Electromagnetic field generation by an ultrashort laser pulse in a rarefied plasma

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We consider the generation of quasistatic fields when a strong laser pulse propagates in a rarefied plasma. We show that the propagation of a circularly polarized pulse is accompanied by the emission of a low-frequency electromagnetic field. We determine the structure of the field and the energy losses connected with the radiative deceleration of the pulse. We compare the magnitudes of the quasistatic electromagnetic fields and the electrostatic wake field.

1. INTRODUCTION

The availability of sources of radiation of subpicosecond length and high power[1,2] has stimulated the study of the interaction between such radiation and matter. In particular, in connection with research on particle acceleration by lasers and on x-ray lasers, great interest has been shown in the study of the passage of short laser pulses through a plasma. So far, most investigations have been connected with a study of the electrostatic wake field resulting from the displacement of plasma electrons from the region occupied by the laser pulse under the action of the ponderomotive force (see, e.g., Ref. 3 and the literature cited there). In the present paper we would like to draw attention to the possibility of generating low-frequency (i.e., with frequencies below the plasma frequency) solenoidal electromagnetic fields by ultrashort laser pulses and indicate the conditions under which this effect may be significant. This effect is connected with the excitation of drag currents in a plasma under the action of the laser field. The drag current leads in the case of a circularly polarized pulse to the generation of electromagnetic fields thanks to the inverse Faraday effect.

The generation of a magnetic field by a long laser pulse (the stationary inverse Faraday effect) has been studied rather completely (see, e. g., Refs. 4 and 5). However, in a sufficiently rarefied plasma, the length of subpicosecond laser pulses may turn out to be less than the time for establishing the electromagnetic field, and its structure will therefore differ from the stationary one. Moreover, if the characteristic time scale of the pulse turns out to be less than the period of the plasma oscillations, there arises together with the quasistatic wake field which appears behind the pulse a radiation field similar to the Cherenkov radiation field of a laser pulse in a dielectric.[6,7] One should, however, emphasize that such an analogy is to a significant extent formal in character, since the phase velocity of the electromagnetic waves in a plasma is always larger than the velocity of light.

The basic equations of the electrodynamics of an ultrashort laser pulse are formulated in the second section, and we also obtain there a general solution for the quasistatic electromagnetic field in the fixed momentum aproximation. In the third section we analyze a rectangular laser pulse. The fourth section is devoted to a study of the generation of an electromagnetic field by a focused Gaussian laser pulse. In the fifth and last section, we compare the amplitudes of the electrostatic and electromagnetic fields, and discuss the possibilities for their application.

2. BASIC EQUATIONS

We consider the propagation of an electromagnetic pulse of frequency ω_0 and wave vector $\mathbf{k}_0$ ($k_0=\omega_0/c$) in a uniform rarefied plasma, assuming that the electron plasma frequency ω_p is appreciably lower than ω_0. Being interested in short pulses, we shall assume the ions to be fixed, playing the role of a neutralizing, uniform background, and the electrons to be cold. We describe their motion by the equations of hydrodynamics, which together with the Maxwell equations have the form:

$$\frac{d\mathbf{p}}{dt}=e\left(\mathbf{E}+\frac{1}{c}[\mathbf{vB}]\right),\quad \frac{dn}{dt}+\operatorname{div}(n\mathbf{v})=0,$$

$$\operatorname{curl}\mathbf{E}=-\frac{1}{c}\frac{d\mathbf{B}}{dt},\quad \operatorname{curl}\mathbf{B}=\frac{4\pi e}{c}n\mathbf{v}+\frac{1}{c}\frac{d\mathbf{E}}{dt}. \tag{1}$$

Here $d/dt=\partial/\partial t+(\mathbf{v}\nabla)$ is the total time derivative; n, $\mathbf{p}$, and $\mathbf{v}$ are the electron density, momentum, and velocity, with $\mathbf{v}=\mathbf{p}c(m^2c^2+p^2)^{-1/2}$; e and m are the electron charge and mass, and c is the speed of light.

