Theory of magnetoacoustic oscillations in metals in a longitudinal magnetic field

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(Submitted 30 June 1993)
Zh. Eksp. Teor. Fiz. **104**, 4107–4120 (December 1993)

Within the framework of the quasiclassical approach, we discuss geometric oscillations in the absorption Γ and propagation velocity s of ultrasonic waves propagating in metals in a longitudinal magnetic field for arbitrary dispersion laws of the metallic electrons. We identify a number of specific features that distinguish these magnetoacoustic oscillations from geometric (Pippard) oscillations in a perpendicular field, and establish that the oscillations in Γ and s for longitudinal waves are due primarily to a strain mechanism associated with electron-phonon interactions and should be sinusoidal in form. For the case of transverse waves the dominant mechanism should be field-dependent as well, in which case the shape of the oscillations can deviate from harmonic. We predict giant geometric oscillations in the regime of interaction between ultrasound and circularly polarized electromagnetic waves.

1. INTRODUCTION

In investigating polarization-related magnetoacoustic effects in single-crystal indium we encountered a new phenomenon:[1] when transverse ultrasonic waves propagate along the fourth-order axis parallel to the magnetic field **H**, we observed an oscillation in the ellipticity $\varepsilon(\mathbf{H})$ which is quasiperiodic in the magnitude of the inverse field $\mathbf{H}^{-1}$. We showed that this phenomenon can be interpreted as magnetoacoustic geometric oscillations in the geometry $\mathbf{q}\parallel \mathbf{H}$ (where **q** is the wave vector), which differs from the geometry that is ordinarily used for observing them.

A survey of the literature showed that no complete theory of geometric oscillations in a longitudinal field exists. Although observed the first geometric oscillations for $\mathbf{q}\parallel \mathbf{H}$ Rayne and Chandrasekhar[2] in 1962, they were unable to interpret their results. The idea that these oscillations could exist related to those observed by Bömmel[3] for $\mathbf{q}\perp \mathbf{H}$ and interpreted by Pippard,[4] was first reported in a paper by MacKinnon *et al.*[5] These authors qualitatively described a physical picture in which ultrasonic absorption could exhibit oscillations. However, they made these assertions in order to interpret narrow high-power absorption peaks in Cd, which in all probability must be due to a completely different effect, i.e., Doppler-shifted cyclotron resonance (DSCR). In a subsequent paper, Daniel and MacKinnon[6] attempted to obtain an expression connecting the period of oscillations with the characteristics of the Fermi surface; however, what they derived were the conditions for observing peaks in the Doppler-shifted cyclotron resonance.

By using a quantum-mechanical model to treat the problem of propagation of longitudinal ultrasonic waves along a magnetic field and a model Fermi surface in the shape of an ellipsoid (one of whose axes is inclined at a certain angle to **H**), Quinn[7] showed that in the collisionless limit the corresponding component of the conductivity tensor contained an oscillatory term. His main result was a correct expression for the period of the geometric oscillations. The same problem, but now within the framework of a simpler quasiclassical theory, was solved by Eckstein,[8] who obtained an expression that was, on the whole, analogous. Later experimental investigations[9–11] confirmed the correctness of the Quinn-Eckstein expressions for the period of the geometric oscillations. As far as we can tell, the last publication on this topic seems to be a paper by Miller[11] that appeared in 1966. This loss of interest in magnetoacoustic oscillations in a longitudinal field is probably connected with the view that the phenomenon is completely analogous to Bömmel-Pippard oscillations.

In this paper, we use a unified quasiclassical scheme to investigate the geometric oscillations that arise when either longitudinal or transverse circularly-polarized modes propagate in a metal with an arbitrary electron dispersion law in a parallel magnetic field. We simultaneously analyze the absorption and change in velocity of these modes, and briefly describe the accompanying polarization effects. We show that magnetoacoustic oscillations in a longitudinal field differ in a number of significant respects from the geometric oscillations that appear for $\mathbf{q}\perp \mathbf{H}$.

2. STARTING RELATIONS

Let us consider propagation of ultrasonic waves along an n-th order axis of rotational symmetry R_n ($n\geqslant 3$) that is parallel to a constant external magnetic field $\mathbf{H}=(0,0,H)$: here $\mathbf{q}\parallel R_n\parallel \mathbf{H}$, where **q** is the wave vector. In this case the normal modes are two circularly-polarized waves and one longitudinal wave. Using the Maxwell equations and the theory of elasticity (described, e.g., in Sec. 5.1 of the review Ref. 12), we can show that the solution to the dispersion equation for characteristic elastic waves has the following form:

1063-7761/93/120985-08$10.00

$$\frac{\Delta q^{(p)}}{q_0^{(p)}}=\frac{i}{2C^{(p)}}\left\{-\omega\alpha^{(p)}+\frac{iH^2|P|}{4\pi}\right.$$
$$\left.+\frac{(\omega\beta^{(p)}+iPcq_0^{(p)}H/4\pi)^2}{\omega\sigma^{(p)}-i|P|(cq_0^{(p)})^2/4\pi}\right\},\qquad(1)$$

where the label p characterizes the polarization of the normal mode, and equals "+", "−", or "l" for circularly and longitudinally polarized modes respectively, while P takes the numerical values ± 1 for $\pm$-polarized waves and zero for longitudinal waves. The quantity $\Delta\mathbf{q}^{(p)}=\mathbf{q}^{(p)}(\mathbf{H})-\mathbf{q}_0^{(p)}$ is a dynamic magnetic-field-dependent correction to the wave vector, with $\mathbf{q}^{(p)}=(0,0,q^{(p)})$ and $\mathbf{q}_0^{(p)}=q^{(p)}(H=0)\equiv\omega/s_0^{(p)}$, where ω is the frequency and $s_0^{(p)}$ is the phase velocity of the normal mode when $\mathbf{H}=0$; $C^{(p)}$ is the quasistatic elastic modulus for $\mathbf{H}=0$, where $C^{(\pm)}=C_{44}$ and $C^{(l)}=C_{33}$; and c is the velocity of light. The specific properties of a given metal, i.e., those that depend on its electronic structure, are taken into account by the three electroacoustic coefficients: $\alpha^{(p)}$ describes the strain-dependent interaction between the elastic and electronic subsystems of the metal, $\beta^{(p)}$ is the strain-dependent conductivity, and $\sigma^{(p)}$ is the electronic conductivity.

