Theory of stimulated Cherenkov emission from sheet relativistic electron beams in a uniform isotropic dielectric medium

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(Submitted 3 August 1993)
Zh. Eksp. Teor. Fiz. **104**, 3940–3954 (December 1993)

The properties of stimulated Cherenkov emission from sheet relativistic electron beams (REBs) in a uniform isotropic dielectric medium are studied. It is shown that in the linear stage of the process there exist growing modes localized near the beam surface which have a transverse electromagnetic energy flux. In the nonlinear stage a mechanism is found for stochastic deceleration of the electrons, which interact successively (by turns) with an ensemble of electromagnetic waves radiated at various angles with respect to the system axis. As a result the efficiency with which the REB energy is converted into electromagnetic wave energy can reach 60%, considerably greater than the values achieved under ordinary conditions in which the transverse structure of the radiation field is produced by the external waveguide geometry. It is shown that this mechanism is promising as a source of powerful short-wavelength (in particular, submillimeter) radiation.

INTRODUCTION

The study of the stimulated emission from electron beams in electrodynamic systems which do not have waveguide properties is of considerable practical interest, primarily as one of the more promising approaches to raising the power and frequency of coherent radiation sources.[1–14] In such systems the spatial structure of the radiation field is determined by the electron beam itself, and as in the situation when individual particles radiate the conditions for synchronous interaction are maintained automatically over a wide range of electron energies because of the variation in the angle at which the wave is radiated. Consequently, the energy conversion efficiency can be considerably greater than the values achieved in ordinary systems in which the spatial structure of the radiation is determined by the external waveguide structures.[15–17] In addition, the analysis of the properties of stimulated emission from electron beams of finite transverse extent in free space and in uniform media, including plasmas (see Refs. 18 and 19), is a fundamental problem of importance not only for laboratory but also for astrophysical applications.

The present work is devoted to the theory of stimulated Cherenkov emission from relativistic electron beams of finite transverse extent moving with superluminous velocity in a uniform isotropic dielectric medium. In Sec. 1 we derive the dispersion relation and show that there exist growing eigenmodes which are localized near the beam surface. In Sec. 2 we use a parabolic equation to describe the evolution of the electromagnetic field and study the linear growing stage of radiation from a sheet electron beam in a semiinfinite problem. Sec. 3 is devoted to the nonlinear stage of the interaction process. A mechanism for stochastic deceleration of electrons in an ensemble of waves emitted at various angles to the system axis is described.

1. DISPERSION RELATION: EIGENMODES OF AN ELECTRON BEAM OF FINITE TRANSVERSE EXTENT IN A DIELECTRIC MEDIUM

Assume that a magnetized relativistic electron beam (REB) whose particles can move only in the z direction is injected into a medium (e.g., a gas) with an index of refraction $n=\sqrt{\varepsilon}$. We treat a two-dimensional model in which it is assumed that the quantities do not depend on the y coordinate and the radiation field has magnetic and electric components H_y and $E_{x,z}$. For a monochromatic process with time dependence $\propto\exp(i\omega t)$ these components are related by the Maxwell equations

$$\frac{\partial E_z}{\partial x}-\frac{\partial E_x}{\partial z}=i\frac{\omega}{c}H_y,\quad \frac{\partial H_y}{\partial z}=-i\frac{\omega}{c}\varepsilon E_x,$$

$$\frac{\partial H_y}{\partial x}=\frac{4\pi}{c}j_z+i\frac{\omega}{c}\varepsilon E_z. \tag{1}$$

By eliminating the quantities E_x and H_y we reduce the system (1) to a single equation

$$\left(\frac{\partial^2}{\partial z^2}+\frac{\partial^2}{\partial x^2}+\frac{\omega^2}{c^2}\varepsilon\right)E_z=\frac{4\pi i\omega}{c^2}\left(j_z+\frac{c^2}{\omega^2\varepsilon}\frac{\partial^2}{\partial z^2}j_z\right). \tag{2}$$

Here j_z is the density of the high-frequency electron current. This quantity can be represented in the form

$$j_z=jf(x), \tag{3}$$

where $f(x)$ is a function describing the transverse beam current profile (here we assumed that the electron beam consists of a slab with thickness b) and j is the amplitude of the current, which can be found by linearizing the electron equation of motion,

$$\left(v_0\frac{\partial}{\partial z}+i\omega\right)^2 j=\frac{i\omega}{4\pi}\omega_{b_\parallel}^2 E_z. \tag{4}$$

Here $v_0=\beta_0 c$ is the unperturbed translational velocity of the particles, $\omega_{b_\parallel}=(4\pi e\rho_0/m\gamma_0^3)^{1/2}$ is the "longitudinal"

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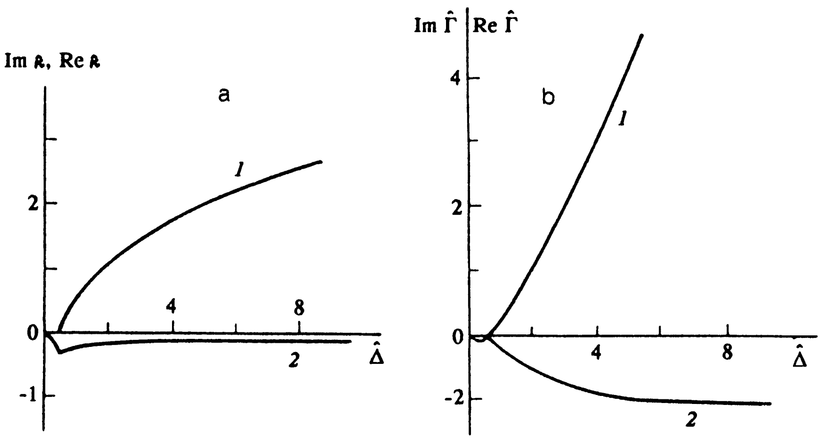

