Sensitivity of the resistivity of a one-dimensional conductor to variations in random-potential realizations

A. M. Satanin

N. I. Lobachevsky Nizhnĭ Novgorod State University, 603003 Nizhny Novgorod, Russia
(Submitted 20 May 1993)
Zh. Eksp. Teor. Fiz. **104**, 3759–3768 (November 1993)

The transfer matrix is used to calculate the resistivity correlation function for two conductor ensembles in which the random fields differ only locally (in regions whose size is of the order of magnitude of the lattice constant or the electron wavelength). The correlation function is shown to depend exponentially on the conductor's length. It is also found to depend on the position of an impurity with altered characteristics. Under an impurity displacement the correlation function oscillates, but it is a monotonic function of variations in the impurity-potential amplitude. A geometric interpretation of the high sensitivity of the resistivity is given based on analysis of the topology of the phase space. Finally, possible manifestations of the sensitivity in experiments with conducting channels are discussed.

1. INTRODUCTION

Mesoscopic conductors are actually quantum interferometers in which a small variation of the random field may drastically alter the interference structure of the coherent state of an electron on scales substantially exceeding the microscopic and, hence, considerably change the conductance of the sample.

On the metallic side of the Anderson transition, when the impurity shift is small, the variation of conductance is a power function of the characteristic sample dimension.[1,2] In a magnetic field the conductance correlation function for a film proves to depend on the position of an impurity in the sample.[3]

With a one-dimensional conductor we should expect an enhancement in the resistivity's sensitivity to random-potential variations. Qualitative considerations clearly show that a diffusing particle "visits" a selected impurity many times. Quantum interference enhances the interaction with the impurity, and a change in the impurity's characteristics has a profound effect on the resistivity's magnitude.

Localization effects in one-dimensional conductors have been thoroughly studied,[4–12] but a detailed investigation of the effect of local perturbations on the various characteristics of conductors has yet to be carried out.

This paper is devoted to the study of the effect of variations of the local characteristics of a random field on the magnitude of the resistivity of a one-dimensional conductor. The T-matrix method is used to calculate the resistivity correlation function for three models: the random-matrix model, the Anderson model, and the point-impurity model. The exponential function is found to depend exponentially on the conductor's length; it oscillates under impurity shifts and is a monotonic function of the variations of the random-potential amplitude. A geometric interpretation is given of the resistivity's sensitivity and demonstrates the universal nature of the discovered behavior.

2. REPRESENTING THE RESISTIVITY CORRELATION FUNCTION IN TERMS OF T-MATRICES

A disordered conductor can be considered a scatterer and characterized by a T-matrix connecting the amplitude of the electron waves to the left and right of the scatterer.[4,5]

The resistivity of a one-dimensional conductor is expressed in terms of a T-matrix via the well-known formula[6,7] (Landauer's formula)

$$\rho=\tfrac{1}{2}\{\tfrac{1}{2}\operatorname{Tr}(TT^{\dagger})-1\}. \tag{1}$$

The structure of the T-matrix follows from conservation of the probability flux and symmetry under time reversal, which we write in the form

$$T^{\dagger}\sigma_z T=\sigma_z, \quad \sigma_x T\sigma_x=T^{*}, \tag{2}$$

where σ_x and σ_z are Pauli matrices. From (2) it follows that

$$T+\begin{pmatrix}\alpha & \beta\\ \beta^{*} & \alpha^{*}\end{pmatrix}, \quad \alpha\alpha^{*}-\beta\beta^{*}=1, \tag{3}$$

that is, the T-matrix depends on three real parameters.

If the conductor is "divided" into N elementary segments, the T-matrix is given by the expression

$$T(N,1)=T_N T_{N-1}\cdots T_n\cdots T_2 T_1. \tag{4}$$

The matrix T_n of an elementary segment can be found by solving the appropriate Schrödinger equation. It obeys (2) and has the structure of (3).

For instance, in the case of a point impurity "lumped" near point x_n,

$$T_n\begin{pmatrix}1-iu_n & -iu_n e^{-2ikx_n}\\ iu_n e^{2ikx_n} & 1+iu_n\end{pmatrix}, \tag{5}$$

where k is the wave vector, and u_n the dimensionless impurity-potential amplitude.