The Fourier components of the electroacoustic tensors α, β, and σ entering into Eq. (1) that follow from solving the Boltzmann equation in the relaxation-time (τ) approximation can be written in the following way:

$$\gamma^{(p)}=\frac{(2^{1-|P|})}{h^2}\int dk_z\sum_{\varkappa}|m_c|\sum_{m=-\infty}^{\infty}$$
$$\times\frac{a_{m,1}^{(p)}\cdot b_{m,2}^{(p)}}{\tau^{-1}-i(q_0^{(p)}\bar{v}_z-\omega-m\Omega)},\qquad(2)$$

where h is Planck's constant, m_c is the cyclotron mass, k_z is the component of the electron wave vector, $\Omega=-eH/(m_c c)$ is the cyclotron frequency, $e<0$ is the electron charge, and $\bar{v}_z$ is the value of the z component of the electron velocity $\mathbf{v}$ averaged over a cyclotron period; the label $\varkappa$ identifies the orbits that exist for a given value of k_z. The quantities $a_{m,1}^{(p)}$ and $b_{m,2}^{(p)}$ are defined to be the components of two vectors $\mathbf{a}$ and $\mathbf{b}$ that can represent either $e\mathbf{v}$ or $\mathbf{w}\equiv(\Lambda_{xz},\Lambda_{yz},\Lambda_{zz})$, where Λ_{ij} are the components of the deformation potential tensor. In order to calculate the electroacoustic coefficients, it is necessary to make the following substitutions in Eq. (2):

$$\text{for }\sigma^{(p)}-\mathbf{a}=\mathbf{b}=e\mathbf{v},$$
$$\text{for }\beta^{(p)}-\mathbf{a}=e\mathbf{v},\quad\mathbf{b}=\mathbf{w},$$
$$\text{for }\alpha^{(p)}-\mathbf{a}=\mathbf{b}=\mathbf{w}.\qquad(3)$$

The quantities $a_{m,1}^{(p)}$ and $b_{m,2}^{(p)}$ are easily written in terms of a common expression. We introduce the following representation for the arbitrary vector $\mathbf{g}$: $\mathbf{g}\equiv\mathbf{g}^{(\pm)}+\mathbf{g}^{(l)}$, where $\mathbf{g}^{(\pm)}\equiv\mathbf{g}_\perp\equiv(g_x,g_y,0)$, and $\mathbf{g}^{(l)}\equiv\mathbf{g}_\parallel\equiv(0,0,g_z)$. Then

$$g_{m,j}^{(p)}=\frac{1}{2\pi}\int_0^{2\pi}|\mathbf{g}^{(p)}|\exp\{i(-1)^{3+P/3-|P|+j+1}$$
$$\times\varphi_g^{(p)}\}\exp\{i(-1)^j[q_0^{(p)}Z(\theta)+m\theta]\}d\theta,\qquad(4)$$

where $\theta=\Omega t$ is the dimensionless time for an electron moving along a cyclotron orbit, and $\varphi_g^{(p)}$ is the angle that specifies the direction of the vector $\mathbf{g}^{(\pm)}$ relative to the x axis or the vector $\mathbf{g}^{(l)}$ relative to the z axis ($\varphi_g^{(l)}=-\varphi_g^{(l)}=0$ or π). The quantity $z(\theta)$ entering into (4) is defined by the expression

$$Z(\theta)=\frac{1}{\Omega}\int_0^{\theta}(v_z-\bar{v}_z)d\theta'\equiv\frac{1}{H}G(\theta).\qquad(5)$$

Recall that the field-dependent electronic corrections to the absorption $\Gamma^{(p)}$ and phase velocity $s^{(p)}$ are given in terms of $\Delta q^{(p)}$ as follows:

$$\Gamma^{(p)}=-\operatorname{Im}\Delta q^{(p)},$$
$$\frac{\Delta s^{(p)}}{s_0^{(p)}}=-\operatorname{Re}\frac{\Delta q^{(p)}}{q_0^{(p)}}.\qquad(6)$$

3. OSCILLATION IN THE ELECTROACOUSTIC COEFFICIENTS FOR q|| H

It is clear from (2)–(4) that any electroacoustic coefficient $\gamma^{(p)}$ is the sum of a series whose elements are labeled by the integer index m. For any $m\neq 0$ the denominator on the right side of (2) is resonant in character, i.e., there exists a value of the field $\mathbf{H}$ for which its magnitude reaches a minimum; the expression as a whole describes Doppler-shifted cyclotron resonance. The element with $m=0$ is nonresonant, and it is just this term that gives rise to the geometric oscillations of the electroacoustic coefficients in the geometry $\mathbf{q}\parallel\mathbf{H}$. Let us consider it in more detail:

$$\gamma_{m=0}^{(p)}=\frac{(2^{1-|P|})}{h^2}\int dk_z\sum_{\varkappa}|m_c|\frac{a_{0,1}^{(p)}\cdot b_{0,2}^{(p)}}{\tau^{-1}-i(q_0^{(p)}\bar{v}_z-\omega)},\qquad(7)$$

$$g_{0,j}^{(p)}=\frac{1}{2\pi}\int_0^{2\pi}|\mathbf{g}^{(p)}|\exp\{i(-1)^{3+P/3-|P|+j+1}$$
$$\times\varphi_g^{(p)}\}\exp\{i(-1)^j[q_0^{(p)}Z(\theta)]\}d\theta.\qquad(8)$$

