2\beta \times[S(1)-S(-1)]\sin\phi\cos(\Phi_1-\Phi_2). \tag{12}$$

For a right-hand elliptical polarization (3), these components are instead

$$\mathbf{F}_1=\mathbf{k}_1(\hbar\gamma_{ba})^{-1}|d_{ba}|^2\{(R_1^2-R_2^2)[S'(1)+S'(-1)] +2R_1R_2[S(1)+S(-1)] \times(\cos\psi_1\cos\psi_2-\sin\psi_1\sin\psi_2)\sin(\Phi_1-\Phi_2)\}, \tag{13}$$

$$\mathbf{F}_2=\mathbf{k}_1(\hbar\gamma_{ba})^{-1}|d_{ba}|^2\beta\{(R_1^2\sigma_1\sin 2\psi_1-R_2^2\sigma_2\sin 2\psi_2) \times[S'(1)-S'(-1)]+2R_1R_2[S(1)-S(-1)] \times(\sigma_1\sin\psi_1\cos\psi_2+\sigma_2\sin\psi_2\cos\psi_1) \times\sin(\Phi_1-\Phi_2)\}, \tag{14}$$

where

$$S(q)=\frac{1}{2J_a+1}\sum_\mu\begin{pmatrix}J_b & J_a & 1\\ \mu & q-\mu & -q\end{pmatrix}^2\frac{\gamma_{ba}}{(\Delta-\Delta_{\mu q})^2+\gamma_{ba}^2} \times\{(\Delta-\Delta_{\mu q})[1-\exp(-\gamma_{ba}t)\cos[(\Delta-\Delta_{\mu q})t]] -\gamma_{ba}\exp(-\gamma_{ba}t)\sin[(\Delta-\Delta_{\mu q})t]\}, \tag{15}$$

$$S'(q)=\frac{1}{2J_a+1}\sum_\mu\begin{pmatrix}J_b & J_a & 1\\ \mu & q-\mu & -q\end{pmatrix}^2\frac{\gamma_{ba}}{(\Delta-\Delta_{\mu q})^2+\gamma_{ba}^2} \times\{\gamma_{ba}[1-\exp(-\gamma_{ba}t)\cos[(\Delta-\Delta_{\mu q})t]] +(\Delta-\Delta_{\mu q})\exp(-\gamma_{ba}t)\sin[(\Delta-\Delta_{\mu q})t]\},$$

$$\Delta_{\mu q}=\mu(\Omega_b-\Omega_a)+q\Omega_a,\quad q=\pm 1,$$
$$\Delta=\omega-\omega_{ba},\quad \phi=\sigma_1\phi_1+\sigma_2\phi_2,\quad \beta=(\mathbf{l}_x\times\mathbf{l}_y)\mathbf{l}_z. \tag{16}$$

The notation for the $3j$ symbol, $\left(\begin{smallmatrix} a & b & c \\ d & e & f\end{smallmatrix}\right)$, and for the reduced dipole moment d_{ba} is taken from Ref. 15.

To pursue the analysis we adopt the condition that the linear polarization vectors in (2) are characterized by angles ϕ_1 and ϕ_2 which lie in the interval $-\pi\leqslant\phi_n\leqslant\pi$, where $n=1,2$. If an angle ϕ_n lies outside this interval, then the substitution $\mathbf{l}_n\to-\mathbf{l}_n$ will put the new angle $\phi'_n=\phi_n-\pi$ in the interval $-\pi\leqslant\phi'_n\leqslant\pi$, and the minus sign which appears on $\mathbf{l}_n$ will lead to a phase shift $\Phi_n\to\Phi_n+\pi$.

If the counterpropagating waves are polarized in a common plane or in mutually perpendicular planes, expressions (11) and (13) are the same, as are (12) and (14). As an example we consider the case $\mathbf{l}_1=\mathbf{l}_x$ and $\mathbf{l}_2=\mathbf{l}_y$. According to (1)–(3), we should then set $\sigma_1=-\sigma_2=1$, $\phi=\pi/2$, $\psi_1=0$, $\psi_2=\pi/2$ in (11)–(14) and make the substitution $\alpha_2\to\alpha_2-\pi/2$. After these changes, the forces in (11) and (13) take the same form, as do (12) and (14).

