Acceleration of charged particles by laser beams

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The acceleration of charged particles by a light field is considered. The latter has a field intensity **E**, the time-averaged gradient of the square or cube of which is nonzero. The functional dependence of the applied energy dK/dz as a function of the particle energy K ($K \gg mc^2$) and the power density P of the laser beams assumes the form $dK/dz = A(P/P_0)(mc^2/K)$ for an accelerating force $\mathbf{F} \propto \nabla E^2$ and $dK/dz = A(P/P_0)^{3/2}(mc^2/K)^2$ for an accelerating force $\mathbf{F} \propto \nabla E^3$. For wavelengths $\lambda \approx 1\ \mu$m the light field is characterized by $P_0 \approx 1.4 \cdot 10^{18}$ W/cm^2, for which the quiver velocity of a nonrelativistic electron would be of order c and the coefficient would satisfy $A \approx 0.5 \cdot 10^{10}$ eV/cm. The possibility is discussed of using either an ordinary Gaussian or a conical (Bessel) beam in the form of short packets accompanying ultrarelativistic particle bunches at an arbitrary angle between the photons and the beam axis. Estimates are given for bounds on the resulting energy of the accelerated particles associated with bremsstrahlung. It is noted that the irradiation of the electron beam by the accompanying light packet permits an effective source of hard bremsstrahlung photons to be achieved.

1. INTRODUCTION

The acceleration of elementary particles by means of high-intensity laser beams has been studied by many authors (see, e.g., Refs. 1–5). Many designs for such devices have been proposed. Acceleration, i.e., the transfer of energy from the field to the particles, occurs when the latter is acted on by a periodic external force with a fixed phase. When the phase takes a different value the opposite process occurs: components of the light field are amplified due to transfer of energy from the particle, giving rise to one form or another of free-electron laser. At first glance the most natural way to produce acceleration would appear to be inverse Cherenkov emission. However, the conservation of energy and momentum when a photon with energy $\hbar\omega$ and momentum $\hbar\mathbf{k}$ is absorbed,

$$K(\mathbf{p}_0+\hbar\mathbf{k}) = K(\mathbf{p}_0) + \hbar\omega, \tag{1}$$

requires that the particle velocity $\partial K/\partial \mathbf{p}$ be greater than the velocity of light in the medium, $|\mathbf{v}| \equiv |\partial K/\partial \mathbf{p}| > c/n$. Here n is the index of refraction, $K(\mathbf{p})$ is the particle kinetic energy as a function of momentum, and $\mathbf{p}_0$ is the initial electron momentum. The rate of acceleration which characterizes the increase in particle energy per unit length is of order

$$\frac{dK}{dz} = e|\mathbf{E}|\sin\theta, \tag{2}$$

where $\theta = \arccos(1/n)$ is the Cherenkov angle, $S(\text{erg/cm}^2\cdot\text{s}) = cn|\mathbf{E}|^2/8\pi$ is the Poynting vector of the light wave, and $e = |e|$ is the absolute value of the particle charge. However, in this case the material must be located in a strong field, which for intensities $P \gg 10^{14}$ W/cm^2 inevitably leads to ionization (and possibly breakdown) of the medium. As a result, the medium changes into plasma, for which $n<1$ generally holds and the Cherenkov condition $v > c/n$ cannot be satisfied. (The restrictions on the inverse Cherenkov effect are discussed in greater detail below in Sec. 2.) For very disperse media $n-1 \ll 1$ the factor $\sin\theta$ in Eq. (2) reduces the value of the right-hand side still further.

Another acceleration technique involves waves propagating in vacuum. This is the induced Compton effect or the Kapitsa–Dirac effect,[6] i.e., acceleration by means of the gradient of the radiation (ponderomotive) force.[7] The synchronization condition follows from the energy and momentum conservation laws in an elementary induced-scattering event, i.e., the absorption of a photon $\hbar\omega_1$ and the induced emission of a photon $\hbar\omega_2$:

$$K(\mathbf{p}_0+\hbar\mathbf{k}_1-\hbar\mathbf{k}_2) = K(\mathbf{p}_0) + \hbar\omega_1 - \hbar\omega_2. \tag{3}$$

For ultrarelativistic particles, condition (3) takes the form

$$\frac{\omega_1}{\omega_2} = \frac{1-(\mathbf{n}_2\mathbf{v})/c}{1-(\mathbf{n}_1\mathbf{v})/c}, \tag{4}$$

where $\mathbf{n}_1 = \mathbf{k}_1/k_1$ and $\mathbf{n}_2 = \mathbf{k}_2/k_2$ are the directions in which the light beams propagate. If one of the light beams propagates in the same direction as the particle beam ($\mathbf{n}_1 = \mathbf{e}_z$, $\mathbf{v} = v\mathbf{e}_z$ where $\mathbf{e}_z$ is the unit vector in the z direction), and

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the second beam proceeds in the opposite direction ($\mathbf{n}_2=-\mathbf{e}_z$), then we obtain good overlap between the beam waist of the laser radiation and the particle trajectories. However, in the ultrarelativistic case $K=mc^2/\sqrt{1-(v^2/c^2)}\approx mc^2/\sqrt{2[1-(v/c)]}\gg mc^2$, the synchronization condition (4) requires beams with a very large frequency ratio:

$$\frac{\omega_1}{\omega_2}=\frac{1}{4}\left(\frac{mc^2}{K}\right)^2. \tag{5}$$

Consequently, in the present work we do not treat acceleration by comoving or oppositely directed waves.

If we use so-called Bessel beams, i.e., conically converging beams (see, e.g., Ref. 4 and Sec. 3 of the present work), then $|\mathbf{vn}|/c<1$ and the frequency ratio ω_1/ω_2 is much greater than unity. Acceleration of relativistic particles by a pair of waves propagating at finite angles θ_1 and θ_2 with respect to the velocity vector $\mathbf{v}$ is treated in Sec. 4.

In Sec. 5 we discuss the possibility of acceleration due to the action of three waves, in which two photons are absorbed and a third is emitted (or vice versa) in a single event:

$$K(\mathbf{p}_0+\hbar\mathbf{k}_1+\hbar\mathbf{k}_2-\hbar\mathbf{k}_3)=K(\mathbf{p}_0)+\hbar\omega_1+\hbar\omega_2-\hbar\omega_3. \tag{6}$$

For this the field in the rest frame of the particle must have a nonvanishing average cube; see Ref. 8.

In Sec. 6 we give estimates of the energy attained by the accelerated particles, which is limited by bremsstrahlung. As will be seen from the results of Secs. 4 and 5, acceleration of relativistic particles by a force $\mathbf{F}\propto\nabla E^3$ is less than that produced by a force $\mathbf{F}\propto\nabla E^2$ by a factor $(mc^2/K)(eE/m\omega c)$. Nevertheless, acceleration by a force $\mathbf{F}\propto\nabla E^3$ has a unique property: it can be achieved using beams of a single frequency, i.e., pulses from a single laser when the angles between the photons and the electron beam axis are appropriately chosen.

