Observation of the asymmetrical two-cone Cherenkov radiation emitted by relativistic protons moving along the binormal in the biaxial crystal triglycine sulfate

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The directivity and polarization properties of Cherenkov radiation produced by the passage of 658 MeV protons along one of the optic axes (binormal) of the biaxial crystal triglycine sulfate (TGS) have been studied. The azimuthal radiation distribution turned out to be highly unusual. First, it was asymmetric relative to the plane passing through the velocity vector of the particle beam perpendicular to the plane of the optic axis (POA). Such Cherenkov asymmetry was not observed in three previously studied cases of the passage of ~660 MeV protons along the principal dielectric axes of the biaxial TGS crystal (i.e., when the particles move along the minor and major bisectrices and also perpendicular to the plane of the optic axes). Secondly, the azimuthal Cherenkov distribution has the form of two symmetric semicones with different cone angles (in a cross section perpendicular to the direction of the proton beam axis these semicones resemble two half-horsehoes inserted one into the other with their arcs facing outward). It also turned out that the outer Cherenkov cone, F_-, is polarized essentially only in the POA while the inner cone, F_+, is polarized in a more complicated manner, i.e., close to $\varphi \sim 180°$ it is in the POA, while at $\varphi \sim 90°$ it is perpendicular to the POA. Expressions are obtained for the directivity of the radiation for any angle φ for such asymmetric two-cone radiation. Comparison of the Cherenkov emission angles calculated from these expressions and those measured experimentally shows good agreement (within the limits of experimental error) for the azimuthal distribution of F_+ waves (inner core), and some systematic increase (by ~0.5°) for the angles of F_- waves (outer cone).

1. INTRODUCTION

Cherenkov radiation has been studied previously[1] in the biaxial crystal triglycine sulfate (TGS) upon passage of 660 MeV protons along the principal dielectric axes ε_1, ε_2 and ε_3, and interesting features were observed. For example, upon passage of protons along the minor besectrix (X_1 axis), the azimuthal distribution $F(\varphi)$ of the intensity of F_-[1)] waves was oval with a uniform distribution, while the F_+ waves had the form of "lobes" lying close to the plane of the optic axes (X_1OX_3) and symmetric relative to it, so that the radiation in this case can, on the whole, be called briefly "oval lobed."

For proton motion along the principal axes X_2 (perpendicular to POA) and X_3 (along the major bisectrix), the Cherenkov intensity distribution in azimuthal angle φ (both the F_+ and F_- waves) was ellipsoidal and nonuniform, so that the radiation in this case could reasonably be called "bielliptical."

The experimental azimuthal distributions, the directivity, and the polarization of the radiation are fully explained by the theories of Muzikář[2] and Obdržálek.[3]

In the case of particles moving along the binormal, there were only two predictions before the present experiments. One,[4] concerning particles moving along the biradial of a biaxial crystal with almost limiting velocity, showed that the radiative intensity in this case increases more steeply than for particle motion in other directions.

The other was the most general prediction about the features of Cherenkov radiation in a biaxial crystal, and was made by Ginzburg[5] more than fifty years ago.[2)]

2. EXPERIMENTAL CONDITIONS

In this experiment, a TGS crystal with dimensions $20\times20\times5$ mm^3 was used; it was cut at the Institute of Crystallography (Russian Academy of Sciences) in such a way that the crystal binormal was perpendicular to the 20×20 mm^2 faces (to an accuracy of ±0.5° for $\lambda=546.1$ nm, while the plane of the optic axes (X_1,X_3) was parallel to the 20×5 mm^2 face, as shown in Fig. 1. The magnitudes of the principal dielectric susceptibilities of this crystal are given by Ivanov and Zotov[6] and in Table I. The accuracy with which the refractive indices n_p, n_m and n_g can be determined from the plots in Ref. 6 is about ±0.002.

The experiment was carried out with a collimated proton beam derived from the reconstructed Joint Institute phasotron. The collimator diameter was 5 mm and its length 700 nm. The mean beam energy was measured by the so-called combined Cherenkov method[7] and was 658 MeV.

