Percolation description of the conductivity of random networks with a broad spectrum of the distribution of resistances

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The hypothesis is put forward that in the percolation description of randomly inhomogeneous media with a continuous broad spectrum of the distribution of resistances it is necessary to assume that the medium is in the smearing regime. A nonlinear equation for the size of the smearing region is obtained. On the basis of this hypothesis and a weak-link model that describes the structure of a two-phase medium near the percolation threshold an analytical expression is found for the critical index of the pre-exponential factor in the effective conductivity of media with a broad spectrum of the distribution of resistances. As examples, media in which the distribution of resistances has an exponential spectrum (high-temperature hopping conductivity) and a power spectrum are considered. In both cases the critical scaling index is equal to $y=(t-q)/2$, where t and q are the critical indices of the conductivity of two-phase media. Good agreement with the known numerical values and with the exact upper and lower boundaries is demonstrated. The critical behavior of the $1/f$ noise in such media is investigated, and the exponent of the exponential and the critical index of the pre-exponential factor in the relative spectral density of the noise are determined.

A large number of diverse problems reduce to the problem of the determination of the conductivity of randomly distributed resistances. For example, the description of the critical behavior of the conductivity, thermopower, Hall coefficient, etc., in percolation systems near the percolation threshold have their origin in this problem. Here there are just two types of resistance—"metallic" resistances r_1 and "dielectric" resistances r_2, with a δ-function distribution function

$$f(r)=p\delta(r-r_1)+(1-p)\delta(r-r_2), \quad r_2 \gg r_1. \tag{1}$$

A large number of works have been devoted to percolation problems (see, e.g., Refs. 1–3).

No less interesting and important is the case when the spectrum of resistances is continuous, while preserving the strong inhomogeneity—the "worst" of all the possible resistances are much greater than the "best." The best-known problem of this type can be formulated, in the simplest case, as follows. Find the effective conductivity σ^e of a random network of resistances

$$R_i=R_0\exp(-\lambda x), \tag{2}$$

where R_i is the resistance of the ith link in the lattice, R_0 is a constant, and x is a random variable with a smooth probability density $D(x)$; henceforth, we shall set $D(x)=1$ for $0<x<1$, and $D(x)=0$ for all other x. Values of $\lambda \gg 1$ correspond to large inhomogeneity. The interest in this problem, with an exponentially large inhomogeneity, arises because the calculation of the hopping conductivity reduces to it in a number of cases.[1] The problem for large inhomogeneity of the power-law type

$$R_i=R_0x^{-\lambda}, \tag{3}$$

can be formulated analogously; in certain cases, this problem can model so-called Swiss-cheese systems.[4] Below, for definiteness, we shall discuss the problem with an exponentially broad spectrum of resistances, and at the end extend this approach to the case of a problem with a power distribution.

A problem with a continuous spectrum of distributions is not directly a percolation problem—there is no percolation threshold such that when it is reached one of the phases forms an infinite cluster, since the phases themselves do not exist. However, there exists a device, proposed in Refs. 5 and 6 (see also Ref. 1), that reduces the problem with continuously distributed resistances to the two-phase percolation problem and makes it possible to determine the principal regular feature—the exponent in σ^e. As regards the pre-exponential factor, debate about this is still continuing.[8–10]

In Ref. 8 the following asymptotic ($\lambda \to \infty$) expression was proposed for the effective conductivity:

$$\sigma^e=\frac{A}{a_0^{d-2}R(x_c)}\left(\frac{D(x_c)}{\lambda}\right)^y, \quad \lambda \gg 1, \tag{4}$$

where A is a constant that depends on the form of the lattice, a_0 is the lattice constant, and the value x_c is defined in terms of the value of the percolation threshold p_c:

