Magnetoplasmons in a system of antidisks

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An antidisk model is proposed. It allows an analytic solution for the spectrum of local plasma oscillations in its vicinity. The spectrum of magnetoplasma oscillations obtained in this way displays a number of characteristic features that have been seen experimentally. A qualitative discussion is given of the change in the magnetoplasmon spectrum between an individual antidisk and a periodic lattice of antidisks.

INTRODUCTION

The different behavior of periodic systems of disks and antidisks in a uniform high-frequency electric field parallel to the plane of the disks (antidisks) is an interesting experimental result which has recently been established. In the former case, the system is excited at frequencies whose dependence on the magnetic field perpendicular to the plane of the system is shown schematically in Fig. 1 (Refs. 1–4). In the latter case, the system is excited at the frequencies shown in Fig. 2 (Ref. 5). This must be regarded as a surprising result since it has become clear (see Sec. 1 below) that the spectra of localized plasmons obtained for individual disks and antidisks with smooth profiles of equilibrium electron density have the same structure and consist of modes such as those shown in Fig. 1a and in Fig. 2. Nevertheless, disks and antidisks have different impedances. Our aim in this paper is to calculate the spectrum of antiplasmons in an individual disk and then use it as a basis for a qualitative discussion of the possible reasons for the impedance difference.

Section 1 presents a solution of the problem of the magnetoplasmon spectrum for an individual antidisk of radius R with an equilibrium electron-density profile $n(r)$ of the form

$$n(r)=\mathrm{const}(r^2/R^2-1)^{1/2},\quad r\geqslant R. \tag{1}$$

Comparison of the resulting spectrum ω_{jm} with the corresponding magnetoplasmon spectrum[6] in an individual disk with the profile

$$n(r)=n_0(1-r^2/R^2)^{1/2},\quad r\leqslant R, \tag{2}$$

shows that these spectra are qualitatively the same.

In Sec. 2 we discuss possible changes in the calculated magnetoplasmon frequencies ω_{jm} when a periodic system of antidisks is considered. Some of the details of this generalization may be significant for the interpretation of experimental data.[1–5]

1. MAGNETOPLASMON SPECTRUM FOR AN INDIVIDUAL ANTIDISK WITH THE *n*(*r*) PROFILE GIVEN BY EQ. (1)

To be specific, suppose that an antidisk arises in the model used in Ref. 7. The electron system in an antidisk is then in the magnetic field produced, on the one hand, by positively-charged donors distributed uniformly in the heterostructure in the 2D layer parallel to the plane of the antidisk and, on the other hand, the external distribution of electrons concentrated in the plane of the antidisk in its exterior within a circle of radius $d<R$. It follows from the results presented in Ref. 7 that, near the boundary, the electron density in the antidisk is proportional to $(r^2/R^2-1)^{1/2}$, which enables us to take the constant in (1) in the form

$$n(\sqrt{2}R)=\frac{2}{\pi}\frac{N_+(2\Sigma-N_+)}{\Sigma-N_+}, \tag{3}$$

where Σ is the density of the external electron system and N_+ is the density of the donors ($\Sigma>N_+$). For $r\gg R$, the correct solution for $n(r)$ must tend to the constant N_+, but this does not happen in the model (1). Nevertheless, this model does not prevent us from obtaining finite magnetoplasmon frequencies ω_{jm} localized near the boundary of the individual antidisk.

Naturally, we assume $R\ll a$ in this Section, where a is the separation between neighboring antidisks in a square 2D lattice consisting of them. Moreover, we shall consider the classical case

$$R\gg a_b,\quad a_b=\kappa\hbar^2/m_e e^2, \tag{4}$$

in which case we neglect quantum corrections to the spectrum of classical plasmons (m_e is the effective electron mass and κ is the dielectric constant).

