# Theory of resonant diffraction in periodic impedance structures

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A modification of perturbation theory is proposed for problems involving resonant diffraction in impedance gratings. The solution is constructed in the form of a ratio of series in powers of the grating amplitudes, and other small parameters are treated exactly. The structure of the general solution in the presence of multiple resonances is analyzed. It is shown that the presence of gratings that rescatter resonant fields back and forth can give rise to high-order resonant fields. The amplitudes of the latter are determined by the lowest-order resonance. The conditions are studied under which the lowest-order resonances can be treated independently. The energy conservation law is analyzed. Conditions are found under which specular reflection is totally suppressed.

The scattering of radiation by an optically irregular surface of a medium with high conductivity exhibits peaks when the period of the grating is on the order of the wavelength of the incident radiation.[1–4] These peaks (which are of the same nature as the well known Wood anomalies) are due to a spatial resonance between the wave diffracted at a grazing angle and a surface electromagnetic wave. In consequence, the specular reflection of the medium may be suppressed and the absorptivity of the surface enhanced. Interest in these questions is related to the production of surface periodic structures by laser radiation.[5–7] Until recently, attention has been devoted mainly to the study of resonant diffraction by gratings having a surface profile, although anomalous phenomena may be associated also with resonant diffraction by gratings with variable permittivity (impedance gratings). This problem has been studied, e.g., by Kapave *et al.*[8,9] in connection with the production of surface structures using radiation. Kats *et al.*[10] treated the general problem of refraction by profile and impedance gratings. The procedure proposed in Ref. 10 allows solutions to be obtained for the fields in the form of a ratio of power series in small parameters: the impedance, the grating amplitudes, and the small normal components of the wave vectors of the resonant spectra.

The present work carries out a detailed treatment of the problem of diffraction by impedance gratings. The general solution method[10] has been improved, as a result of which the series of the modified perturbation theory are expansions only in the grating amplitudes, with the other small parameters being taken into account exactly. This makes it easier to ascertain the accuracy of the approximation, and also yields the exact solution in the limiting case of an unmodulated surface. We treat the problem of the suppression of specular reflection as an application.

## 1. FORMULATION OF THE PROBLEM AND GENERAL SOLUTION

We assume that a highly conducting medium with a modulated impedance,[1)]

$$\xi=\xi_0+\sum_g \xi_g \exp(i\mathbf{gr}), \tag{1.1}$$

$|\xi|\ll 1$, $\xi'>0$, $\xi''<0$ is subjected to an incident plane electromagnetic wave

$$\mathscr{E}_i=\mathbf{E}\exp[i(\mathbf{k}_t\mathbf{r}+k_z z-\omega t)]. \tag{1.2}$$

Here and in what follows the subscript $t$ indicates the tangential component. We choose a system of coordinates in which the $z$ axis is in the inward direction normal to the surface and the $x$ axis lies in the plane of incidence. The electromagnetic field $\mathscr{E}$, $\mathscr{H}$ on the surface of the highly conductive medium satisfies the Leontovich boundary condition

$$\mathscr{E}_t=\xi[\mathscr{H}\mathbf{e}_z], \tag{1.3}$$

where $\mathbf{e}_z$ is the unit vector in the $z$ direction (the inward normal). We seek an electric field $\mathscr{E}$ in the half-space $z<0$ in the form of a superposition of the incident wave and the diffraction spectrum:

$$\mathscr{E}\equiv\mathscr{E}_i+\sum_q \mathscr{E}_q,\quad \mathscr{E}_q\equiv\mathbf{E}_q\exp[ik(\boldsymbol{\alpha}_q\mathbf{r}-\beta_q z)-i\omega t], \tag{1.4}$$

where

$$\{\mathbf{q}\}=\left\{\sum p_i\mathbf{g}_i,\, p_i\in\mathbb{Z}\right\},\quad \boldsymbol{\alpha}_q=(\mathbf{k}_t+\mathbf{q})/k,$$

$$\beta_q=(1-\alpha_q^2)^{1/2},$$

with $\mathrm{Re}\,\beta_q$, $\mathrm{Im}\,\beta_q\geqslant 0$, and $\mathbf{q}=0$ corresponds to the specularly reflected wave. The magnetic field can be expressed in terms of the electric field by means of the relation $\mathscr{H}=-ik^{-1}\,\mathrm{rot}\,\mathscr{E}$. We substitute expressions (1.1) and (1.4) in (1.3) and identify terms with the same spatial structure $\propto\exp(ik\boldsymbol{\alpha}_q\mathbf{r})$. As a result we find a system of linear equations for the amplitudes of the diffracted waves:

$$\sum_{q'}[T_{qq'}\mathbf{E}_{q't}+\mathbf{V}_{qq'}E_{q'z}]=-T_q\mathbf{E}_t-\mathbf{V}_qE_z, \tag{1.5}$$

where

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$$T_{qq'}=t_q\delta(\mathbf{q}-\mathbf{q}')+\tilde{t}_{qq'},\quad T_q=\delta(\mathbf{q})-\beta_0\xi_q,$$

$$t_q=1+\xi_0\beta_q,\quad \tilde{t}_{qq'}=\beta_{q'}\xi_{q-q'}[1-\delta(\mathbf{q}-\mathbf{q}')],$$

$$\mathbf{V}_{qq'}=\alpha_{q'}\xi_{q-q'},\quad \mathbf{V}_q=\alpha_0\xi_q,\quad \delta(\mathbf{q})=\begin{cases}1, & \mathbf{q}=0,\\ 0, & \mathbf{q}\neq 0.\end{cases}\tag{1.6}$$

In addition to Eqs. (1.5) the amplitudes $\mathbf{E}_q$ must also satisfy the transverse wave condition div $\mathscr{E}=0$, from which we have

$$\alpha_q\mathbf{E}_{qt}-\beta_qE_{qz}=0.\tag{1.7}$$

Since the matrix $T_{qq'}$ contains the nonsingular diagonal part $t_q\delta(\mathbf{q}-\mathbf{q}')$, we can eliminate the tangential field components $\mathbf{E}_{qt}$ from Eqs. (1.5) and (1.7), using a regular expansion in the grating amplitudes $\xi_g$. Note that in contrast to Ref. 10, the main part $t_q\delta(\mathbf{q}-\mathbf{q}')$ of the matrix $T_{qq'}$ includes a constant impedance component $\xi_0$, so that when the matrix $T_{qq'}$ (and similar matrices) is inverted $\xi_0$ is treated exactly and the expansion is carried out only in powers of the amplitudes $\xi_g$ of the impedance gratings.