A relativistic electron traverses the interaction length $L=10$ cm in a time of order $T=L/c=3\cdot 10^{-10}$ s. From energy considerations, it is essentially impossible to maintain this extraordinarily high laser radiation power density over such a long time. An ultrashort light pulse (a "flash") of length $\tau_0\ll L/c$ (e.g., $\tau_0=10^{-13}$ s) can stay with a bunch of ultrarelativistic electrons over the entire distance L when they are comoving. However, for acceleration schemes involving forces $\mathbf{F}\propto\nabla E^2$ and $\mathbf{F}\propto\nabla E^3$ the electron beam must propagate at an angle with respect to the photons. The question is then whether a light "flash" of obliquely incident photons $\tau_0=10^{-13}$ s long can stay with a relativistic electron bunch. The answer turns out to be affirmative, and the corresponding treatment, which is presented in Sec. 3, yields a favorable estimate for the total energy required for the pulse.

2. ABSENCE OF ACCELERATION TO FIRST ORDER IN THE VACUUM FIELD

The direct Cherenkov effect consists of energy transfer from a particle to the field under the assumption that the resonance $\omega=\mathbf{kv}$ holds between the particle velocity and the corresponding projection of the wave phase velocity. When phase resonance occurs, either stimulated Cherenkov emission or the inverse Cherenkov effect can occur, depending on the phase of the field along the particle trajectory. Since this phase is constant on the particle trajectory, energy is transferred even to first order in the field amplitude.

The failure of the resonance condition $\omega=\mathbf{kv}$ is obvious in vacuum for a plane monochromatic wave $E\propto\exp(-i\omega t+i\mathbf{kr})$. For a light pulse with complicated spatial structure it is less obvious. In some treatments published on the subject of laser acceleration of particles, complicated light-field configurations and pulses are proposed, for the description of which recourse to approximate expressions is unavoidable. Although the approximate expressions themselves contribute little relative error, using them can lead to erroneous results regarding energy exchange between the field and particle.

Since in the present work we will be discussing irradiating beams with fairly complicated space–time dependence, it is important for us to demonstrate that no energy exchange takes place to first order in the field for arbitrary light beams.

The energy δK transferred from the field to a particle with charge e moving on the trajectory $\mathbf{r}=\mathbf{r}(t)$ is equal to

$$\delta K=e\int_{-\infty}^{\infty}\mathbf{E}_{\rm real}[\mathbf{r}(t)]\,\frac{d\mathbf{r}}{dt}\,dt, \tag{7}$$

since the magnetic field performs no work. (Here $\mathbf{E}_{\rm real}$ is the instantaneous real intensity of the field of the light wave.) Assume that the unperturbed motion of the particle is given:

$$\mathbf{r}(t)=\mathbf{r}_0+\mathbf{v}_0 t. \tag{8}$$

Then to first order we find an expression for δK:

$$\delta K^{(1)}=e\int_{-\infty}^{\infty}\mathbf{v}_0\mathbf{E}(\mathbf{r}_0+\mathbf{v}_0 t)dt+O(E^2). \tag{9}$$

We present an example of an approximate prescription for the field which yields a clearly incorrect result for the energy transfer to first order. Specifically, let us consider the usual Gaussian approximation for a focused laser beam:

$$\mathbf{E}_{\rm real}(\mathbf{r},t)=\frac{\mathbf{e}_x}{2}\left\{\frac{E_0}{1+iz/ka^2}\exp\left[-i\omega t+ikz-\frac{x^2+y^2}{2a^2(1+iz/ka^2)}\right]+\text{c.c.}\right\}, \tag{10}$$

where $k=\omega/c=2\pi/\lambda$; here c is the velocity of light in vacuum, a is the beam half-width at the beam waist, measured at the point where the intensity is a factor e^{-1} lower than the maximum value;[1)] $\Delta x=\Delta y=a$, and it is assumed that the angular divergence $\delta\theta=1/ka$ in the far field is much less than one radian. We find δK for the trajectory

$$\mathbf{r}_0=0,\quad \mathbf{v}_0=v_0\mathbf{e}_x, \tag{11}$$

i.e., for a particle passing through the focal beam waist perpendicular to the central direction of the beam. Substituting (8) and (10) in (9) we find

$$\delta K^{(1)}=\sqrt{\frac{\pi}{2}}\,ae(E_0+E_0^*)\exp\left(-\frac{c^2}{2v_0^2\delta\theta^2}\right). \qquad (12)$$

We estimate $v_0=0.97c$, so that the initial kinetic energy of the particle is

$$K_0=mc^2\left[\frac{1}{\sqrt{1-v_0^2/c^2}}-1\right]\approx 3.1m_0c^2.$$

From Eq. (12) we see that the Gaussian beam (10) contains elementary plane waves with $k_x=\omega/v_0=1.03(\omega/c)$, which satisfy the Cherenkov condition $\omega=\mathbf{k}\mathbf{v}_0$. Assume that the angular divergence $\delta\theta$ of the beam is of order $\delta\theta\approx 0.3$ rad $\approx 17°$, a very reasonable value. Then (12) yields

$$\delta K^{(1)}=\sqrt{\frac{\pi}{2}}\,ae(E_0+E_0^*)\exp(-5.9)$$

$$\approx 2.7\cdot 10^{-3}\sqrt{\frac{\pi}{2}}\,ae(E_0+E_0^*).$$

In the present section we show that to first order in the field, the energy δK vanishes identically. This can be proved most simply using the Fourier expansion of the field in vacuum:

$$\mathbf{E}(\mathbf{r},t)=\frac{1}{2}\int \mathbf{E}(\mathbf{k})\exp(i\mathbf{k}\mathbf{r}-ic|\mathbf{k}|t)d\mathbf{k}+\text{c.c.} \qquad (13)$$

Substituting (8) and (13) in (7), we find

$$\delta\mathbf{K}^{(1)}=e\pi\int \mathbf{v}_0\mathbf{E}(\mathbf{k})\exp(i\mathbf{k}\mathbf{r}_0)\delta(\mathbf{k}\mathbf{v}_0-c|\mathbf{k}|)d\mathbf{k}+\text{c.c.} \qquad (14)$$

Since $|\mathbf{v}_0|<c$ for the particle, the argument of the δ function vanishes nowhere, and hence $\delta K^{(1)}=0$. The vanishing of the argument of $\delta(x)$ in Eq. (14) corresponds to the Cherenkov condition, which must hold in a medium with index of refraction $n>c/v_0$ ($\omega=c|\mathbf{k}|/n<kv_0$), but not in vacuum.