The problem of the magnetoplasmon spectrum can now be formulated as follows. The electrostatic potential $\varphi(r,z)$ in

$$\varphi(x,y,z,t)=\varphi(r,z)e^{i\omega t}, \tag{5}$$

is an even function of z that is harmonic everywhere except for the part of the $z=0$ plane ($r\geqslant R$) occupied by the antidisk and satisfying the boundary conditions

$$\left.\frac{-\partial\varphi}{\partial z}\right|_{z=+0}=\begin{cases}2\pi e\delta n/\kappa, & r\geqslant R \quad \text{(6a)}\\ 0, & \mathbf{r}<R, \quad \text{(6b)}\end{cases}$$

where $\delta n(r,\theta,t)=\delta n^0(r,\theta)\exp(-i\omega t)$ is the additional oscillatory term in the electron density that arises during magnetoplasmon excitation,

$$e\delta n=(i\omega)^{-1}\,\mathrm{div}\,\mathbf{j};\quad j_i=\sigma_{ik}(r,\omega)E_k,$$

j_i is the two-dimensional current in the antidisk,

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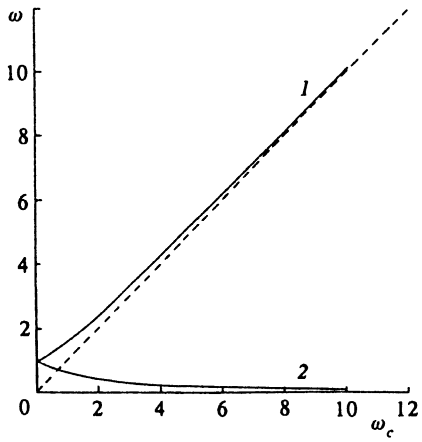

FIG. 1. Magnetoplasmon spectrum of an antidisk in dimensionless units for $j=|m|-1$; $m=1$, $j=0$(*1*—$|\omega^-|$,*2*—ω^+).

$$E_k=-(\partial_k\varphi)_{z=0,r>R},\quad \sigma_{ik}(r,\omega)=\sigma_{ik}(\omega)(r^2/R^2-1)^{1/2}$$

is the two-dimensional conductivity tensor, which in this approximation is independent of δn and is proportional to $n(r)$, $\sigma_{ik}(\omega)$ are constants that are effectively the components of the conductivity tensor at $r=\sqrt{2}R$ when the equilibrium electron density is equal to the constant $n(\sqrt{2}R)$ in (1), and the factor $(r^2/R^2-1)^{1/2}$ is due to the distribution given by (1).

The harmonic potential φ is conveniently written in terms of the coordinates (σ,τ,θ) of an oblate ellipsoid of revolution, so that the variables of the above problems can be separated:

$$r=R[(1+\sigma^2)(1-\tau^2)]^{1/2},$$
$$z=R\sigma\tau;\quad 0\leqslant\sigma<\infty,\quad 0\leqslant\tau\leqslant 1. \tag{7}$$

A single-valued solution of the Laplace equation written in terms of these coordinates and satisfying the condition $\varphi\to 0$ for $r\to\infty$ can take the form of the two combinations

$$\varphi^{\pm}=P_\nu^{\pm|m|}(\tau)Q_\nu^{\pm|m|}(i\sigma)e^{im\theta-i\omega t}, \tag{8}$$

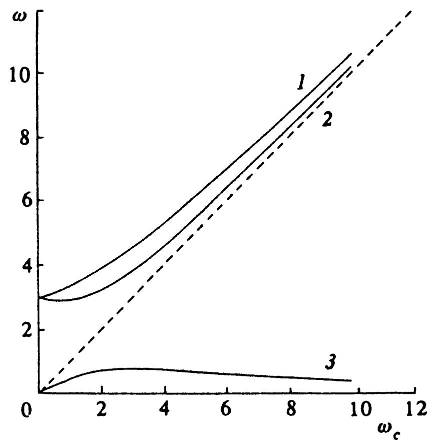

FIG. 2. Magnetoplasmon spectrum of an antidisk in dimensionless units for $j<|m|-1$; $m=3$, $j=0$(*1*—$|\omega_-|$,*2*—ω_+,*3*—ω_L)

where $P_\nu^\mu(\tau)$ and $Q_\nu^\mu(i\sigma)$ are the associated Legendre functions of the first and second kind, respectively, m is an integer, and $\nu>-0.5$.