Equations (13) and (14) cover several cases of the polarizations of waves (1). If both elliptically polarized waves have a left-hand polarization, as in (4), we need to make the following substitution in (13) and (14):

$$\psi_n\to\pi/2+\psi_n,\quad n=1,2. \tag{17}$$

If one of the elliptically polarized waves has a left-hand polarization, then substitution (17) is made for the corresponding parameter ψ_1 or ψ_2. In the case of a right-hand circular polarization we have one of the following: $\psi_1=\pi/4$ and $\psi_2\neq\pi/4$, $\psi_1\neq\pi/4$ and $\psi_2=\pi/4$, $\psi_1=\psi_2=\pi/4$. For a left-hand circular polarization we make the substitution $\psi_1\to3\pi/4$ or $\psi_2\to3\pi/4$, respectively. If one of the waves in (1), e.g., the first, is linearly polarized, then we should set $\psi_1=0$ for $\mathbf{l}_1=\mathbf{l}_x$ according to (3). In the case of an orthogonal polarization $\psi_1=\pi/2$, $\sigma_1=1$, and $\mathbf{l}_1=i\mathbf{l}_y$, we need to make the substitution $\alpha_1\to\alpha_1+\pi/2$. The polarization unit vector then takes a canonical form, $\mathbf{l}_1=\mathbf{l}_y$, and the factor of i is incorporated in a constant phase shift.

For one traveling wave, $R_1=R$ and $R_2=0$, with a linear polarization (2) or an elliptical polarization (3) or (4), in the steady state, under the condition

$$1\ll\gamma_{ba}t, \tag{18}$$

the RPF in (10) in the absence of a magnetic field is of the same form:

$$\mathbf{F}=\frac{2\mathbf{k}_1|d_{ba}|^2R^2\gamma_{ba}}{3(2J_a+1)\hbar(\Delta^2+\gamma_{ba}^2)}. \tag{19}$$

In the case of a linearly polarized standing wave,

$$\mathbf{k}_1/k_1=\mathbf{l}_z,\quad \mathbf{l}_1=\mathbf{l}_2,\quad R_1=R_2=R,\quad \alpha_1=\alpha_2,\quad \mathbf{H}=0,$$

in the steady-state regime in (18) for the RPF in (10), we find, according to (11) and (12),

$$\mathbf{F}=\frac{4\mathbf{k}_1|d_{ba}|^2R^2\Delta}{3(2J_a+1)\hbar(\Delta^2+\gamma_{ba}^2)}\sin(2k_1z). \tag{20}$$

In the case $J_a=0$, the RPFs in (19) and (20) are the same as the results of Refs. 1–5, 16 and 1, 2, 9, 16, respectively, if we set $\mathbf{d}^2=|d_{ba}|^2/3$ and switch to the case of a weak field $\mathbf{E}$ in those cases. Those previous results were derived for a two-level atom without consideration of a level degeneracy with respect to angular-momentum projections.

Finally, for counterpropagating waves which are polarized in a common plane, the RPF in (10) in region (18) with $\mathbf{H}=J_a=0$ and $d^2=|d_{ba}|^2/3$ is the same as the result derived in Ref. 16 without consideration of level degeneracy.

3. RADIATION-PRESSURE FORCE NEAR ZEEMAN RESONANCES

Equations (11)–(14) are useful for studying the behavior of RPF (10) near a resonance $\Delta=0$ at the central transition frequency ω_{ba}, in which case Zeeman sublevels (5) with positive and negative angular-momentum projections appear in these equations in equivalent ways. In a study of the RPF near resonances involving Zeeman sublevels, however, this equivalency is disrupted. In this case it is necessary to decompose RPF (10) into terms corresponding to circular waves which are natural modes for propagation in a direction collinear with $\mathbf{H}$.

For this purpose we use a right-hand Cartesian coordinate system in which the unit pseudoscalar β has a positive value $\beta=1$, and the unit vector $\mathbf{l}_z$ along the z axis is parallel to the quantization axis. The expansion of RPF (10) in natural modes then takes the following form in the case of counterpropagating waves with linear polarization (2):

$$\mathbf{F}=\mathbf{k}_1(\hbar\gamma_{ba})^{-1}|d_{ba}|^2\{(R_1^2-R_2^2)[S'(1)+S'(-1)]$$
$$+2R_1R_2[S(1)\sin(2\mathbf{k}_1\mathbf{r}-\phi-\alpha)$$
$$+S(-1)\sin(2\mathbf{k}_1\mathbf{r}+\phi-\alpha)]\}, \tag{21}$$

where

$$\Phi_1-\Phi_2=2\mathbf{k}_1\mathbf{r}-\alpha,\quad \alpha_1-\alpha_2=\alpha.$$

The expansion of RPF (10) in natural modes for counterpropagating waves with a right-hand elliptical polarization as in (3) is written in a different way:

$$\mathbf{F}=\mathbf{k}_1(\hbar\gamma_{ba})^{-1}|d_{ba}|^2\{Q_1S'(1)+Q_2S'(-1)$$
$$+2[Q_3S(1)+Q_4S(-1)]\sin(\Phi_1-\Phi_2)\}, \tag{22}$$