From the definition of $z(\theta)$ it follows that this function is periodic with period 2π. If we do not treat the trivial case $v_z(\theta)=\text{const}$, the number of minima of this function on an interval $0\leqslant\theta\leqslant 2\pi$ coincides with the number of maxima, i.e., the number of extrema R is even. In what follows, we will refer to points on a cyclotron path corresponding to extremal values of $z(\theta)$ as turning points. Points of this kind are actually in a system of coordinates moving with velocity $\bar{v}_z$ in the direction $\mathbf{H}$. If for any neighboring pair of points, one of which θ_N is a maximum and one θ_{N+1} a minimum, we have

$$q_0^{(p)}|Z(\theta_N)-Z(\theta_{N+1})|\gg 2\pi,\qquad(9)$$

the expression under the integral sign in (8) is a rapidly oscillating function of the argument θ. In this case in order to estimate the integral we can apply the method of stationary phase (see Ref. 13). As a result we obtain

$$g_{0,j}^{(p)}=\left(\frac{|\Omega|}{2\pi q_0^{(p)}}\right)^{1/2}\sum_{N=1}^{R}\frac{|g^{(p)}(\theta_N)|}{|\partial v_z/\partial\theta|_N^{1/2}}\times\exp\{i(-1)^j\Phi_g^{(p)}(\theta_N)\},\qquad(10)$$

where

$$\Phi_g^{(p)}(\theta_N)=q_0^{(p)}Z(\theta_N)-\varphi_g^{(p)}(\theta_N)(-1)^{3+P/3-|P|}+\frac{\pi}{4}(-1)^N.\qquad(11)$$

The label N in Eqs. (10)–(12) implies that this quantity is evaluated at the corresponding turning point. Here and in what follows, turning point labels are chosen so that $N=1$ corresponds to a maximum of the function $z(\theta)$, while the remaining extrema are labeled by the successive cyclotron trajectories.

The inequality (9) implies that as an electron moves between two neighboring extremal points it traverses a distance (in the z direction) that greatly exceeds the ultrasonic wavelength $2\pi/q_0^{(p)}$. As the discussions that follow will show, this corresponds to treating oscillations whose labels are rather large.

When (4) is included, the expression for the product of the electroacoustic coefficient and ω [it is in this combination that the coefficient enters into (1)] takes the form

$$\omega\gamma_{m=0}^{(p)}=\frac{|e|2^{-|P|}}{\pi h^2 c}\left(\frac{H}{q_0^{(p)}}\right)\int_{\Pi\Phi}dk_z\sum_{\varkappa}\sum_{k=1}^{R}\sum_{l=1}^{R}\times\frac{|\mathbf{b}^{(p)}(\theta_k)|}{|\partial v_z/\partial\theta|_k^{1/2}}\frac{|\mathbf{a}^{(p)}(\theta_l)|}{|\partial v_z/\partial\theta|_l^{1/2}}$$
$$\times\exp i\left[q_0^{(p)}\Delta Z_{kl}-(-1)^{3+P/3-|P|}(\varphi_b^{(p)}(\theta_k)-\varphi_a^{(p)}(\theta_l))+\frac{\pi}{4}((-1)^k-(-1)^l)\right]$$
$$\times\left\{\omega\tau\left[1-i\omega\tau\left(\frac{\bar{v}_z}{s_0^{(p)}}-1\right)\right]^{-1}\right\}.\qquad(12)$$

Here

$$\Delta Z_{kl}=Z(\theta_k)-Z(\theta_l)\equiv\Delta G_{kl}/H\qquad(13)$$

is the distance between turning points k and l in the direction $\mathbf{H}$, measured in the moving system of coordinates described above.

The oscillatory component of $\omega\gamma_{m=0}^{(p)}$ is associated with elements of the sum in the integrand in Eq. (12) for which $k\neq 1$. The periodic variation of these elements with the inverse magnetic field is associated with the spatial dispersion parameter $q_0^{(p)}\cdot\Delta z_{kl}\sim q_0^{(p)}/H$. To within a factor of 2π, this is the ratio of the length of the electron path in the z direction to the wavelength. The argument of the exponential contains a phase correction that is independent of magnetic field but depends on polarization. The appearance of ellipticity and rotation of the plane of polarization of a transverse ultrasonic wave that is initially linearly polarized during propagation is associated with this correction. Furthermore, the characteristics $|e\mathbf{v}^{(p)}|$ and $|\mathbf{w}^{(p)}|$ of the electrons at the turning points, and the characteristic time $|\partial\mathbf{v}_z/\partial\theta|\cdot q_0^{(p)}/\mathbf{H}$ during which an electron remains in the vicinity of such a point, are contained under the integral sign in Eq. (12).

In addition, we should note the factor in curly brackets in Eq. (12), which distinguishes a group of electrons that interact efficiently with the ultrasonic wave. These electrons are located within a belt on the Fermi surface centered around the cyclotron path specified by the value $\bar{v}_z=s_0^{(p)}$. The half-width of the belt (with respect to $\bar{v}_z$) is determined by the temporal dispersion parameter:

$$\Delta\bar{v}_z=2s_0^{(p)}/\omega\tau.\qquad(14)$$

In the limit $\omega\tau\to\infty$ the real part of the last factor in (12) coincides with a δ function of the argument $\bar{v}_z-s_0^{(p)}$ to within a factor independent of k_z, while the imaginary part coincides with the derivative of this function. Thus, only those carriers that move in the z direction with a velocity whose value averaged over a cyclotron period equals the velocity of sound are effective; the period, amplitude, and phase of the oscillating components of $\omega\gamma^{(p)}$ will be determined by the characteristics of these turning point orbits alone. Note that this circumstance points to an important difference between magnetoacoustic oscillations in a longitudinal field and oscillations in a normal field, for which the oscillations are determined by electrons with extremal orbits.