For the Anderson model we have[4]

$$T_n=QM_nQ^{-1}, \tag{6}$$

 1063-7761/93/110803-05$10.00

where

$$M_n=\begin{pmatrix}\varepsilon_n-\varepsilon-1 & \\ 1 & 0\end{pmatrix},\quad Q=\begin{pmatrix}1 & 1\\ e^{ikd} & e^{-ikd}\end{pmatrix}, \tag{7}$$

with d the lattice constant and ε_n the random site energy (for the Anderson model the resonance tunneling integral is taken as the unit of energy).

Let us consider two ensembles of conductors. The first is characterized by a set of random parameters with a respective distribution function; say, the set of parameters (u_n,x_n) in the case (5) or the ε_n in the case (6). Suppose that the second ensemble differs from the first in that the parameters of the scattering potential have been locally altered but are functionally related to the respective parameters of the potential of the first ensemble. This relation occurs if the positions of the diffusing atoms are correlated. Thus, in the approach considered, a change of the local characteristics of the random field leads to the transformation $T_n\to T_n'$. According to (4), this changes the total T-matrix and the conductor's resistivity, $\rho\to\rho'$. The statistical relation between the resistivities of the conductors is characterized by the correlation function

$$K=\langle(\rho-\rho')^2\rangle. \tag{8}$$

Using Eq. (1) and the identity

$$\mathrm{Tr}(A)\mathrm{Tr}(B)\equiv\mathrm{Tr}(A\otimes B),$$

with $A\otimes B$ the direct product of matrices, we can write (8) in the form

$$K=\tfrac{1}{16}\langle[\mathrm{Tr}(TT^\dagger\otimes TT^\dagger)+\mathrm{Tr}(T'T'^\dagger\otimes T'T'^\dagger -2\mathrm{Tr}(TT^\dagger\otimes T'T'^\dagger)]\rangle. \tag{9}$$

To calculate the correlation function we must average in (9) over the conductor ensemble with allowance for the statistical dependence of the characteristics of individual scatterers.

3. THE RANDOM-MATRIX MODEL

We calculate the correlation function in the weak-scattering approximation, where the mean free path l is large compared to the electron wavelength ($l\gg k^{-1}$). If the segment size exceeds k^{-1} considerably, the phases of the complex-valued α_n and β_n become random quantities and are uniformly distributed in the interval $(0,2\pi)$. The random-matrix model introduced in this manner reflects correctly all localization effects in a one-dimensional conductor.[4–10] Within this model it is natural to assume that a variation in the segment parameters leads to the transformation $T_n\to T_n'$. We assume for the simplicity that

$$T_n'=vT_n, \tag{10}$$

where matrix v satisfies the same conditions (2) as the T-matrix, that is,

$$T=\begin{pmatrix}a & a\\ b^* & a^*\end{pmatrix}.$$

Transformation (10) simply means that variations in segment n are characterized by three real numbers. Below we show that this relationship holds true also in more realistic situations, where the local parameters of the potential vary.

The ideal of further calculations is illustrated using the last term in (9),

$$\langle\mathrm{Tr}(TT^\dagger\otimes T'T'^\dagger)\rangle. \tag{11}$$

We write the matrices T and T' in the form of products:

$$T=T_RT_nT_L,\quad T'=T_RT_n'T_L, \tag{12}$$

where T_L contains the matrices of the segments numbered 1 to $n-1$, and T_R the matrices of the segments numbered $n+1$ to N.

We write (11) as

$$\langle\mathrm{Tr}(T_n\otimes T_n'U_LT_n^\dagger\otimes T_n^\dagger U_R)\rangle, \tag{13}$$

where

$$U_L=\langle T_LT_L^\dagger\otimes T_LT_L^\dagger\rangle,\quad U_R=\langle T_R^\dagger T_R\otimes T_R^\dagger T_R\rangle.$$

To calculate U_L and U_R we represent U_m for m segments in the form

$$U_m=A_mI\otimes I+B_m(\sigma_x\otimes\sigma_x+\sigma_y\otimes\sigma_y). \tag{14}$$

The recurrence formula

$$U_{m+1}=\langle T_{m+1}\otimes T_{m+1}U_mT_{m+1}^\dagger\otimes T_{m+1}^\dagger\rangle$$

yields

$$A_{m+1}=(1+2\Delta)A_m+2\Delta B_m,\qquad B_{m+1}=\Delta A_m+(1+\Delta)B_m, \tag{15}$$

where $\Delta=2(\bar\rho+\overline{\rho^2})$, with $\bar\rho$ and $\bar\rho+\overline{\rho^2}$ the first and second resistivity moments of an elementary segment. The solution to (15) is

$$A_m=\tfrac{1}{3}(2\lambda^m+1),\quad B_m=\tfrac{1}{3}(\lambda^m-1), \tag{16}$$

where $\lambda=3\Delta+1$.