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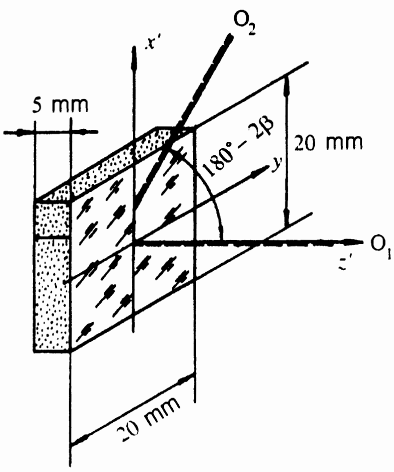

FIG. 1. Orientation of the binormals O_1 and O_2 of the TGS crystal relative to its cross section (the particle beam is directed along the binormal O_1). The crystal dimensions are given in mm.

A special Cherenkov photocell was used to record the whole azimuthal radiation distribution emitted by protons in the crystal, its method of operation being shown in Fig. 2. The proton beam, *2*, goes through the collimator, *1* which is in optical contact ("SISS compound S17" grease) with a thick (12 mm along the axis) plano-parabolic lens, *4*, with $f_D=(22.7\pm0.1)$ mm and refractive index $n_D=1.512\pm0.001$.

The Cherenkov radiation *5* emitted in the crystal was focused by the lens, in the focal plane of which a plane NC19 "ORWOCOLOR" 18×24 cm^2 photographic film, *8*, of sensitivity 19 DIN or a plate of the "Isoorthochrome" type (90 GOST sensitivity) was located. In polarization experiments a polychlorvinyl polaroid, *6*, was used. In Charenkov radiation *7* generated in the lens itself, *4*, also showed up on the photographic film in the form of a regular ring and served as a reference for determining the angles of Cherenkov emission in the crystal.

3. DIRECTIVITY PROPERTIES OF CHERENKOV RADIATION PARTICLE MOTION ALONG THE BINORMAL

1. Angles of Cherenkov radiation in the plane of the optic axes (POA)

It can be shown by plotting according to Pafomov's method[8] that the angles of Cherenkov radiation in this case for $F_\pm$ waves have the form

$$\mathrm{tg}\theta^{\pm}=\beta_0 X_{1,2}, \tag{1}$$

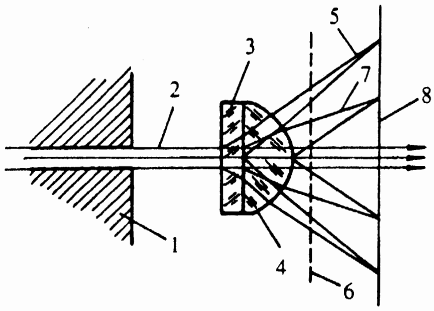

FIG. 2. Method of operation of the Cherenkov photocell: *1*—collimator (beam shaper); *2*—photon beam; *3*—TGS crystal; *4*—focusing (and, simultaneously, reference) lens; *5*—radiation from TGS crystal; *6*—flat thin polaroid; *7*—Cherenkov radiation from the lens (4); *8*—photographic plate or color negative film.

where β_0 is the proton velocity at the center of the TGS plate (calculated from measurements of the mean energy),

$$X_{1,2}=\frac{b}{a}\pm\frac{\sqrt{b^2-ac}}{a},$$

where

$$a=n_x^2\cos^2\beta+n_z^2\sin^2\beta,$$

$$b=p\sin\beta\cos\beta(n_z^2-n_x^2),$$

$$c=p^2n_x^2\sin^2\beta+p^2n_z^2\cos^2\beta-n_x^2n_z^2,$$

$$p=\frac{1}{\beta_0},\quad \sin\beta=\frac{n_z}{n_y}\sqrt{\frac{n_y^2-n_x^2}{n_z^2-n_x^2}},$$

under the conditions $n_z>n_y>n_x$.