where

$$Q_1=R_1^2(\cos\psi_1+\sigma_1\sin\psi_1)^2-R_2^2(\cos\psi_2+\sigma_2\sin\psi_2)^2, \tag{23}$$

$$Q_2=R_1^2(\cos\psi_1-\sigma_1\sin\psi_1)^2-R_2^2(\cos\psi_2-\sigma_2\sin\psi_2)^2, \tag{24}$$

$$Q_3=R_1R_2(\cos\psi_1+\sigma_1\sin\psi_1)(\cos\psi_2+\sigma_2\sin\psi_2), \tag{25}$$

$$Q_4=R_1R_2(\cos\psi_1-\sigma_1\sin\psi_1)(\cos\psi_2-\sigma_2\sin\psi_2). \tag{26}$$

For the left-hand elliptical polarization in (4), we make substitution (17) in Eqs. (23)–(26), taking note of the comments made for other possible polarizations of the waves.

It can be seen from (22) that for a first wave propagating along $\mathbf{l}_z$ with a right-hand circular polarization, $\sigma_1=1$ and $\psi_1=\pi/4$, and for a second wave propagating opposite $\mathbf{l}_z$ with a left-hand circular polarization, $\sigma_2=-1$ and $\psi_2\to 3\pi/4$, we have $Q_2=Q_4=0$. For these waves, a resonance on Zeeman sublevels is therefore realized at the frequency

$$\omega=\omega_{ba}+M_b(\Omega_b-\Omega_a)+\Omega_a \tag{27}$$

for the terms in $S'(1)$ and at frequencies

$$\omega=\omega_{ba}+M_{ba}(\Omega_b-\Omega_a)+\Omega_a\pm\gamma_{ba} \tag{28}$$

for the terms in $S(1)$. These resonances correspond to transitions $J_b\to J_a$ characterized by the quantum numbers

$$M_b-M_a=1, \quad -J_b\leqslant M_b\leqslant J_b.$$

Zeeman resonances play an extremely important role in a strong magnetic field:

$$(\delta E_a/\hbar)^2\gg\Omega_a^2\gg\gamma_{ba}^2, \quad (\delta E_b/\hbar)^2\gg\Omega_b^2\gg\gamma_{ba}^2, \tag{29}$$

where δE_a and δE_b are the fine splitting of the lower and upper levels. Near resonances (27) and (28), the quantities $S(1)$ and $S'(1)$ in Eqs. (11)–(14), (21), and (22) are larger than $S(-1)$ and $S'(-1)$ by a factor of Ω_a^2/γ_{ba}^2 or Ω_b^2/γ_{ba}^2. The latter can thus be omitted. In addition, in the sum over the index μ in $S(1)$ and $S'(1)$ we should retain only the resonance terms.

Correspondingly, for a first wave propagating along $\mathbf{l}_z$ with a left-hand circular polarization, $\sigma_1=1$ and $\psi_1\to 3\pi/4$, and for a second wave propagating opposite $\mathbf{l}_z$, with a right-hand circular polarization, $\sigma_2=-1$ and $\psi_2=\pi/4$, we have $Q_1=Q_3=0$. For these waves a resonance thus occurs at the frequency

$$\omega=\omega_{ba}+M_b(\Omega_b-\Omega_a)-\Omega_a \tag{30}$$

for the terms in $S'(-1)$ and at frequencies

$$\omega=\omega_{ba}+M_b(\Omega_b-\Omega_a)-\Omega_a\pm\gamma_{ba} \tag{31}$$

for the terms in $S(-1)$. These resonance correspond to transitions $J_b\to J_a$ characterized by the quantum numbers

$$M_b-M_a=-1, \quad -J_b\leqslant M_b\leqslant J_b.$$

Near resonances (30) and (31) in a strong magnetic field, (29), the quantities $S(-1)$ and $S'(-1)$ in Eqs. (11)–(14), (21), and (22) are larger than $S(1)$ and $S'(1)$ by a factor of Ω_a^2/γ_{ba}^2 or Ω_b^2/γ_{ba}^2. The latter can thus be ignored, and in the sum over the index μ in $S(-1)$ and $S'(-1)$ we need to retain only the resonance terms.