We can provide an explanation for the erroneous result (12). The Gaussian expression (10) corresponds to the parabolic approximation to the exact wave equation, in which the exact value k_z^{exact} of the wave vector is replaced by the parabolic approximation k_z^{parab}:

$$k_z^{\text{exact}}=\pm\sqrt{\left(\frac{\omega}{c}\right)^2-k_x^2-k_y^2},\quad k_z^{\text{parab}}=\frac{\omega}{c}-\frac{c}{2\omega}(k_x^2+k_y^2). \qquad (15)$$

In Fig. 1 we show that the Cherenkov condition $\omega=\mathbf{k}\mathbf{v}_0$ cannot be satisfied in the exact wave equations. But if we carelessly use the parabolic approximation, then in a Gaussian beam the individual components with a plane wave front will be found to satisfy this condition. We have derived the expression (14) for arbitrary particle velocity $\mathbf{v}_0$ with $|\mathbf{v}_0|<c$, arbitrary beam focusing, and arbitrary momentum profile. The only restriction is that Maxwell's equations hold throughout all space.

If the medium is spatially nonuniform and (or) refracting, the result $\delta K^{(1)}=0$ is no longer correct. However,

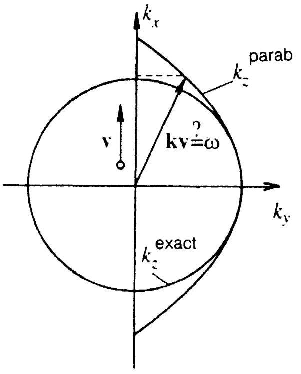

FIG. 1. Angular distribution of wave vectors corresponding to the exact wave equation and the parabolic approximation. The difference between the two distributions can give rise to substantial errors when the motion of a particle perpendicular to a Gaussian beam is treated.

it is impossible to obtain large values of $\delta K^{(1)}$ using only remote lenses and low power densities applied to them. It is necessary either to direct intense light beams into the media (with all the problems that stem therefrom, involving breakdown of the medium) or to use schemes in which energy transfer begins to second or third order in the field amplitudes (see below). Another possibility is to use static external fields (which in the electron rest frame will obviously no longer be static; see, e.g., Ref. 5).

The main conclusion of the present section is that to transfer energy to first order from the electromagnetic field to a free elementary particle, it is necessary to modify the field by means of some medium (uniform or nonuniform). Other schemes which purport that energy $\delta K^{(1)}$ is transferred to the particles to first order in E in vacuum are fundamentally incorrect.

3. FOCUSING OF A "FLASH" MOVING WITH THE SPEED OF LIGHT

In a number of acceleration schemes considered below it is necessary to use a high-density wave over a relatively long distance $\geqslant 10$ cm. We will be concerned with accelerating very short ultrarelativistic particle bunches with $\tau_0\sim 10^{-11}$–10^{-14} s. A wave comoving with the particles can be represented by an equally short light pulse and can stay with the particles over the whole interaction length L. Then the specified power density would be required only over a very short time τ_0, and the total energy of the pulse would be modest. But if the photons propagate at an angle to the particle beam or in the opposite direction, then it appears at first glance that the light pulse must have duration $\tau_{\text{opt}}\propto L/2c$, which exceeds the length of the accelerated particle bunch by several orders of magnitude. In the present section we give a solution of the Maxwell equations in vacuum describing a wave packet with very special properties. Specifically, the optical field at each time is focused only in a region of length $\Delta z=u\tau_0$ parallel to the z axis, which gives rise to considerable economy in the total

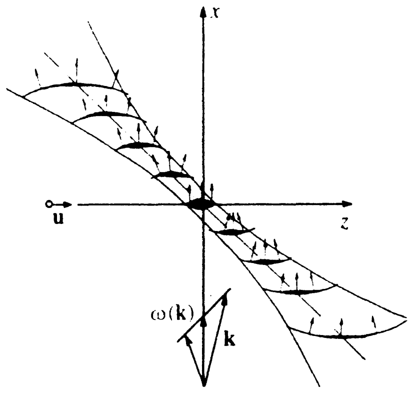

FIG. 2. Setup for a two-dimensional "flash" accompanying a relativistic beam of accelerated particles. The flash propagates in the z direction along with the particles.

power of the optical pulse. Hence the "flash" moves along the z axis with constant speed $dz/dt=u$, moving together with the particle bunch.

Two important facts should be noted. First, the speed u with which the flash moves can be arbitrary: it can be less than, greater than, or equal to the velocity of light c. Second, the photons can move at an arbitrary angle θ_0 to the z axis (see Fig. 2). The parameters of the light packet are determined by its frequency–angle spectrum.

The d'Alembert equation describing the evolution of each Cartesian component of the field in vacuum takes the form

$$\frac{1}{c^2}\frac{\partial^2 E}{\partial t^2}-\Delta E=0. \qquad (16)$$

We begin by directing our attention to the two-dimensional problem $E=E(x,z,t)$. It can easily be verified that the following expression for the field is an exact solution of Eq. (16):

$$E(x,z,t)=\exp(-i\omega_0 t+ik_{0z}z+ik_{0x}x)f\left(t-\frac{z}{u},x\right), \qquad (17)$$

$$f(t',x)=\int_{-\omega_0}^{\infty}\exp\left\{-i\Omega t'+ix\left[\sqrt{\left(\frac{\omega_0+\Omega}{c}\right)^2-\left(k_{0z}+\frac{\Omega}{u}\right)^2}-k_{0x}\right]\right\}F(\Omega)d\Omega, \qquad (18)$$

where $F(\Omega)$ is an arbitrary function and

$$k_{0z}=\frac{\omega_0}{c}\cos\theta_0,\quad k_{0x}=\frac{\omega_0}{c}\sin\theta_0. \qquad (19)$$

The frequency of the light packets (17)–(19) depends on angle. The parameter θ_0 characterizes the average angle at which photons intersect the z axis (see Fig. 2).

Let us consider the most interesting case, when the length τ_0 of the intensity envelope of the pulse on the z axis is substantially greater than the light period, $\omega_0\tau_0\gg 1$. Then from the uncertainty relation, the spectral density $|F(\Omega)|^2$ is significantly different from zero only for $|\Omega|\leqslant\tau_0^{-1}\ll\omega_0$, so the lower limit in the integral of Eq. (18) can be replaced by $-\infty$. As always, the simplest formulas result for Gaussian packets:

$$f(t',x=0)=E_0\exp\left(-\frac{t'^2}{2\tau_0^2}\right)=\frac{E_0\tau_0}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\exp\left(-\frac{\Omega^2\tau_0^2}{2}\right)\times\exp(-i\Omega t')d\Omega. \qquad (20)$$

For these, due to the large value of the parameter $\omega_0\tau_0$, the envelope $f(t',z)$ can be calculated by the method of stationary phase:

$$f\left(t-\frac{z}{u},z\right)=\frac{E_0}{(1+ix/x_0)^{1/2}}\exp\left[-\frac{(t-z/u-vx/u)^2}{2\tau_0^2(1+ix/x_0)}\right], \qquad (21)$$

$$v=\operatorname{tg}\beta=\frac{1-(c/u)\cos\theta_0}{\sin\theta_0}\frac{u}{c},\quad x_0=\frac{\omega_0}{c}\frac{u^2\tau_0^2\sin^3\theta_0}{[1-(u/c)\cos\theta_0]^2}. \qquad (22)$$