$$\int_{x_c}^{1} D(x)dx=p_c, \tag{5}$$

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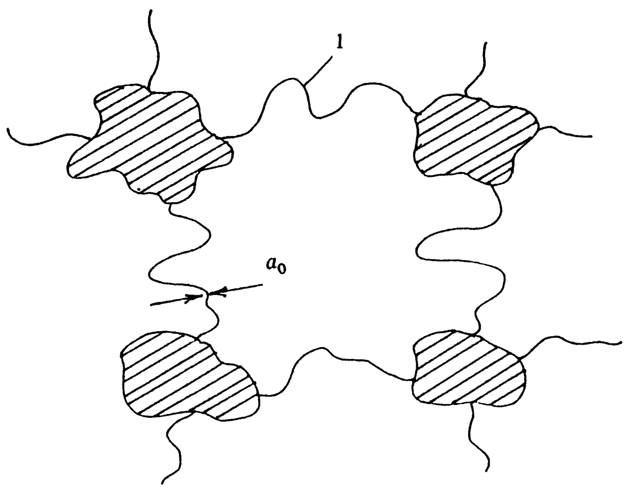

FIG. 1. Structure of a medium above the percolation threshold; *1*) a bridge—a set of "singly connected bonds" that consists of the good-conductor phase. The good-conductor phase is shaded.

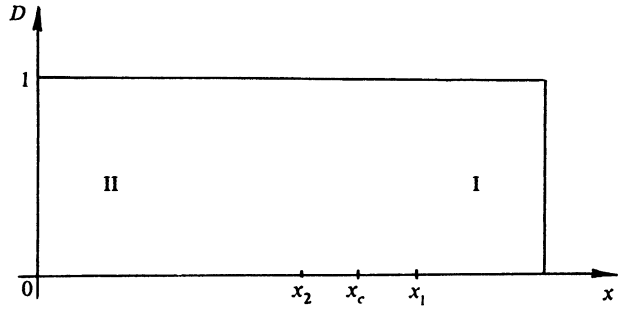

FIG. 2. Form $D=D(x)$ of the distribution of the random variable x that determines the magnitude of the link resistance via $R_i=R_0\exp(-\lambda x)$: I) region of low ("metallic") resistances; II) region of high ("dielectric") resistances.

where all resistances with $x>x_c$ are conventionally regarded as "metallic" and those with $x<x_c$ as "dielectric."

The quantity that determines the behavior of the preexponential factor is the critical index y. The analytical expression obtained for y in Ref. 9 does not fully agree with the numerical values and upper and lower boundaries y_+ and y_- obtained in Refs. 7 and 8.

In this paper we propose an approach which makes it possible to obtain an analytical expression for the critical index y and the analogous indices for other forms of broad continuous distributions of resistances. The basis of this approach is the weak-link model (WLM) (Refs. 11–14) and the hypothesis that for a correct percolation description of randomly inhomogeneous media with a broad spectrum of the distribution of resistances it is necessary to assume that they are in the smearing regime.[1)] A simplified approach is given in Ref. 15.

In Sec. 1 of the paper we consider the critical behavior of the σ^e of a network with an exponentially broad spectrum of resistances, while in Sec. 2 we consider the case of a power spectrum. In Sec. 3 we give an analysis of the results and a comparison with the numerical values and upper and lower boundaries known from the literature. In Sec. 4 we consider the critical behavior (associated with the fourth moment of the current distribution) of the relative spectral density of the $1/f$ noise.

1. AN EXPONENTIALLY BROAD DISTRIBUTION OF RESISTANCES

To determine the critical index y we shall assume first that for $x\to x_c (x>x_c)$ the medium is above the percolation threshold, i.e., if we were to regard all the resistances with $R\leqslant R(x_c)$ as the "metallic" phase, and all the others as the "dielectric" phase, we would obtain the standard structural picture of the nodes-links-blobs (NLB) type[16–19] (Fig. 1), with a definite bridge length [a set of single connected bonds (SCB)]. It is the resistance of these bridges that determines the entire resistance in the correlation volume.