We shall consider Eq. (1) assuming that the oscillatory velocity of the electrons in the electric field E_0 of the laser pulse is small compared to the speed of light:

$$\mathbf{v}_E=\frac{eE_0}{m\omega_0}\ll c. \tag{2}$$

This weak-relativity condition makes it possible to write all hydrodynamic quantities and fields as a sum of three harmonics: at the fundamental frequency ω_0, at the doubled frequency $2\omega_0$, and at "zero" frequency (quasistatic quantities changing over times considerably longer than the period of the laser radiation). This corresponds to the quadratic approximation in the amplitude of the electromagnetic field and takes into account such effects as strictional nonlinearity, relativistic nonlinearity, generation of the second harmonic of the frequency of the laser pulse, and quasistatic drag current. Eliminating quantities at the

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second harmonic, we arrive at a system of equations for the scalar (Ψ) and vector ($\mathbf{A}$) potentials and the amplitude E_0 of the laser pulse:

$$\frac{\partial^2\Psi}{\partial t^2}+\omega_p^2\Psi=-\frac{\omega_p^2}{\omega_0^2}e\frac{|\mathbf{E}_0|^2}{4m}, \tag{3}$$

$$\frac{\partial^2\mathbf{A}}{\partial t^2}-c^2\Delta\mathbf{A}+\omega_p^2\mathbf{A}=i\frac{ec\omega_p^2}{4m\omega_0^3}(\mathbf{E}_0\,\mathrm{div}\,\mathbf{E}_0^*-\text{c.c.}), \tag{4}$$

$$2i\left(\omega_0\frac{\partial\mathbf{E}_0}{\partial t}+c^2(\mathbf{k}_0\nabla)\mathbf{E}_0\right)+c^2\Delta_\perp\,\mathbf{E}_0-c^2\mathbf{k}_0(\mathbf{k}_0\mathbf{E}_0)$$
$$=\omega_p^2\left(\frac{\delta n}{n_0}\mathbf{E}_0-\frac{e^2|\mathbf{E}_0|^2}{4m^2\omega_0^2c^2}\mathbf{E}_0+i\frac{e}{mc\omega_0}[\mathbf{E}_0\mathbf{B}]\right.$$
$$\left.+\frac{\mathbf{k}_0}{\omega_0}(\mathbf{v}\mathbf{E}_0)\right). \tag{5}$$

We assume Ψ, $\mathbf{A}$, and $\mathbf{E}_0$ to be slowly changing in comparison with the period $2\pi/\omega_0$ of the laser pulse and its wavelength $2\pi/k_0$. In Eq. (5), $\Delta_\perp$ is the transverse Laplacian with respect to the direction of propagation ($\mathbf{k}_0$), and the potential and nonpotential components of the electron velocity $\mathbf{V}$ are respectively determined by the continuity equation and the equation of conservation of generalized vorticity:

$$\mathrm{div}\,\mathbf{v}+\frac{1}{n_0}\frac{\partial\delta n}{dt}=0,\quad \mathbf{v}+\frac{eA}{mc}=0, \tag{6}$$

while the perturbation of the electron density, $\delta n=-\Delta\Psi/4\pi e$ and the quasistatic magnetic field $\mathbf{B}=\mathrm{curl}\,\mathbf{A}$ can be expressed in terms of the slowly varying potentials.

The generation of the quasistatic magnetic field is, according to Eq. (4), determined by the nonlinear drag current:[5]

$$\mathbf{j}_{NL}=\frac{e\omega_p^2}{4m\omega_0^3}i(\mathbf{E}_0\,\mathrm{div}\,\mathbf{E}_0^*-\text{c.c.}). \tag{7}$$

If we understand by τ and L the characteristic time and length scales for changes in the magnetic field, the use of Eq. (7) for describing magnetic field generation assumes, in accordance with Ref. 5, that

$$L\gg v_T\tau, \tag{8}$$

where v_T is the electron thermal velocity.

Condition (8) determines the boundary of applicability of the hydrodynamic description for slow motions resulting from the time- and coordinate-dependence of the field amplitude E_0. Moreover, the longitudinal spatial ($L_\parallel$) and time (τ) scales are correlated, $L_\parallel\sim c\tau$, for a pulse propagating in a rarefied plasma. This makes it possible to verify the validity of (8) for $v_T\ll c$, if by L we mean the longitudinal size $L_\parallel$ of the pulse, which corresponds to electromagnetic beams of a rather large transverse size with a radius R larger than the pulse length. In the opposite case, $R<L_\parallel$, (8) constrains the pulse length, which must be small compared to R/v_T. In practice this constraint is not overly stringent. For instance, in the case of a laser pulse with $R\sim10\ \mu$m, propagating in a plasma with a temperature of a few hundred eV, (8) is satisfied for pulse lengths less than 1 ps.