From this expression we see that the spatial variation of the intensity at a specified point (x,z) also is Gaussian, but the duration increases as

$$\tau(x)=\tau_0\sqrt{1+(x/x_0)^2}. \qquad (23)$$

In particular, for $u=c$ and $\theta_0=90°$, the magnitude of x_0 is related to the size $\Delta z=c\tau_0$ of the flash on the axis by

$$x_0=\frac{\omega_0}{c}(\Delta z)^2. \qquad (24)$$

Just this kind of relation holds between the length of the beam waist (in this case, x_0) and its radius (Δz) when a Gaussian beam of photons propagating in the x direction is focused. We emphasize that for $u=c$ and $\tau_0\omega_0\gg 1$, we will have $\Delta z\gg c/\omega_0$, since $x_0\gg\Delta z$. This implies a moderate convergence (or divergence) angle $\Delta\theta$ for the focused photons relative to one another.

The spatial distribution of the intensity $|E(x,z,t)|^2$ at a given time t consists of a region inclined with respect to the z axis by the angle $-\beta$, where $v=\tan\beta$ is defined by Eq. (22). The width Δz of this region is smallest on the z axis, and away from it increases as

$$\Delta z(x)=u\tau_0\sqrt{1+(x/x_0)^2}. \qquad (25)$$

If we assume that the packet is unbounded in time, $-\infty<t<+\infty$, then the high-intensity region moves in the z direction with velocity u. A bounded packet, $0<t<T$, where $T=L/u\gg\tau_0$, has a beginning and an end; they both naturally move with velocity

$$v_z=c\cos\theta_0,\quad v_x=c\sin\theta_0. \qquad (26)$$

A curious circumstance should be noted. For the most interesting case $u=c$, the angle β of the orientation of the instantaneous optical energy distribution in space is equal to

$$\beta=\theta_0/2. \qquad (27)$$

In other words, the planes of constant amplitude $x=-z\tan\beta$ (for $t=$const) differ from the planes of constant phase; the latter are perpendicular to the direction of the wave vector (19) and hence are characterized by the equation $x=-z\tan\theta_0$. But if they were the same, then the translational velocity of the beam in the z direction would be $u=c/\cos\theta_0$.

In the three-dimensional case it is convenient to represent the solution in the form of Bessel packets; for $x=r\cos\varphi$, $y=r\sin\varphi$ we have

$$E_m(r,\varphi,z,t)=\exp(im\varphi-i\omega_0t+ik_{0z}z)f_m\left(r,t-\frac{z}{u}\right), \quad (28)$$

$$f_m(r,t')=\int_{-\omega_0}^{\infty}F_m(\Omega)J_m\left[r\sqrt{\left(\frac{\omega_0+\Omega}{c}\right)^2-\left(k_{0z}+\frac{\Omega}{u}\right)^2}\right]\times\exp(-i\Omega t')d\Omega. \quad (29)$$

As in the two-dimensional case, for $\tau_0\omega_0\gg1$ and $u\sim c$ the extent of the focal region $\Delta z=u\tau_0$ is much larger than a wavelength. Hence near the z axis itself the Bessel function can be taken outside the integrand:

$$E_m(r,\varphi,z,t)=E_m\exp(-i\omega_0t+ik_{0z}z+im\varphi)\times\exp\left[-\frac{(t-z/u)^2}{2\tau_0^2}\right]J_m\left(r\frac{\omega_0}{c}\sin\theta_0\right). \quad (30)$$

In order to be specific we have taken a Gaussian profile here for the time dependence of the envelope.

We now determine how large the energy of the light packet must be in order that a field of amplitude E_0 accompany an ultrarelativistic particle bunch of length $\sim\tau_0$ over the interaction length $L\gg c\tau_0$.

If the two are to propagate together we can use a laser pulse of the same length τ_0, since in the ultrarelativistic case the particle velocity vector is almost the same as the photon velocity vector. A doubly Gaussian packet (in space and time) has the form

$$E(x,z,t)=\frac{E_0\exp[-i\omega_0t+i(\omega_0/c)z]}{\sqrt{1+iz/z_1}}\times\exp\left[-\frac{x^2}{2a_0^2(1+iz/z_1)}\right]\exp\left[-\frac{(t-z/c)^2}{2\tau_0^2}\right], \quad (31)$$

$$E(x,y,z,t)=\frac{E_0\exp[-i\omega_0t+i(\omega_0/c)z]}{1+iz/z_1}\times\exp\left[-\frac{x^2+y^2}{2a_0^2(1+iz/z_1)}\right]\exp\left[-\frac{(t-z/c)^2}{2\tau_0^2}\right], \quad (32)$$

$$z_1=ka_0^2,\quad a_0=\Delta r, \quad (33)$$

where the radius Δr of the focal constriction is written a_0. If we require that the length Δz of the focal constriction be the same as the interaction length L, then we obtain a condition on the radius a_0 of the constriction:

$$a_0=\sqrt{Lc/\omega_0}. \quad (34)$$

Then in the two- and three-dimensional cases, respectively, the total energy $Q[J]$ carried by the light pulse is equal to

$$Q_{\text{comou}}^{(2D)}=10^{-7}\frac{c|E_0|^2}{8\pi}\sqrt{\pi}a_0\Delta y\sqrt{\pi}\tau_0=10^{-7}\frac{c|E_0|^2}{8}\tau_0\Delta y\sqrt{\frac{L\lambda_0}{2\pi}}, \quad (35)$$

$$Q_{\text{comou}}^{(3D)}=10^{-7}\frac{c|E_0|^2}{8\pi}\pi a_0^2\sqrt{\pi}\tau_0=10^{-7}\frac{c|E_0|^2}{8}\tau_0\sqrt{\pi}\frac{L\lambda_0}{2\pi}. \quad (36)$$

Here 10^{-7} is the factor for converting from ergs to Joules and E_0 and c are assumed to be given in CGSE units; for the two-dimensional problem Δy is the beam thickness in the second transverse coordinate; we assume $\Delta y\gg a_0$.