If the frequency of the linearly polarized waves lies near resonances (27) and (28), then the RPF in (21) in steady state (18) for a strong magnetic field, (29), becomes

$$\mathbf{F}=\frac{\mathbf{k}_1|d_{ba}|^2}{(2J_a+1)\hbar}\begin{pmatrix} J_b & J_a & 1 \\ M_b & 1-M_b & -1\end{pmatrix}^2 \times\left[\frac{(R_1^2-R_2^2)\gamma_{ba}}{\Delta_1^2+\gamma_{ba}^2}+\frac{2R_1R_2\Delta_1}{\Delta_1^2+\gamma_{ba}^2}\sin(2\mathbf{k}_1\mathbf{r}-\phi-\alpha)\right], \tag{32}$$

where

$$\Delta_q=\omega-\omega_{ba}-M_b(\Omega_b-\Omega_a)-q\Omega_a, \quad q=\pm 1.$$

Near the resonances in (30) and (31) we have, in place of (32),

$$\mathbf{F}=\frac{\mathbf{k}_1|d_{ba}|^2}{(2J_a+1)\hbar}\begin{pmatrix} J_b & J_a & 1 \\ M_b & -1-M_b & 1\end{pmatrix}^2 \times\left[\frac{(R_1^2-R_2^2)\gamma_{ba}}{\Delta_{-1}^2+\gamma_{ba}^2}+\frac{2R_1R_2\Delta_{-1}}{\Delta_{-1}^2+\gamma_{ba}^2}\sin(2\mathbf{k}_1\mathbf{r}+\phi-\alpha)\right]. \tag{33}$$

For a standing wave, $R_1=R_2$, and $\alpha=0$, and for the atomic transition $J_b=1\to J_a=0$ with $M_b=-1$ and $\mathbf{l}_z\|\ \mathbf{H}$, the quantity in (33) is the same as the result of Ref. 10, if we switch to a weak field $\mathbf{E}$ in the latter result.

For elliptically polarized waves, RPF (22) in steady-state regime (18), for a strong magnetic field, (29), is

$$\mathbf{F}=\frac{\mathbf{k}_1|d_{ba}|^2}{(2J_a+1)\hbar}\begin{pmatrix} J_b & J_a & 1 \\ M_b & 1-M_b & -1\end{pmatrix}^2 \times\left[\frac{Q_1\gamma_{ba}}{\Delta_1^2+\gamma_{ba}^2}+\frac{2Q_3\Delta_1}{\Delta_1^2+\gamma_{ba}^2}\sin(\Phi_1-\Phi_2)\right] \tag{34}$$

near resonances (27) and (28) and

$$\mathbf{F}=\frac{\mathbf{k}_1|d_{ba}|^2}{(2J_a+1)\hbar}\begin{pmatrix} J_b & J_a & 1 \\ M_b & -1-M_b & 1\end{pmatrix}^2 \times\left[\frac{Q_2\gamma_{ba}}{\Delta_{-1}^2+\gamma_{ba}^2}+\frac{2Q_4\Delta_{-1}}{\Delta_{-1}^2+\gamma_{ba}^2}\sin(\Phi_1-\Phi_2)\right] \tag{35}$$

near resonances (30) and (31).

According to (28) and (31), the second terms in (32)–(35), containing Δ_1 and Δ_{-1}, each have two resonances, at nearly equal frequencies, differing by $2\gamma_{ba}$. Corresponding to the resonant frequencies ω in (28) and (29), which contain $+\gamma_{ba}$ and $-\gamma_{ba}$, are the following respective factors in (32)–(35):

$$\Delta_{\pm 1}/(\Delta_{\pm 1}^2+\gamma_{ba}^2)=1/2\gamma_{ba},$$

$$\Delta_{\pm 1}/(\Delta_{\pm 1}^2+\gamma_{ba}^2)=-1/2\gamma_{ba}.$$

If the waves in (1) are polarized in a common plane or in mutually perpendicular planes, then Eqs. (32) and (34) are the same, as are (33) and (35). For example, let us assume $\mathbf{l}_1=\mathbf{l}_y$ and $\mathbf{l}_2=\mathbf{l}_x$. We should then set $\sigma_1=1$, $\psi_1=\pi/2$, and $\Psi_2=0$ in (34) and (35) and make the substitution $\alpha_1\to\alpha_1+\pi/2$. Equations (34) and (35) then take the form of (32) and (33), respectively, with $\phi=\pi/2$.

The substitution $\mathbf{H}\to-\mathbf{H}$ changes the numerical values of the resonant frequencies in (27), (28), (30), and

(31), since Ω_a and Ω_b change sign. This point must be taken into account in calculating the RPFs in (32)–(35).