Substituting U_L and U_R with coefficients determined through (16) into (13) and allowing for (10), we can easily carry out the last averaging. Transforming the other terms in (9) in a similar manner, we obtain

$$K=\tfrac{1}{3}\{[1-\tfrac{1}{3}(2V_0+V_1)+\tfrac{3}{4}(V_0^2-1)]\lambda^N-\tfrac{1}{3}(V_0-V_1)\times(\lambda^{N-n}+\lambda^n)+\tfrac{1}{3}[1-\tfrac{1}{3}(2V_1+V_0)]\}, \tag{17}$$

where

$$V_0=\tfrac{1}{2}\mathrm{Tr}(v^\dagger v),\quad V_1=\tfrac{1}{4}\mathrm{Tr}(\sigma_xv^\dagger\sigma_xv+\sigma_yv^\dagger\sigma_yv).$$

From Eq. (17) it follows that for large N the correlations are exponentially high,

$$K\propto e^{\Lambda N},$$

and the growth exponent

$$\Lambda=\ln(1+3\Delta)$$

coincides with that of the resistivity's dispersion.[7,8]

For matrices v of class (2) we have $V_0\geqslant V_1$, and the contribution to the correlation function from the term that depends on the position of the segment with altered param-

eters is always negative, that is, the correlations decrease as the altered segment is displaced toward the edges of the conductor.

The random-matrix model becomes exact when the length of the segment is equal to, or is an integral multiple of, half of the electron wavelength. Suppose that each such segment contains only one point impurity. Scattering by the impurity is described by (5). If the impurity coordinates are uniformly distributed over the segments, the phases β are uniformly distributed over the interval $(0, 2\pi)$. When the position of the impurity in the number n segment changes, that is, $x_n \to x_n + \delta n$, so will phase β, which can be accounted for by selecting the elements of matrix v in the form

$$a = 1 + u^2(1 - e^{i2k\delta x}), \quad b = (u^2 - iu)(e^{-i2k\delta x} - 1).$$

The correlation function, only in the case where the scattering by impurities is weak ($u^2 \ll 1$), is given by

$$K = \frac{4u^2}{9} \sin^2(k\delta x)\left[2\lambda^N - (\lambda^{N-n} + \lambda^n) - \frac{4}{3}\right]. \tag{18}$$

If the length of a segment is of the order of several half-waves, the function K oscillates upon variations of the position of the impurity in this segment. Clearly, oscillations occur when the impurity is displaced, while changes of the impurity-potential amplitude result in monotonic variation of K. Section 5 discusses these properties in greater detail.

4. THE ANDERSON MODEL

In the Anderson model the T-matrix is given by (6). Let us calculate the resistivity correlation function for the case where the Fermi energy coincides with the center of an unperturbed band ($\varepsilon = 0$). Instead of the matrices T_n we can employ M_n [see Eq. (6)], since Q does not appear any more in (1). To simplify the calculations we assume that the distribution function for the site energies is symmetric, $P(\varepsilon) = P(-\varepsilon)$. A random-parameter local variation, $\varepsilon_n \to \varepsilon_n'$, changes M_n to M_n'. As in Sec. 3, we write the average

$$\langle \mathrm{Tr}(MM^T \otimes M'M'^T) \rangle \tag{19}$$

that enters into the correlation function in the form

$$\langle \mathrm{Tr}(M_n \otimes M_n' U_L M_n^T \otimes M_n'^T U_R) \rangle. \tag{20}$$

To calculate U_L and U_R we must solve the following recurrence equation

$$U_{m+1} = \langle M_{m+1} \otimes M_{m+1} U_m M_{m+1}^T \otimes M_{m+1}^T \rangle. \tag{21}$$