Thus, from (1.5) we have

$$\mathscr{E}_{qt}=-\sum_{q'}T^{-1}_{qq'}\left(T_{q'}\mathbf{E}_t+\mathbf{V}_{q'}E_z+\sum_{q''}\mathbf{V}_{q'q''}E_{q''z}\right),\tag{1.8}$$

where

$$T^{-1}=\sum_{k=0}^{\infty}(-1)^k t^{-1}(\tilde{t}t^{-1})^k,\quad t^{-1}_{qq'}=t_q^{-1}\delta(\mathbf{q}-\mathbf{q}').\tag{1.9}$$

Then using the transverse-wave condition (1.7) we obtain a system of equations for the $z$ components $E_{qz}\equiv E_q$ of the diffracted wave amplitudes:

$$\sum_{q'}D_{qq'}E_{q'}=F_q,\tag{1.10}$$

where

$$D_{qq'}=d_q\delta(\mathbf{q}-\mathbf{q}')+\tilde{d}_{qq'},\quad d_q=(\beta_q+\xi_0)/t_q,$$

$$\tilde{d}_{qq'}=\alpha_q\sum_{q''}T^{-1}_{qq''}\mathbf{V}_{q''q'}-\alpha_q^2\xi_0\delta(\mathbf{q}-\mathbf{q}')/t_q,$$

$$F_q=-\alpha_q\sum_{q'}T^{-1}_{qq'}(T_{q'}\mathbf{E}_t+\mathbf{V}_{q'},E_z).\tag{1.11}$$

As before in the case of the matrix $T_{qq'}$, in $D_{qq'}$ we identify the portion $d_q\delta(\mathbf{q}-\mathbf{q}')$ which does not depend on the amplitudes of the impedance gratings and can then be treated exactly. The matrix elements $\tilde{d}_{qq'}$ are expressed in the form of series in $\xi_g$ and are small by virtue of the smallness of the grating amplitudes: $|\tilde{d}_{qq'}|\ll 1$, so that all off-diagonal elements of the matrix $D_{qq'}$ are small. The diagonal elements $D_{qq}$ will be small, however, only if $|\beta_q|\ll 1$. The corresponding diffracted waves consist of a set of resonant waves,[4,6] denoted in what follows by the subscript $r$. In contrast, the nonresonant waves (subscript $n$) include fields for which $|\beta_q|\sim 1$, and consequently $|D_{qq}|\sim 1$. As a result, the system of equations (1.10) breaks up into two subsystems. Since the diagonal elements of the nonresonant subsystem matrix satisfy $|D_{nn}|\sim 1$, the matrix $D_{nn'}$ can be inverted by means of a regular procedure:

$$D^{-1}_{nn'}=\sum_{l=0}^{\infty}(-1)^l\{d^{-1}(\tilde{d}d^{-1})^l\}_{nn'}.\tag{1.12}$$

We use (1.12) to express the nonresonant fields in terms of the resonant fields:

$$E_n=\sum_{n'}D^{-1}_{nn'}\left(F_{n'}-\sum_{r}D_{n'r}E_r\right).\tag{1.13}$$

Then substituting $E_n$ into the resonant subsystem we obtain a finite system of equations for the $z$ components of the resonant field amplitudes:

$$\sum_{r'}G_{rr'}E_{r'}=P_r,\tag{1.14}$$

where

$$G_{rr'}=D_{rr'}-\sum_{n,n'}D_{rn}D^{-1}_{nn'}D_{n'r'},$$

$$P_r=F_r-\sum_{n,n'}D_{rn}D^{-1}_{nn'}F_{n'}.\tag{1.15}$$

The elements $G_{rr'}$ and the column vector on the right-hand side of (1.14) take the form of power series in the grating amplitudes $\xi_g$, and solutions of the system (1.14) are given in the form of a ratio of the corresponding power series. The coefficients of these series are functions of the small parameters $\xi_0$ and $\beta_r$, treated exactly in Eqs. (1.14) and (1.15). In contrast to the previous approach,[10] where the expansion was carried out with respect to all the small parameters, this result corresponds to partial summation of the power series in Ref. 10 over the small parameters $\beta_r$ and $\xi_0$.

The approximate solution of the system corresponds to retaining a finite number of terms of the series in the numerator and denominator. This makes it possible to go correctly to the limit of scattering by a plane unmodulated boundary with finite conductivity ($\xi=\xi_0$, $\xi_g=0$), unlike Ref. 10, where the limiting solution ($\xi_g\to 0$) corresponds to a perfectly conducting material ($\xi_0=0$).

## 2. STRUCTURE OF THE GENERAL SOLUTION: THE ROLE OF THE RESONANT AND NONRESONANT GRATINGS

Assume that the system contains $n$ different resonant fields.[2)] The expression for the resonant field with index $m$ takes the form

$$E_{r_m}=\Delta_m/\Delta,\quad m=1,\dots,n,\tag{2.1}$$

where $\Delta$ and $\Delta_m$ are the determinant of the matrix $G_{rr'}$ and the auxiliary determinants of the system (1.14), found by replacing the $m$-th column with the column of free terms $P_{r_k}$. The denominator $\Delta$ determines the general resonant dependence of the fields (2.1) on the amplitudes and directions of the gratings, and the condition $\Delta=0$ determines the number of eigenmodes when periodic structures are present on the boundary. The numerator $\Delta_m$ describes the