4. VECTOR PROPERTIES OF THE RADIATION-PRESSURE FORCE NEAR THE RESONANCE $\omega=\omega_{ba}$

The results in (10)–(14) can be used to study the vector properties of the RPF which follow from the symmetry for the atom in the external fields. For this purpose we note that the quantities in (15) and (16) depend on not only the time t but also the quantities q, **H**, and Δ. We accordingly use the notation

$$S(q)=S(q,\mathbf{H},\Delta), \quad S'(q)=S'(q,\mathbf{H},\Delta).$$

The substitution $\mathbf{H}\to-\mathbf{H}$ does not alter the direction of the quantization axis selected in (5), so the quantities Ω_a and Ω_b change sign. However, in the case of the substitutions $\Omega_a\to-\Omega_a$ and $\Omega_b\to-\Omega_b$ we should change the summation index, $\mu\to-\mu$, in (15) and (16). As a result we find the important equations

$$S(q,-\mathbf{H},\Delta)=S(-q,\mathbf{H},\Delta), \qquad S'(q,-\mathbf{H},\Delta)=S'(-q,\mathbf{H},\Delta). \tag{36}$$

In the case of the substitution $\Delta\to-\Delta$ we should also change the summation index, $\mu\to-\mu$, in (15) and (16); as a result we find the new equations

$$S(q,\mathbf{H},-\Delta)=-S(-q,\mathbf{H},\Delta), \qquad S'(q,\mathbf{H},-\Delta)=S'(-q,\mathbf{H},\Delta). \tag{37}$$

Using (36) and (37), we find

$$S(q,-\mathbf{H},0)=-S(q,\mathbf{H},0), \quad S'(q,-\mathbf{H},0)=S'(q,\mathbf{H},0).$$

It also follows from (36) and (37) that the sum $S(1)+S(-1)$ is an even function of **H** and an odd function of Δ. The difference $S(1)$–$S(-1)$ is an odd function of **H** and an even function of Δ. The sum $S'(1)+S'(-1)$ is an even function of **H** and Δ. The difference $S'(1)+S'(-1)$ is an odd function of **H** and Δ. These results, which follow from the symmetry for the atom in the external fields, underlie the vector properties of the RPF.

Let us first examine the behavior of RPF (10) near a resonance at the central frequency of the transition, $\Delta=0$, for which Zeeman sublevels with positive and negative angular-momentum projections appear symmetrically in (10)–(14). This symmetry gives rise to the following characteristic behavior.

The inversion $\mathbf{l}_x\to-\mathbf{l}_x$, $\mathbf{l}_y\to-\mathbf{l}_y$, and $\mathbf{l}_z\to-\mathbf{l}_z$ leads to the changes $\Omega_a\to-\Omega_a$ and $\Omega_b\to-\Omega_b$, which are accompanied by transformation (36). Accordingly, the differences $S(1)$–$S(-1)$ and $S'(1)$–$S'(-1)$ change sign. In addition, the sign of the unit pseudovector β changes, so RPF(10), with (11)–(14), behaves as a polar vector.

The first term in (10), $\mathbf{F}_1$, is an even function of **H**, and it is nonzero if $\mathbf{H}=0$. The direction of $\mathbf{F}_1$ in the case $\alpha_1=\alpha_2$ is determined by the vector $\mathbf{k}_1$; in the case $R_1=R_2$ it also depends on the sign of Δ. The second term in (10), $\mathbf{F}_2$, is an odd function of **H**, and it vanishes in the case $\mathbf{H}=0$. With increasing magnetic field **H** in the region

$$|\Delta_{\mu q}|\ll\max(|\Delta|,\gamma_{ba}),$$

the force $\mathbf{F}_2$ increases in proportion to **H**; it then changes to a different behavior, tending toward zero with a further increase in the magnetic field, in the region

$$|\Delta_{\mu q}|\gg\max(|\Delta|,\gamma_{ba}).$$

This behavior of the force $\mathbf{F}_2$ can be seen clearly in steady state (18), for which the characteristic differences are

$$S(1)-S(-1)=2(\mathbf{H}\mathbf{l}_z)\mu_B S_0/\hbar, \tag{38}$$

$$S'(1)-S'(-1)=2(\mathbf{H}\mathbf{l}_z)\mu_B S'_0/\hbar, \tag{39}$$

where

$$S_0=\frac{\gamma_{ba}}{2J_a+1}\sum_\mu\begin{pmatrix}J_b & J_a & 1\\ \mu & 1-\mu & -1\end{pmatrix}^2 \times\frac{[\mu(g_b-g_a)+g_a](\Delta^2-\Delta_{\mu1}^2-\gamma_{ba}^2)}{[(\Delta-\Delta_{\mu1})^2+\gamma_{ba}^2][(\Delta+\Delta_{\mu1})^2+\gamma_{ba}^2]},$$

$$S'_0=\frac{2\Delta\gamma_{ba}^2}{2J_a+1}\sum_\mu\begin{pmatrix}J_b & J_a & 1\\ \mu & 1-\mu & -1\end{pmatrix}^2 \times\frac{\mu(g_b-g_a)+g_a}{[(\Delta-\Delta_{\mu1})^2+\gamma_{ba}^2][(\Delta+\Delta_{\mu1})^2+\gamma_{ba}^2]}.$$