The solution to (21) is sought in the form

$$U_m = A_m I \otimes I + B_m \sigma_x \otimes \sigma_x + C_m \sigma_y \otimes \sigma_y + D_m(\sigma_z \otimes I + I \otimes \sigma_z). \tag{22}$$

From Eq. (21) follow the recurrence equations for the coefficients:

$$A_{m+1} = \overline{(\tfrac{1}{2}\varepsilon^2 + 1)^2} A_m + \overline{\varepsilon^2} B_m + \tfrac{1}{2}\overline{(\varepsilon^2 - 1)} C_m + \overline{\varepsilon^2(\tfrac{1}{2}\varepsilon^2 + 1)} D_m, \tag{23}$$

$$B_{m+1} = \overline{\varepsilon^2} A_m + B_m + \overline{\varepsilon^2} C_m + 2\overline{\varepsilon^2} D_m, \tag{24}$$

$$C_{m+1} = \tfrac{1}{4}\overline{(\varepsilon^2 - 1)^2} A_m + \overline{\varepsilon^2} B_m + \tfrac{1}{4}\overline{(\varepsilon^2 - 1)^2} C_m + \overline{\varepsilon^2(\varepsilon^2 - 1)} D_m, \tag{25}$$

$$D_{m+1} = \tfrac{1}{2}\overline{\varepsilon^2(\tfrac{1}{2}\varepsilon^2 + 1)} A_m + \overline{\varepsilon^2} B_m + \tfrac{1}{4}\overline{\varepsilon^2(\varepsilon^2 - 1)} C_m + \overline{(2\varepsilon^4 - 1)} D_m, \tag{26}$$

where the bar stands for averaging with $P(\varepsilon)$. It is convenient to write the correlation function as follows:

$$K = \tfrac{1}{4}[\overline{(\varepsilon - \varepsilon')^2}(F_{N-n}B_{n-1} + F_{n-1}B_{N-n}) \times [\tfrac{1}{4}(\varepsilon^2 - \varepsilon'^2)^2 F_{N-n}F_{n-1}]], \tag{27}$$

where $F_n = A_n + C_n + 2D_n$. To obtain an explicit expression for K we must solve the recurrence equations (23)–(26) and use (27). Simple formulas can be obtained in the cases of small ($\overline{\varepsilon^4} \gg \overline{\varepsilon^2} \gg 1$) and great ($\overline{\varepsilon^4} \ll \overline{\varepsilon^2}$) disorder.

In the case of small disorder it is sufficient to retain in Eqs. (23)–(26) only the terms linear in $\overline{\varepsilon^2}$. The eigenvalues of the matrix of the recurrence equations are

$$\lambda_1 = 1 + \sqrt{3}\,\overline{\varepsilon^2}, \quad \lambda_2 = 1 - \sqrt{3}\,\overline{\varepsilon^2}, \quad \lambda_3 = 1, \quad \lambda_4 = -1.$$

If we now find the eigenvectors of this matrix with required accuracy, we can write the general solution to Eqs. (23)–(26) as a linear combination of these eigenvectors. Then, knowing the solution at an arbitrary recurrence step, we can find K via (27). After straightforward cumbersome calculations we arrive at

$$K = \frac{1}{18}\overline{(\varepsilon - \varepsilon')^2}\left[\left(\frac{\sqrt{3}+1}{2}\right)^2 [\sqrt{3}\lambda_1^{N-1} - (\lambda_1^{N-n} + \lambda_1^{n-1})] - \left(\frac{\sqrt{3}-1}{2}\right)^2 [\sqrt{3}\lambda_2^{N-1} - (\lambda_2^{N-n} + \lambda_2^{n-1})] + 1\right]. \tag{28}$$

This expression is valid for an arbitrary ratio of localization length to conductor length. The correlation function depends on the position of the site with an altered parameter value. As with the example discussed in Sec. 3, correlations diminish when the selected site is shifted to the edge of the sample. For large values of N, terms with $\lambda_1 (\lambda_1 > 1)$ become significant.