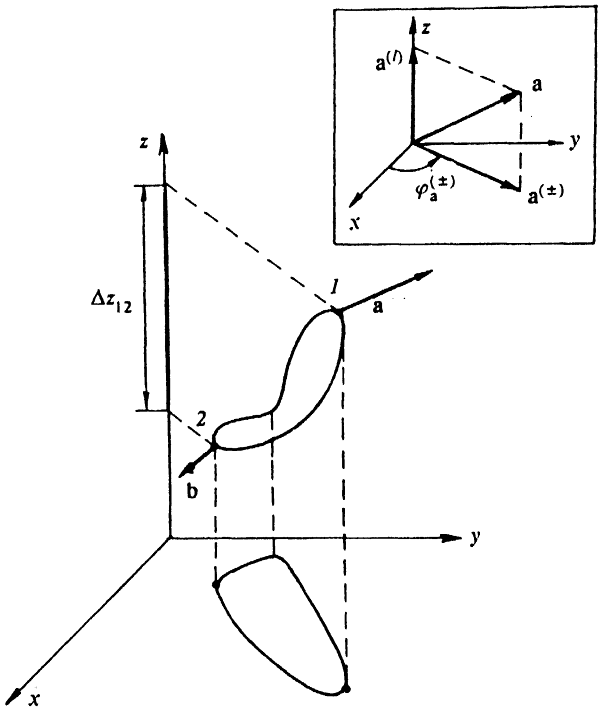

FIG. 1. Path of effective electrons in a system of coordinates moving with the velocity of sound. We show the projection of paths on the plane xy and the z axis, and also the vectors **a** and **b** at turning points *1* and *2*. In the inset we show the decomposition of the vector **a** into $\mathbf{a}^{(l)}$ and $\mathbf{a}^{(\pm)}$, and the angles $\varphi_a^{(\pm)}$.

As the "effectiveness belt" (14) widens, the integration over k_z can cause a radical decrease in the amplitude of oscillations of $\omega\gamma^{(p)}$, since in this case oscillatory contributions with different periods are being summed. However, for $\omega\tau \ll 1$ we can apply the method of stationary phase repeatedly and verify that the Pippard principle of discarding other effective orbits begins to apply if there is a sufficiently large group of carriers with an extremal value of Δz_{kl}, then the contribution from this group becomes dominant. In this situation it is the extremal group alone that determines the frequency and phase of oscillation of the electroacoustic coefficients. Thus, a decrease in $\omega\tau$ not only decreases the oscillation amplitude significantly, but can also lead to a change in its period.

When there are several effective groups of electrons and (or) more than two inequivalent turning points are present on the cyclotron path, we will observe several series of oscillations with different periods, phases, and amplitudes.

4. GEOMETRIC OSCILLATIONS IN THE ABSORPTION AND VELOCITY OF ULTRASONIC WAVES

Let us consider how harmonic oscillations of the electroacoustic coefficients give rise to more complicated oscillations in the absorption and velocity of ultrasonic waves, and consider which of the interaction mechanisms (strain- or field-induced) is dominant for waves of different polarizations.

4.1. Longitudinal waves

It is clear from Eq. (12) that the amplitude of oscillations of the electroacoustic coefficients is determined by the values of the components of the vectors **v** and (or) **w** at the turning point, along with other factors. For longitudinal waves these components are Λ_{zz} and v_z, and for an effective orbit $v_z = s_0^{(l)}$. Since $s_0^{(l)}$ is three orders of magnitude smaller than the characteristic value of the Fermi velocity v_F, we should expect that the oscillating components of the electroacoustic coefficients associated with this value of v_z will be considerably smaller in absolute value than the components due to the fundamental and higher harmonics of the Doppler-shifted cyclotron resonance, which are determined by values of v_z on the order of v_F. This circumstance allows us to write the expression for $\Delta q^{(l)}$ in the form

$$\frac{\Delta q^{(l)}}{q_0^{(l)}} = \frac{i}{2C^{(l)}} \left\{ -\omega\alpha^{(l)} + \frac{(\omega\beta_b^{(l)})^2}{\omega\sigma_b^{(l)}} \times \left[1 + \frac{2\beta_{m=0}^{(l)}}{\beta_b^{(l)}} - \frac{\sigma_{m=0}^{(l)}}{\sigma_b^{(l)}} \right] \right\}, \tag{15}$$

where $\beta_b^{(l)}$ and $\sigma_b^{(l)}$ are the fundamental components of $\beta^{(l)}$ and $\sigma^{(l)}$ calculated using Eq. (2) and excluding terms with the label $m=0$.

Based on what we have said above, it is difficult to believe that $|\beta_{m=0}^{(l)}/\beta_b^{(l)}|$ will be larger than 10^{-3} in order of magnitude, while $|\sigma_{m=0}^{(l)}/\sigma_b^{(l)}|$ likewise should not exceed 10^{-6}. At the same time, there is no requirement that $\Lambda_{zz}(\theta_N)$ be small compared to characteristic values of Λ_{zz}.

Consequently, when an experiment involving longitudinal fields leads to the observation of oscillations in the absorption or velocity of ultrasound that significantly exceed 10^{-3} of the characteristic value of $\Delta q^{(l)}$, these oscillations are most likely due to the strain interaction. Electromagnetic interactions lead to detectable effects only for $|\Lambda_{zz}(\theta_N)/\Lambda_{zz}| \leqslant 10^{-3}$; however, in this case $|\beta_{m=0}^{(l)}/\beta_b^{(l)}|$ should not be greater than or of order 10^{-6}, and geometric oscillations in the absorption or velocity of ultrasound will be observable only for anomalously large values of $|\omega\beta_{m=0}^{(l)}/\omega\sigma_b^{(l)}|$.