FIG. 1. Real and imaginary parts of the transverse (a) and longitudinal (b) wave numbers versus the mismatch parameter $\hat{\Delta}$ for Cherenkov synchronization for an amplified localized mode ($n=4$) in the case of a thin electron beam.

plasma frequency, ρ_0 is the beam charge density, and $\gamma_0=(1-v_0^2/c^2)^{-1/2}$ is the relativistic mass factor.

To derive the dispersion relations, assuming the system is unbounded in the z direction, we look for a solution of Eqs. (2) and (4) in the form

$$E_z=Ce^{-ihz}\begin{cases}\cos(gx) & \text{(symmetric mode)}\\ \sin(gx) & \text{(antisymmetric mode)}\end{cases}$$

inside the beam and

$$E_z=De^{-ihz}e^{-i\varkappa|x|}$$

outside the beam. Here C and D are arbitrary constants,

$$g=(k^2\varepsilon-h^2)^{1/2}\left(1-\frac{\omega_{b_\parallel}^2}{\varepsilon(\omega-hv_0)^2}\right)^{1/2}, \tag{5}$$

$$\varkappa=(k^2\varepsilon-h^2)^{1/2} \tag{6}$$

are the transverse wave numbers inside and outside the electron beam respectively, and $k=\omega/c$. From the boundary conditions at the limits of the electron beam ($x=\pm b/2$) we find the dispersion relations

$$\mathrm{tg}\,(gb/2)=i\varkappa/g \quad \text{symmetric mode,} \tag{7}$$

$$\mathrm{ctg}\,(gb/2)=-i\varkappa/g \quad \text{antisymmetric mode.} \tag{8}$$

We begin by investigating the case of a thin sheet electron beam

$$|gb/2|\sim|\varkappa/g|\ll 1. \tag{9}$$

In this case there exists only one symmetric mode, for which we have[9,12] from Eq. (7) together with (5) and (6)

$$(\omega-hv_0)^2=\frac{i\varkappa}{2\varepsilon}\,\omega_{b_\parallel}^2\,b. \tag{10}$$

We next study the waves propagating at a small angle with respect to the electron translational motion

$$h=k\sqrt{\varepsilon}(1-\Gamma), \quad |\Gamma|\ll 1. \tag{11}$$

In this case the dispersion relation can be rewritten in the form

$$(\hat{\Delta}-\hat{\Gamma})^2=i\hat{\Gamma}^{1/2} \quad \text{or} \quad (\hat{\Delta}-\hat{\varkappa}^2)^2=i\hat{\varkappa}, \tag{12}$$

where we have written $\hat{\Delta}=\Delta/G$, $\Delta=(hv_0-\omega)/\omega n\beta_0$ is the initial mismatch in the Cherenkov condition with respect to a wave propagating rigorously parallel to the system axis; $\hat{\varkappa}=\varkappa/(kn\sqrt{2G})$, $\hat{\Gamma}=\Gamma/G$ ($\hat{\Gamma}=\hat{\varkappa}^2$); and

$$G=\left(\frac{\omega_{b_\parallel}^2\,b}{\sqrt{2}\omega cn}\right)^{2/3} \tag{13}$$

is the gain.

The waves are amplified and aligned by the electron beam when there is a normal wave which grows in the longitudinal direction, ($\mathrm{Im}\,\hat{\Gamma}<0$), localized in the transverse direction, ($\mathrm{Im}\,\hat{\varkappa}<0$), and having a flux of electromagnetic energy away from the beam toward the periphery, ($\mathrm{Re}\,\hat{\varkappa}>0$). For $\hat{\Delta}=0$ the solutions of (12) are given by the relations

$$\hat{\varkappa}_n=\exp\left[i\left(\frac{\pi}{6}+\frac{2\pi(n-1)}{3}\right)\right], \quad n=1\text{–}3, \quad \hat{\varkappa}_4=0$$

$$\hat{\Gamma}_n=\exp\left[i\left(\frac{\pi}{3}+\frac{4\pi(n-1)}{3}\right)\right], \quad n=1\text{–}3, \quad \hat{\Gamma}_4=0, \tag{14}$$

and none of the four normal modes satisfies the properties listed above. A growing localized mode exists for $\hat{\Delta}>\hat{\Delta}_*=3\cdot 2^{1/3}/8$ (for more detail see Ref. 10). The dependence on $\hat{\Delta}$ of the real and imaginary parts of the transverse and longitudinal wave numbers for this wave (which we specify by labeling it $n=4$) are shown in Fig. 1.