For the very simple form of $D(x)$ that we have chosen here $[D(1)=1, 0\leqslant x\leqslant 1]$ the distance $\tau=(p-p_c)/p_c$ from the percolation threshold (p is the concentration of "metallic" resistances) is related to x and x_c as follows (Fig. 2):

$$x=x_c+(1-x_c)|\tau|. \tag{6}$$

Taking into account the form of $D(x)$ and the inequality $\lambda\gg 1$ we can estimate the average resistance of a bridge as follows:

$$\langle R\rangle_1=\int_{x_1}^{1} R(x)P_1(x)dx\approx\frac{R(x_1)}{1-x_1}\cdot\lambda^{-1},\quad (x_1>x_c), \tag{7}$$

where the renormalization $P_1(x) = D(x)/\int_{x_1}^{1}D(x)dx$ of the distribution is due to fulfillment of the condition that the largest bridge resistance is $R(x_1)$. The bridge resistance (and, consequently, the resistance of the entire correlation volume) is equal to

$$R_1=N_1\langle R\rangle_1, \tag{8}$$

where N_1 is the number of SCB in the bridge.

We now assume that for $x\to x_c (x<x_c)$ the medium is below the percolation threshold; i.e., if we were to color all the resistances with $R<R(x_2)$ black, we would obtain a picture dual to the preceding case (see Fig. 3)—the entire resistance is concentrated in a thin layer, of thickness equal to one link length a_0, between "metallic" clusters. The average resistance (the links in the layer are parallel to each other) can be estimated as follows:

$$\langle 1/R\rangle_2=\int_0^{x_2}[P_2(x)/R(x)]dx\approx 1/\lambda x_2 R(x_2), \tag{9}$$

where the renormalization $P_2(x) = D(x)/\int_0^{x_2}D(x)dx$ of the distribution is due to the fact that the smallest resistance in the layer is $R(x_2)$. The resistance of the layer (and, consequently, of the entire correlation volume) is equal to

$$R_2=1/(N_2\langle 1/R\rangle_2), \tag{10}$$

where N_2 is the number of so-called single disconnected bonds (SDB).[21]

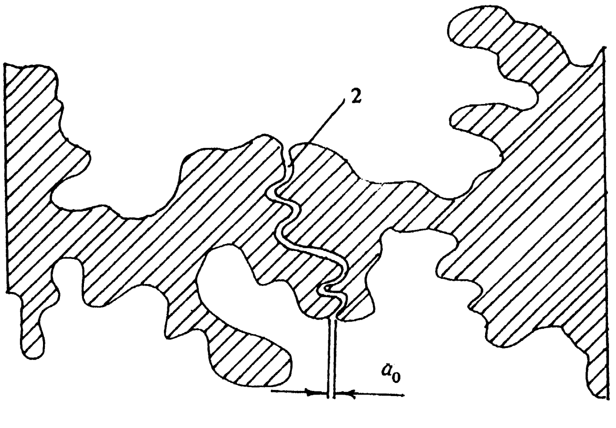

FIG. 3. Structure of a medium below the percolation threshold: *2*) a layer—a set of "singly disconnected bonds" that consists of the poor-conductor phase. The good-conductor phase is shaded.

The numbers N_1 and N_2 of SCB and SDB have a power dependence on the distance $\tau=(x-x_c)/(1-x_c)$ [see (6)] from the percolation threshold p_c:

$$N_i\sim|\tau|^{-\alpha_i},\quad \alpha_i>0. \tag{11}$$

The values α_i are determined differently in the different models of the percolation structure. In the NLB model the index $\alpha_1=1$ (Refs. 16, 1, 21–23), and, according to Refs. 24 and 20, $\alpha_2=1$ as well. According to the WLM,[11–14] the α_i are expressed in terms of the well known indices ξ_R and ξ_G describing the behavior of the resistance $\langle R_\xi\rangle\sim\tau^{-\xi_R}$ and conductance $\langle G_\xi\rangle\sim|\tau|^{-\xi_G}$ averaged over the correlation length ξ:

$$\alpha_1=\xi_R=t-\nu(d-2),\quad \alpha_2=\xi_G=q+\nu(d-2), \tag{12}$$

where t and q are the critical indices of the effective conductivity of a two-phase medium above and below the percolation threshold. In such two-phase systems, with the conductivities of the phases satisfying $\sigma_1\gg\sigma_2$, as is well known,[25] near the percolation threshold we have

$$\sigma^e\approx\sigma_1\tau^t,\quad (\tau>0),\quad \sigma^e\approx\sigma_2|\tau|^{-q}(\tau<0). \tag{13}$$