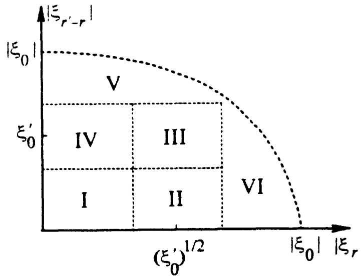


FIG. 1. Regions of strong (sharp) and weak resonance and different strengths are interrelated, depending on the grating amplitudes (for metallic media, $\xi_0' \ll |\xi_0''|$): I) the region of small grating amplitudes ($|\xi_r| \ll (\xi_0')^{1/2}$, $|\xi_{r'-r}| \ll \xi_0'$), where perturbation theory is applicable and the resonances are weak; II) the region of strong resonances ($|\xi_r| \sim (\xi_0')^{1/2}$) and weak coupling between them ($|\xi_{r'-r}| \ll \xi_0'$); III) the region of sharp ($|\xi_r| \sim (\xi_0')^{1/2}$) strongly coupled ($|\xi_{r'-r}| \sim \xi_0'$) resonances; IV) the region of strongly coupled ($|\xi_{r'-r}| \sim \xi_0'$) but weak ($|\xi_r| \ll (\xi_0')^{1/2}$) resonances; V, VI) regions of strong intermode ($|\xi_{r'-r}| \gg \xi_0'$) and intramode ($|\xi_r| \gg (\xi_0')^{1/2}$) nonlinearity, respectively, which give rise to broadening and weakening of the resonances. The amplitudes of the gratings that are not shown are assumed to be vanishingly small.

contribution of the different scattering channels to the specified resonant field $E_{r_m}$ (through the factors $P_{r_k}$). In the general case it follows from (1.11), (1.12), (1.14), and (1.15) that $\Delta_m$ takes the form

$$\Delta_m=\Gamma_m\xi_{g_m}+\sum_{s\geq 2}\Gamma_{m;l_1,\ldots,l_s}\delta\left(\mathbf{q}_m-\sum_{k=1}^{s}\mathbf{g}_{l_k}\right)\prod_{k=1}^{s}\xi_{l_k}, \quad (2.2)$$

where the coefficients satisfy $\Gamma_m$, $\Gamma_{m;l_1,\ldots,l_s}\sim 1$. Here the first term corresponds to the occurrence of the field $E_{r_m}$ in the first diffraction order, and each successive term corresponds to the $s$th order, $s=2, 3, \ldots$ .

We begin by noting the characteristic scales of the parameters that enter into the problems. The magnitude of the resonant denominator largely depends on their relative size. By definition, the resonant field corresponds to $|\beta_r|\ll 1$. Specifically, for vanishingly small modulation amplitudes, the resonances correspond to minima in $|\beta_{r_m}+\xi_0|$: $\min|\beta_{r_m}+\xi_0|=\xi_0'$. To study the resonances in the small-amplitude limits $|\xi_g|$, $|\xi_g|^2\ll|\beta_r+\xi_0|$, the general perturbation theory suffices. Strong resonant effects like total suppression of specular reflection, however, appear only for relatively large grating amplitudes: the contribution of the corresponding terms ($\sim\xi_g,\xi_g^2,\ldots$) to the coefficients $G_{rr}$ and $G_{rr'}$ (given explicitly below) must be comparable with the parameter $\beta_r+\xi_0$, which is independent of the modulation amplitudes. Such values of the grating amplitudes correspond to sharp strongly coupled resonances (see Fig. 1), and the modified perturbation theory presented here serves to describe the fields in this range of amplitudes.

Note that by virtue of the conditions $\xi'>0$ and $\xi''<0$ (the first of which corresponds to the physical requirement that the medium be passive and the second to the requirement that the magnetic dispersion be small), the grating amplitudes must satisfy the conditions

$$\sum_g|\xi_g+\xi_{-g}^*|<\xi_0,\quad \sum_g|\xi_g-\xi_{-g}^*|<|\xi_0''|. \quad (2.3)$$

In the case $\xi_{-g}=\xi_g$, [which corresponds to the real and imaginary parts of the impedance being in phase, $\xi=\xi_0+2\xi_g\cos(\mathbf{gr})$], these inequalities go over to the form

$$\sum_g|\xi_g'|<\xi_0'/2,\quad \sum_g|\xi_g''|<|\xi_0''|/2. \quad (2.4)$$

If we have $\xi_0'\gg|\xi_0''|$, which usually holds for insulators, then the general perturbation theory usually suffices to describe resonant diffraction, since the contribution of the terms due to intermode nonlinearity (the term $\sim\xi_{r-r'}$ in $G_{rr'}$) and intramode nonlinearity (the terms $\sim\xi_g^2$ in $G_{rr'}$ and $G_{rr}$) is small compared with the main terms $G_{rr}\sim\beta_r+\xi_0$ in the diagonal matrix elements.

The contribution of these terms can, however, be large in the special case of grazing incidence, when by virtue of the other resonant structure (due to inclusion of the resonant fields of the specular component), anomalous phenomena occur at much smaller modulation amplitudes (a similar situation arises also for grazing incidence on a relief grating[11]).

In the opposite limiting case $|\xi_0''|\gg\xi_0'$ (metals, semiconductors), to which special attention will be devoted, we should expect anomalous resonant diffraction only under the condition[3] $|(\xi')_g|\ll|(\xi'')_g|$, i.e., for media in which the imaginary part of the impedance varies. This follows from the form of the matrix elements $G_{rr'}$ and $G_{rr}$.

Specifically, when the parameter $\beta_r$ is real we have $|\beta_r+\xi_0|>|\xi_0''|$, and consequently it is large in comparison with the terms that are nonlinear in $\xi_g$, which in turn implies that general perturbation theory is applicable. In the most interesting case of imaginary $\beta_r$, when we have $\min|\beta_r+\xi_0|=\xi_0'\ll|\xi_0''|$ (since $\beta_r''>0$ and $\xi_0''<0$), the sharp anomalously strong resonances correspond to amplitudes $|\xi_g|\sim(\xi_0')^{1/2}$ of the resonance gratings and amplitudes $|\xi_{r-r'}|<\xi_0'$ of the gratings between resonances. Because $|(\xi')_g|<\xi_0'$, these conditions can be satisfied only when the amplitude of the variation in the imaginary part of the impedance is large: $|(\xi'')_g|\gg|(\xi')_g|$.

Thus, the largest resonances of the fields correspond to amplitudes $|\xi_g|\sim(\xi_0')^{1/2}$, $|\xi_{r-r'}|\sim\xi_0'$ in the impedance variations (mainly in the imaginary part of the impedance). Increasing the amplitudes above these values smears out the resonances; reducing the amplitudes reduces the resonant fields and weakens the coupling between them (Fig. 1, regions V, VI, and I respectively).