The boundary condition given by (6b), which corresponds to the absence of surface charges at $z=0$, $r<R$ [in terms of the above ellipsoidal coordinates, this region lies on the line $\sigma=0$ and (6b) reduces to the requirement that $\partial Q_\nu^{\pm|m|}(i\sigma)/\partial\sigma|_{\sigma=0}=0$] is satisfied if ν is a nonnegative integer $\nu=j\geqslant 0$ and j is an integer such that $j+m$ is an odd number. However, in that case, on the $z=0$ plane in the interior of the antidisk, i.e., when $\tau=0$, we have $\varphi^+\equiv 0$, since $P_j^{+|m|}(0)=0$, so that $\omega\equiv 0$ as well. It follows that the solution φ^+ in (8) must be rejected.

On the other hand, φ^- is not identically zero on the $z=0$ plane in the interior of the disk if $j<|m|$. We must recall here that the set of indices of Legendre functions in which we are interested here makes it unnecessary to have the factor $\Gamma(\nu+\mu+1)$ in the definition of $\widetilde{Q}_\nu^\mu(\xi)$ [see, for example, Ref. 8; $\Gamma(x)$ is the gamma-function]; it was introduced for the sake of convenience in writing the different relationships between the Legendre functions. Henceforth, we shall take $\widetilde{Q}_\nu^\mu(\xi)$ to be the usual associated Legendre function of the second kind without the factor $\Gamma(\nu+\mu+1)$ in its definition. The function $\varphi(\sigma,\tau,\theta,t)$ has the following form in our case:

$$\varphi=P_j^{-|m|}(\tau)\widetilde{Q}_j^{-|m|}(i\sigma)e^{im\theta-i\omega t}; \tag{9}$$

where j,m are integers, $0\leqslant j<|m|$, and $j+m$ is an odd number.

It will be useful to have the following list of the first few functions in (6):

$$P_0^{-1}(\tau)=[(1-\tau)/(1+\tau)]^{1/2},$$

$$P_2^{-3}(\tau)=[(1-\tau)/(1+\tau)]^{3/2}\times[1-3(1-\tau)/4+3(1-\tau^2)/20]/6,$$

$$P_0^{-3}(\tau)=[(1-\tau)/(1+\tau)]^{3/2}/6;$$

$$\widetilde{Q}_0^{-1}(i\sigma)=i(1+\sigma^2)^{-1/2},$$

$$\widetilde{Q}_2^{-3}(i\sigma)=-i(1+\sigma^2)^{-3/2},$$

$$\widetilde{Q}_0^{-3}(i\sigma)=i[(1+\sigma^2)^{-1/2}-4(1+\sigma^2)^{-3/2}/3].$$

If we substitute (9) into the boundary condition (6a), we obtain an equation for the number of magnetoplasmons:

$$i\sigma_{xx}[m^2-j(j+1)]=-\omega\kappa RL_{jm}/2\pi-m\sigma_{xy}, \tag{10}$$

$$L_{jm}=-\left(\frac{1}{P_j^{-|m|}(\tau)}\frac{dP_j^{-|m|}(\tau)}{d\tau}\right)\Bigg|_{\tau=0}=2\frac{\Gamma((|m|+j)/2+1)\Gamma((|m|-j+1)/2)}{\Gamma((|m|+j+1)/2)\Gamma((|m|-j)/2)}.$$