If we interchange the counterpropagating waves in the case of a linear polarization as in (2),

$$R_1\leftrightarrow R_2,\quad \mathbf{k}_1\leftrightarrow\mathbf{k}_2,\quad \sigma_1\leftrightarrow\sigma_2,\quad \phi_1\leftrightarrow\phi_2,\quad \alpha_1\leftrightarrow\alpha_2, \tag{40}$$

and also in the case of elliptical polarizations (3) and (4),

$$R_1\leftrightarrow R_2,\quad \mathbf{k}_1\leftrightarrow\mathbf{k}_2,\quad \sigma_1\leftrightarrow\sigma_2,\quad \psi_1\leftrightarrow\psi_2,\quad \alpha_1\leftrightarrow\alpha_2, \tag{41}$$

the force $\mathbf{F}_1$ does not change, while $\mathbf{F}_2$ changes sign. The meaning here is that transformations (40) and (41) are equivalent to the replacement $\mathbf{H}\to-\mathbf{H}$.

A special case in transformation (40) is that of waves with $R_1=R_2$ and $\alpha_1=\alpha_2$, polarized in a common plane, $\phi=0$ or $\phi=\pm\pi$. In this case the substitution $\mathbf{k}_1\leftrightarrow\mathbf{k}_2$ means an interchange of the counterpropagating waves, which is accompanied by a change in the sign of $\mathbf{F}_2$. On the other hand, the interchange of the waves in this case does not alter the physical system or the force $\mathbf{F}_2$. The only vector which does not change upon a change in sign is $\mathbf{F}_2=0$. It follows that $\mathbf{F}_2$ is an odd function of ϕ near the points $\phi=0,\pm\pi$. Since this assertion follows from the symmetry, it holds for an arbitrary intensity of the waves and for an arbitrary saturation.

It follows from (40) and (41) that at a fixed Δ the direction of $\mathbf{F}_2$ is determined by the directions of the wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$, of the first and second waves in (1); here the polarizations, amplitudes, and phase shifts of these waves are taken into account. It can also be asserted that the direction of $\mathbf{F}_2$ is determined by the polar vector $\beta\mathbf{H}$, since the substitution $\mathbf{H}\to-\mathbf{H}$ changes the sign of the force $\mathbf{F}_2$. In particular, in steady state (18) the force $\mathbf{F}_2$ is proportional to $\beta\mathbf{H}$ according to (12), (14), (38), and (39), since the following equalities hold:

$\sigma_2=-\sigma_1$, $(\mathbf{k}_1/k_1)(\mathbf{H}\mathbf{l}_z)\sigma_1=\mathbf{H}$.

The forces in (11), (13), and (14) contain terms which are even and odd in Δ, while the force in (12) is an even function of Δ. Even functions of Δ have a resonance at the frequency $\omega=\omega_{ba}$, while odd functions of Δ have two resonances, at approximately equal frequencies, $\omega=\omega_{ba}+\gamma_{ba}$ and $\omega=\omega_{ba}-\gamma_{ba}$. Here the directions of the corresponding vector terms are opposite.

If the waves are polarized in a common plane, we have $\mathbf{F}_2=0$. However, $\mathbf{F}_1$ vanishes only if $R_1=R_2$ and $\Delta=0$. If the waves are polarized in orthogonal planes, we have $\mathbf{F}_2\neq0$. Here we have $\mathbf{F}_1\neq0$ if $R_1\neq R_2$ for arbitrary Δ, or for $\Delta\neq0$ and arbitrary R_1 and R_2.

If the waves have identical amplitudes, $R_1=R_2=R$, and are strictly at resonance with the central frequency of the transition, $\Delta=0$, the RPF in (10) contains only the second term, $\mathbf{F}=\mathbf{F}_2$, and is given by

$$\mathbf{F}=-2\mathbf{k}_1(\hbar\gamma_{ba})^{-1}|d_{ba}|^2R^2\beta[S(1)-S(-1)] \times\sin\phi\cos(\Phi_1-\Phi_2) \quad (42)$$

for linear polarization (2) and

$$\mathbf{F}=2\mathbf{k}_1(\hbar\gamma_{ba})^{-1}|d_{ba}|^2R^2\beta[S(1)-S(-1)] \times(\sigma_1\sin\psi_1\cos\psi_2+\sigma_2\sin\psi_2\cos\psi_1)\sin(\Phi_1-\Phi_2) \quad (43)$$

for right-hand polarization (3). In the case of the left-hand polarization in (4), we need to take (17) into account.