The asymptotic behavior of the wave numbers for large positive values of the mismatch in the synchronization condition ($\hat{\Delta}\gg 1$) is given by the relations

$$\hat{\varkappa}_4=\hat{\Delta}^{1/4}-\frac{\sqrt{2}}{4}\hat{\Delta}^{-1/4}(1+i),$$

$$\hat{\Gamma}_4=\hat{\Delta}-\frac{\sqrt{2}}{4}\hat{\Delta}^{1/4}(1+i). \tag{15}$$

According to (15), the growth rate increases monotonically as a function of $\hat{\Delta}$: $|\mathrm{Im}\,\hat{\Gamma}|\sim\hat{\Delta}^{1/4}/\sqrt{2}$, or in dimensional variables

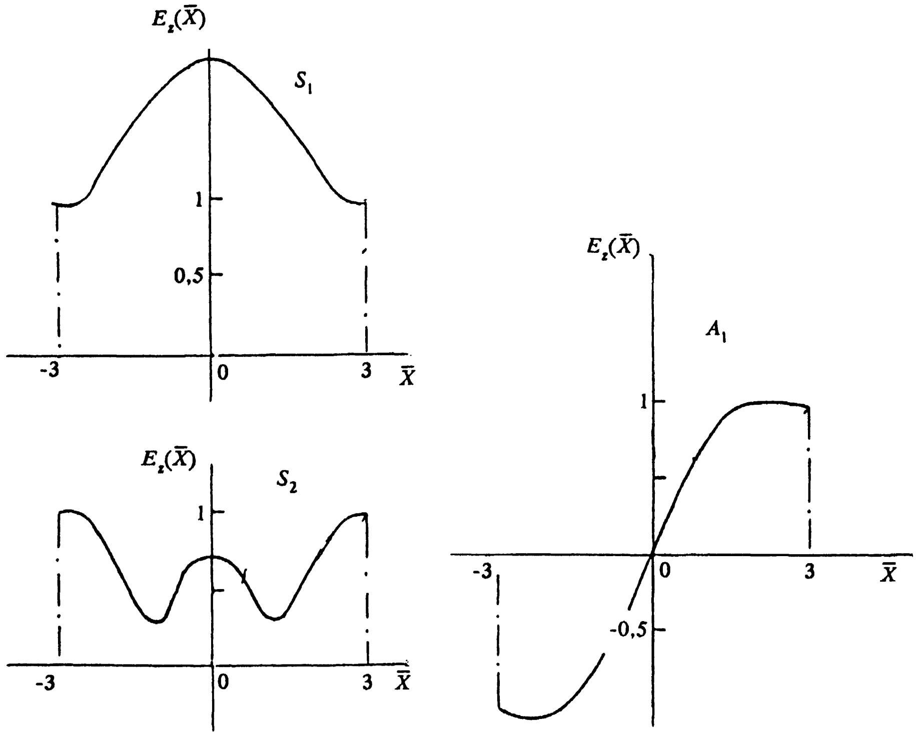

FIG. 2. Transverse (in the direction $\bar{X}=knx\sqrt{2\Delta}$) distribution of the longitudinal electric field amplitude for symmetric and antisymmetric modes, S_m and A_m respectively, in the case of a thick electron beam; here $\bar{B}=6$.

$$|\mathrm{Im}\, h|=\left(\omega_{b_\parallel}^2\, b\,\frac{\omega n}{c^3}\right)^{1/2}\frac{(n\beta_0-1)^{1/4}}{2^{3/4}}. \tag{16}$$

This is because the angle $\psi=\mathrm{Re}\,\varkappa/h\simeq\sqrt{2G}\hat{\Delta}^{1/4}$, at which the wave is radiated increases as a function of $\hat{\Delta}$. Thus, for the phase velocity of this wave in the z direction to equal the particle translational velocity we must have

$$v_{\mathrm{ph}}=\frac{c}{n(1-\mathrm{Re}\,\Gamma)}\approx\frac{c}{n(1-\Delta)}\approx v_0. \tag{17}$$

Accordingly, as the angle ψ increases the amplitude of the longitudinal component of the electric field acting on the electrons increases: $|E_z/E_x|\sim|\varkappa/k|$, which is responsible for the increase in the growth rate.

For $\hat{\Delta}\gg 1$ the aligning effect of the electron beam becomes negligibly small: $|\mathrm{Im}\,\hat{\varkappa}|\sim\hat{\Delta}^{-1/4}$. This allows us to assume that in this limit the eigenmode is a superposition of two waves emitted in opposite directions from the beam at the Cherenkov angle $\cos\psi=1/n\beta_0$.

In the regime of greatest practical interest, $\hat{\Delta}\gg 1$, we can easily use the dispersion relations (7) and (8) to describe a thick electron beam also. In this case, noting that $\Gamma=\Delta+\Lambda$ holds, where $|\Lambda|\ll 1$, we transform expressions (5) and (6) for transverse wave numbers into the form

$$\varkappa=nk\sqrt{2\Gamma}\approx nk\sqrt{2\Delta},$$

$$g=nk\sqrt{2\Gamma}\left(1-\frac{\omega_{b_\parallel}^2}{\omega^2 n^2(\Delta-\Gamma)}\right)^{1/2}$$

$$\approx nk\sqrt{2\Delta}(1-I/\Lambda^2)^{1/2}, \tag{18}$$

where $\bar{I}=\omega_{b_\parallel}^2/(\omega^2 n^2)$. This enables us to reduce the dispersion relations to the form[1)]

$$\mathrm{tg}\,(\bar{g}\bar{B}/2)=i\bar{g}\quad\text{symmetric mode},$$

$$\mathrm{ctg}\,(\bar{g}\bar{B}/2)=-i\bar{g}\quad\text{antisymmetric mode}, \tag{19}$$