For wavelength $\lambda=600$ nm (at which the measurements of the radiation angles were carried out) and $n_z=1.576$, $n_y=1.551$, $n_x=1.482$, calculation according to Eq. (1) for a velocity $\beta_0=0.809$ gives values of the Cherenkov angles:

$\theta^{(-)}=38°14.5'$ ($\varphi=0°$, i.e., in a plane passing through the beam axis (POA), $\theta^{(+)}=-34°25'$ (the minus sign corresponds to $\varphi=180°$, i.e., toward the beam), and $\Delta\theta=\theta^{-}-|\theta^{+}|=3°49.5'$ determines the asymmetry of the F_- and F_+ waves in this plane.[2)]

The radiation angles of the F_- waves in the POA for $\varphi=0°$ and $\varphi=180°$ should be identical (as follows, for example, from graphical constructions), and the directivity condition for them is of the form

TABLE I.

λ, nm	$\varepsilon_1=\varepsilon_x=n_p^2$	$\varepsilon_2=\varepsilon_y=n_m^2$	$\varepsilon_3=\varepsilon_z=n_g^2$
400	1.5034^2	1.5784^2	1.6067^2
500	1.490^2	1.561^2	1.587^2
546.1	1.489^2	1.556^2	1.582^2
600	1.482^2	1.551^2	1.576^2
650	1.479^2	1.547^2	1.572^2

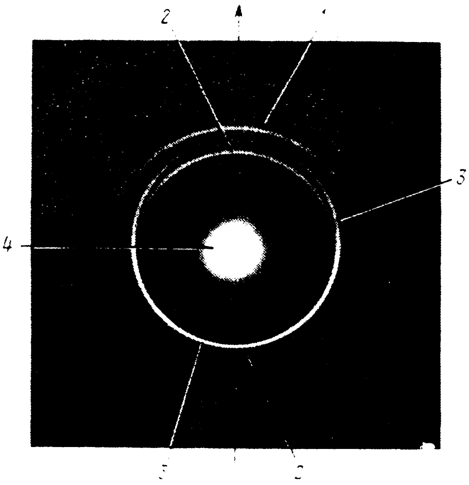

FIG. 3. Photograph of Cherenkov radiation emitted by protons propagating along the binormal of a crystal of TGS (arrow indicates the plane of the optical axes, $\varphi=0°$).

$$\cos\theta^{\pm}=1/n_y\beta_0\,. \qquad (2)$$

For $n_y=1.5509$ and $\beta_0=0.809$, $\theta^{-}=37°06.5'$.

2. Cherenkov emission angles in the plane perpendicular to the POA ($\varphi=90°$)

The directivity properties in this case are determined by the following equations (a particular case of the general formula (Eq. (4) for $\varphi=90°$).

$$\mathrm{tg}^2\theta^{+,-}=U/p^2\,, \qquad (3)$$

where

$$U=-\frac{B'}{2\varepsilon_y}\pm\frac{\sqrt{(B')^2-4D'\varepsilon_y}}{2\varepsilon_y};$$

$$B'=n_{z'}^2\left(\varepsilon_y-\frac{B}{n_z^2}+\varepsilon_z\cos^2\beta+\varepsilon_x\sin^2\beta\right),$$

$$D'=n_{z'}^4(\varepsilon_x\sin^2\beta+\varepsilon_z\cos^2\beta)$$
$$-n_{z'}^2(A\sin^2\beta+C\cos^2\beta)+D,$$

$$A=\varepsilon_x(\varepsilon_y+\varepsilon_z),\quad B=\varepsilon_y(\varepsilon_x+\varepsilon_z),$$

$$C=\varepsilon_z(\varepsilon_x+\varepsilon_y),\quad D=\varepsilon_x\varepsilon_y\varepsilon_z,$$

$$n_{z'}^2=\frac{1}{\beta_0^2}=p^2,\quad n_{z'}^4=p^4.$$

3. Expression for Cherenkov directivity in an arbitrary plane φ

A rather lengthy calculation leads to a general equation for the angles $\theta^{\pm}$ in a biaxial crystal for any φ:

$$A_0'(\varphi)\,\mathrm{tg}^4\,\theta^{\pm}(\varphi)+B_0'(\varphi)\,\mathrm{tg}^3\,\theta^{\pm}(\varphi)$$
$$+C_0'(\varphi)\,\mathrm{tg}^2\,\theta^{\pm}(\varphi)+D_0'(\varphi)\,\mathrm{tg}\,\theta^{\pm}(\varphi)+\varepsilon_0=0, \qquad (4)$$

where

$$A_0'(\varphi)=p^4(\varepsilon_x\cos^2\beta\cos^2\varphi+\varepsilon_y\sin^2\varphi$$
$$+\varepsilon_z\sin^2\beta\cos^2\varphi),$$

$$B_0'(\varphi)=2p^4(\varepsilon_x-\varepsilon_z)\sin\beta\cos\beta\cos\varphi,$$

$$C_0'(\varphi)=p^2\{\varepsilon_x p^2(\cos^2\varphi+\sin^2\varphi\sin^2\beta)+\varepsilon_y p^2\sin^2\varphi$$
$$+\varepsilon_z p^2(\cos^2\varphi+\sin^2\varphi\cos^2\beta)$$
$$-A\cos^2\varphi\cos^2\beta-B\sin^2\varphi$$
$$-C\sin^2\beta\cos^2\varphi\};$$

$$D_0'=2p^2\{p^2(\varepsilon_x-\varepsilon_z)-A+C\}\sin\beta\cos\beta\cos\varphi,$$

$$\varepsilon_0=p^2\sin^2\beta(\varepsilon_x p^2-A)+p^2\cos^2\beta(\varepsilon_z p^2-C)+D,$$

$$D=\varepsilon_x\varepsilon_y\varepsilon_z$$

and where, in turn

$$A=\varepsilon_x(\varepsilon_y+\varepsilon_z),\quad C=\varepsilon_z(\varepsilon_x+\varepsilon_y),$$

$$B=\varepsilon_y(\varepsilon_x+\varepsilon_z),\quad D=\varepsilon_x\varepsilon_y\varepsilon_z,\quad \mathrm{tg}^2\beta=\frac{n_z^2(n_y^2-n_x^2)}{n_x^2(n_z^2-n_y^2)},$$

and φ is reckoned from the POA to that part toward which the second binormal O–O_2 (Fig. 1) is directed.

We note that although Eq. (4) for $\varphi=0°$ (POA) is a somewhat different expression than Eq. (1), they give the same numerical results.

4. EXPERIMENTAL RESULTS

1. Cherenkov directivity in TGS

A photograph of Cherenkov radiation obtained in this experiment (Fig. 2) is shown in Fig. 3, where the outer and inner Cherenkov cones are visible, having the form of two horseshoes lying one within the other asymmetrically, with "arcs" directed outward, i.e., the outside "arc" (the outer Cherenkov cone—the cone above the beam) is farther from the Cherenkov ring[3)] (emitted from the lens) than the inner "arc" (the inner Cherenkov cone—the cone below the beam).

The asymmetry of the Cherenkov cones can be seen more clearly from the photometric curves with the help of an automatic microdensitometer[9] for cross sections at $\varphi=0°$ and $\varphi=90°$, shown in Fig. 4. (The central peak in these curves is the proton beam).

Experimental results at Cherenkov angles $\theta^{\pm}$ for different angles φ were obtained by measuring the "diameters" of these distributions on photographic color negatives with the help of an Abbe comparator (Carl Zeiss, Jena) in the vicinity of $\lambda\sim(600\pm25$ nm), which together with the diameter of the ring from the reference lens enabled the radii $R^{\pm}(\varphi)$ of the external and internal cones to be determined for a given φ. These radii were then translated into the corresponding angles using a calibration curve obtained, in turn, with a special device enabling a parallel light beam from an "LNG=76" laser to be directed to the plano surface of the reference lens (see Fig. 2) at various angles with an accuracy of $\pm0.001°$.