Note that all the previously published theoretical work on geometric oscillations in a longitudinal field[7,8] involved the longitudinal ultrasound itself and was based on analysis of the components of the conductivity tensor. This body of work does makes a number of features of the phenomenon understandable; nevertheless, as we showed above, its contribution to the understanding of the electroacoustic coefficients turns out to be minimal. In the majority of cases of practical interest, analysis of the oscillating corrections to the wave vector requires the use of the expression

$$\frac{\Delta q_{\rm osc}^{(l)}}{q_0^{(l)}} = \frac{-i}{2C^{(l)}} \omega\alpha_{m=0}^{(l)}. \tag{16}$$

We can analyze in more detail how the parameters $\omega\tau$ and q_0/H affect the amplitude of the oscillations in the experimentally observed quantities when the geometric oscillations in the absorption and velocity of ultrasound are determined by the electroacoustic coefficient $\alpha^{(l)}$ alone. For this we consider a model of the Fermi surface for which the oscillatory contributions to $\alpha^{(l)}$ are due to n equivalent sheets, each of which has a plane of mirror symmetry parallel to **H**. The intersection of each sheet with the plane $k_z = \text{const}$ is simply connected, and the cyclotron orbits contain the minimum number of turning points ($R=2$). Then the expression for the oscillatory contribution to $\Delta q^{(l)}$ has the form

$$\frac{\Delta q_{\rm osc}^{(l)}}{q_0^{(l)}} = \frac{-i}{C_{33}} \frac{|e|}{c\pi h^2} \int dk_z \sum_{\varkappa} \left(\frac{q_0^{(l)}}{H}\right)^{-1} \frac{\Lambda_{zz}^2(\theta_1)}{|\partial v_z/\partial\theta|_1} \times (1+\sin q_0^{(l)}\Delta Z_{12})\omega\tau \left\{ 1 - i\omega\tau\left(\frac{\bar{v}_z}{s_0^{(l)}} - 1\right)\right\}^{-1}. \tag{17}$$

Here we have used the relations $\Lambda_{zz}(\theta_1) = \Lambda_{zz}(\theta_2)$ and $v_z(\theta_1) = v_z(\theta_2)$, which follow from the conditions of mirror symmetry. As we said above, the parameter $\omega\tau$ determines the width of the belt of effective electrons. If we assume that $\omega\tau$ is so large that the quantities m_c, $\Lambda_{zz}(\theta_1)$, and $|\partial v_z/\partial\theta|_1$ can be treated as constants within this belt, while $h/2\pi \cdot dk_z = M \cdot d\bar{v}_z$, where M is a constant with the dimensions of mass, then Eq. (16) can be written in the form:

$$\frac{\Delta q_{osc}^{(l)}}{q_0^{(l)}}=\frac{-i}{C_{33}}\frac{|e|n}{c\pi h^2}\frac{\Lambda_{zz}^2(\theta_1)}{|\partial v_z/\partial\theta|_1}\left(\frac{q_0^{(l)}}{H}\right)^{-1}\frac{2\pi M s_0^{(l)}}{h}$$

$$\times\int_{-\infty}^{\infty} d(\tilde{v}\omega\tau)\frac{\left(1+\sin\left[\left(\frac{q_0^{(l)}}{H}\right)\Delta G_{12}\right]\right)}{1-i(\tilde{v}\omega\tau)}, \qquad (18)$$

where we have introduced the renormalized velocity $\tilde{v}=\bar{v}_z/s_0^{(l)}-1$, while ΔG_{12} is determined from Eq. (13). If we write this quantity in the form of a sum $G_{12}^{\text{eff}}+F_{12}(\tilde{v})$, where G_{12}^{eff} is the value of ΔG_{12} evaluated on an effective orbit, then Eq. (18) takes the form

$$\frac{\Delta q_{osc}^{(l)}}{q_0^{(l)}}=-iB\left(\frac{H}{q_0^{(l)}}\right)\left\{\int_{-\infty}^{\infty}\frac{d(\tilde{v}\omega\tau)}{1-i(\tilde{v}\omega\tau)}+\sin\left(\frac{q_0^{(l)}}{H}G_{12}^{\text{eff}}\right)\right.$$

$$\times\int_{-\infty}^{\infty}\frac{d(\tilde{v}\omega\tau)}{1-i(\tilde{v}\omega\tau)}\cos\left(\frac{q_0^{(l)}}{H}F_{12}(\tilde{v})\right)$$

$$+\cos\left(\frac{q_0^{(l)}}{H}G_{12}^{\text{eff}}\right)\int_{-\infty}^{\infty}\frac{d(\tilde{v}\omega\tau)}{1-i(\tilde{v}\omega\tau)}$$

$$\left.\times\sin\left(\frac{q_0^{(l)}}{H}F_{12}(\tilde{v})\right)\right\}, \qquad (19)$$

where we have introduced the notation

$$B=\frac{2M}{C_{33}}\frac{|e|ns_0^{(l)}}{ch^3}\times\frac{\Lambda_{zz}^2(\theta_1)}{|\partial v_z/\partial\theta_1|}.$$

In analyzing Eq. (19), we can see that if the function $F_{12}(\tilde{v})$ is even or odd, the second term in the curly brackets is purely real; for even $F_{12}(\tilde{v})$ the third component is also purely real. In this case only oscillations in the ultrasonic absorption will arise. For odd $F_{12}(\tilde{v})$, however, the third term becomes pure imaginary, i.e., it describes geometric oscillations in the velocity of sound. It is apparent that geometric oscillations in the absorption and sound velocity will exist simultaneously when $F_{12}(\tilde{v})$ has no properties of evenness or oddness.