The dependences (13) are valid for $\Delta\ll|\tau|\ll1$, where Δ is the region of smearing. At a finite ratio σ_2/σ_1 (the ratio σ_2/σ_1 is the analog of the external field in the theory of second-order phase transitions, the effect of which depends on the magnitude of the order parameter) the discrete percolation-threshold point disappears and the transition "is smeared out." The order of magnitude of the region of smearing can be estimated by equating the values of σ^e above and below the percolation threshold:[25]

$$\sigma_1\Delta^t\approx\sigma_2\Delta^{-q}, \tag{14}$$

whence follow both the expression for the region of smearing and the value of the effective conductivity in the region of smearing:

$$\Delta\approx(\sigma_2|\sigma_1)^{1/t+q},\quad \sigma^e\approx\sigma_1(\sigma_2/\sigma_1)^{t/t+q}. \tag{15}$$

We shall return to the problem with a continuous distribution of resistances. Substituting (7) and (9) into (8) and (10) and going over from R to the conductivity $\sigma_i^e=1/R_i\xi^{d-2}$ (where $\xi\sim a_0|\tau|^{-\nu}$ is the correlation length, with critical index ν, and d is the dimensionality of the problem), we obtain

$$\sigma_1^e=\frac{1-x_1}{R(x_1)a_0^{d-2}}\tau^{\alpha_1+\nu(d-2)}\cdot\lambda, \tag{16}$$

$$\sigma_2^e=\frac{|\tau|^{-\alpha_2+\nu(d-2)}}{x_2R(x_2)a_0^{d-2}}\lambda^{-1}. \tag{17}$$

In analyzing (16) and (17) one should note that they are unsatisfactory for at least two reasons. First, in the form in which they are written they contradict the hypothesis of the universality of the critical index y—the dependences on λ in (16) and (17) are directly opposite. Second, the value taken by τ (and hence, according to (6), by x_1 and x_2) is not clear. We recall that, according to the method proposed in Refs. 5 and 6 (see also Ref. 1), the distance τ from the percolation threshold is introduced by convention and is not a free parameter whose value can be chosen at will, as can be done in two-phase systems by changing the concentration.

These contradictions can be removed by assuming that at the point of completion (or breaking) of the percolation cluster the system is in the smearing regime. We note that whereas in two-phase systems the size of the region of smearing can equal zero (either the metal is a perfect conductor or the dielectric is a perfect insulator), in the problem with a continuous distribution of resistances the size of the region of smearing has an entirely definite, nonzero value. By analogy with (14) we obtain an equation for the region of smearing:

$$\sigma_1^e(\tau=\Delta)=\sigma_2^e(\tau=\Delta). \tag{18}$$

In the brief communication Ref. 15 it was assumed for simplicity that in (16) and (17) $x_1=x_2=x_c$, which, with the use of (18), made it possible to remove the contradiction and determine approximately the critical index y. Here, we take into account that, according to (6),

$$x_1=x_c+(1-x_c)\Delta,\quad x_2=x_c-(1-x_c)\Delta, \tag{19}$$

i.e., that $x_1\neq x_2$.