If we now use the conductivity tensor[1)] obtained in the single-electron collisionless approximation

$$\sigma_{xx}=\frac{i\omega n(2^{1/2}R)e^2}{m_e(\omega^2-\omega_c^2)},\quad \sigma_{xy}=\frac{\omega_c n(2^{1/2}R)e^2}{m_e(\omega^2-\omega_c^2)},$$
$$\omega_c=\frac{eH}{m_e c} \tag{11}$$

and if we normalize all the frequencies to $\Omega_j^m=[2\pi n(2^{1/2}R)e^2/\kappa m_e RL_{jm}]^{1/2}$, we find from (10) that

$$\omega^2-[\omega_c^2+m^2-j(j+1)]=-m\omega_c/\omega. \tag{12}$$

The behavior of the roots of this equation as functions of ω_c is different for $j=|m|-1$ and $j<|m|-1$. When $j=|m|-1$ holds, Eq. (12) has the root $\omega^{(0)}_{|m|-1,m}=\omega_c\,\text{sign}(m)$ and also the pair of roots

$$\omega^{\pm}_{|m|-1,m}=[-\omega_c/2\pm(\omega_c^2/4+|m|)^{1/2}]\text{sign}(m). \tag{13}$$

The first of these roots, i.e., $\omega^{(0)}_{|m|-1,m}=\omega_c\,\text{sign}(m)$, has arisen as a result of the multiplication of both sides of (10) by $\omega^2-\omega_c^2$ between (10) and (12), and must be rejected. The behavior of $\omega^{\pm}_{|m|-1,m}$ as a function of ω_c is shown in Fig. 1 for $m=1$ (the solution ω^- is replaced in the figure by its numerical value).

Next, it is readily verified that, for any ω_c and $j<|m|-1$, Eq. (12) has three real roots with the following asymptotic behavior:

$$\begin{aligned}
\omega_c\to 0:\quad &\omega_+=A^{1/2}-\frac{m\omega_c}{2A}, &\omega_c\to\infty:\quad &\omega_+=\omega_c+\frac{(A-m)}{2\omega_c},\\
&\omega_-=-A^{1/2}-\frac{m\omega_c}{2A}, & &\omega_-=-\omega_c-\frac{(A+m)}{2\omega_c},\\
&\omega_L=m\omega_c/A, & &\omega_L=m/\omega_c,
\end{aligned} \tag{14}$$

where $A=m^2-j(j+1)$.

It is also useful to determine the position of the extrema of $\omega_+(H)$ for $m>0$, $\omega_-(H)$ for $m<0$, and $\omega_L(H)$. This is done by solving (12) for ω_c and then finding the position of the extrema from the condition $\partial\omega_c/\partial\omega\to\infty$. Hence

$$(\omega_+^{\min}|_{m>0})^2=(\omega_-^{\max}|_{m<0})^2=(A/2)(1+\sqrt{1-m^2/A^2}), \tag{15a}$$

$$(\omega_L^{\max}|_{m>0})^2=(\omega_L^{\min}|_{m<0})^2=(A/2)(1-\sqrt{1-m^2/A^2}). \tag{15b}$$

The dependence of the oscillation frequencies ω on ω_c for this case is shown in Fig. 2 for the lowest numerical value of m (equal to 3) for which this case is possible (we have taken $m=3$ and replaced ω_- with its numerical value).

The spectrum of MP oscillations was derived for the antidisk using the equilibrium electron-density distribution (1) which increases without limit as $r\to\infty$. Nevertheless, the oscillatory additional term δn in the electron density, which by (6a) and (9) is given by

$$\delta n\propto\frac{\tilde{Q}_j^{-|m|}(i[r^2/R^2-1]^{1/2})}{(r^2/R^2-1)^{1/2}}, \tag{16}$$

has the asymptotic form $(r/R)^{-j-2}$ for large distances $r\gg R$. Because of this, the behavior of the equilibrium concentration at these distances should not significantly affect the magnetoplasmon frequencies (however, see Sec. 2). We also note that δn diverges as $r\to R$. This is due to the use of the above approximate formulation of the eigenvalue problem for the magnetoplasmon frequencies for which the boundary of the electron gas in the antidisk is fixed. In fact, this boundary moves in the case of real MP oscillations. The function $R(\theta,t)$ can be taken into account, but only within a nonlinear theory.