where $\bar{g}=g/(nk\sqrt{2\Delta})=(1-I/\Lambda^2)^{1/2}$. Equations (19) give the complex transverse wave numbers for modes with different numbers of field variations as functions of the reduced beam thickness $\bar{B}=nkb\sqrt{2\Delta}$. Consequently, the transverse dependence of the field amplitude inside the slab is determined (see Fig. 2). Note that outside the slab the field amplitude in this approximation is constant in the x direction. The growth rates for the modes can then be found from the simple relation

$$\Lambda^2=\frac{I}{\bar{g}^2-1}. \tag{20}$$

The growth rates S_m and A_m for the symmetric and antisymmetric modes are shown as functions of the reduced electron beam thickness in Fig. 3. It can be seen that as the beam thickness increases the growth rate of a mode first rises, reaches a maximum, and then begins to decrease. Hence as the thickness increases there is a continual increase in the index of the mode at which the maximum growth rate is reached. The maximum growth rate corresponds to $|\bar{g}|\approx 1$. For modes with high indices $m\gg 1$ this condition is satisfied if $gb\approx m\pi$ holds, i.e., a whole number of half wavelengths laid down across the thickness of the beam, corresponding to the transverse wave number $g=nk\sqrt{2\Delta}$.

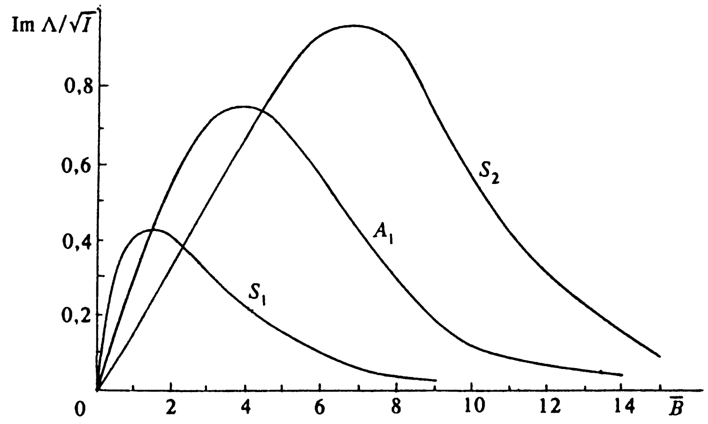

FIG. 3. Growth rates S_m and A_m for the symmetric and antisymmetric modes respectively versus the reduced electron beam thickness $\bar{B}$.

2. THE PARABOLIC EQUATION AND THE AMPLIFICATION OF AN INITIAL PERTURBATION IN A SEMI-UNBOUNDED SYSTEM

As shown by the above analysis, the field emitted by an REB can describe a set of waves propagating at small angles with respect to the system axis, i.e., it can be represented as a quasioptical wave beam

$$E_z=A(z,x)e^{-iknz}. \tag{21}$$

Substituting (21) in (2) and assuming correspondingly $j_z=j(z)\exp(-iknz)$, we find using Eq. (4) a system of equations in which the evolution of the field amplitude $A(x,z)$ is described by an inhomogeneous parabolic equation (cf. Refs. 2 and 10):

$$i\frac{\partial^2 a}{\partial X^2}+\frac{\partial a}{\partial Z}=2i\frac{\partial j}{\partial Z}f(x), \tag{22}$$

$$\left(\frac{\partial}{\partial Z}-i\hat{\Delta}\right)^2 j=ia, \tag{23}$$

where $Z=knGz$, $X = kn\sqrt{2G}x$, $a=eA/(mc\omega\gamma_0^3\beta_0G^2)$.

The boundary conditions for Eqs. (22) and (23) including a possible initial modulation of the beam density j_0 and velocity χ_0 can be represented in the form

$$a|_{Z=0}=a_0(X),\quad j|_{Z=0}=j_0,\quad \left.\frac{\partial j}{\partial Z}\right|_{Z=0}=\chi_0. \tag{24}$$

Using Eqs. (22)–(24), let us treat the problem of the evolution of an initial perturbation specified in the form of a modulation in the electron beam density ($j_0\neq 0$, $a_0=\chi_0=0$). Here we restrict ourselves to the case of a thin sheet electron beam: $f(X)=\delta(X)$, where $\delta(X)$ is the delta function.

Using the Laplace transform in the longitudinal coordinate, $\phi_p=L[\phi(Z)]$, we have from (22) and (23) for the transformed functions

$$i\frac{d^2a_p}{dX^2}+pa_p=2i(pj_p-j_0)\delta(X), \tag{25}$$

$$(\hat{\Delta}-ip)^2j_p+(\bar{p}-2i\hat{\Delta})j_0=-ia_p. \tag{26}$$

Taking into account the boundary condition which follows from Eq. (25),

$$\left\{\frac{da_p}{dX}\right\}_{X=0}=2i(pj_p-j_0),$$

from (25) and (26) we find for the transform of the field

$$a_p=\frac{i\hat{\Delta}^2j_0}{p+i\sqrt{ip(\hat{\Delta}-ip)^2}}e^{-\sqrt{ip}|X|}. \tag{27}$$