Substituting (16), (17), and (19) into (18), we obtain the following nonlinear equation for Δ:

$$e^{-2(1-x_c)\Delta\lambda}=(x_c-(1-x_c)\Delta)(1-x_c-(1-x_c)\Delta)\times\Delta^{\alpha_1+\alpha_2}\lambda^2. \tag{20}$$

Taking into account that $\Delta\ll1$ (but $\Delta\lambda\gg1$), and neglecting the term $(1-x_c)\Delta$ in comparison with x_c and $1-x_c$, from (20) we obtain

$$\Delta\approx\frac{\ln[x_c(1-x_c)\lambda^{2-(\alpha_1+\alpha_2)}]}{2(1-x_c)\lambda}+O(\ln\ln\lambda^{2-(\alpha_1+\alpha_2)}). \tag{21}$$

Now, both from (16) $[\sigma_1^e=\sigma_1^e(\tau=\Delta)]$ and from (17) $[\sigma_2^e=\sigma_2^e(\tau=\Delta)]$ we obtain an expression for the effective conductivity:

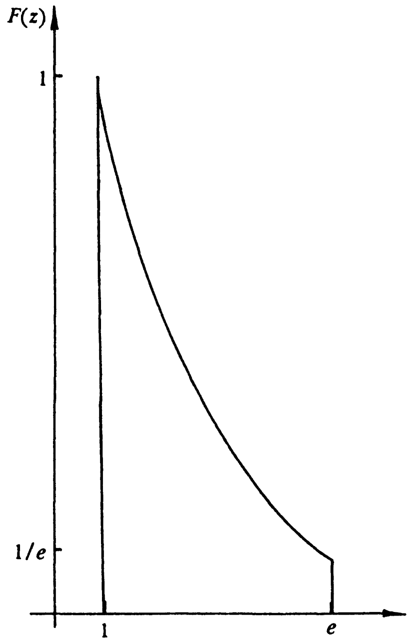

FIG. 4. Distribution $F(z)$ of the random variable z for which resistances specified in the power-law manner (3) take the exponential form (2).

$$\sigma^e \approx \frac{A}{a_0^{d-2} R(x_c)} \lambda^{-y}, \tag{22}$$

where A depends only weakly on λ [$A \sim (\ln \lambda)^{\alpha_1+\alpha_2+\nu(d-2)}$] and the critical index y is equal to

$$y = \frac{\alpha_1 - \alpha_2 + 2\nu(d-2)}{2}. \tag{23}$$

An analysis of this expression is given in Sec. 3.

2. A POWER-LAW DISTRIBUTION OF RESISTANCES

We now consider a random network of resistances with a power-law dependence (3) of the resistance on the random variable x. We choose the distribution $D(x)$ to be the same as before: $D(x)=1$ for $0 \leqslant x \leqslant 1$. The expression (3) can be brought to the exponential form (2) by the change of variable $x = \ln z$, but the distribution of the random variable z now has a form different from that of $D(x)$ (Fig. 4).

In terms of the variable x the determination of σ^e is analogous to the preceding determination. For example, the average bridge resistance and layer resistance (with allowance for the fact that $\lambda \gg 1$) are equal to

$$\langle R \rangle_1 \approx \frac{R_0}{(1-x_1)\lambda x_1^{\lambda-1}}, \quad \left\langle \frac{1}{R} \right\rangle_2 \approx \frac{x_2^\lambda}{R_0 \lambda}, \tag{24}$$

while the equation for the region of smearing is

$$\left(\frac{1 - \dfrac{1-x_c}{x_c}\Delta}{1 + \dfrac{1-x_c}{x_c}\Delta} \right)^\lambda \approx \lambda^2 \Delta^{\alpha_1+\alpha_2}, \tag{25}$$