The amplitude of oscillations A_{osc}, in addition to other factors, must be a function of the temporal dispersion parameter $q_0^{(l)}/H$ and the quantity G_{12}^{eff}, which differ only by a factor from the spatial dispersion parameter $q_0^{(l)}\cdot\Delta Z^{\text{eff}}$. We can also verify this based on analysis of Eq. (18). For example, let us consider the power-law function

$$F_{12}=K(\tilde{v})^{\eta}, \qquad (20)$$

where K and η are constants. It can be shown that in this case

$$A_{osc}=\left(\frac{H}{q_0^{(l)}}\right)f\left(\frac{q_0^{(l)}}{H}(\omega\tau)^{-\eta}\right). \qquad (21)$$

Clearly the case of most interest (especially when $\omega\tau\gg 1$) is the linear function $F_{12}=K\cdot\tilde{v}$. In this case

$$\frac{\Delta q_{osc}^{(l)}}{q_0^{(l)}}=-i\pi B\frac{H}{q_0^{(l)}}\left\{1+\exp\left(-|K|\frac{q_0^{(l)}}{H}(\omega\tau)^{-1}\right)\right.$$

$$\times\left[\sin\left(\frac{q_0^{(l)}}{H}G_{12}^{\text{eff}}\right)+i\,\text{sign}(K)\right.$$

$$\left.\left.\times\cos\left(\frac{q_0^{(l)}}{H}G_{12}^{\text{eff}}\right)\right]\right\}. \qquad (22)$$

Here we wish to point out that the oscillations disappear as $\omega\tau\to 0$, and have their largest amplitude as $\omega\tau\to\infty$, in complete agreement with the qualitative discussion given above regarding the role of the parameter $\omega\tau$.

4.2. Transverse circularly polarized waves

Our description of the propagation of circularly polarized modes, in contrast to the case of longitudinal waves, is distinguished by the variety of forms which the geometric oscillations can exhibit. This is primarily connected with the substantially increased role played by the electromagnetic terms in Eq. (1), due, first of all, to the lack of any requirement of smallness of the corresponding components of electron velocity at the turning point (in this case $v^{(p)}\equiv v_\perp\cong v_F$); and, secondly, to the more complicated form of the electromagnetic term, which allows a substantial difference between $q^{(+)}$ and $q^{(-)}$ where the ultrasound interacts with electromagnetic waves:

$$\frac{\Delta q^{(\pm)}}{q_0^{(\pm)}}=\frac{i}{2C_{44}}\left\{-\omega\alpha^{(\pm)}+\frac{(\omega\beta^{(\pm)})^2}{\omega\sigma^{(\pm)}-i(cq_0^{(\pm)})^2/4\pi}\right\}. \qquad (23)$$

Taking into account inequality (9), we limit ourselves to the region of fairly weak fields, neglecting the terms $H^2/4\pi$ and $cq_c^{(p)}H/4\pi$ in Eq. (1).

Furthermore, we should take into account the phase shifts

$$\Delta\varphi_{ba}^{(\pm)}(\theta_k,\theta_l)=(-1)^{3+P/3-|P|}[\varphi_b^{(p)}(\theta_k)-\varphi_a^{(p)}(\theta_l)]$$

$$\equiv\pm\Delta\varphi_{ba}(\theta_k,\theta_l), \qquad (24)$$

entering into Eq. (12), which clearly depend on the polarization. Inclusion of these phase shifts causes the quantities $g_{0,j}^{(\pm)}$ and the corresponding contributions to $\gamma_{m=0}^{(\pm)}$ from any orbits that possess rotational symmetry to vanish [see Eqs. (10)-(11)]; this conclusion is a special case of the rule for discarding harmonics in the Doppler-shifted cyclotron resonance, which was first formulated by Kotkin.[14] When the values of $\gamma_{m=0}^{(\pm)}$ are nonzero, these phase shifts lead to geometric oscillations in the ellipticity ε and angle of rotation ϕ of the plane of polarization of the ultrasound, which are determined by the following expressions:

$$\varepsilon=\tanh\left(\frac{\Gamma^+-\Gamma^-}{2}L\right),$$

$$\phi=\frac{\omega(1/s^+-1/s^-)}{2}L, \qquad (25)$$

where L is the distance traversed by ultrasound in the sample.

When the oscillations are due primarily to the strain-induced interaction mechanism between ultrasound and electrons, or when the oscillatory contributions to the electroacoustic coefficients are comparatively small, the geometric oscillations in the absorption and velocity of sound are sinusoidal in form (as in the case of longitudinal waves). Under other conditions, anharmonic oscillations may be observed.

4.2.1. Harmonic geometric oscillations. In order to describe harmonic oscillations in the propagation parameters for transverse ultrasound ($\Gamma^{(\pm)}$, $s^{(\pm)}$, ε, and ϕ), let us consider a multiply-connected Fermi surface containing, besides the sheets with nth order rotational symmetry, $2n$ other sheets, each of which has a single symmetry element—a mirror-reflection plane parallel to the k_z axis (so that n sheets are centered in the region $k_z>0$, while the other n sheets are obtained from them by the operation of inversion). In this case the rotational symmetry of the Fermi surface as a whole is preserved by transforming one sheet into another by rotation through angles that are multiples of $2\pi/n$ (recall that $n\geqslant 3$). It is with these low-symmetry sheets that the geometric oscillations are associated. For simplicity we limit ourselves to the case where the cyclotron orbits for effective electrons have two turning points apiece, while the total contribution of charge carriers to $\gamma^{(\pm)}_{m\neq 0}$ in the region where the geometric oscillations exist is a comparatively slowly-varying function of the field H. As in the previous section, within the belt of effective electrons the quantities m_c, $(w)_N$, $(v)_N$, $\varphi_g^{(\pm)}(\theta_N)$, and $|\partial v_z/\partial\theta|_1$ can be treated as constants, and we will assume $h/2\pi\cdot dk_z=M\cdot d\bar{v}_z$, and $\Delta G_{12}=G_{12}^{\text{eff}}+K\bar{v}$. With these assumptions, the contribution from sheet I, centered in the half-plane $k_z>0$, takes the form