In the case of strong disorder we must retain in Eqs. (23)–(26) terms proportional to $\overline{\varepsilon^4}$. In this approximation the coefficient B_n is small and can be dropped, while $A_n = C_n = D_n = \overline{\varepsilon^4}$, and

$$K = \overline{(\varepsilon^2 - \varepsilon'^2)^2}\,\overline{(\varepsilon^4)^{N-1}}. \tag{29}$$

The meaning of this expression can easily be understood since for $\overline{\varepsilon^4} \ll \overline{\varepsilon^2}$ the resistivity behaves like

$$\rho \simeq \varepsilon_1^2 \varepsilon_2^2 \cdots \varepsilon_n^2 \cdots \varepsilon_N^2, \tag{30}$$

from which Eq. (29) immediately follows. The correlation function is independent of the site with an altered param-

eter, since in the strong-disorder approximation the localization range of the wave function ($\sim a/\ln\overline{\varepsilon^4}$) is actually of the order of the lattice constant and interference effects are suppressed.

5. A POINT IMPURITY

The characteristics of the scattering potential in a one-dimensional conducting channel may vary because of diffusion hopping of an impurity along the channel and because of substitution by an impurity of another type. To model such processes and establish how they affect the resistivity we study the T-matrix variation brought about by displacement of a scattering center by δx and by variation of the dimensionless amplitude of the potential, $u \to u'$. In the first case it is expedient to write

$$T_n' = e^{\dagger}(\delta x) T_n e(\delta x), \tag{31}$$

where

$$e(x) = \begin{pmatrix} e^{ikx} & 1 \\ 1 & e^{-ikx} \end{pmatrix},$$

and in the second case,

$$T_n' = e^{\dagger}(x_n) \begin{pmatrix} 1-iu' & -u' \\ iu' & 1+iu' \end{pmatrix} e(x_n). \tag{32}$$

Theoretically, calculation of the correlation function in these cases can be done according to the above scheme. Since the matrix U generally contains five tensor structures, it is possible to analyze the resulting equations only numerically. To obtain results that are easily interpreted, we assume the scattering weak and the impurity displacement small compared to the mean impurity separation. Here, if $kl \ll 1$, the parameter $k\delta x$ may vary in a wide range.

Within the framework of the above assumptions, the sections to the left and right of an impurity can be examined by the method of Sec. 3. Combining (31) with a combination of the type (13) and determining the coefficients of U via (16), we can easily determine the correlation function for a displaced impurity:

$$K = \frac{4u^2}{9} \sin^2(k\delta x)\{[4(1+u^2)+2u^2\cos^2(k\delta x)]\lambda^{N-1} - [1+u^2+2u^2\cos^2(k\delta x)](\lambda^{N-n}+\lambda^{n-1}) + 2u^2\cos^2(k\delta x) - 1\}. \tag{33}$$

We see that the correlation function oscillates under an impurity displacement. In contrast to the two-dimensional case,[1,2] the oscillation amplitude does not diminish under the displacement of an impurity. The presence of oscillations is explained by the fact that the wave function of a weakly localized state is extremely sensitive to the position of the scatterer. Indeed, for $kl \gg 1$ the wave function rapidly changes over a distance of the order of k^{-1}, while its envelope changes over a distance of the order of unity. If the conductor's dimension L is much greater than l, the electron state can be interpreted as a well-localized mode in a cavity with a high Q-factor. When the impurity is displaced by a distance of the order of k^{-1}, the localization condition for this mode changes, which leads to a strong variation of the resistivity. Note that oscillations occur in any realization; for this reason K oscillates. If the sections to the left and right of the impurity are described by the exact matrices

$$T_{L,R} = \begin{pmatrix} \sqrt{1+\rho_{L,R}}\,e^{i\phi_{L,R}} & \sqrt{\rho_{L,R}}\,e^{i\theta_{L,R}} \\ \sqrt{\rho_{L,R}}\,e^{-i\theta_{L,R}} & \sqrt{1+\rho_{L,R}}\,e^{-i\phi_{L,R}} \end{pmatrix} \tag{34}$$

and the impurity is described by matrix (31), we can easily write an expression for the square of the difference of resistivities and see that the following always holds true:

$$(\rho-\rho')^2 = \sin^2(k\delta x)\, W(k\delta x), \tag{35}$$

where $W(x)$ is a function depending on the resistivities and phases to the left and right of the selected impurity. For example, if $u \ll 1$,

$$W(x) = [(L_0R_1 + L_1R_0)\cos x + (L_0R_2 + L_2R_0)\sin x]^2,$$

where

$$L_j = \tfrac{1}{2}\mathrm{Tr}(T_L^{\dagger}\sigma_j T_L), \quad R_j = \tfrac{1}{2}\mathrm{Tr}(T_R^{\dagger}\sigma_j T_R),$$

with $j = 0,1,2$. Thus, the correlation function always contains an oscillation factor, which does not vanish as a result of averaging.