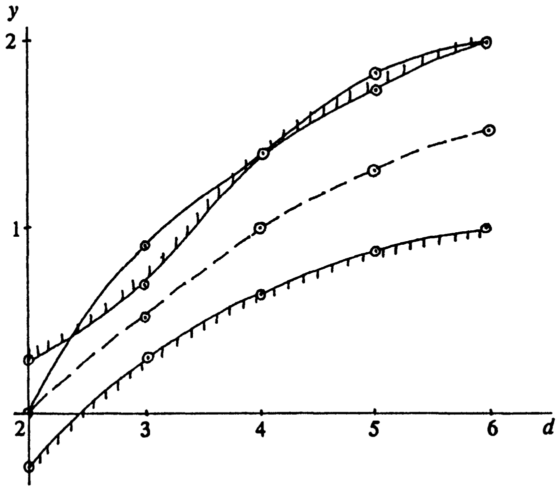

FIG. 5. Dependence of the critical index y on the dimensionality d of the problem. The lines with shading are the upper and lower boundaries,[8,9] the solid line is the critical index of LeDoussal,[10] and the dashed line is the critical index $y=(t-q)/2$ in the weak-link model. For the numerical values of the critical indices of the two-phase media we have taken $\nu_2=4/3$, $t_2=q_2=1.29$; $\nu_3=0.9$, $t_3=1.7$, $q_3=0.7$; $\nu_4=0.7$, $t_4=2.4$, $q_4=0.35$; $\nu_5=0.6$, $t_5=2.7$, $q_5=0.14$; $\nu_6=0.5$, $t_6=3$, $q_6=0$, where the subscript denotes the dimensionality of the problem. For convenience the dependence of y on d is presented as a continuous line.

the solution of which has the form

$$\Delta \approx \frac{\ln \lambda^{\alpha_1+\alpha_2-2}}{2 \dfrac{1-x_c}{x_c} \lambda}. \tag{26}$$

Substituting (26) into the expression for the resistance of the correlation volume and going over to the effective conductivity σ^e, we obtain

$$\sigma^e \approx \frac{A}{a_0^{d-2} R_0} x_c^\lambda \lambda^{-y}, \tag{27}$$

where A depends logarithmically on λ and y coincides with (23).

3. ANALYSIS OF THE EXPRESSIONS OBTAINED

The analytical expression for the critical index y has been obtained on the basis of the assumption that the system is in the smearing regime. The numerical value of y will depend on the model used for choosing α_1 and α_2. When the α_i are chosen using the NLB models ($\alpha_i = 1$) the index y coincides with that in the paper of LeDoussal:[10]

$$y(\alpha_1 = 1, \alpha_2 = 1) = y_{LD} = \nu(d-2). \tag{28}$$

Comparison of y_{LD} with the numerical values in Ref. 9 and the upper and lower boundaries in Refs. 8 and 9 shows insufficiently good agreement (see Fig. 5).

When the α_i are chosen using the WLM,[11–14] we have

$$y_{WLM} = y(\alpha_1 = \xi_R, \quad \alpha_2 = \xi_G) = (t-q)/2, \tag{29}$$

which agrees well with Refs. 8 and 9. It is easy to see that since the upper boundary y_+ and lower boundary y_- are equal to $y_+ = t-1$ and $y_- = 1-q$ (Refs. 8, 9) the value obtained for y_{WLM} is their average $y_{WLM} = (y_+ + y_-)/2$.

We now show that y_{LD} is a particular case of y_{WLM} (23). For this it is sufficient to resort to the values of t and q that follow from the simplest variants of the models with SCB and SDB. We shall consider first the case $\tau>0$. On the one hand, according to Refs. 1, 16, and 21–23, $N_1 \sim \tau^{-1}$, and, on the other, the average value of the resistance of a network of size equal to the correlation length is $\langle R_\xi \rangle \sim N_1$. Comparing these values with the familiar expression $\langle R_\xi \rangle \sim \tau^{-\xi_R}$ $[\xi_R = t - \nu(d-2)]$, for the critical index t we obtain $t^* = 1 + \nu(d-2)$. This result, based on the model of a single-cable network, turns out to be inaccurate in a number of cases (see the discussion of t^* in the book Ref. 1), and this has stimulated a number of papers on the refinement of the structure of a percolation cluster (see, e.g., Ref. 26). Analogous arguments for the case $\tau<0$ (Refs. 24 and 20) lead to $q^* = 1 - \nu(d-2)$. This result, based on SDB, is also inaccurate in a number of cases; e.g., for $d>4$ we have $q^*<0$, unlike the value $q>0$ generally adopted for all dimensionalities. The structure of the percolation cluster for $\tau<0$ is investigated in Ref. 27.