Expanding the denominator of Eq. (27) in partial fractions

$$\frac{1}{p+i\sqrt{ip(\hat{\Delta}-ip)^2}}=p^{-1/2}\sum_{n=1}^{4}\frac{s_n}{\sqrt{p}-\sqrt{i\hat{\varkappa}_n}},$$

where $s_n=[1+4i\hat{\varkappa}_n(\hat{\Delta}-\hat{\varkappa}_n)]^{-1}$, we reduce the inverse Laplace transform

$$a=\int_{\Omega-i\infty}^{\Omega+i\infty}a_p\exp(ipZ)dp$$

to a sum of standard integrals.[20] As a result we find for the amplitude of the longitudinal field component

$$a(X,Z)=ij_0\hat{\Delta}^2\sum_{n=1}^{4}s_n\exp(-i\hat{\varkappa}_n|X|+i\hat{\Gamma}_nZ)\times\left(1-\Phi\left\{\frac{\sqrt{i}|X|}{2\sqrt{Z}}-\hat{\varkappa}_n\sqrt{iZ}\right\}\right), \tag{28}$$

where

$$\Phi(u)=2/\sqrt{\pi}\int_0^u\exp(-\tau^2)d\tau$$

is the probability integral and $\hat{\varkappa}_n$ are the roots of the dispersion relation (12).

Using the asymptotic form of the probability integral for large values of the argument,

$$\Phi(u)=1-\frac{1}{\sqrt{\pi}}\frac{\exp(-|u|^2)}{u},\quad |u|\to\infty,\quad |\arg u|<\frac{3\pi}{4},$$

we find from Eq. (28) the following approximate expression for the amplitude of the radiated field at distances from the input cross section substantially greater than the inverse growth rate:

$$a \xrightarrow[Z\gg 1]{} 2ij_0\hat{\Delta}^2 s_4 \exp(-i\hat{\varkappa}_4|X|+i\hat{\Gamma}_4 Z). \quad (29)$$

Thus for sufficiently long interaction lengths parallel to the beam surface the electromagnetic field develops as a growing localized wave. This conclusion can be extended to initial perturbations of arbitrary form, e.g., when the initial profile of the wave beam amplitude is given (nonincreasing in the limit $|X|\to\infty$).

3. NONLINEAR INTERACTION STAGE

To study the nonlinear stage of electron beam emission in a dielectric medium we must replace the linearized equations of motion (4) with the original full equations. Under the assumption that the electron energy is in the ultrarelativistic range $\gamma\gg 1$, by making use of the relation $v\approx c(1-\gamma^{-2}/2)$ we can reduce the self-consistent system of equations to the form

$$i\frac{\partial^2\alpha}{\partial\xi^2}+\frac{\partial\alpha}{\partial\zeta}=2iIf(\xi)\frac{\partial J}{\partial\zeta},\quad J=\frac{1}{\pi}\int_0^{2\pi}e^{-i\theta}d\theta_0, \quad (30)$$

$$\frac{\partial\mathscr{E}}{\partial\zeta}=\mathrm{Re}[\alpha e^{i\theta}],\quad \frac{\partial\theta}{\partial\zeta}=\mathscr{E}^{-2}-1+\tilde{\Delta} \quad (31)$$

with boundary conditions

$$\alpha|_{\zeta=0}=\alpha_0(\xi),\quad \mathscr{E}|_{\zeta=0}=1,$$

$$\theta|_{\zeta=0}=\theta_0+r\cos\theta_0,\quad \theta_0\in[0,2\pi]. \quad (32)$$

Here we have written $\xi=kx\gamma_0^{-1}n$, $\zeta=nkz\gamma_0^{-2}/2$, and $\mathscr{E}=\gamma/\gamma_0$ is the electron energy scaled with respect to the initial value; $\theta=\omega t-knz$ is the electron phase with respect to the synchronous wave; $\tilde{\Delta}=\Delta 2\gamma_0^2$, $I=\omega_{b_\parallel}^2\gamma_0^4/(\omega^2n^2)$; r is a parameter describing the initial modulation of the electron beam density, and $\alpha=2e\gamma_0A/(mc\omega)$.

The energy conversion efficiency (the electronic Q) is given by the relation

$$\eta=\frac{1}{1-\gamma_0^{-1}}\left(1-\frac{1}{2\pi B}\int_0^{2\pi}\int_{-B/2}^{B/2}f(\xi)\mathscr{E}d\theta_0 d\xi\right), \quad (33)$$

where $B=knb\gamma_0^{-1}$.

In the quasioptical approximation used here we can assume that the amplitudes of the transverse field components are related by $H_y\approx nE_x$. For the scaled values of these quantities,

$$h_y=\frac{2e\gamma_0}{mc\omega}H_y,\quad e_x=\frac{2e\gamma_0}{mc\omega}nE_x$$

the coordinate dependence can be represented using Eqs. (1b,c) in the form

$$\left.\begin{matrix}h_y(\xi,\zeta)\\ e_x(\xi,\zeta)\end{matrix}\right\}=I\int_0^{\xi}Jf(\xi')d\xi'+i\int_0^{\xi}\alpha d\xi'. \quad (34)$$

The energy conservation law in this system can be written in the form

$$\frac{dP_z}{d\zeta}=-4I\int_{-B/2}^{B/2}f(\xi')\mathrm{Re}(\alpha J^*)d\xi'=8I\frac{d\eta}{d\zeta}, \quad (35)$$

where $P_z=\mathrm{Re}\int_{-\infty}^{\infty}e_xh_y^*d\xi'$ is the power transported by the electromagnetic field in the longitudinal direction. The first relation in Eq. (35) is the integral of the excitation equations (30), and the second is the integral of the equations of motion (31).