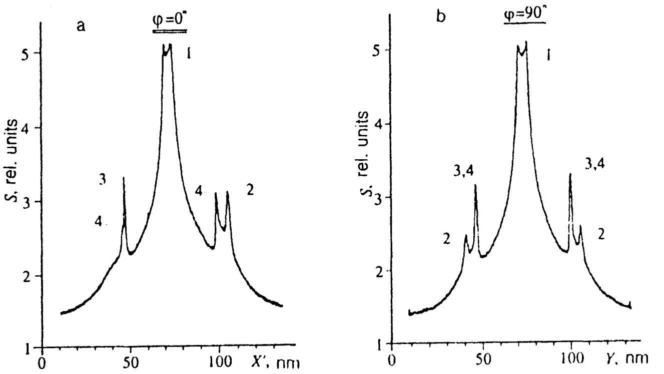

FIG. 4. Photometric curves of the negative Cherenkov image (shown in Fig. 3); (a) cross section in the plane of the optical axes $\varphi=0$; (POA is X_1OX_3); *1*—peak from proton beam; *2*—outer Cherenkov cone; *3*—inner Cherenkov cone; *4*—Cherenkov cone from the reference lens (ring). (b) Cross section in the plane perpendicular to POA $\varphi=90°$ (Z'OY plane) *1*—proton beam, *2*—outer Cherenkov cone; *3,4*—cone fusion from inner TGS and the reference lens. (The density of blackening S in relative units is the ordinate in both figures).

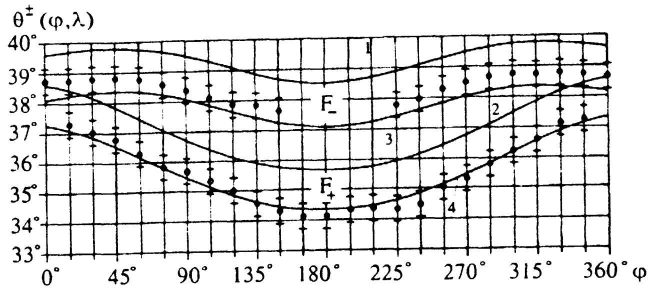

FIG. 5. Azimuthal Cherenkov directivity ($\theta^{\pm}(\varphi)$ for F_+ and F_- waves resulting from proton motion with $\beta=0.81$ along the binormal of the TGS crystal: (a) solid curves—theoretical (Eq. 4); *1* and *3*) for $\lambda=400$ nm, *2* and *4*) for $\lambda=600$ nm; b) ●—experimental values of $\theta^-(\varphi)$ for the outer cone $F_-(\varphi)$ at $\lambda\simeq600$ nm, ○—the same for the inner cone $F_+(\varphi)$.

The measured correspondence between the angles in the reference lens from the laser and the distances to the photographic film was fit by a polynomial (for the photocell used)

$$D=C_1+C_2\theta_\lambda(\varphi)+C_3\theta_\lambda^2(\varphi)+C_4\theta_\lambda^3(\varphi)+C_5\theta_\lambda^4(\varphi), \tag{5}$$

where

$C_1=0.14785,\quad C_2=1.032364,$

$C_3=0.024577,\quad C_4=-0.00088487,$

$C_5=0.000013745,$

D is the diameter in mm and $\theta(\varphi)$ is in degrees.

The angles $\theta_\lambda^{\pm}(\varphi)$ determined from the calibration curve correspond to the Cherenkov radiation emitted in the crystal and refracted at the crystal–lens interface. To convert to the angles $\theta^{\pm}$ in the crystal, refraction at this boundary is taken into account:

$$\sin\theta^{\pm}(\varphi)=\frac{n_L\sin\theta_L^{\pm}(\varphi)}{n^{\pm}(\varphi)}, \tag{6}$$

where

$$n^{\pm}(\varphi)=\frac{p}{\cos\theta^{\pm}(\varphi)},\quad p=\frac{1}{\beta_0}.$$

The experimental angles $\theta(\varphi)$ in the TGS crystal were also determined using an empirical formula (verified previously in a separate experiment, in which several radiators with different n were simultaneously penetrated by a ~660 MeV proton beam), using the relation between the radii $R_{\rm TGS}^{\pm}$ and the radius R_L of the Cherenkov ring from the lens (see Ref. 1).