If the impurity-potential amplitude varies, then

$$K = \tfrac{1}{3}\{[2(u-u')^2+7(u^2-u'^2)^2]\lambda^{N-1} - [(u-u')^2 - 2(u^2-u'^2)^2](\lambda^{N-n}+\lambda^{n-1}) + (u^2-u'^2)^2\}. \tag{36}$$

Here there are no oscillations, but the correlations function depends on the position of the impurity. It is worth noting that there are contributions that depend on the fourth power of the potential. These cannot be found if one remains within the perturbation-theory framework.

6. CONCLUSION

The above calculations indicate that the resistivity of mesoscopic conductors is highly sensitive to variations in the random-field realizations. In the case of weakly localized states the explanation is that small local variations of the random field alter the conditions of localization of the wave function, which leads to dramatic changes in resistivity. Qualitatively such a sensitivity is due to the multiplicative nature of the resistivity, so that variation of separate factors leads to a sizeable variation in the product of the random quantities. It is important that the correlation function depends on the position of the impurity, whose characteristics have changed. Displacement of the impurity leads to oscillations in the correlation function, and the oscillation period is determined by the wavelength of an electron with the Fermi energy.

These conclusions are drawn without resorting to perturbation-theory techniques. Many are model-independent. They can be interpreted purely geometrically

on the basis of analysis of the structure of the phase space of the system. Instead of matrix T_m we introduce three real variables:

$$x_m=\tfrac{1}{2}\mathrm{Tr}(T_m^\dagger\sigma_x T_m),$$

$$y_m=\tfrac{1}{2}\mathrm{Tr}(T_m^\dagger\sigma_y T_m),$$

$$t_m=\tfrac{1}{2}\mathrm{Tr}(T_m^\dagger T_m),$$

which represent the state of the conductor at the mth recurrence step. From the properties of the matrix it follows that

$$t_m^2-x_m^2-y_m^2=1. \tag{37}$$

We can express the resistivity ρ_m in terms of these variables as

$$\rho_m=\tfrac{1}{2}(t_m-1).$$

Addition of a segment leads to displacement of the representative point over the hyperboloid (37):

$$(t_m,x_m,y_m)\to(t_{m+1},x_{m+1},y_{m+1})$$

(the equations can be found in Refs. 14 and 15). A conductor partitioned into N segments is represented by a trajectory on the hyperboloid. The terminal point of the trajectory corresponds to the resistivity $\rho_N=\frac{1}{2}(t_N-1)$, and an ensemble of conductors is represented by a set of such trajectories. In the event of a local variation of random-field parameters the trajectories of two ensembles coincide up to segment number n, but after the nth step they undergo a small distortion and part exponentially fast in view of noncompactness of the phase space. The growth exponent Λ is a Lyapunov exponent, which characterizes a takeoff process, and the correlation function K determines the rms distance between the projections of the terminal points on the t axis. Thus, the sensitivity and dependence of correlations on the impurity's position must occur in systems with one-dimensional topology.

A convenient object for observing the sensitivity of resistivity is the quasi-one-dimensional conducting channel based on the GaAs/AlAs heterostructure with planar electrodes.[16] With a channel width of roughly 100 Å, a single-channel current flow is obtained. To vary the realizations, a small potential difference must be applied to the planar electrodes to cause a shift in the conducting channel relative to the fixed impurities and defects. In this manner a representative sampling of the conductor ensemble can easily be obtained. Local variations of the potential can be carried out with an additional electrode. Similar effects will occur in single-mode optical fiber, where the backscattering coefficient acts as resistivity.

I would like to express my gratitude to Hermán Luno Acosta, V. Ya. Demikhovskiĭ, and N. M. Makarov for fruitful discussions. The results of Sec. 3 were obtained in collaboration with D. D. Tsiteladze and were reported earlier at an international workshop.[17]

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Translated by Eugene Yankovsky

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