Equations (3)–(5) enables us to study the self-consistent evolution of a laser pulse and of the quasistatic rotational electromagnetic field and longitudinal wake field generated by it.[3] We restrict ourselves here to using Eq. (4) to study the excitation of a magnetic field and a rotational electric field in the approximation where the intensity distribution of the laser pulse is fixed.

The physical reason for the generation of a magnetic field is the inverse Faraday effect, which appears in the case of an elliptically polarized pulse and which produces the nonvanishing nonlinear current (7). The latter occurs due to the spatial inhomogeneity of the pulse,[1)] which is of limited extent in the longitudinal and transverse directions. We shall restrict ourselves below to considering axisymmetric electromagnetic pulses propagating along the z axis. Equation (4) then determines one component, $A_\varphi=A$, of the vector potential and, correspondingly, two components, $B_r=-\partial A/\partial z$ and $B_z=(1/r)(\partial(rA)/\partial r)$, of the magnetic field. Writing the field of the laser pulse in the form $\mathbf{E}_0=(\mathbf{e}_x+i\lambda\mathbf{e}_y)E_0(r,z-v_gt,t)+\text{c.c.}$, we have the following equation for the vector potential:

$$\frac{1}{c^2}\frac{\partial^2A}{\partial t^2}-\left(\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial A}{\partial r}+\frac{\partial^2A}{\partial z^2}\right)+\left(\frac{\omega_p^2}{c^2}+\frac{1}{r^2}\right)A=-\frac{\partial B_0}{\partial r}, \tag{9}$$

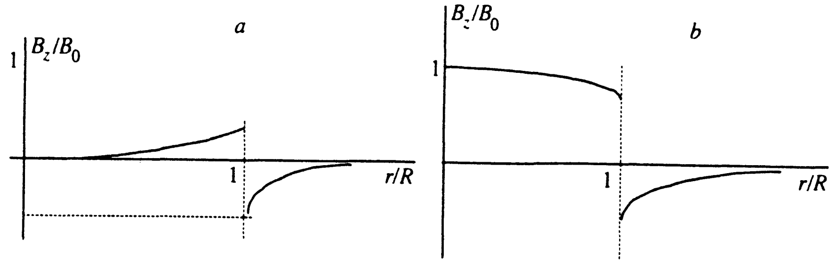
where $B_0\equiv(e\lambda\omega_p^2/4m\omega_0^3c)E_0^2(r,z-v_gt,t)$ is the characteristic magnitude of the magnetic field, λ is the degree of ellipticity of the radiation ($|\lambda|\leqslant1,\lambda=0$: plane polarization, $\lambda=1$: circular polarization), and v_g is the group velocity of the laser pulse. It is convenient to change in what follows to a frame of reference moving with the pulse, introducing the variable $\xi=z-v_gt$. Moreover, it is natural to assume that the pulse length is appreciably longer than the wavelength, $L\gg2\pi c/\omega_0$, and that its shape changes slowly compared to its length $\tau=L/c$. Equation (9) for A can then be simplified:

$$\frac{2}{c}\frac{\partial^2A}{\partial\xi\partial t}+\frac{1}{r}\frac{\partial}{\partial r}r\frac{\partial A}{\partial r}-\left(k_p^2+\frac{1}{r^2}\right)A=-\frac{\partial B_0}{\partial r}, \tag{10}$$

where $k_p=\omega_p/c$. Equation (10) is linear and its solution can be written in general form. To do this we take the Fourier–Bessel transform[8] with respect to the radial variable,

$$A(s,\xi,t)=\int_0^\infty drrJ_1(sr)A(r,\xi,t),$$

where J_1 is a Bessel function, and the two-sided Laplace transform with respect to time. As a result we obtain a first-order equation in the axial variable ξ, the solution of which, with the boundary condition that there is no field in front of the pulse, has the form:



$$A(s,\xi,p)=\frac{sc}{2p}\int_0^\infty d\xi' B_0(s,\xi+\xi',p)$$

$$\times\exp\left(-c\xi'\frac{s^2+k_p^2}{2p}\right), \quad (11)$$

where $B_0(s,\xi,p)=\int dt\int dr rJ_0(sr)B_0(r,\xi,t)$.