The data for $\theta^{\pm}(\varphi)$ obtained by the two methods were averaged and are shown together with calculated values [obtained from Eq. (4)] in Fig. 5. the errors in the angles $\theta_E^{\pm}$ were determined by factors shown in Table II. The rms error was $\pm17'$, which is shown in Fig. 5. To within the experimental errors, the experimental results agree with

TABLE II. Rms error $\Delta\theta^{\pm}(\varphi)=\pm17'$.

No.	Uncertainties	$\Delta\theta$, arcmin
1	Refractive index of TGS, $\Delta n=\pm0.002$	±6.0
2	Mean energy of proton beam, $\Delta E=\pm0.6$ MeV	±0.1
3	Uncertainty in the location of $\lambda\simeq600$ nm in measurement on a comparator, $\Delta\lambda\simeq\pm25$ nm	±6.0
4	Error in averaging measurements of $\theta^{\pm}(\varphi)$ by two methods	±15

the theoretical (the lack of experimental points for F_- near $\varphi=180°$ and F_+ for $\varphi=0°$ ($\varphi=360°$) is the result of the F_+ and F_- intensity falling to zero near these angles). However, the systematic excess of the measured radiation angles θ_E^- compared with the calculated curve is noteworthy, and may be related to the fact that the crystal shear plane (for $\lambda=546.1$ nm) may deviate from perpendicularity to the binormal by $\sim \pm 0.5°$.

The less smooth variation of the experimental points for the $F_+(\varphi)$ dependence is also to be noted, and is due to the difficulties in measuring the angles $\theta^+(\varphi)$ in the region where the Cherenkov cone F_+ and the cone from the reference lens overlap in the range $\varphi \sim 90°$–$150°$ and $\varphi \sim 270°$–$330°$.

2. Polarization properties of the Cherenkov radiation

To verify these properties, a polaroid was inserted between the reference lens and the photographic film. On orienting it in the position to transmit the electric vector of the radiation in the POA, it turned out that the F_- waves were preferentially polarized in this plane; the F_+ waves were also polarized, but only in the plane passing through the beam direction and perpendicular to the POA, i.e., in the interval $\varphi \sim 180° \pm 90°$.

Rotation of the polaroid through an angle $\varphi=90°$ led to transmission of F_+ waves over the whole interval $\varphi \simeq 90° \pm 60°$. The outer F_- cone was thus polarized essentially only in the POA, while the inner F_+ cone was more complicated, i.e., both in the POA and perpendicular to the POA.

5. CONCLUSIONS

Basic properties of Cherenkov radiation have been studied in this work—azimuthal directivity and polarization properties; we have also obtained a qualitative picture of the azimuthal distribution of Cherenkov intensity for the motion of 658 MeV protons along the binormal of the biaxial crystal triglycine sulfate. A new feature of the Cherenkov radiation was found in such crystals—its asymmetric angular distribution relative to the plane through the binormal perpendicular to the plane of the optic axes.

Formulae were obtained describing the asymmetric distribution of the double-cone Cherenkov radiation and their agreement with the experimental results was within the limits of experimental errors.

The lack of a complete theory predicting the azimuthal intensity distribution of F_+ and F_- Cherenkov waves and its polarization properties for such a case is noteworthy.

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[1)] F_+ and F_- waves denote Cherenkov radiation with mutually perpendicular polarization.

[2)] The plus and minus signs both for the angles θ and the F_+ and F_-

[3)] Nonequality of the radiation intensity over φ in the ring from the isotropic lens is connected with the nonequality of the protons distribution in the beam and with the coloring of the lens substance in the point of intersection.

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