$$\omega(\gamma^{(\pm)}_{m=0})^I=E|\mathbf{a}_\perp(\theta_1)|\,|\mathbf{b}_\perp(\theta_1)|\frac{H}{q_0^{(\pm)}}\Bigg\{\cos\Delta\varphi_{ba}(\theta_1,\theta_1)+Di\,\text{sign}(K)\exp\Bigg[-i\,\text{sign}(K)\times\left(\frac{q_0^{(\pm)}}{H}G_{12}^{\text{eff}}\mp\Delta\varphi_{ba}(\theta_1,\theta_2)\right)\Bigg]\Bigg\},\qquad(26)$$

where

$$D=\exp\left(-|K|\frac{q_0^{(\pm)}}{H}(\omega\tau)^{-1}\right),$$

$$E=\frac{4\pi|e|s_0^{(\pm)}}{ch^3}\frac{M}{|\partial v_z/\partial\theta|_1}.\qquad(27)$$

In these expressions we have taken into account the properties of the vectors $\mathbf{a}$ and $\mathbf{b}$ that follow from the mirror symmetry.

Continuing, let us consider the contribution to $\gamma^{(\pm)}_{m=0}$ from sheet II, which is obtained by inversion of sheet I with respect to the center of the Brillouin zone. The effective orbit on the second sheet is a cyclotron orbit formed by inverting the corresponding orbit from sheet I, for which $\bar{v}_z=-s_0^{(\pm)}$. Therefore, the periods of oscillation associated with sheets I and II are, in general, not equal to one another. However, because the quantity $s_0^{(\pm)}$ is small compared to the Fermi velocities, we may expect that this difference in periods is small, and neglect it in analyzing oscillations with comparatively small labels. Within the model we are discussing here, the value of $(G_{12}^{\text{eff}})^{\text{II}}=G_{12}^{\text{eff}}-2K$ holds sheet II, where G_{12}^{eff} and K are the same quantities that appear in Eq. (25). Furthermore, according to the rule for assigning labels [see Sec. 3, the commentary to Eq. (11)], it is necessary to assign a label of 2 to the tuning point obtained as the result of inverting the point $N=1$, and conversely. In what follows we will include the fact that the constant K changes sign as we go from sheet I to sheet II. Taking into account the equations

$$\mathbf{v}(\mathbf{k})=-\mathbf{v}(-\mathbf{k}),$$

$$\mathbf{w}(\mathbf{k})=\mathbf{w}(-\mathbf{k}),$$

which follow from the inversion symmetry of the Fermi surface and the relations implied by reflection symmetry of the orbits, we can write the following properties of the phase corrections:

$$(\Delta\varphi_{vv}^{(\pm)})^{\text{I}}=-(\Delta\varphi_{vv}^{(\pm)})^{\text{II}},$$

$$(\Delta\varphi_{ww}^{(\pm)})^{\text{I}}=-(\Delta\varphi_{ww}^{(\pm)})^{\text{II}},$$

$$(\Delta\varphi_{vw}^{(\pm)})^{\text{I}}=\pi-(\Delta\varphi_{vw}^{(\pm)})^{\text{II}}.$$

After this we can obtain expressions for the electroacoustic coefficients, including contributions from all $2n$ sheets:

$$\omega(\sigma^{(\pm)}_{m=0})=|e\mathbf{v}_\perp(\theta_1)|^2\frac{nEH}{q_0^{(\pm)}}\left\{2+2D\exp(\mp i\Delta\varphi_{vv})\times\sin\left(\frac{q_0^{(\pm)}}{H}G_{12}^{\text{eff}}\right)\right\},\qquad(28)$$

$$\omega(\alpha^{(\pm)}_{m=0})=|\mathbf{w}_\perp(\theta_1)|^2\frac{nEH}{q_0^{(\pm)}}\left\{2+2D\exp(\pm i\Delta\varphi_{ww})\times\sin\left(\frac{q_0^{(\pm)}}{H}G_{12}^{\text{eff}}\right)\right\},\qquad(29)$$

$$\omega(\beta^{(\pm)}_{m=0})=|\mathbf{w}_\perp(\theta_1)|\,|e\mathbf{v}_\perp(\theta_1)|\frac{nEH}{q_0^{(\pm)}}2D\times i\exp\times(\pm i\Delta\varphi_{-vw})\cos\left(\frac{q_0^{(\pm)}}{H}G_{12}^{\text{eff}}\right),\qquad(30)$$

where the angular differences $\Delta\varphi_{ab}$ are defined for a sheet with positive K and are functions of the arguments θ_1 and θ_2. Thus, including inversion symmetry leads us to the following phase relations: oscillations of $\text{Re}\,\alpha^{(+)}$ are almost in phase with the oscillations in $\text{Re}\,\alpha^{(-)}$, while the imaginary parts of $\alpha^{(+)}$ and $\alpha^{(-)}$ oscillate 180° out of phase. The same assertions are valid for $\sigma^{(\pm)}$ as well. For $\beta^{(\pm)}$ the relations are inverted: the real parts of $\beta^{(+)}$ and

$\beta^{(-)}$ are 180° out of phase, while the imaginary parts are in phase. Including the small difference in effective values of G_{12}^{eff} for sheets I and II, we find that a phase shift appears in the phase relations we have listed above; this shift is still more noticeable as the oscillation label increases.

Equation (30) allows us to analyze the phase relations for geometric oscillations in the absorption and velocity of circular waves based on Eqs. (6) and (23). If the oscillating corrections to the electroacoustic coefficients are relatively small, it is not difficult to obtain an expansion analogous to Eq. (15), but with the replacement of $\sigma_b^{(l)}$ by $\sigma_b^{(l)} - i(cq_0^{(\pm)})^2/4\pi$.