To numerically simulate the nonlinear stage of the interaction it is convenient to use the integral representation for the solution of the parabolic equation. For this purpose, after the substitutions $\theta=\tilde{\Delta}\zeta+\vartheta$, $\alpha=\tilde{\alpha}\exp(-i\tilde{\Delta}\zeta)$, we reduce the system of equations (30)–(31) to the form

$$\tilde{\alpha}=-\frac{\tilde{I}}{\sqrt{\pi i}}\int_0^{\zeta}\int_{-B/2}^{B/2}f(\xi')\frac{\exp[-i\tilde{\Delta}(\zeta-\zeta')]}{(\zeta-\zeta')^{1/2}}$$

$$\times\exp\left(-i\frac{(\xi-\xi')^2}{4(\zeta-\zeta')}\right)\left(\frac{d\tilde{J}}{d\zeta'}+i\tilde{\Delta}\tilde{J}\right)d\xi'd\zeta', \quad (36)$$

$$\frac{\partial\mathscr{E}}{\partial\zeta}=\mathrm{Re}[\tilde{\alpha}e^{i\vartheta}],\quad \frac{\partial\vartheta}{\partial\zeta}=\mathscr{E}^{-2}-1, \quad (37)$$

where $\tilde{J}=1/\pi\int_0^{2\pi}e^{-i\vartheta}d\theta_0$.

We first consider the case of a thin sheet beam $f(\xi)=B\delta(\xi)$. In this limit Eq. (36) simplifies to

$$\tilde{\alpha}=-\frac{\tilde{I}}{\sqrt{\pi i}}\int_0^{\zeta}\frac{\exp[-i\tilde{\Delta}(\zeta-\zeta')]}{(\zeta-\zeta')^{1/2}}\exp$$

$$\left(-i\frac{\xi^2}{4(\zeta-\zeta')}\right)\left(\frac{d\tilde{J}}{d\zeta'}+i\tilde{\Delta}\tilde{J}\right)d\zeta', \quad (38)$$

where $\tilde{I}=IB$. Accordingly, the integration algorithm reduces to a combination of the solution of the equations of motion (37) and the integral representation (38) evaluated at $\xi=0$, i.e., in the plane of the electron motion. The transverse distribution of the longitudinal and transverse components can then be found when $\tilde{J}(\zeta)$ is known by using Eqs. (38) and (34).

Small modulation of the beam density ($r=0.1$) have been used as the initial condition. The numerical simulation supports the conclusion drawn in Sec. 2, to the effect that for $\tilde{\Delta}>\tilde{\Delta}_*$ sufficiently far from the input cross section ($\zeta\approx 5$ in Fig. 4a) the structure of the radiated field in the linear amplification regime is close to that of a localized eigenmode ($n=4$), regardless of the initial conditions. Thus, in the linear stage of the interaction the electron beam amplifies and aligns the radiation. This continues until the system goes over to the nonlinear stage.

Since the alignment process is accompanied by a partial leakage of electromagnetic energy from the electron channel ($\mathrm{Re}\,\hat{\varkappa}>0$), in the nonlinear stage ($\zeta>5$) when the growth of the field amplitude near the axis saturates, the wave beam broadens. The explanation for this is that

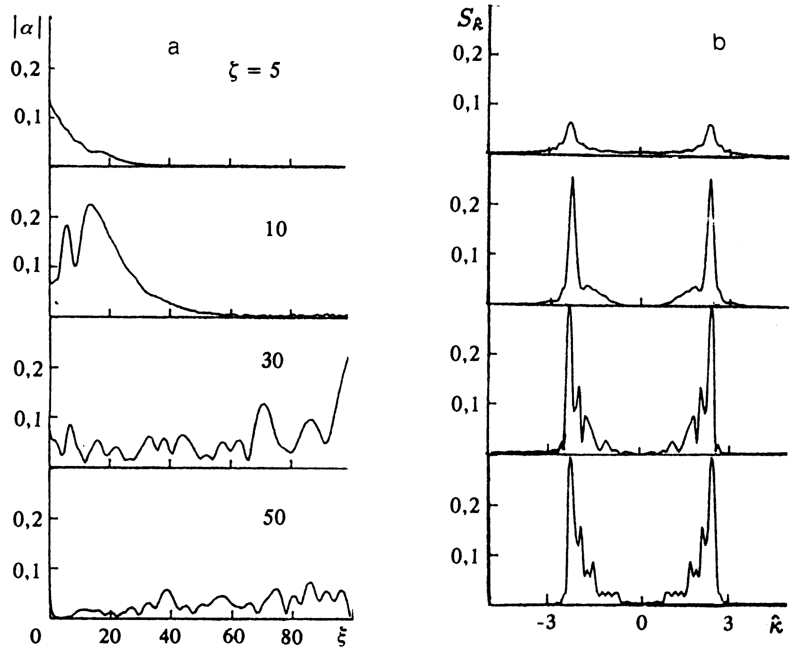

FIG. 4. Evolution of (a) the transverse distribution of the magnitude of the wave beam amplitude and (b) the angular spectrum of the field for a thin electron beam; here $\tilde{I}=0.1$, $\tilde{\Delta}=7$.

rays emitted by electrons in the previous cross sections reach the periphery. As a result, the electromagnetic energy gradually radiates away into the external space (see Fig. 4a).