The preceding analysis applies primarily to the case of single-resonance scattering. Let us consider the interaction between resonances of different order when radiation is diffracted from the simplest system consisting of two gratings $\xi_r$ and $\xi_{r'-r}$ such that scattering by the $\xi_r$ grating gives rise to a resonant field $E_r$, which subsequently scatters from the $\xi_{r'-r}$ grating between resonances and gives rise to

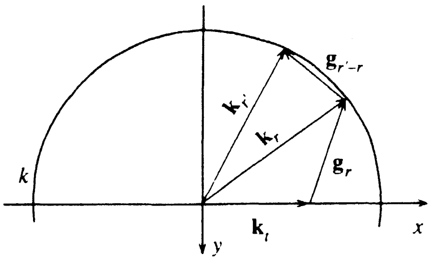


FIG. 2. Diagram of scattering by the gratings $\xi_r$ and $\xi_{r'-r}$ with the formation of first- and second-order resonant diffraction fields.

the resonant field $E_{r'}$ in the second diffraction order[4] (Fig. 2). The coefficients of the resonant system are

$$G_{rr}=\beta_r+\xi_0+\sum_{\substack{g=r,\\ r'-r}} A_g\xi_g\xi_{-g}, \quad G_{rr'}=\xi_{r-r'},$$

$$P_r=C\xi_rE, \quad P_{r'}=C'\xi_r\xi'_{r'-r}E, \tag{2.5}$$

where we have retained the leading terms in the grating amplitudes (the coefficients $A_g$, $C$, and $C' \sim 1$ are determined by the geometry and do not depend on the grating amplitudes; we exhibit their explicit form in specific situations in Sec. 3).

The solution of Eqs. (1.14) for the amplitudes of the resonant fields assumes the form

$$E_r=(G_{r'r'}P_r-G_{rr'}P_{r'})/\Delta, \tag{2.6}$$

$$E_{r'}=(G_{rr}P_{r'}-G_{r'r}P_r)/\Delta, \tag{2.7}$$

where $\Delta=G_{rr}G_{r'r'}-G_{rr'}G_{r'r}$. In (2.5) and (2.6), the first terms in the numerator describe the direct channel for scattering of incident radiation into the given resonant field, and the second terms describe the contribution of the neighboring resonance as a result of rescattering by the inter-resonance grating $\xi_{r-r'}$. From (2.6) we see, however, that near sharp resonances the effect of the field $E_{r'}$ of the second diffraction order on the field $E_r$ of the first diffraction order is small in comparison with the contribution of the direct scattering channel. The contributions of the direct and indirect scattering channels are comparable only when the inter-resonance grating amplitude is large, $|\xi_{r'-r}| \sim (\xi'_0)^{1/2}$, when the resonances have become broader and weaker. Thus, in the most interesting region $|\xi_r| \sim (\xi'_0)^{1/2}$, $|\xi_{r'-r}| \sim \xi'_0$ the field $E_r$ has the form $E_r \approx G_{r'r'}P_r/\Delta \sim \xi_0\xi_rE/\Delta$. Note that a second-order resonance affects only the structure of the denominator $\Delta$.

In contrast to this, the effect of the first-order resonance (the field $E_r$) on the second-order resonance (the field $E_{r'}$) is more important and can exceed the contribution of the direct channel for scattering of the incident field (successive scattering from the gratings $\xi_r$ and $\xi_{r'-r}$). In fact, in the strong-coupling limit ($|\xi_{r'-r}| \sim \xi'_0$), the first term in (2.7) is of order $\xi'_0\xi_r\xi_{r'-r}E \sim (\xi'_0)^2\xi_rE$, significantly smaller than the contribution of the second term: $\xi_{r-r'}\xi_rE \sim \xi'_0\xi_rE$. Consequently, the magnitude of the resonance of the second-order diffraction $E_{r'} \approx -G_{r'r}P_r/\Delta$ is completely determined by the first-order resonance; furthermore, in the strong-coupling limit we are considering ($|\xi_{r'-r}| \sim \xi'_0$) these resonant fields are found to be of the same order of magnitude: $E_{r'} \sim \xi'_0\xi_rE/\Delta \sim E_r$.

The field amplitudes $E_r$ and $E_{r'}$ become equalized due to the strong coupling between resonances mediated by the inter-resonance grating $\xi_{r-r'}$. It is easy to see that a similar effect occurs when the resonances are coupled, e.g., by a pair of gratings $\xi_{r'-g}\xi_{g-r}$ with effective amplitude $|\xi_{r'-g}\xi_{g-r}| \sim \xi'_0$. This coupling can be treated by going to the next term in the expansion of the coefficient $G_{rr'}$ in the grating amplitudes; its general form is $\Sigma_g B_g\xi_{r-g}\xi_{g-r'}$.

Thus, strong coupling destroys the hierarchy of resonances, and in each individual case it is important to specify and treat this coupling correctly. From the preceding discussion, it is also clear that although the amplitudes of the strongly coupled resonances of different orders are similar in magnitude, there is still a difference between them. This is because a higher-order diffraction resonance is determined by the lower-order diffraction resonance, and not the other way around.

## 3. EXAMPLES

We will apply these results to the simplest situations with a small number of resonances in the first-order diffraction spectrum. We give explicit expressions for the coefficients of the system (1.14), restricting ourselves to the treatment of terms up to second order in the grating amplitudes $\xi_g$ in the coefficient $G_{rr'}$, and the first-order parts of the free terms:

$$G_{rr'}=t_r^{-1}\Big\{\beta_r(1+\beta_r\xi_0)\delta(\mathbf{r}-\mathbf{r}')+(\alpha_r\alpha_{r'})\xi_{r-r'} + \sum_n \beta_n^{-1}([\alpha_n\alpha_r][\alpha_n\alpha_{r'}] -(\alpha_r\alpha_{r'}))\xi_{r-n}\xi_{n-r'}\Big\} \tag{3.1}$$

$$P_r=t_r^{-1}\Big\{-(\alpha_r\mathbf{E})(\delta(\mathbf{r})-\beta_0\xi_r[1-\delta(\mathbf{r})])+[\alpha_r\mathbf{H}]_z\xi_r +(\alpha_r\alpha_0)(\alpha_0\mathbf{E})\xi_r\sum_n \delta(\mathbf{n})/(\beta_0+\xi_0)\Big\}. \tag{3.2}$$

Here, as was stipulated previously, $r$ and $n$ denote respectively $q_r$ and $q_n$. From (3.1) and (3.2), it follows that in the coefficients of Eqs. (1.14) the factor $t_r=1+\beta_r\xi_0$ cancels out exactly, so that in what follows we can omit it. Consequently, as can easily be seen, the solution of the resonance system of equations in leading order contains no expansions of the form $\xi_0\beta_r$ (which appear in higher orders).