Taking the inverse transform with respect to the radial variable and time, we get the following expression, which describes the space and time dependence of the vector potential of the quasistatic field:

$$A(r,\xi,t)=\frac{1}{2\pi i}\int_{-i\infty+\sigma}^{i\infty+\sigma} dp e^{pt}\int_0^\infty ds sJ_1(sr)\frac{sc}{2p}\int_0^\infty d\xi' B_0$$

$$\times(s,\xi+\xi',p)\exp\left(-c\xi'\frac{s^2+k_p^2}{2p}\right), \quad (12)$$

where the integration contour for the variable p passes in the complex p plane vertically upwards to the right of all singularities of the integrand ($\sigma>0$).

Equation (12) describes the nonlocal relationship of the electromagnetic field (A) to its source (B_0). A local relationship between A and B_0 arises for smoothly inhomogeneous beams, $k_pR\gg 1$, over sufficiently long times and distances from the leading front of the beam, $k_p^2\xi ct\gg 1$. Under those conditions, Eq. (12) becomes the well-known expression for the stationary inverse Faraday effect: $A=-k_p^2\partial B_0/\partial r$.

3. DYNAMICS OF ELECTROMAGNETIC FIELD GENERATION BY A SHORT RECTANGULAR LASER PULSE

We can see the basic features of the dynamics of the electromagnetic field from the example of a uniform pulse of length L and radius R appearing in the plasma at time $t=0$. The source then takes the form

$$B_0(r,\xi,t)=B_0\theta(R-r)\theta(t)[\theta(-\xi)-\theta(-\xi-L)], \quad (13)$$

where $\theta(x)$ is the Heaviside step function. The field in the plasma can be written as a superposition of the fields of two pulses of "positive" ($B_1=B_0$) and "negative" ($B_2=-B_0$) polarity, shifted along the ξ axis by a distance L with respect to one another:

$$A(\xi,r,t)=A_0(\xi,r,t)-A_0(\xi+L,r,t), \quad (14a)$$

where A_0 is the field of an infinitely long pulse:

$$A_0(\xi,r,t)=B_0R\int_0^\infty \frac{sds}{s^2+k_p^2}J_1(sR)J_1(sr)$$

$$\times[1-J_0(a\sqrt{s^2+k_p^2})]. \quad (14)$$

Here $a=\sqrt{-2ct\xi}$, and A_0 is nonzero only for $\xi<0$. We note that the dependence on the longitudinal coordinate occurs in (14) solely in the form of the product ξt. The potential A_0 thus reaches for large times, $k_pa\gg 1$, a stationary expression which depends only on the radial coordinate:

$$A_{0,st}(r)=B_0R\int_0^\infty \frac{sds}{s^2+k_p^2}J_1(sR)J_1(sr)$$

$$=\frac{B_0R}{\pi}\int_0^\pi d\varphi\cos\varphi K_0(k_p\rho), \quad (15)$$

where $\rho=\sqrt{r^2+R^2-2rR\cos\varphi}$. In the steady state, only the axial component of the magnetic field $B_z(r)$ remains, which is localized along the length of the pulse: $-L<\xi<0$.

It follows from (15) that for a broad pulse, $k_pR\gg 1$, the vector potential is nonvanishing only in a ring of width $\sim k_p^{-1}$ near the edge of the beam. The magnetic fields inside the beam and outside it are then in opposite directions. Such a configuration corresponds to the magnetic field of a solenoid which has the surrounding plasma at its surface (Fig. 1a). The jump in the field at $r=R$ is connected with the step form of the pulse, due to which the drag current turns out to be localized on its surface.

It follows from Eq. (15) that for a narrow pulse, $k_pR\ll 1$, the field $B_z(r)$ is nonvanishing and essentially uniform [$B_z(r)\cong B_0$] inside the pulse ($r<R$), and vanishes outside it. $B_z(r)$ varies appreciably only near the beam (Fig. 1b). We note that in this case the magnetic field outside the beam also changes sign, but that its magnitude is smaller by a factor $(k_pR)^2$ than the field inside the beam.