Let us now estimate the energy required for obliquely propagating waves moving with particles with velocity $u=c$ over a distance L. In the two-dimensional problem this is done most easily by evaluating the x component of the Poynting vector in the $x=0$ cross section and integrating it over time. Specifically, all the photons involved in the process of accelerating particles intersect the z axis at the same time t at some point $(x=0,z)$. As a result we have

$$Q^{(2D)}=10^{-7}\frac{c|E_0|^2}{8\sqrt{\pi}}\tau_0L\Delta y\sin\theta. \quad (37)$$

We see that in the planar (2D) case, oblique copropagation requires substantially higher (by a factor $\sqrt{L/\lambda}$) light energy. It is noteworthy that in the case of Bessel focusing of a three-dimensional (3D) beam, the energy required for copropagation in the oblique case $\theta_0\neq0$ turns out to be essentially the same as for parallel copropagation. For this calculation we use the asymptotic representation of the Bessel functions:

$$J_m(v)\approx\sqrt{\frac{1}{2\pi v}}\left\{\exp\left[i\left(v-m\frac{\pi}{2}-\frac{\pi}{4}\right)\right]+\exp\left[-i\left(v-m\frac{\pi}{2}-\frac{\pi}{4}\right)\right]\right\}. \quad (38)$$

Then we must take the integral at some $r=r_0$ over the surface $2\pi r_0L$ of the cylinder, and over time of that part of the radial component of the Poynting vector which corresponds only to the convergent part of the wave (or equivalently, only the divergent part). The result naturally is independent of the formally introduced parameter r_0 (for $r_0\gg m\lambda_0/2\pi\sin\theta_0$) and takes the form

$$Q_{\text{Bess}}^{(3D)}(m)=10^{-7}\frac{c|E_m|^2}{8\pi}\frac{L\lambda_0}{2\pi}\sqrt{\pi}\tau_0. \quad (39)$$

It is interesting to note that the required energy turns out to be independent of the angle θ_0; this result, however, was obtained neglecting the vector nature of the electromagnetic wave (the polarization factors).

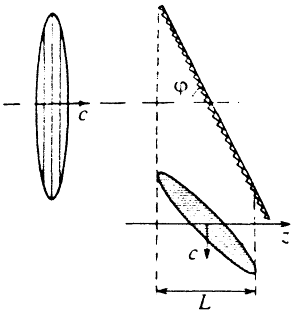

FIG. 3. Use of a diffraction grating to produce a light flash moving with the speed of light c. The period of the lines in the diffraction grating and the angle φ of the cone are chosen so that after reflection the photons propagate perpendicular to the z axis, while the angle β with which the distribution slopes in space is equal to 45°.

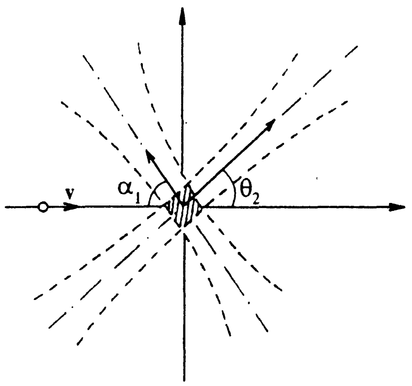

FIG. 4. Scheme for accelerating ultrarelativistic particles using a pair of waves of different frequencies through the ponderomotive force $\mathbf{F}\propto\nabla E^2$.

In the present work it is not our purpose to construct, much less to optimize, the system of prisms, lenses, diffraction gratings, etc., required to shape such packets from ordinary ultrashort pulses. In order to illustrate the possibilities in principle we present only one specific example of a design of this sort (Fig. 3). This design solves the problem of shaping a photon packet propagating perpendicular to the z axis so that the irradiated region moves in the $-z$ direction with the velocity of light c.

We assumed that the original ultrashort pulse consists of a spatial bunch (a relatively flat disk of thickness $c\tau_0$) moving in the z direction. The wavefront surfaces in this bunch are perpendicular to the z axis, and naturally the displacement velocity of the bunch is equal to $\mathbf{v}=\mathbf{e}_z c$. If we neglect transverse diffraction, the bunch moves as a whole with this velocity, the amplitude–phase profile remaining unchanged.

Now assume that this bunch is reflected from a conical diffraction grating with axis in the z direction and vertex angle $\varphi=\arctan 2\approx 63°$. We assumed that the lines of the diffraction grating have a period which is a multiple of $\Lambda=\lambda_0\sqrt{5}$, where λ_0 is the central wavelength of the radiation. Under these conditions, the diffraction grating directs photons precisely toward the z axis. It can easily be verified that the irradiated region will then propagate on the axis (for $x=y=0$) according to $z=\text{const}-ct$, as required. It is clear that a small deviation from these values either of the period Λ or the angle φ or both simultaneously enables us to obtain any desired velocity $u\approx c$ for the flash.

The angle φ of the conical diffraction grating is also easily found for the general case, in which it is necessary to produce convergence of the photons at an angle θ_0 to the z axis (in the previous example we took $\theta_0=90°$), with the spot velocity on the axis equal to $v_z=-u$. For this we must satisfy the relation

$$\mathrm{tg}\,\varphi=\left(1+\frac{c}{u}\right)\frac{\sin\theta_0}{1-(c/u)\cos\theta_0}. \qquad (40)$$

Without going into detail regarding the explicit expression for the period Λ in the general case, we note that the grating should be constructed with grazing angle for efficient conversion of the beam energy into the required diffraction order.

Let us summarize the results obtained in the present section. First, there are Bessel packets in which the power is focused on the z axis in the form of an illuminated spot (a flash) moving with arbitrary velocity $v_z=u$, including in particular $u=c$, and for which the photons propagate at an arbitrary angle θ_0 to the axis. Second, there exist designs consisting of conical diffraction gratings (and possibly a number of other elements), which enable us to obtain such packets from ordinary planar packets. Third, the light energy needed to accompany a relativistic particle bunch of length τ_0 over a distance L in the case of Bessel packets propagating obliquely is found to be given by essentially the same expression (39) as for focused beams propagating parallel to the bunch. Here, however, the oblique propagation of the photons relative to the axis gives rise to a huge ($\sim(1-v^2/c^2)^{-1/2}$) blue Doppler shift of the light frequency when we transform to the rest frame of the ultrarelativistic particles.

4. ACCELERATION BY THE PONDEROMOTIVE FORCE $F\propto\nabla E^2$

Let us discuss the design in which two laser beams of the same polarization propagate at angles θ_1 and θ_2 to the z axis, along which the accelerating particles move (Fig. 4). We start with the case of two plane waves. Since the resulting force is proportional to $E_1E_2^*$, the transition to two Bessel packets is achieved by integrating over all the components of the packets. Consequently, the transverse component of the force averages to zero and the longitudinal component will be determined for the individual components with a planar wave front by replacing E_1 and E_2 in the formulas with the corresponding field strengths on the beam axis. Here we assume that both packets move together with an ultrarelativistic particle over the entire interaction length L.