An important property of the nonlinear stage of Cherenkov emission in free space filled with a dielectric is that the deceleration of the particles becomes stochastic in the limit $\tilde{\Delta}\gg 1$. The reason is that at sufficient distances $\zeta\gg 1$ from the initial cross section the radiated field can be represented as an ensemble of plane waves with random phases propagating at various angles with respect to the direction of translational motion of the particles (see the angular spectrum of the field, $S_\varkappa = 1/\sqrt{2}\int_{-\infty}^{\infty}\alpha\exp(i\hat{\varkappa}\xi)d\xi$ shown in Fig. 4b). The phase velocity of these waves is given by

$$v_{\rm ph}=\frac{c}{n\cos\psi}.$$

The slowest component $v_{\rm ph}^{\rm min}=c/n$, of the wave packet obviously corresponds to an electromagnetic wave propagating exactly along the system axis, $\psi=0$.

The time development of the angular spectrum of the radiation shows that, while in the initial linear stage of the interaction waves are radiated primarily at angles ψ such that their phase velocity in the direction parallel to the system axis is close to the unperturbed translational velocity of the particles (the spectral maximum in Fig. 4b at $\zeta\approx 5$ corresponds to $\hat{\varkappa}\approx\hat{\Delta}^{1/2}$), as the electrons slow down, waves satisfying $v_{\rm ph}(\psi)<v_0$ fill up the angular spectrum. Hence under conditions such that the initial translational electron velocity v_0 is considerably larger than $v_{\rm ph}^{\rm min}$ so that $\tilde{\Delta}\gg 1$ holds, the electrons can interact synchronously successively with different parts of the packet of radiated waves until the average (over the ensemble) particle translational velocity is comparable with $v_{\rm ph}^{\rm min}$. After that, the energy exchange essentially terminates (see Fig. 5). Assuming that the average final velocity of the electrons is equal to $v_{\rm ph}^{\rm min}$, we find the following estimate for the electronic Q:

$$\eta=1-\frac{1}{(\tilde{\Delta}-1)^{1/2}}. \tag{39}$$

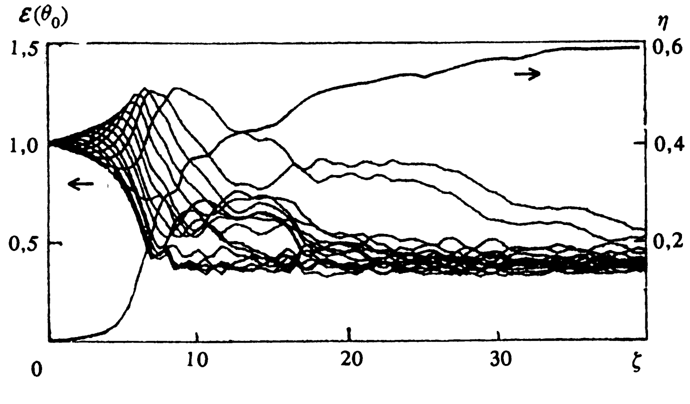

FIG. 5. The quality η and electron energy $\mathscr{E}$ (for different input phases θ_0) versus the longitudinal position for the case of a thin electron beam with $\tilde{I}=0.1$, $\tilde{\Delta}=7$.

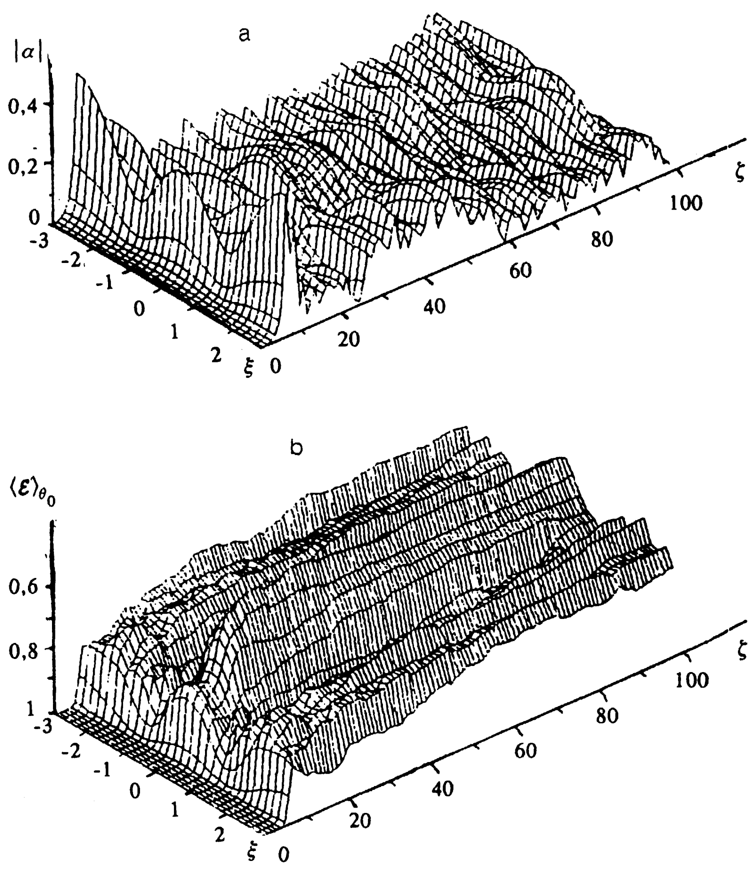

FIG. 6. Variation with the longitudinal coordinate ζ of (a) the field amplitude $|a|$ and (b) the electron energy $\langle\mathscr{E}\rangle_{\theta_0}$ averaged over the input phases for the case of a thick electron beam with $I=0.25$, $\bar{B}=6$.