Substituting t^* and q^* into the general expression (23) for y we obtain

$$y(t=t^*, \quad q=q^*) = y_{LD}. \tag{30}$$

For the critical dimensionality $d_c=6$ the critical indices t and q are known exactly[27] $[t(d_c)=t^*(d_c)=3]$, but $q^*(d_c)=-1$, which does not coincide with the exact value $q(d_c)=0$. The latter evidently permits us to assert that $y(d_c)=(3-0)/2=1.5$ (and not $y_{LD}=2$, as stated in Ref. 10).

The results of this approach are not changed if one chooses the different notation adopted in, e.g., Ref. 1:

$$R_i = R_0 \exp \xi, \tag{31}$$

where R_i, as before, is the resistance of the ith link, and ξ is a random variable with constant probability density in the interval $[-\xi_0, \xi_0]$. A large inhomogeneity is specified by the condition $\xi_0 \gg 1$. In this notation, as is easily shown,

$$\sigma^e \sim \frac{1}{R(\xi_c)} \xi_c^{-y}, \tag{32}$$

where ξ_c is related to the percolation threshold p_c by $p_c = (\xi_c + \xi_0)/2$. The critical index y has the same value [(23) and (29)] as before.

It is evident that the problem with a power-law distribution of resistances can serve as a suitable basis of a model for the description of the effective conductivity of island metallic films[29] on a weakly conducting substrate. In this case the substrate can be interpreted as the second, poorly conducting phase, and the resistance of the metallic islands can be neglected in the first approximation. Then the resistance between two islands will be determined by the shape of the boundaries of the island near the contact, and, if we assume that this shape is smooth and almost a circle, we obtain $R \sim x^\alpha$ (α=const), as in the Swiss-cheese model.[4]

We note also that an investigation of the optical properties of metallic island films on a nonconducting substrate[30] points to the need to examine the properties of the structure over distances of the order of the correlation length, and this, when "translated into the concentration language," implies a region of smearing.

4. RELATIVE SPECTRAL DENSITY OF THE 1/f NOISE

The amplitude $\mathscr{S}$ of the relative spectral density (RSD) of the $1/f$ noise[31] is one of the important characteristics of materials. A large number of papers are devoted to its study in macroscopically inhomogeneous media (see Refs. 12, 20, and 32–35, and the literature cited therein). As is well known, in two-phase media near the percolation threshold the effective amplitude $\mathscr{S}^e$ of the RSD of the $1/f$ noise behaves in a critical manner. The problem of the determination of $\mathscr{S}^e$ in macroscopically inhomogeneous media is formulated analogously to the problem of the determination of σ^e; from the given local values $\mathscr{S}_1$ and $\mathscr{S}_2$ of the RSD in the two phases one determines the $\mathscr{S}^e$ of the whole sample. Here, just as in the case of the problem of the determination of σ^e, there exist rules for calculating the $\mathscr{S}^e$ of parallel- and series-connected resistances R_i:

$$\mathscr{S}^e = \sum_i \left(\frac{R_i}{R^e}\right)^2 \mathscr{S}_i, \quad \mathscr{S}^e = \sum_i \left(\frac{R^e}{R_i}\right)^2 \mathscr{S}_i, \tag{33}$$

where R^e is the total resistance and $\mathscr{S}_i$ is the RSD of the resistance R_i; the first relation is for resistances connected in series, and the second for resistances connected in parallel.

It is often more convenient to use the specific relative spectral density C of the $1/f$ noise. This is related to $\mathscr{S}$ as follows:

$$C = \mathscr{S}_V \cdot V, \tag{34}$$

where $\mathscr{S}_V$ is the RSD of the $1/f$ noise of a homogeneous sample of volume V.

In terms of C the expression for the effective RSD of the $1/f$ noise has the form

$$C^e = \frac{\langle C(r)(E(r) j(r))^2 \rangle}{(\langle E(r) \rangle \langle j(r) \rangle)^2}, \tag{35}$$

where $\mathbf{j}$ is the current density and $\mathbf{E}$ is the field intensity.