The modes $|\omega^-_{|m|-1,m}|$ in (13) and $|\omega_-|$ for $m>0$ or ω_+ for $m<0$ in (14) begin with finite values for $H=0$ and tend monotonically to ω_c as H increases. This is in complete analogy with the behavior of the spectrum of the MP oscillations of a uniformly charged plane. However, the modes $|\omega_-|$ for $m<0$ and ω_+ for $m>0$ have a negative H dipersion for low magnetic fields, as observed in Ref. 5. A specific dependence on the magnetic field is also found for the modes $\omega^+_{|m|-1,m}$ and ω_L, which behave as H^{-1} when $H\to\infty$; this corresponds to the behavior of magnetoplasmons localized near the antidisk boundary. Moreover, as H decreases, ω_L approaches its maximum and linearly tends to zero as $H\to 0$. The function $\omega_L(H)$ was found in Ref. 5 to have a similar form.

Similar behavior of magnetoplasmon frequencies as functions of the magnetic field has also been found in the case of a disk.[6] The significant formal difference between the two cases is that, for a disk with a fixed azimuthal number m, there is an infinite number of modes with radial indices $j\geqslant|m|$, $j+m$ even, whereas for an antidisk the number of modes is limited by the conditions $j<|m|$, $j+m$ odd. In particular, modes with $m=0$, i.e., pure radial oscillations, are absent in the case of the antidisk.

2. A PRIODIC SYSTEM OF ANTIDISKS

The above antidisk model has advantages such as an equilibrium profile $n(r)$ (1) that is close to the real profile[7] for $r\gtrsim R$ and the possibility of a complete solution of the problem of MP oscillations, but it also has the qualitative disadvantage that its local plasma oscillations do not exhibit wave attenuation. When the problem of collective excitations is correctly formulated, this type of attenuation of local modes, whose eigenvalues lie in the continuous spectrum of unlocalized excitations (in our case, unlocalized excitations of a uniformly charged plane), is always present and is usually relatively large (see, for example, Ref. 10). As far as oscillations in a periodic system of antidisks are concerned, here we may expect the superposition of two effects. First, the periodic modulation of the initially uniformly charged plane should cause the initially continuous plasma spectrum ω_q to break up and Brillouin forbidden bands (gaps) to appear. Such effects were predicted in Ref. 11 and were observed in Ref. 9. Second, the emission of local modes acquires new features. Coherence effects suppress some of the local eigenmodes whose frequencies fall into portions of the continuous spectrum. If, however, a local mode is found within a Brillouin gap, its existence becomes allowed and, on the contrary, wave attenuation is suppressed. It is therefore clear that a periodic system of antidisks has certain selective properties with respect to local modes. Some of these modes are renormalized and amplified, and some are suppressed.

We shall now illustrate these ideas by considering a square lattice of identical linear oscillators of mass M and eigenfrequency ω_0, placed on a conducting plane at $z=0$. We shall consider the case where the amplitude of these oscillators is much less than the lattice period a.

The set of equations describing the interaction between the oscillator lattice and the electron system takes the form (assume for simplicity that $H=0$):

$$m_e\dot{v}(x,t)=eE(x,t)+\frac{1}{2}\sum_{n_1n_2}F_{n_1n_2}(t)\delta(x-n_1a_1-n_2a_2), \quad (17)$$

$$\delta\dot{n}(x,t)+n_s \operatorname{div} v(x,t)=0, \quad (18)$$

$$E(x,t)=-\operatorname{grad}\varphi(x,z,t), \quad \Delta\varphi|_{z\neq 0}=0, \quad (19)$$

$$-\frac{\partial\varphi}{\partial z}\Big|_{z=+0}+\frac{\partial\varphi}{\partial z}\Big|_{z=-0}=\frac{4\pi e\delta n}{\kappa}, \quad (20)$$

$$M\ddot{r}_{n_1n_2}(t)+kr_{n_1n_2}(t)=-F_{n_1n_2}(t), \quad (21)$$

where x is the 2D electron coordinate, $v(x,t)=\dot{x}(t)$, $r_{n_1n_2}$ is the radius vector of the nth oscillator with origin at the corresponding n-th site, a_1 and a_2 are the basis vectors of the square lattice, and n_s is the electron concentration.