The field dynamics are described by the second term in Eq. (14); it is convenient to write it in the form:

$$A_{0,N}(\xi,r,t)=\frac{B_0R}{\pi}\int_0^\pi d\varphi\cos\varphi\int_0^\infty \frac{sds}{s^2+k_p^2}J_0(s\rho)$$

$$\times J_0(a\sqrt{s^2+k_p^2}).$$

The potential $A_{0,N}$, together with the quasistatic field B_z, also determines the radiation field $B_r=-\partial A/\partial\xi$ and $E_\varphi=\partial A/\partial\xi-(1/c)(\partial A/\partial t)$:

$$B_r=\frac{\partial A_{0,N}}{\partial a}\sqrt{\frac{ct}{-2\xi}},\quad E_\varphi=-\frac{\partial A_{0,N}}{\partial a}\left(\sqrt{\frac{-\xi}{2ct}}-\sqrt{\frac{ct}{-2\xi}}\right), \quad (16)$$

where

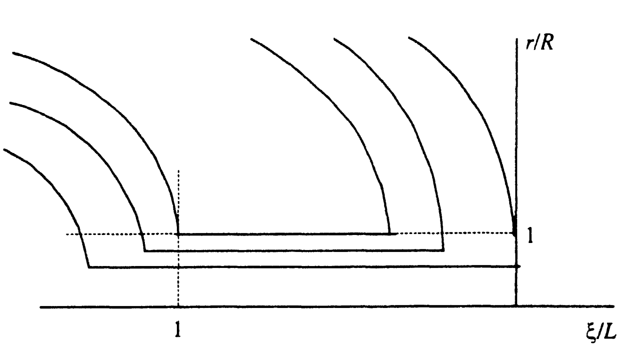


FIG. 2.

$$\frac{\partial A_{0,N}}{\partial a}=\frac{B_0R}{\pi a}\int_0^{\pi} d\varphi \cos\varphi\theta(a-\rho)J_0(k_p\sqrt{a^2-\rho^2}). \tag{17}$$

One can see from this last Eq. (17) that the fields B_r and E_φ are nonvanishing only inside a conical layer of width $2R$ with its center at the leading edge of the beam (Fig. 2):

$$|a-R|<r<a+R. \tag{18}$$

For a broad beam, $k_pR\gg 1$, these fields are also, like the field B_z, localized near the edge of the beam in a ring of width $\Delta r\sim k_p^{-1}$. These fields therefore turn out to be negligible at distances $\Delta\xi\sim R$ from the head of the beam.

A qualitatively different situation is realized for a narrow beam, $k_p^{-1}R\ll 1$. The radiation field occupies here a considerable volume of plasma outside the beam $r\leqslant 1/k_p^2R$. In that case in the region (18) we have $k_p^2(a^2-\rho^2)\ll 1$, and for $\partial A_{0,N}/\partial a$ we obtain

$$\frac{\partial A_{0,N}}{\partial a}\cong\frac{B_0R}{\pi a}\left[1-\left(\frac{r^2+R^2-a^2}{2rR}\right)^2\right]^{1/2}, \tag{19}$$

whence it follows that B_r and E_φ reach a maximum for $r^2=a^2-R^2$, with $\partial A_{0,N}/\partial a$ given by Eq. (19).

In contrast to B_z, the B_r and E_φ fields transfer energy. The associated Poynting vector $S_z=(c/8\pi)(\partial A/\partial a)^2[1-(ct/\xi)]$ describes energy losses from the laser pulse to radiation of the electromagnetic field. The total radiative energy flux $Q=\int 2\pi r S_z(r)dr\cong(caB_0^2R^3/4\pi^2\xi^2)$ increases for $a\gg R$ with time and reaches a maximum value $Q_{\max}\cong cB_0^2/8\pi^2k_p^2$ for $a\geqslant r_{\max}=1/k_p^2R$. It is easy to show that radiative losses from a laser pulse are always small in a rarefied plasma: $Q_{\max}\ll E_0^2R^2$, which corresponds to the fact that the radiation fields are small compared to the field of the laser pulse itself.