The tangential components of the fields (both resonant and nonresonant) can be expressed in terms of the $z$ components $E_{qz}$ using the formula [see (1.8)]

$$\mathbf{E}_{qt}=t_q^{-1}\Big(-(\delta(\mathbf{q})-\beta_0\xi_q)\mathbf{E}_t+\beta_0\xi_q(1-\delta(\mathbf{q}))\mathbf{E}_t/t_q$$

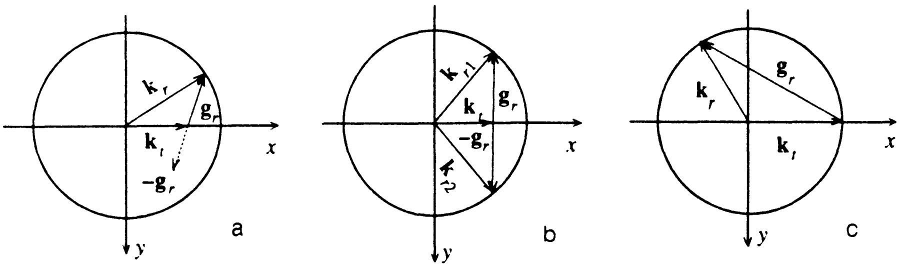


FIG. 3. Wave vector diagrams for a) single-resonance scattering, b) two-resonance scattering (degenerate case), and c) resonant scattering in the case of grazing incidence.

$$-\alpha_0\xi_q E_z-\sum_{\mathbf{q}'}\alpha_{q'}\xi_{q-q'}E_{q'z}\Big). \tag{3.3}$$

As a rough approximation, we can use in place of (3.3) the simple expression

$$\mathbf{E}_{qt}=-\delta(\mathbf{q})\mathbf{E}_t-\sum_r \alpha_r\xi_{q-r}E_r, \tag{3.4}$$

where we have taken into account the contribution of the resonant fields while retaining only the leading term from the nonresonant fields. An unsatisfactory aspect of this formula is that in the limiting case $\xi_g=0$, we find $\mathbf{E}_{0t}=-\mathbf{E}_t$ for the amplitude of the specular wave ($\mathbf{q}=0$). This is equivalent to a perfectly conducting surface or to the leading term in the expansion for a surface with finite but large conductivity ($|\xi_0|<1$). Let us consider some specific examples.

### 1. Single-resonance case

We have a single grating

$$\xi-\xi_0=\xi_g\exp(i\mathbf{gr})+\xi_{-g}\exp(-i\mathbf{gr}), \tag{3.5}$$

such that $|\mathbf{k}_t+\mathbf{g}|\approx k$ (Fig. 3a), and there is one resonant field corresponding to it. The system of equations (1.14) consists of a single equation from which we find

$$E_g=P_g/G_{gg}, \tag{3.6}$$

where

$$P_g=2\xi_g[\boldsymbol{\alpha}_g\mathbf{H}]_z,$$

$$G_{gg}=\beta_g+\xi_0-(\beta_0^{-1}+\beta_{2g}^{-1})(1-[\boldsymbol{\alpha}_0\boldsymbol{\alpha}_g]^2)\xi_g\xi_{-g}. \tag{3.7}$$

By direct substitution we see that the energy conservation law in this case takes the form (see Appendix)

$$\beta_0(|\mathbf{E}|^2-|\mathbf{E}_0|^2)-\beta_g'|E_{gz}|^2=\xi_0'|\mathbf{H}_{gt}|^2+\mathrm{Re}\{(\xi_g^*+\xi_{-g})(((\mathbf{H}+\mathbf{H}_0)_t^*,\mathbf{H}_{gt})+(\mathbf{H}_{2gt},\mathbf{H}_{gt}^*))\}, \tag{3.8}$$

where

$$\mathbf{H}_{gt}\approx[\boldsymbol{\alpha}_g\mathbf{e}_z]E_{gz},$$

$$\mathbf{H}_{0t}=\mathbf{H}_t+([\mathbf{e}_z\boldsymbol{\alpha}_g]+\boldsymbol{\alpha}_0(\mathbf{e}_z[\boldsymbol{\alpha}_0\boldsymbol{\alpha}_g]))\xi_{-g}E_{gz}/\beta_0,$$

$$\mathbf{H}_{2gt}=([\mathbf{e}_z\boldsymbol{\alpha}_g]+\boldsymbol{\alpha}_{2g}(\mathbf{e}_z[\boldsymbol{\alpha}_{2g}\boldsymbol{\alpha}_g]))\xi_gE_{gz}/\beta_{2g}. \tag{3.9}$$