For example, for $\tilde{\Delta}=7$ we have from Eq. (39) $\eta\approx 60\%$, which agrees well with the results of the numerical simulation (Fig. 5).

Thus, the results obtained above enable us to conclude that when a sheet REB emits Cherenkov radiation in a uniform dielectric a higher efficiency can be achieved for the energy release than in conventional systems in which the transverse field structure is determined by the external shape of the waveguide (in uniform types of relativistic Cherenkov amplifiers[15-17] the value of Q is less than 30%). It is also important to emphasize that for radiation in a uniform dielectric the value of Q depends weakly on the beam current, whereas to achieve a high Q in conventional designs it is necessary to have a very specific (fairly high) current density.

Let us investigate the nonlinear stage of radiation from a thick electron beam in the limit $\tilde{\Delta}\gg 1$. In this regime the integral representation (36) for the radiated field simplifies and can be reduced to the form

$$\alpha=-\tilde{\Delta}^{1/2}I\int_{-B/2}^{B/2}\exp(-i\tilde{\Delta}^{1/2}|\xi-\xi'|)\tilde{J}\times\left(\xi,\zeta-\frac{|\xi-\xi'|}{(2\tilde{\Delta})^{1/2}}\right)d\xi'. \tag{40}$$

The transition to (40) corresponds to going to a representation of the radiated field as a superposition of waves propagating exactly at the Cherenkov angle $\cos\psi=1/n\beta_0$. In the limit $\tilde{\Delta}\gg 1$ we can disregard the variation of the radiation angle, even when the energy of the ultrarelativistic electrons changes substantially.

If the layer thickness is small on the scale of the inverse growth rate, $B\ll(2\tilde{\Delta})^{1/2}$, then Eq. (40) permits the following simplification:

$$\alpha=-I\int_{-\bar{B}/2}^{\bar{B}/2}\exp(-i|\bar{\xi}-\bar{\xi}'|)\tilde{J}(\bar{\xi},\zeta)d\bar{\xi}', \tag{41}$$

where $\bar{\xi}=\tilde{\Delta}^{1/2}\xi$, $\bar{B}=\tilde{\Delta}^{1/2}B$. Simultaneous solution of (41) and the equations of motion (37) shows that in the initial linear stage of the interaction a field profile develops inside the electron beam which corresponds to the structure of an eigenmode having the maximum growth rate for the specified beam thickness. In the simulated case $\bar{B}=6$ this is the second symmetric mode S_2 (compare Fig. 6a for $\zeta=10$ with Fig. 2). Then the transverse structure of the field becomes complicated (stochastic) in the nonlinear stage. The distribution of the average (over the input phases) electron energy along the slab is shown in Fig. 6b. It is evident that the presence of maxima and minima in the amplitude of the *hf* field causes the energy to be exchanged nonuniformly in different parts of the electron beam. In particular, there exist regions near the field minima at which the energy release is quite small. Consequently, the value of Q integrated over the transverse cross section of the electron beam is less than 50% (see Fig. 7).

In conclusion we show that the radiation from a sheet REB in a dielectric can be used to obtain powerful electro-

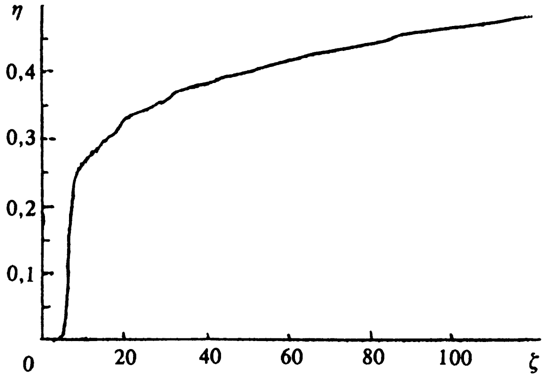

FIG. 7. Efficiency η with which energy is converted as a function of longitudinal position for a thick electron beam with $I=0.25$, $\bar{B}=6$.

magnetic waves in the short-wavelength range. Here we demonstrate this possibility for the case of submillimeter radiation. Assume that the high-current REB has electron energy $\mathscr{E}=1$ MeV, current density $\sim 10^4$ A/cm^2, and thickness $b=1$ mm, and that it is injected into a channel in a dielectric with index of refraction $n=1.2$. Assume that the radiation wavelength is $\lambda \sim 0.5$ mm (for the beam modulator in this case we can use, e.g., a gyrotron[21]). These values of the parameters correspond to $I=0.25$, $\bar{B}=6$. Then, using Eq. (16) and Fig. 7, we find that the growth rate is equal to $|\mathrm{Im}\, h| \sim 10$ cm^{-1}, and the efficiency with which beam energy is converted into electromagnetic radiation energy amounts to 40% at a distance ~ 10 cm from the input cross section. The radiated power per centimeter of transverse cross section may exceed 400 MW.

[1)]From the practical standpoint it is important to note that the same modes also exist when an electron beam propagates through a dielectric in a channel of width b.

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Translated by David L. Book