We note that although the characteristic frequency of the radiation field is small compared to the laser frequency, it is nonetheless large compared to the electron plasma frequency, and it decreases with distance from the leading edge of the pulse. At distances $r\sim r_{\max}$, this frequency is comparable to the plasma frequency, and as a result the radiation field is screened at distances $r>r_{\max}$.

According to Eq. (13), at the trailing edge of the laser pulse there arises yet another radiation cone in which the fields B_r and E_φ have opposite signs, but the Poynting vector turns out to be the same as in the cone arising from the leading edge of the pulse. The cones from the leading and trailing edges do not overlap if the pulse length is greater than its radius, $L>R$. In the opposite case, $L\ll R$, the radiation field is weakened due to the interference of the radiation fields produced by the leading and trailing edges. The attenuation coefficient of the fields (16) in a short pulse has the form $(L/r)\sqrt{ct|\xi|}$.

4. GENERATION OF ELECTROMAGNETIC FIELDS BY A FOCUSED GAUSSIAN LASER PULSE

The time dependence of the fields generated shows a different character when a laser pulse is focused in a plasma by a lens of optical power $1/f$. In that case, the pulse appears in the plasma not instantaneously (as was assumed in Sec. 3), but propagates in the plasma smoothly, changing its characteristics with time. We shall assume that the pulse has a Gaussian shape:

$$B_0(r,\xi,t)=B_0\frac{R_0^2}{R^2(t)}\exp\left\{-\frac{r^2}{R^2}-\frac{\xi^2}{L^2}\right\}, \tag{21}$$

where L is the length of the pulse, $R_0=f/k_0$ is the minimum radius of the laser beam in the focal plane, f is the ratio of the focal length of the lens to its diameter, and $R^2(t)=R_0^2+c^2t^2/f^2$ is the instantaneous radius of the beam; we take as $t=0$ the moment the pulse passes through the middle of the caustic. The potential of the field in the plasma then has the form, according to (12),

$$A(r,\xi,t)=\frac{B_0f}{4i\sqrt{\pi}}\int\frac{dp}{p}\int dss J_1\left(s\frac{r}{R_0}\right)\times\exp\left\{-\frac{s^2}{4}+p^2+ps\frac{ct}{fR_0}\right\}\int d\xi'\times\exp\left\{-\frac{(\xi+\xi')^2}{L^2}-k_0\xi'\frac{s^2+k_p^2R_0^2}{2ps}\right\}. \tag{22}$$

Using the fact that the pulse is long compared to its wavelength ($k_0L\gg 1$), we can drop the term ξ'^2/L^2 in the argument of the exponential. We then get the simpler expression

$$A(r,\xi,t)=\frac{1}{2}B_0R_0\exp\left\{-\frac{\xi^2}{L^2}\right\}\times\int ds\frac{sJ_1\left(s\frac{r}{R_0}\right)\exp\left\{-\frac{s^2}{4}\left(1+\frac{t^2}{\tau^2}\right)\right\}}{k_p^2R_0^2+s^2\left(1-\frac{2\xi t}{k_0L^2\tau}\right)}, \tag{23}$$

where $\tau=(c/R_0f)^{-1}$ is the time it takes the pulse to pass through the Rayleigh length fR_0. The assumption that the pulse parameters vary slowly with time, used in writing down Eq. (10), corresponds to the condition $L\ll c\tau\equiv fR_0$. We start the analysis of the nature of the excited fields with the case of a dense plasma ($k_pR_0\gg 1$). In that case, the potential (23) yields the quasistationary solution for the electromagnetic fields

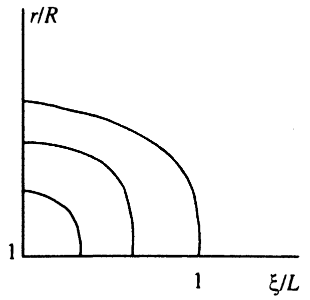


FIG. 3.

$$A(r,\xi,t)=\frac{2B_0R_0^2}{k_p^2R^4}\,r\exp\left\{-\frac{\xi^2}{L^2}-\frac{r^2}{R^2}\right\}. \tag{24}$$