### 2. Two-resonance case

This corresponds to the presence of two resonant fields, which may result from having two resonant gratings. Two resonant fields also arise from scattering by the single grating (3.5) in the degenerate case $(\mathbf{gk}_t)\to 0$, when $|\mathbf{k}_t\pm\mathbf{g}|\approx k$ (Fig. 3b). In these cases, the resonances can also be coupled directly by the grating $\xi_{r-r'}$ or $\xi_{2r}$ respectively. The system of equations (1.14) for these two cases is formally the same, but the coefficients are different because the set of intermediate nonresonant fields taking part in rescattering processes is different. Here we give the solution for the degenerate case $\mathbf{g}\perp\mathbf{k}_t$, $\beta_{-g}=\beta_g$:

$$E_{jg}=2\xi_{jg}[\boldsymbol{\alpha}_0\mathbf{H}]_z/D_-+2j\xi_{jg}[\boldsymbol{\chi}\mathbf{H}]_z/D_+, \tag{3.10}$$

where $j=\pm1$, $\boldsymbol{\chi}=\mathbf{g}/k$, and

$$D_\pm=\beta_g+\xi_0-(\beta_0^{-1}+\beta_{2g}^{-1})(1-[\boldsymbol{\alpha}_0\boldsymbol{\alpha}_g]^2)\xi_g\xi_{-g}\pm\beta_0^{-1}((\boldsymbol{\alpha}_g\boldsymbol{\alpha}_{-g})+[\boldsymbol{\alpha}_0\boldsymbol{\alpha}_g]^2)\xi_g\xi_{-g}. \tag{3.11}$$

For a specularly reflected wave, we find from (3.4) using (3.10)

$$\mathbf{E}_{0t}=-\mathbf{E}_t-4\xi_g\xi_{-g}(\boldsymbol{\alpha}_0[\boldsymbol{\alpha}_0\mathbf{H}]_z/D_-+\boldsymbol{\chi}[\boldsymbol{\chi}\mathbf{H}]_z/D_+). \tag{3.12}$$

For normal incidence ($\boldsymbol{\alpha}_0=0$, $\boldsymbol{\alpha}_{\pm g}=\pm\boldsymbol{\chi}_g$), it follows from (3.10) and (3.12) that

$$E_{\pm g}=2\xi_{\pm g}[\boldsymbol{\alpha}_{\pm g}\mathbf{H}]_z/D,$$

$$\mathbf{E}_{0t}=-\mathbf{E}_t-4\xi_g\xi_{-g}\boldsymbol{\alpha}_g[\boldsymbol{\alpha}_g\mathbf{H}]_z/D, \tag{3.13}$$

where $D=\beta_g+\xi_0-(2\beta_0^{-1}+\beta_{2g}^{-1})\xi_g\xi_{-g}$.

### 3. Grazing incidence

This case should actually be regarded as a two-resonance case, since one of the first-order diffraction waves is resonant with the specularly reflected wave (Fig. 3c). In this case the coefficients of Eqs. (1.14) take the form

$$G_{00}=\beta_0+\xi_0,\quad G_{gg}=\beta_g+\xi_0,$$

$$G_{g0}=(\boldsymbol{\alpha}_0\boldsymbol{\alpha}_g)\xi_g,\quad G_{0g}=(\boldsymbol{\alpha}_0\boldsymbol{\alpha}_g)\xi_{-g},$$

$$P_0\equiv-(\beta_0-\xi_0)[\boldsymbol{\alpha}_0\mathbf{H}]_z,$$

$$P_g=2[\boldsymbol{\alpha}_g\mathbf{H}]_z\xi_g-\xi_g(\boldsymbol{\alpha}_0\boldsymbol{\alpha}_g)[\boldsymbol{\alpha}_0\mathbf{H}]_z. \tag{3.14}$$

Consequently, the desired fields are

$$E_0=D_0^{-1}(-(\beta_0-\xi_0)(\beta_g+\xi_0)[\boldsymbol{\alpha}_0\mathbf{H}]_z-\xi_g\xi_{-g}(\boldsymbol{\alpha}_0\boldsymbol{\alpha}_g)\times(2[\boldsymbol{\alpha}_g\mathbf{H}]_z-(\boldsymbol{\alpha}_0\boldsymbol{\alpha}_g)[\boldsymbol{\alpha}_0\mathbf{H}]_z)),$$

$$E_g=D_0^{-1}\xi_g\{(\beta_0+\xi_0)(2[\boldsymbol{\alpha}_g\mathbf{H}]_z-(\boldsymbol{\alpha}_0\boldsymbol{\alpha}_g)[\boldsymbol{\alpha}_0\mathbf{H}]_z)$$

$$+(\beta_0-\xi_0)(\boldsymbol{\alpha}_0\boldsymbol{\alpha}_g)[\boldsymbol{\alpha}_0\mathbf{H}]_z\}, \quad (3.15)$$

where $D_0=(\beta_0+\xi_0)(\beta_g+\xi_0)-(\boldsymbol{\alpha}_0\boldsymbol{\alpha}_g)^2\xi_g\xi_{-g}$. Note that the expression for $\mathbf{E}_0$ remains valid even in the limiting case of an unmodulated surface with finite conductivity. The single-resonance case with one resonant grating and many nonresonant gratings, the two-resonance case with coupling, the case of a single second-order resonance $[(\xi_r)_{\text{eff}}\sim\Sigma_g\xi_g\xi_{r-g}]$, etc., can be treated in a similar fashion.

## 4. SUPPRESSION OF SPECULAR REFLECTION

As is well known,[6,12] resonant profile gratings give rise to anomalies in the reflection and absorption of light by the surface. This phenomenon is related to resonant diffraction, as a result of which the energy of the incident radiation is channeled into resonant diffraction spectra propagating at grazing incidence along the surface, and is ultimately absorbed by the medium. Here we study the suppression of specular reflection[12] in connection with diffraction by impedance gratings. In the single-resonance case, we have from (3.4), (3.6), and (3.7)

$$\mathbf{E}_{0t}=-G_{gg}^{-1}\{\mathbf{E}_t(\beta_g+\xi_0+(\beta_0^{-1}-\beta_{2g}^{-1})(1-[\boldsymbol{\alpha}_0\boldsymbol{\alpha}_g]^2)\xi_g\xi_{-g})+2\xi_g\xi_{-g}(\beta_0[\boldsymbol{\alpha}_g[\boldsymbol{\alpha}_g\mathbf{E}_t]]+\beta_0^{-1}(\boldsymbol{\alpha}_0\boldsymbol{\alpha}_g)[\boldsymbol{\alpha}_0[\boldsymbol{\alpha}_g\mathbf{E}_t]])\}. \quad (4.1)$$

From (4.1) it follows that in order to achieve total suppression of specular reflection ($\mathbf{E}_{0t}=0$), the light must be linearly polarized with

$$\boldsymbol{\alpha}_g \parallel \mathbf{E}_t,$$

$$\beta_g+\xi_0+(\beta_0^{-1}-\beta_{2g}^{-1})(1-[\alpha_0\alpha_g]^2)\xi_g\xi_{-g}=0. \quad (4.2)$$

The first condition determines the direction of the grazing (resonant) wave, and the second determines the period and amplitude of the modulation.