If the waves have identical polarizations (linear, right-hand or left-hand circular, or elliptical with $\psi_1=\psi_2$), then the corresponding RPF (42) or (43), vanishes. For waves with a right-hand or left-hand elliptical polarization, with $\psi_1\neq\psi_2$, the quantity in (43) is nonzero.

If we fix the directions of $\mathbf{k}_1$ and $\mathbf{k}_2$ and also the phase difference $\alpha_1-\alpha_2$ and interchange polarization planes ($\phi\to-\phi$) or if we interchange the right- and left-hand circular polarizations or the right-hand (left-hand) polarization and the left-hand (right-hand) polarization in the case of elliptically polarized waves with arbitrary ψ_1 and ψ_2, the RPF in (42) changes sign, while that in (43) does not change.

In contrast, if the parameters characterizing the polarizations of the waves, ϕ_1, ϕ_2, ψ_1, and ψ_2, are kept the same, but the directions of the wave vectors $\mathbf{k}_1$ and $\mathbf{k}_2$ are reversed, with the simultaneous substitution $\alpha_1-\alpha_2\to-(\alpha_1-\alpha_2)$, the RPF in (42) does not change, while the vector in (43) changes sign.

If the directions of $\mathbf{k}_1$ and $\mathbf{k}_2$ and also the phase difference $\alpha_1-\alpha_2$ are kept the same, and if the right-hand polarizations of the elliptically polarized waves with arbitrary ψ_1 and ψ_2 are replaced by left-hand polarizations with the help of (17), then the RPF in (43) does not change.

The reason for the properties found here for RPF (10) is that the Zeeman levels with positive and negative angular momentum projections participate on an equal basis in the interaction of the atom with the waves, so the basic equations, (10)–(14), enter the picture symmetrically. This symmetry is disrupted in a strong magnetic field, (29), near the Zeeman resonances (27), (28), (30), and (31), since the sum over μ in (15) and (16) is dominated by the resonance terms alone; the other terms can be discarded. Ignoring the small terms disrupts basic equations (36) and (37), which led to the vector properties found for RPF (10).

The range of applicability of the vector properties found here for the RPFs is thus near the resonance at the central frequency of the transition, $\Delta=0$, in a weak magnetic field, while in a strong magnetic field, (29), it is far from the resonances involving Zeeman sublevels. It also incorporates the strict resonance $\Delta=0$ for an arbitrary magnetic field.

5. VECTOR PROPERTIES OF THE RADIATION-PRESSURE FORCE NEAR ZEEMAN RESONANCES

In a strong magnetic field, (29), near Zeeman resonances, the RPF has some different properties, which can be summarized as follows.

For linearly polarized counterpropagating waves, the dependence of the RPFs in (32) and (33) on the angle ϕ, between the polarization planes, near Zeeman resonances differs from that in the case of the strict resonance, $\Delta=0$, which is represented by Eq. (42).

Near Zeeman resonances, it is no longer possible to decompose the RPF into parts which are even and odd in $\mathbf{H}$ or Δ, either in the case of a linear polarization, (32) and (33), or in the case of circular or elliptical polarizations, (34) and (35). Instead, the RPFs in (32)–(35) break up into two terms, the first of which is conserved, while the second changes sign under the simultaneous substitutions $\mathbf{H}\to-\mathbf{H}$ and $\Delta\to-\Delta$.

If waves with $R_1=R_2$ have identical polarizations (linear, right-hand or left-hand circular, or elliptical with $\psi_1=\psi_2$), the corresponding RPFs in (32)–(35) are nonzero near Zeeman resonances, in contrast with the case of the exact resonance, $\Delta=0$.

If we fix R_1 and R_2 and also the parameters characterizing the polarizations of the waves, ϕ_1, ϕ_2, ψ_1, and ψ_2, and if we change the directions of the wave vectors, $\mathbf{k}_1\to-\mathbf{k}_1$ and $\mathbf{k}_2\to-\mathbf{k}_2$, while making the simultaneous substitution $\alpha_1-\alpha_2\to-(\alpha_1-\alpha_2)$, then the contributions of the resonances involving Zeeman sublevels (27) and (28) and also (30) and (31) trade places with respect to the new counterpropagating waves as the frequency ω is scanned. As a result, RPFs (32) and (34) are replaced by (33) and (35), and vice versa. This result is an important distinction from the cases in (42) and (43).