The field lines of the excited magnetic field are shown in Fig. 3. It is clear that the field lines are closed and the maximum magnetic field strength is reached on the beam axis ($r=0$):

$$B_{\max}=\frac{4B_0R_0^2}{k_p^2R^4}.$$

The effect of the excited fields being nonstationary is small and reveals itself in higher orders of the small parameter $(k_pR)^{-1}\ll1$. In the case of a rarefied plasma ($k_pR_0\ll1$), the nonstationary corrections are also small with respect to the parameter $(k_pL)^{-1}\ll1$:

$$A(r,\xi,t)=B_0\frac{R_0^2}{r}\exp\left\{-\frac{\xi^2}{L^2}\right\}\left(1-\exp\left\{-\frac{r^2}{R^2}\right\}\right)\times\left(1+\frac{2\xi t}{k_0L^2\tau}\right). \tag{25}$$

It is clear from Fig. 4 that the maximum of the magnetic field, $B_{\max}\cong2B_0R_0^2/R^2$, is then reached on the beam axis. The field lines of the magnetic field are not closed (they are closed when we take into account higher-order corrections $\sim(k_pL)^{-1}\ll1$), and there is a magnetic field outside the pulse as well.

When the pulse passes through the caustic ($|t|\ll\tau$) at distances $r\ll k_p^{-1}$, the solution (25) is close to stationary.

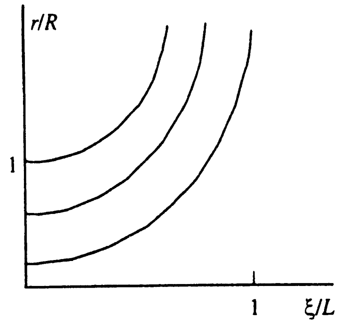


FIG. 4.

Nonstationary effects may be important outside the caustic for $|t|\gg f^2L/c$. The total energy flux transferred by the B_r and E_φ fields is

$$Q=-\alpha Q_0, \tag{26}$$

where

$$\alpha\cong\frac{\lambda^2}{2}\left(\frac{v_E}{c}\frac{\omega_p^2}{\omega_0^2}\right)^2\left(\frac{R_0}{L}\right)^4\left(\frac{\xi}{R}\right)^2\begin{cases}\dfrac{1}{f^4}, & k_pR_0\gg1,\\[2mm] \dfrac{\ln2}{8}, & k_pR_0\ll1,\end{cases}$$

and Q_0 is the energy flux of the incident pulse (21). It is clear from (26) that the radiative energy losses by the laser pulse are always small. The maximum value of Q is reached for $k_pR_0\ll1$ in the caustic at the leading and trailing edges of the pulse ($|\xi|\sim L$), where we have $\alpha\sim(v_E/c)^2(\omega_p/\omega_0)^4$.

5. CONCLUSION

The analysis given here shows that a laser pulse in a plasma is a source not only of an electrostatic field, but of a rotational electromagnetic field as well. The latter is localized along a length c/ω_p around the laser pulse, and is approximately ω_0/ω_p times weaker than the maximum electrostatic field. However, the rotational and potential fields depend in different ways on the laser pulse parameters, which makes it possible easily to distinguish one from the other. Whereas the wake field is efficiently excited only by a short laser pulse of length $L\leqslant c/\omega_p$, the magnetic field strength on the beam axis is essentially independent of its length.

In the case of a narrow beam, $R\ll c/\omega_p$, the wake field is concentrated within a cylinder with a radius equal to the beam radius behind its trailing edge, whereas the electromagnetic field is concentrated in a disk of radius $\sim c/\omega_p$ and thickness of the order of the pulse length.

The electromagnetic field of a laser pulse may turn out to be a convenient means of remote detection of the passage of ultrashort laser pulses through a plasma. For instance, a laser pulse of wavelength 0.5 μm, energy $\sim$1 J, and length $\sim$1 ps in a plasma of density 10^{18} cm^{-3} produces an electric pulse of amplitude 10^{-3} V/cm, which can easily be detected.

The axial magnetic field produced by a laser pulse may turn out to be useful for laser acceleration of electrons, confining them to the pulse axis. We note that the dependence of the magnetic field on the shape of the laser pulse makes it possible to produce special magnetic field configurations that provide for particle containment within a required region of space for a fairly long time.

[1)]The rotational drag current (7) also results from plasma density irregularities,[4] which, however, we will not consider here.

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