It follows from the structure of the resonant denominator in (3.6) that the resonant field (and hence the specularly reflected wave) can undergo a substantial change only if $|\xi_g| \sim (\xi_0')^{1/2} \gg \xi_0'$. On the other hand, $\xi'>0$ for inactive media, so that we must have $|(\xi')_g| < \xi_0'$. It follows that resonant phenomena in the reflection and absorption of light are related to the modulation of the imaginary part of the impedance. We will therefore assume that $|(\xi')_g| \ll |(\xi'')_g|$ and $\xi_g\approx i(\xi'')_g$, $\xi_{-g} \approx i(\xi'')_g^*$. Then from the second condition in (4.2), it follows that the conditions for total suppression of specular reflection restrict the grating period and modulation amplitude according to

$$\beta_g''=-\xi_0''+(1-[\boldsymbol{\alpha}_0\boldsymbol{\alpha}_g]^2)|\xi_g|^2/\beta_{2g}'',$$

$$|\xi_g|^2=\beta_0\xi_0'/(1-[\boldsymbol{\alpha}_0\boldsymbol{\alpha}_g]^2). \quad (4.3)$$

Under these conditions, total suppression of specular reflection can occur, and all of the incident radiation will be completely absorbed, as can be seen from the conservation law (A3).

Now let us consider elliptically polarized waves incident on a resonant grating:

$$\mathbf{E}=|\mathbf{E}|\{\mathbf{e}_s e^{i\delta}\sin\psi+\mathbf{e}_p\cos\psi\},$$

$$\mathbf{e}_s=[\mathbf{e}_z\boldsymbol{\alpha}_0]/\alpha_0, \quad \mathbf{e}_p=[\mathbf{e}_s\mathbf{k}]/k, \quad 0\leqslant\psi<\pi, 0\leqslant\delta<\pi, \quad (4.4)$$

where $\delta$ is the phase shift between the $s$ and $p$ components of the incident wave. We assume that the resonance conditions (4.2) are satisfied. Then from (4.1) and (4.3), we can find an expression for the amplitude of the specularly reflected wave:

$$\mathbf{E}_0=[\mathbf{E}\boldsymbol{\alpha}_g]_z((\boldsymbol{\alpha}_0\boldsymbol{\alpha}_g)\mathbf{e}_{0s}+\beta_0(\mathbf{e}_z[\boldsymbol{\alpha}_0\boldsymbol{\alpha}_g])\mathbf{e}_{0p})/(1-[\boldsymbol{\alpha}_0\boldsymbol{\alpha}_g]^2)\alpha_0, \quad (4.5)$$

where $\mathbf{e}_{0s}\equiv\mathbf{e}_s$, $\mathbf{e}_{0p}\equiv[\mathbf{e}_s\mathbf{k}_0]/k$, and $\mathbf{k}_0$ is the wave vector of the reflected wave. In the case of linear polarization, as noted previously, total suppression of specular reflection ($\mathbf{E}_0=0$) results when $\boldsymbol{\alpha}_g \parallel \mathbf{E}_t$.[5)] In the general case of an elliptically polarized incident wave the suppression is not complete, since the complexity of $\mathbf{E}$ implies $[\boldsymbol{\alpha}_g\mathbf{E}_z]\neq 0$. However, in this case we can suppress either component ($s$ or $p$) of the reflected wave (for $\boldsymbol{\alpha}_g \perp \boldsymbol{\alpha}_0$ or $\boldsymbol{\alpha}_g \parallel \boldsymbol{\alpha}_0$, respectively). In addition, we can suppress any linear combination of the $s$ and $p$ components. Then the reflected light will be linearly polarized with an arbitrarily prescribed direction of polarization $\mathbf{m}=m_s\mathbf{e}_s+m_p\mathbf{e}_{0p}$ (the vector $\mathbf{m}$ lies in the plane perpendicular to the wave vector $\mathbf{k}_0$). The orientation of this diffraction grating can be found from the condition $\mathbf{m}\mathbf{E}_0=0$, from which by using (4.5) we find

$$\operatorname{tg}\varphi=\alpha_{gy}/\alpha_{gx}=-m_s/\beta_0 m_p, \quad (4.6)$$

where $\varphi=\widehat{\boldsymbol{\alpha}_0\boldsymbol{\alpha}_g}$. It is easy to see that the orientation $\boldsymbol{\alpha}_g$ in the plane $z=0$ corresponds to projection of the vector $\mathbf{m}$ onto this plane.

Extrema of the reflected-wave amplitude $|\mathbf{E}_0|$ correspond to resonant diffraction in the direction $\varphi$ determined by

$$\operatorname{tg}\varphi=(-1\pm(1+\cos^2\delta\operatorname{tg}^2 2\psi)^{1/2}/\beta_0\cos\delta\operatorname{tg}2\psi. \quad (4.7)$$

It can be shown that the directions $\boldsymbol{\alpha}_g$ given by the conditions (4.5) are the same as the projections of the principal axes of the polarization ellipse onto the plane of the grating. Then the amplitude of the specularly reflected wave is equal to

$$|\mathbf{E}_0|^2=\tfrac{1}{2}|\mathbf{E}|^2(1\pm(1-\sin^2\delta\sin^2 2\psi)^{1/2})/2, \quad (4.8)$$

where the plus sign obviously corresponds to the minimum, and the minus sign to the maximum. In the case of linear polarization ($\delta=0$), we have from (4.7) and (4.8)

$$\operatorname{tg}\varphi=\pm(\operatorname{tg}\psi)^{\pm1}/\beta_0,$$

$$|\mathbf{E}_0|^2_{\text{extr}}=|\mathbf{E}|^2(1\mp1)/2; \quad (4.9)$$

the minimum value is $|\mathbf{E}_0|=0$.