To accelerate particles in the longitudinal direction, it is necessary that one of the beams be preferentially oppositely directed ($\alpha_1 = 180° - \theta_1 < 90°$) and the other preferentially comoving ($\theta_2 < 90°$). In order to be specific we concentrate on the case $\omega_2 = 2\omega_1$, i.e., a laser beam and its second harmonic are present. Then the angles θ_2 and α_1 are related by Eq. (4), so that we have

$$1 + \frac{v}{c}\cos\alpha_1 = 2\left(1 - \frac{v}{c}\cos\theta_2\right), \tag{41}$$

which in the ultrarelativistic limit yields

$$\cos\alpha_1 + 2\cos\theta_2 = 1. \tag{42}$$

Condition (4) together with (41) implies that in the particle rest frame the frequencies $\omega_1' = (\omega_1 - \mathbf{k}_1\mathbf{v})(1 - v^2/c^2)^{-1/2}$ and $\omega_2' = (\omega_2 - \mathbf{k}_2\mathbf{v})(1 - v^2/c^2)^{-1/2}$ of the two waves are identical, $\omega_1' = \omega_2' = \omega'$, i.e., the interference pattern is stationary in this system. Then the quadratic ponderomotive force[7] in the rest frame is equal to

$$\mathbf{F}' = -\frac{\partial U_{\text{eff}}}{\partial \mathbf{r}'}$$

$$U_{\text{eff}} = -\frac{e^2}{4m\omega_1\omega_2}\{\mathbf{E}_1\mathbf{E}_2^*\exp[i(\mathbf{k}_1' - \mathbf{k}_2')\mathbf{r}'] + \mathbf{E}_1^*\mathbf{E}_2\exp[-i(\mathbf{k}_1' - \mathbf{k}_2')\mathbf{r}']\}, \tag{43}$$

$$F_z' = \frac{e^2}{2mv}\frac{(\omega_2 - \omega_1)}{\omega_1\omega_2}\sqrt{1 - \frac{v^2}{c^2}}\,|\mathbf{E}_1\mathbf{E}_2^*|\sin\Phi. \tag{44}$$

This force depends on the phase $\Phi = -(\omega_2 - \omega_1)z_0/v\sqrt{1 - v^2/c^2} + \varphi_1 - \varphi_2$ and the interference pattern for the point toward which the particle is moving. Without going into details about the phase interactions, we take the maximum possible value for this force, corresponding to $\sin\Phi = 1$.

Over a time dt' in the proper coordinate system, the particle momentum changes from zero to a value $dp_z' = F_z'dt'$, and the energy K' to first order in dt' remains unchanged. Then in the laboratory coordinate system, the energy K increases by

$$dK = v_z dp_z = v_z\frac{dp_z'}{\sqrt{1 - v^2/c^2}} \approx \frac{F_z' v_z dt'}{\sqrt{1 - v^2/c^2}} = F_z' v_z dt = F_z' dz,$$

i.e.,

$$\frac{dK}{dz} = F_z' = \frac{e^2}{2m\omega_1\omega_2}\frac{(\omega_2 - \omega_1)}{v}\sqrt{1 - v^2/c^2}\,|\mathbf{E}_1\mathbf{E}_2|. \tag{45}$$

Expression (45) has been written in the most general case of arbitrary frequencies ω_1, ω_2 and angles θ_1, θ_2 satisfying the condition (41) for arbitrary values of the parameter v/c. In the ultrarelativistic case ($v \approx c$) we have

$$\frac{dK}{dz} \approx \left(\frac{mc^2}{K}\right)\frac{e^2(\omega_2 - \omega_1)}{2\omega_1\omega_2 mc}|\mathbf{E}_1\mathbf{E}_2|. \tag{46}$$

Let us find an approximate value for the case $\omega_2 = 2\omega_1$, $\lambda_1 = 2\pi c/\omega_1 = 1\ \mu\text{m}$, $10^{-7}c|\mathbf{E}_{1,2}|^2/8\pi \approx 10^{18}\ \text{W/cm}^2$. At this intensity the field strength is equal to $E \approx 0.9 \cdot 10^8$ CGSE $= 2.65 \cdot 10^{10}$ cm and the quiver velocity of a nonrelativistic electron is $|v_{\text{nonr}}| \approx c$, so that even for an electron at rest the iteration in powers of $|v_{\text{nonr}}/c|$ used to derive expression (43) for the force is valid. For electrons with $v \approx c$ we finally obtain

$$\frac{dK}{dz} \approx \sqrt{\frac{P_1}{P_0}\frac{P_2}{P_0}\frac{mc^2}{K}}A, \tag{47}$$

where $P_0 = 1.4 \cdot 10^{18}\ \text{W/cm}^2$, $A \sim mc^2/\lambda \sim 0.5 \cdot 10^{10}\ \text{V/cm}$.

If we take the length of the light packets on the axis equal to $\tau_0^{(1,2)} = 10^{-13}$ s, then out of the whole electron bunch only a group of this length will be accelerated. The energy of both Bessel packets required to move with the beam over a distance $L = 10$ cm with a local power density $P_0 = 10^{18}\ \text{W/cm}^2$ is equal to

$$W_1 = W_2 = \frac{c|E_0|^2}{8\sqrt{\pi}}\lambda_0 L\tau_0 \approx 150\ \text{J}. \tag{48}$$

Thus, using Bessel packets enables us to use the ponderomotive force $\mathbf{F} \propto \nabla E^2$ to achieve acceleration for relatively modest energies of the light pulses. It is significant that on account of the oblique incidence of the light on the beam axis, the phase condition can easily be satisfied owing to the oblique wave propagation. In addition, for an ultrarelativistic beam, the phase condition can be satisfied with moderate (e.g., $\omega_2/\omega_1 \approx 2$) ratios of the frequencies ω_1 and ω_2. Bessel beams are thus a substantial improvement over waves focused parallel to the particle trajectories: in the latter case it would be necessary to take $\omega_1/\omega_2 \sim (mc^2/K)^2$.

5. ACCELERATION PRODUCED BY A FORCE $\mathbf{F} \propto \nabla E^3$

One shortcoming of the scheme based on acceleration by the ponderomotive force $\mathbf{F} \propto \nabla E^2$ is the need to use beams with a large frequency ratio. Let us consider the situation in which there is a high-quality laser producing ultrashort ($\tau \sim 10^{-13}$ s) pulses of very high energy, but where for some reason it is difficult to convert them into comparably energetic beams at different frequencies. Then acceleration by the ponderomotive force $\mathbf{F} \propto \nabla E^2$ is impossible, since the condition $\omega_1(-\mathbf{n}_1\mathbf{v}/c) = \omega_2(1 - \mathbf{n}_2\mathbf{v}/c)$ cannot be satisfied for $\omega_1 = \omega_2$ in any geometry or for any value of v/c.

We would like to direct our attention toward the possibility of achieving particle acceleration for the case in which waves with the same central frequency ω_0 take part. This possibility is based on the existence of a force proportional to ∇E^3, predicted and calculated by Baranova and Zel'dovich.[8] Although Ref. 8 alludes to the possibility of using this force to accelerate elementary particles, no specific treatment of the acceleration problem (geometry, frequencies, estimates, etc.) is given. The present section is devoted to this task.

The synchronization condition for three-wave action on a moving electron takes the form

$$\omega_1 - \mathbf{k}_1\mathbf{v} + \omega_2 - \mathbf{k}_2\mathbf{v} = \omega_3 - \mathbf{k}_3\mathbf{v}. \tag{49}$$

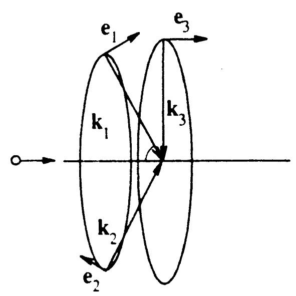

FIG. 5. Scheme for accelerating relativistic particles with three monochromatic waves using the force $\mathbf{F} \propto \nabla E^3$.