Thus, e.g., if pure strain oscillations predominate, we should observe in-phase geometric oscillations in the absorption of circular ultrasonic waves, while the oscillatory corrections to the velocity of the ultrasound should be out of phase by 180°. In this situation oscillations appear in the angle of rotation of the plane of polarization, see Eq. (25). However, when the electromagnetic contributions to the geometric oscillations predominate, another situation may be realized in which 180° out-of-phase oscillations are observed in the absorption and in-phase oscillatory corrections to the velocity of the circular modes. This should result in the appearance of geometric oscillations in the ellipticity. Here we must keep in mind that oscillations of ε and ϕ may be associated not only with dephasing of the oscillations of Γ^+ and Γ^-, or s^+ and s^-, but also with differences in their amplitudes, which should be observed where ultrasound interacts with circularly-polarized electro-magnetic waves.[15–17]

We note that in-phase geometric oscillations of the absorption have been observed in Sb (Ref. 11), while oscillations of ε caused by the 180° out-of-phase oscillations of Γ^+ and Γ^- were observed in In (Ref. 1).

4.2.2. Anharmonic oscillations. When the oscillatory corrections are accompanied by contributions from Doppler-shifted cyclotron resonance ($\gamma_{m\neq 0}^{(\pm)}$) to the electroacoustic coefficients, we should observe geometric oscillations more complicated than those described above.

Adopting a common terminology, we may group anharmonic geometric oscillations into a minimum of two types, both of them due to an electromagnetic interaction mechanism. The first type is associated with the fact that the coefficient $\beta^{(\pm)}$ enters into Eq. (23) to second order, and leads to the appearance of geometric oscillations with doubled frequency. The second type of oscillation is due to the form of the denominator of the field-dependent term, which has a resonant character. The conditions for this resonance are

$$(cq_0^{(\pm)})^2/4\pi - \text{Im}(\omega\sigma^{(\pm)}) = 0,$$

$$|\text{Re}(\sigma^{(\pm)})/\text{Im}(\sigma^{(\pm)})| < 1. \tag{31}$$

The oscillatory contribution to the Hall conductivity Im $\sigma^{(\pm)}$ makes the system periodically approach the resonance conditions and then depart from them. During the approach to resonance the quantity exhibits sharp anomalies. Therefore, in this case peculiar geometric oscillations in the resonance polarization should appear, which are distinguished, first of all, by their large amplitude (greatly exceeding the amplitude of geometric oscillations in the nonresonance polarization), and, secondly, by their unusual nonsinusoidal shape. However, when it happens that the amplitude of oscillation $\sigma_{m=0}^{(p)}$ is already large enough to rigorously satisfy the resonance condition (31) (this takes place twice per period), the giant oscillations in $\Delta q_0^{(p)}/q_0^{(p)}$ acquire a characteristic two-horned shape, i.e., a dip appears at the location of the first maximum. Since the amplitude of oscillations of this kind can be comparable to the characteristic level of the electronic contributions to the wave vector $\mathbf{q}^{(p)}$, and can even exceed it, we may apply the term "giant" to these geometric oscillations.

Note that giant geometric oscillations can appear only in a non-Pippard geometry and only for transverse waves.

Since these giant geometric oscillations should appear for only one (resonant) polarization, under these conditions we should expect strong polarization effects.

5. CONCLUSIONS

Thus, the following basic results follow from our quasiclassical treatment of the problem of propagation of ultrasonic waves in a longitudinal magnetic field along an axis of symmetry of at least third order in a metal with an arbitrary dispersion law for electrons.

1. Oscillatory components appear in the electroacoustic coefficients that are determined by electronic properties, i.e., the values of the velocity $\mathbf{v}$ and the vector $\mathbf{w}$ made up of components of the strain potential tensor, evaluated in the vicinity of turning points on effective cyclotron paths. For $\omega\tau \gg 1$ the effective paths are those of charge carriers whose average velocities equal the velocity of sound, while for $\omega\tau \ll 1$ these paths are those whose size ΔZ_{kl} along the magnetic field between two turning points is extremal.

2. We find that the specific features of geometric oscillations in a longitudinal field compared to oscillations with $\mathbf{q}^{(p)} \perp \mathbf{H}$ are most clearly manifest for $\omega\tau \gg 1$. In this case the width of the belt of effective electrons is determined by the temporal dispersion parameter $\omega\tau$, while the period of oscillations is determined by the spatial dispersion $q_0^{(\pm)}\Delta Z_{kl}$.

3. We have shown that oscillations in the absorption and velocity of longitudinal ultrasound are due primarily to the strain mechanism for electron-phonon interaction, and should be sinusoidal in form.

4. For geometric oscillations involving transverse circularly-polarized waves, the dominant interaction mechanism can be either strain- or field-induced. For the latter type we should expect the shape of the oscillations to deviate significantly from harmonic, and we listed two reasons for this distortion in shape. The first is the dominant influence of the strain-induced conductivity in generating the oscillations, while the second is oscillations in the conductivity in the region of interaction of the ultrasound with electromagnetic waves. In this case the distortions in the shape of the oscillations should be observed for only one polarization. We also predict the possible existence of giant geometric oscillations.

5. We have shown that oscillations of the electroacoustic coefficients that describe the propagation of transverse waves have phase corrections that depend on polarization. In the expressions for the wave vectors, these coefficients are combined in such a way that not only a phase difference but also a difference in amplitudes can appear in the oscillations of the absorption and (or) velocity of waves of different circular polarizations. This circumstance leads to the appearance of oscillatory components in the ellipticity and angle of rotation of the plane of polarization of the ultrasound.

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Translated by Frank J. Crowne