It follows from (33) and (35) that to determine the RSD of the $1/f$ noise it is necessary to know not only the spatial distribution of the phases but also the distribution of the currents in the system. Models of the NLB or WLM type, describing the principal elements of the structure, make it possible to estimate the behavior of the RSD of the $1/f$ noise in systems with continuously distributed resistances.

First it is necessary to formulate an assumption analogous to the hypothesis of Ref. 31, according to which $C=\alpha/\sigma$, where $\alpha \approx$const is the universal Hooge constant. We shall assume that in our model system with an exponential spread of resistances the specific local RSD of the $1/f$ noise depends as follows on the random variable x:

$$C(x) = \frac{\alpha}{\sigma(x)}, \quad \alpha = \text{const}. \tag{36}$$

The subsequent calculation is implemented analogously to the calculation of σ^e. First it is necessary to determine the RSD of the $1/f$ noise above the percolation threshold. According to (33) (the resistances in the bridge are arranged in series),

$$\mathscr{S}_1 = \sum (R(x)/R^e)^2 \mathscr{S}(x), \tag{37}$$

where R^e is the resistance of the whole sample. Use of (36), which can be formulated in the form

$$\mathscr{S}(x) = \alpha R(x)/a_0^2, \tag{38}$$

brings (37) to the form

$$\mathscr{S}_1 = \alpha\left(\sum R^3(x)\right)/a_0^2(R^e)^2. \tag{39}$$

The sum in (39) is determined analogously to (7) and (8): $\Sigma R^3(x) = N_1 \langle R^3 \rangle_1$. Calculating $\langle R^3(x)\rangle_1$ and substituting into (39), we obtain for an exponential distribution of resistances (here we have already taken into account that the system is in the smearing regime):

$$\mathscr{S}_1 \sim \alpha R(x_c)\lambda^{m_1}, \tag{40}$$

where the critical index m is equal to

$$m_1 = 2 - \frac{\alpha_2 + 3\alpha_1}{2}. \tag{41}$$

The calculation below the percolation threshold is performed analogously; in this case,

$$\mathscr{S}_2 \sim \alpha R(x_c)\lambda^{m_2}, \tag{42}$$

where

$$m_2 = -\frac{\alpha_2 - \alpha_1}{2} + \nu d. \tag{43}$$

In the NLB models ($\alpha_1 = \alpha_2 = 1$) we have $m_1 = m_2 = 0$. According to the WLM, we have $m_2 > m_1$, and, in the final expression for $\mathscr{S}^e$ it is necessary to retain only (42). Going over to the specific relative spectral density $C^e = \mathscr{S}^e \Delta^{-\nu}$ of the $1/f$ noise, we obtain

$$C^3 \sim \alpha R_0 e^{-\lambda x_c}\lambda^m, \tag{44}$$

where

$$m \geqslant \frac{\alpha_2 - \alpha_1}{2} + \nu d. \tag{45}$$

Taking into account the values (12) of the α_i in the WLM, from (45) we obtain

$$m \geqslant \frac{t-q}{2} + 2\nu. \tag{46}$$

The expressions (44) and (45), (46) fully determine the critical behavior of the RSD of the $1/f$ noise in a model system with an exponentially broad spectrum of the distribution of resistances in terms of the "Hooge hypothesis" (36) (we note that it would be possible to choose any other hypothesis). According to (44)–(46) the RSD of the $1/f$ noise depends exponentially strongly on λ. Unlike the critical index y, which determines the pre-exponential factor of the effective conductivity σ^e, the critical index m is not equal to zero in the two-dimensional case. We note that the local "Hooge hypothesis" $C(x)\sigma(x) = \text{const}$ is no longer fulfilled for the whole sample:

$$C^e\sigma^e \sim \lambda^{2\nu} \gg 1. \tag{47}$$

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[1)] By the smearing region Δ we mean the magnitude of the "smearing" of the transition from one critical behavior to the other (the percolation transition is an analog of a second-order phase transition); another name for this quantity, also adopted in the literature, is the width of the transition. For details, see Refs. 14 and 15.

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