In the case $\omega_1 = \omega_2 = \omega_3$ this condition assumes the form of an equation for the three angles θ_1, θ_2, and θ_3 between the wave vectors and the direction of the velocity $\mathbf{v} = v\mathbf{e}_z$:

$$\frac{v}{c}(\cos\theta_1 + \cos\theta_2 - \cos\theta_3) = 1. \quad (50)$$

Hence it follows immediately that the electron velocity cannot be less than $v = c/3$, i.e., the kinetic energy must satisfy $T = K - mc^2 \geqslant 0.06mc^2 \approx 33$ KeV. An expression for the effective third-order potential $U^{(3)}$ was obtained in Ref. 8 using the gauge $\mathbf{A}(\mathbf{R},t) \neq 0$, $\varphi(\mathbf{R},t) \equiv 0$. Here we employ this gauge in the electron rest frame, so that in the laboratory coordinate system the fields will be described by vector and scalar potentials $\mathbf{A}$ and φ.

We start by considering the general case of three plane waves with vector amplitudes $\mathbf{E}_\alpha \exp(i\mathbf{k}_\alpha \mathbf{r} - i\omega t)$, where

$$\mathbf{k}_\alpha = \frac{\omega}{c}\mathbf{n}_\alpha, \quad (51)$$

$$\mathbf{n}_\alpha = \mathbf{e}_z \cos\theta_\alpha + (\mathbf{e}_x \cos\varphi_\alpha + \mathbf{e}_y \sin\varphi_\alpha)\sin\theta_\alpha. \quad (52)$$

Below, in order to be specific, we consider a particular beam geometry. Let the waves E_1 and E_2 be elementary components of the same Bessel packet with angle $\theta_1 = \theta_2 = \arccos(c/2v)$. We also assume that the wave E_3 has angle $\theta_3 = 90°$, i.e., in the laboratory coordinate frame it is incident perpendicularly on the axis (see Fig. 5). (Here, by the way, the electron kinetic energy must be at least $0.15\ mc^2 = 85$ keV.) In this geometry, the force $\mathbf{F}'$ acting in the electron rest frame vanishes when averaged over time both in first and second order in the field amplitude $\mathbf{E}$. Hence the third-order term is the first nonvanishing contribution:

$$F_i' = -\frac{\partial U'^{(3)}}{\partial x_i}, \quad (53)$$

$$U'^{(3)} = -\frac{e^3}{2m^2c^3}\left(\int_{-\infty}^{t} A_i' dt'\right)\frac{\partial \mathbf{A}'^2}{\partial x_i}. \quad (54)$$

The vector potential $\mathbf{A}'$ in (54) consists of three components:

$$\mathbf{A}' = \mathbf{A}_1' + \mathbf{A}_2' + \mathbf{A}_3', \quad (55)$$

each of which can be expressed in terms of the field amplitude $\mathbf{E}_\alpha = E_\alpha \mathbf{e}_\alpha$ in the laboratory coordinate frame:

$$\mathbf{A}_\alpha' = -\frac{ic}{2\omega_\alpha'}\left[\frac{\mathbf{e}_\alpha - \mathbf{e}_z(\mathbf{e}_z\mathbf{e}_\alpha)}{\mathbf{e}_z\mathbf{n}_\alpha}\frac{\mathbf{e}_z\mathbf{n}_\alpha - v/c}{\sqrt{1 - v^2/c^2}} + \mathbf{e}_z(\mathbf{e}_z\mathbf{e}_\alpha)\right]$$
$$\times [E_\alpha \exp(i\omega_\alpha' t' + i\mathbf{k}'\mathbf{r}') - E_\alpha^* \exp(i\omega_\alpha' t' - i\mathbf{k}'\mathbf{r}')]. \quad (56)$$

We are interested in the relativistic case $v/c \approx 1$, $\theta_1 = \arccos(c/3v) \approx 60°$. The leading terms in the small parameter $mc^2/K \approx 1/\sqrt{1-(v/c)^2}$ in $F' = dK/dz$ take the form

$$\frac{dK}{dz} = -\frac{e^3}{2m^2c^2\omega^2}\frac{c}{v}\left(\frac{mc^2}{K}\right)^2 \frac{(\cos\theta - v/c)^3}{\cos^2\theta(1 - v/c\cos\theta)^3}$$
$$\times |E_1E_2E_3| \sin(\Delta\mathbf{k}'\mathbf{r}' + \varphi)\mathbf{e}_{1,\perp}\,\mathbf{e}_{2,\perp}, \quad (57)$$

where $\Delta\mathbf{k}'$ is the wave vector of the grating formed by the cube of the field:

$$\Delta\mathbf{k}' = \mathbf{k}_{1,\perp} + \mathbf{k}_{2,\perp} + \mathbf{e}_z\frac{\omega}{v}\sqrt{1 - \frac{\omega^2}{c^2}}, \quad (58)$$

and we have introduced the unit polarization vectors orthogonal to the electron velocity:

$$\mathbf{e}_{\alpha,\perp} = \mathbf{e}_\alpha - \mathbf{e}_z(\mathbf{e}_z\mathbf{e}_\alpha). \quad (59)$$

To order of magnitude, dK/dz given by (57) differs from the first-order force $F = eE_3$ by two dimensionless factors. The first of these, $(mc^2/K)^2$, falls off rapidly as a function of particle energy. The second is equal to $(e^2E_1E_2/m\omega^2)/mc^2$, i.e., the ratio of the quiver energy $\sqrt{E_1E_2}$ in the field to the rest energy mc^2, evaluated for a nonrelativistic electron with the given frequency ω. Note that the Lorentz transformation for the frequencies and fields in the transition to the electron rest frame does not change the ratio E'/ω', so that

$$\langle (v_{\text{quiv}}'/c)^2 \rangle \approx (e^2E^2/m\omega^2)/mc^2. \quad (60)$$

For perturbation theory to be applicable, the parameter $\langle (v_{\text{quiv}}'/c)^2 \rangle$ must be less than unity.

Consider a Bessel packet consisting of plane wave components which form a cone with the axis having angle $\theta = 60°$. By virtue of the transversality condition, the various azimuthal components of the beam do not have the same three-dimensional vector for the polarization of the field $\mathbf{E}$. We select an azimuthal dependence of the polarization vectors for the fields $\mathbf{E}_1$, $\mathbf{E}_2$ which is optimum for the acceleration force $\mathbf{F} \propto \nabla E^3$:

$$\mathbf{e}_\alpha = \frac{-\mathbf{e}_x \cos\theta + \mathbf{e}_z \sin\theta\cos\varphi_\alpha}{\sqrt{\cos^2\theta + \sin^2\theta\cos^2\varphi_\alpha}}. \quad (61)$$

Then after averaging over azimuth we find

$$\langle \mathbf{e}_{1,\perp}\,\mathbf{e}_{2,\perp} \rangle \to 0.47. \quad (62)$$

The right-hand side of Eq. (57) must have an additional factor of 0.47, and E_1 and E_2 in (57) are taken to be the amplitudes which would result for conical focusing of a scalar Bessel wave of the same energy.