## 5. CONCLUSION

In the present work, we have thus used the example of an impedance-modulated surface to demonstrate a technique for solving the problem of diffraction by surface periodic structures under conditions such that Wood resonances occur. We have examined the structure of the general solution, and studied the simplest particular cases. In contrast to our previous work,[10] a modified perturbation theory is developed here which expands only in the amplitudes of the impedance structures, but not in all small parameters, which is a more attractive approach. We have considered the total suppression of specular reflection, and studied the conditions under which it occurs. Finally, we have formulated the energy conservation law in a general form for resonant diffraction, and have given its particular form for the single-resonance case.

## APPENDIX: ENERGY CONSERVATION

The flux of radiative energy $S_0=c/8\pi)[\mathscr{E}\mathscr{H}^*]\mathbf{e}_z$ through the surface $z=0$ can be expressed using the Leontovich boundary condition in terms of the magnetic field at the surface:

$$S_0=\frac{c}{8\pi}(\xi'|\mathscr{H}_t|^2_{z=0})_{q=0}. \tag{A1}$$

This flux is equal to the energy flowing into the metal from outside and dissipating in it. On the other hand, the absorbed flux can be expressed in terms of the fields in the far zone as the difference between the energy fluxes arriving at and departing from the surface:

$$S_0=\frac{c}{8\pi}\left(\beta_0\left|\mathbf{E}\right|^2-\sum_q \beta'_q\left|\mathbf{E}_q\right|^2\right). \tag{A2}$$

We show that the fields which solve Eqs. (1.5) satisfy a conservation law that follows from (A1) and (A2):

$$(\xi'|\mathscr{H}_t|^2_{z=0})_{q=0}=\beta_0|\mathbf{E}|^2-\sum_q \beta'_q|\mathbf{E}_q|^2. \tag{A3}$$

Note that in the special case $\xi=0$, it follows from (A3) that energy is conserved in diffraction by a perfectly reflecting surface (cf. Ref. 13). We transform the left side of Eq. (A3):

$$\begin{aligned}(\xi'|\mathscr{H}_t|^2_{z=0})_{q=0}&=\sum_g \xi_g(|\mathscr{H}_t|^2_{z=0})_{-g}/2\\ &\quad+\sum_g \xi^*_{-g}(|\mathscr{H}_t|^2_{z=0})_{-g}/2\\ &\equiv(I+I^*)/2.\end{aligned} \tag{A4}$$

Using the relation $\mathbf{H}_q=[\boldsymbol{\alpha}_q-\beta_q\mathbf{e}_z,\mathbf{E}_q]$ between the amplitudes of the electric and magnetic fields, we have

$$\begin{aligned}I=\sum_g \xi_g\sum_{q'}&(\boldsymbol{\alpha}_{q'}E_{q'z}+\beta_{q'}\mathbf{E}_{q't}+\delta(\mathbf{q}')\mathbf{A})(\boldsymbol{\alpha}_{g+q'}E_{g+q',z}\\ &+\beta_{g+q'}\mathbf{E}_{g+q',t}+\delta(\mathbf{q}'+\mathbf{g})\mathbf{A})^*,\end{aligned} \tag{A5}$$

where $\mathbf{A}=\boldsymbol{\alpha}_0E_z-\beta_0\mathbf{E}_t$.

We pass from summation over $g$ to summation over $q=q'+g$, noting that according to (1.6),

$$\boldsymbol{\alpha}_{q'}\xi_{q-q'}=\mathbf{V}_{qq'},\quad \beta_{q'}\xi_{q-q'}=T_{qq'}-\delta(\mathbf{q}-\mathbf{q}').$$

Then taking into account (1.5) and performing some simple manipulations, we find

$$\begin{aligned}I=\beta_0|\mathbf{E}|^2-&\sum_q(\beta^*_q|E_{qz}|^2+\beta_q|\mathbf{E}_{qt}|^2)\\ &+2i\beta_0\,\mathrm{Im}(\mathbf{E}^*_t\mathbf{E}_{0t}+E_zE^*_{0z}),\end{aligned} \tag{A6}$$

from which we can show by using (A4) that (A3) is correct.

We emphasize that if solutions for the fields are found exactly, then the relation (A3) holds identically. In practice, however, we have at our disposal only approximate expressions for the fields, determined by the solution of Eqs. (1.14) for the $z$ component of the resonant fields and relations (1.8) and (1.13). Then the conservation law in the form (A3) will hold only approximately, with an error that depends on the accuracy of the solutions for the fields. Note, however, that in the particular case of single-resonance scattering and for the solution in leading order, the conservation holds to within terms of order $|\xi_g|^2$ [cf. Eq. (3.8)].

[1)]The summation in (1.1) is over different pairs of oppositely directed ($\mathbf{g}$ and $-\mathbf{g}$) wave vectors, where in the general case we have $\xi_g\neq\xi_{-g}$ and $\xi_{-g}\neq\xi^*_g$.

[2)]This condition does not impose any restriction on the number of gratings $\xi_g$ taking part, and has the following literal meaning: there exist exactly $n$ vectors $\mathbf{q}$ [cf. Eq. (1.4)] such that $|\mathbf{k}_t+\mathbf{q}|\approx k$ holds. Actually, however, in view of the assumed smallness of the grating amplitudes, the presence of high-order resonances $\mathbf{q}=\Sigma p_i\mathbf{g}_i$ with $\Sigma|p_i|p_i>2$ is generally unimportant (see below).

[3)]Note that, generally speaking, we have $(\xi')_g\neq(\xi_g)'$, $(\xi'')_g\neq(\xi_g)''$. Here and below for estimates it is convenient to use the Fourier amplitudes of the real part $\xi'(\mathbf{r})$ and the imaginary part $\xi''(\mathbf{r})$ of the impedance. This contrasts with the general representation (1.1), which contains the complex harmonics $\xi(\mathbf{r})=\xi'(\mathbf{r})+i\xi''(\mathbf{r})$.

[4)]We assume that the resonant field $E_{r'}$ is absent or small in first-order diffraction because the corresponding grating $\xi_{r'}$ is absent or small (it is not shown in Fig. 2).

[5)]The analogous condition for relief gratings takes the form $\mathbf{g}\parallel\mathbf{H}_t$, where $\mathbf{H}_t$ is the tangential component of the magnetic field in the incident wave.


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