The motions of the oscillators and the electron system are coupled by the requirement that

$$v(x,t)|_{x=n_1a_1+n_2a_2}=\dot{r}_{n_1n_2}(t). \quad (22)$$

If we suppose that all the time-dependent variables have their time dependence described by the factor exp $(-i\omega t)$, and if we transform to the Fourier components, we find from (17)–(21) that

$$F_{n_1n_2}(t)=F(t)=\text{const}\, e^{-i\omega t}, \quad (23)$$

$$r_{n_1n_2}(t)=-\frac{F(t)/k}{\omega^2/\omega_0^2-1}, \quad \omega_0^2=k/M, \quad (24)$$

$$v_G(t)=i\frac{\omega F(t)}{(\omega^2-\omega_G^2)m_e}, \quad \omega_G^2=\frac{2\pi e^2n_s}{\kappa m_e}G, \quad (25)$$

where G are the reciprocal-lattice eigenvectors of the oscillator system ($G=|\mathbf{G}|$).

If we now use the condition given by (22), we obtain the dispersion relation

$$\sum_G \frac{1}{\omega^2-\omega_G^2}=\frac{(m_e/M)}{\omega_0^2-\omega^2}. \quad (26)$$

It is clear that the solution for ω given by (26) can be real, but does not exist for all ω_0. The necessary condition for a real solution is

$$\omega_0\simeq\omega_G. \quad (27)$$

CONCLUSION

We have calculated the spectrum of antiplasmons in an individual antidisk. The qualitative difference between the impedance of a system of disks and a system of antidisks can only be explained if we assume that the mode ω_{01} in the system of antidisks is suppressed by the mechanism described in Sec. 2. This suppression does not occur in a system of disks.

As far as the $m=3$ mode is concerned, it is not the uniform external electric field that excites this mode in a periodic system of antidisks with quadratic symmetry, but the secondary fields produced in the square lattice of antidisks.

It is also useful to examine some of the qualitative details of the spectrum reported in Ref. 5. Our results provide a qualitative explanation of the behavior of the magnetoplasmon frequencies as functions of the magnetic field. However, the gap in the calculated spectrum is greater than the observed gap. For a quantitative comparison with experimental data, it is convenient to use the ratio of characteristic frequencies of magnetoplasmon modes, which is independent of the parameters that appear in the definition of the normalizing quantity Ω_j^m. These characteristic frequencies—given in Ref. 5—are the frequency $\omega_+(H=0)$ of mode 1 (Fig. 2) and the maximum frequency $\omega_L^{\max}$ of mode 3. Experimental data for a lattice with a period of 300 nm and antidisk diameter of 100 nm give

$$\frac{\omega_+(H=0)}{\omega_L^{\max}}=2.73 \text{ (exper.)}.$$

The calculated result is

$$\frac{\omega_+(H=0)}{\omega_L^{\max}}=5.92 \text{ (calc.)}.$$

A different equilibrium electron-density profile in the antidisk will have to be used to obtain better agreement between calculated and experimental results.

[1)]In principle, it is best to use for σ_{ik} the experimental values of this tensor near the frequencies in which we are interested, i.e., in the infrared. However, we know of no systematic data on $\sigma_{ik}(\omega)$ in this frequency range. We only know of reports[2,3,5] that the characteristic properties of the static Hall effect are strongly suppressed in the infrared. We shall therefore confine our attention in specific calculations to the determination of σ_{ik} reported in Ref. 9.

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