After an interchange of the right-hand circular and left-hand circular polarizations of the waves, while $\mathbf{k}_1$, $\mathbf{k}_2$, R_1, R_2, and $\alpha_1-\alpha_2$ are fixed, we find that as the frequency ω of the new counterpropagating waves is scanned the RPFs in (34) and (35) trade places near Zeeman resonances. This property of the RPFs has no analog in the case of the strict resonance, $\Delta=0$.

In the time-varying case, the factors γ_{ba} and $\Delta_{\pm1}$ in the numerator of the fractions in the first and second terms in (32)–(35) should be replaced by the expressions in curly brackets in (16) and (15), respectively, with $\mu = M_b$. Accordingly, the properties found for the RPFs hold for an arbitrary time evolution of the changes in the RPFs.

6. DISCUSSION

The vector properties and quantitative characteristics of the radiation-pressure force near resonances have been found for low wave intensities, corresponding to a slight saturation. However, individual aspects of the behavior of the RPF remain the same for arbitrary intensities, regardless of the degree of saturation. For example, the decomposition of the RPF into parts even and odd with respect to **H** near a resonance at the central frequency of the transition, $\Delta = 0$, for a weak magnetic field and exactly at resonance, $\Delta = 0$, for an arbitrary value of **H** is a consequence of the symmetry of the Zeeman sublevels with positive and negative angular-momentum projections in the interaction with the counterpropagating waves. Accordingly, this property of the RPF remains in force for arbitrary intensities of the waves, as has been verified experimentally.[11] In addition, in a sufficiently weak magnetic field the part of the RPF odd in **H** increases in proportion to **H**, regardless of the intensities of the waves. Finally, a change in the sign of the part of the RPF odd in **H** upon an interchange of the counterpropagating waves is a consequence of the same symmetry, so this property remains in force at arbitrary intensities. The latter property of the RPF in the case of the strict resonance, $\Delta = 0$, with $R_1 = R_2$ and $\alpha_1 = \alpha_2$, leads to the important conclusions that the RPF vanishes for counterpropagating waves which are polarized in a common plane, $\phi = 0, \pm\pi$, and it changes sign upon a change $\phi = -\phi$ in the angle between the polarization planes of the waves with arbitrary intensities, in agreement with experiment.[11]

The sharp change in the properties of the RPF in a strong magnetic field upon the switch from the strict resonance at the central frequency of the transition, $\Delta = 0$, to resonances involving Zeeman sublevels is a consequence of the symmetry, which gives rise to natural modes for light waves propagating in the direction collinear with **H**. Accordingly, this sharp change in the properties of the RPF will be observed for arbitrary intensities of the counterpropagating waves, although the nature of the change may vary with the intensities of these waves. For example, the ϕ dependence of the RPF in a strong magnetic field as described by (32) and (42) changes when the intensities of the waves are raised substantially. The angular dependence of the RPF in the case of strong saturation is also affected significantly by a term discarded from Eq. (8) which describes the arrival of the atom at the lower level due to spontaneous emission in the upper level. While this term is inconsequential in first-order perturbation theory, in the case of saturation it makes a contribution to the RPF which is on the same order of magnitude as the other terms on the right side of Eq. (8). Calculations carried out in second-order perturbation theory support this assertion.

A method of rectifying the radiation force in the case of two counterpropagating light waves with low intensities and arbitrary angle between the polarization planes is considered in Ref. 17 for atomic transitions with angular momenta 1/2 and 3/2 and with identical gyromagnetic factors. The results in this article can be used to generalize the method of Ref. 17 to include arbitrary polarizations of counterpropagating light waves and for atomic transitions with arbitrary angular momentum and different gyromagnetic factors.

Most of the experiments on the RPF use atoms with a nonzero nuclear spin. The hyperfine structure of the levels of such atoms seriously complicates the equations of nonlinear-optics phenomena, as can be seen in the example of four-wave mixing in a magnetic field.[18] However, despite this complication of nonlinear-optics phenomena, the properties of the RPF which stem from the symmetry remain the same when there is a hyperfine level structure. The behavior of the RPF is quite different from that of an atom without a nuclear spin, because of resonances involving components of the hyperfine structure. The reason is that the small value of the hyperfine splitting of the levels of resonant atoms in the upper level in a magnetic field usually leads to a Paschen–Back regime, while in the lower level one of the following regimes is established: a Zeeman regime, a Paschen–Back regime, or an intermediate regime (see, for example, Refs. 18 and 19). In addition, in the case of strong saturation there is a decrease in the populations of the resonance sublevels because of a transition of the atom to nonresonance sublevels of the hyperfine structure of the lower level. The frequency dependence of the RPF and the related properties of the RPF for atoms with a hyperfine structure thus require a separate study.

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