6. BREMSSTRAHLUNG

Apart from all the disadvantages associated with breakdown of a medium subject to powerful laser radiation, particle acceleration by means of the inverse Cherenkov effect has one important advantage. In the frame attached to the moving particle, the laser field is static by virtue of the Cherenkov condition $\omega' = (\omega - \mathbf{kv})/\sqrt{1-v^2/c^2} = 0$ itself. Consequently, there are no oscillations and thus no waves reradiated by the electron.

In contrast, irradiation of an ultrarelativistic electron with light in vacuum generally yields $\omega' \gg \omega$; this will happen, at least when the angle θ between the photon wave vector and the electron velocity is not too small. As a result, in the rest frame the electron is subjected to a strong high-frequency field. Its radiation in this system can be treated as Thomson scattering. The order of magnitude of the intensity of this radiation is

$$\frac{dW'}{dt'}\left[\frac{\text{erg}}{\text{s}}\right]=\frac{2\langle \ddot{\mathbf{d}}'^2\rangle}{3c^3}=\frac{e^4|\mathbf{E}'|^2}{3m^2c^3}\approx\frac{e^4|\mathbf{E}|^2}{3m^2c^3}\frac{K}{mc^2}. \quad (63)$$

In the rest frame of the particle, the dipole radiation has the usual directionality and frequency ω'. When we go back to the laboratory coordinate system we have for the frequency $\omega_{\text{rad}}(\mathbf{n}')$ and the direction $\mathbf{n}_{\text{rad}}(\mathbf{n}')$ of the emitted photons as a function of the direction $\mathbf{n}'$ of emission in the proper coordinate system

$$\omega_{\text{rad}}=\omega'\frac{1+\mathbf{vn}'/c}{\sqrt{1-v^2/c^2}} \quad (64)$$

$$\mathbf{n}_{\text{rad}}=\frac{\omega'}{\omega_{\text{rad}}}\left(\mathbf{n}'_{\perp}+\frac{\mathbf{n}'\mathbf{v}/v+v/c}{\sqrt{1-v^2/c^2}}\frac{\mathbf{v}}{v}\right), \quad (65)$$

i.e., $\omega_{\text{rad}}\sim(K/mc^2)^2\omega$. This law is well established. The particle energy losses in its rest frame correspond to the momentum losses in the laboratory system, i.e., bremsstrahlung due to Thomson scattering:

$$F_{\text{brem}}=\frac{dp}{dt}=\frac{v}{c^2\sqrt{1-v^2/c^2}}\frac{dW'}{dt'}\frac{dt'}{dt}\approx\frac{e^4|E|^2}{3m^2c^4}\left(\frac{K}{mc^2}\right)^2. \quad (66)$$

The above comments are included for the sake of completeness. To within factors ~ 1 they are equivalent to the treatment in Sec. 76 of Landau and Lifshitz,[9] which is devoted to bremsstrahlung in the relativistic case, where for $K \gg mc^2$ an expression is given for the component of force F_z:

$$F_z=-\frac{2e^2}{3m^2c^4}[(E_x-H_y)^2+(E_y+H_x)^2]\left(1-\frac{v^2}{c^2}\right)^{-1}. \quad (67)$$

This formula from Ref. 9 is written assuming $\mathbf{v}=v\mathbf{e}_z$, and the fields are taken in the laboratory coordinate system. For the comoving wave with $\theta_0\approx 0$ the relation $E_x\approx H_y$, $E_y\approx -H_x$ holds, and the decelerating force is small. This is equivalent to a strong red Doppler shift in the transition to the particle rest frame. In contrast, for all other angles $\theta_0\neq 0$, we have $|E_x-H_y|\sim E_x$, $|E_y+H_x|\sim E_y$, and we arrive at the estimate (66). Note, however, that the nonrelativistic expression for the force associated with Thomson scattering (see Sec. 78 of Landau and Lifshitz[9]) in our case is inapplicable precisely because we are considering the limit $(1-v^2/c^2)^{-1}\gg 1$. Let us find the threshold value K_0 of the energy for which the decelerating force balances the accelerating force $\mathbf{F}\propto\nabla E^2$ or $\mathbf{F}\propto\nabla E^3$. For $\mathbf{F}\propto\nabla E^2$ we have

$$\frac{K_0}{mc^2}=\left(\frac{\lambda}{2\pi r_{\text{cl}}}\right)^{1/3}, \quad (68)$$

where λ is the effective wavelength of the laser transition and $r_{\text{cl}}=e^2/mc^2$ is the classical particle radius, with $r_{\text{cl}}\approx 3\cdot 10^{-13}$ cm for an electron. It is evident that when the wavelength is $\lambda\approx 1\ \mu$m, the maximum achievable energy is rather severely limited by the force of radiative deceleration to $K\sim 200$ MeV for electrons.

In the case of acceleration by a force $\mathbf{F}\propto\nabla E^3$ a similar limit holds:

$$\frac{K_0}{mc^2}\approx\left(\frac{\lambda}{2\pi r_{\text{cl}}}\right)^{1/4}. \quad (69)$$

This restriction is quite discouraging for ponderomotive acceleration. On the other hand, these estimates imply that the conversion of strongly focused laser packets into γ radiation by means of spontaneous Thomson scattering on ultrarelativistic electron beams is very promising. Collective scattering by multiple electrons should be treated specifically.

7. CONCLUSION

In the present work we have derived estimates for the acceleration of charged particles (primarily electrons) by ponderomotive forces that vary as $\mathbf{F}\propto\nabla E^2$ and $\mathbf{F}\propto\nabla E^3$ in the field of powerful laser pulses.

We have shown that the momentum carried off by scattered waves restricts the achievable energy to a level $K\sim mc^2(\lambda/r_{\text{cl}})^{1/3}\sim 200$ MeV when the acceleration is caused by a force $\mathbf{F}\propto\nabla E^2$. In the case $\mathbf{F}\propto\nabla E^3$ a similar restriction yields $K\sim 50$ MeV. This restriction is about 2000 times less severe for protons, i.e., for them the achievable energy is 2000 times higher.

We propose to use Bessel wave packets which accompany a particle beam over a large distance L with their high-intensity focal "flash," moving with arbitrary velocity v (aside from the ultrarelativistic case $1-v^2/c^2\ll 1$).

After we had finished this work the JETP referee directed our attention to a closely related paper by Steinhauer and Kimura.[10]

[1)]The quantities Δx, $\delta\theta$, Δz, etc., that appear here and in what follows are determined according to the same criterion.

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