Polarization dependences of photoinduced variations in magnetocrystalline anisotropy in $Y_3Fe_5O_{12}$ for pulsed excitation

V. G. Veselago, R. A. Doroshenko, and S. G. Rudov

Institute of General Physics, Russian Academy of Sciences, 117942 Moscow, Russia
(Submitted 3 June 1993)
Zh. Eksp. Teor. Fiz. **105**, 638–647 (March 1994)

A new method of magnetooptical spectroscopy is proposed for studying magnetic dielectrics, which enables the contribution of particular optical transitions to the spin–orbit interaction to be determined. The method is based on measurement of the variation in the component of the magnetization vector of a magnetic dielectric perpendicular to an external magnetic field under the action of a light pulse, for different orientations of the external magnetic field relative to the crystallographic axes of the specimen. The capabilities of the method are demonstrated for the case of $Y_3Fe_5O_{12}$. The photoinduced increase in the cubic anisotropy energy in undoped $Y_3Fe_5O_{12}$ was found to be 10^3 erg/cm^3; in doped specimens, photoinduction of uniaxial magnetic anisotropy was observed along or perpendicular to the plane of polarization of the light, depending on the dopant. The experimental results are discussed within the framework of the traditional phenomenological model of photomagnetism. A number of parameters of the photoinduced anisotropic centers responsible for the phenomenon in $Y_3Fe_5O_{12}$ were obtained.

1. INTRODUCTION

The effect of light on a number of its magnetic characteristics has been observed in the ferromagnetic dielectric $Y_3Fe_5O_{12}$: the dynamic magnetic susceptibility and coercive force, ferromagnetic resonance, dichroism,[1–4] parameters of the domain structure and magnetocrystalline anisotropy.[5–10] Photomagnetic effects were observed upon illuminating $Y_3Fe_5O_{12}$ containing low concentrations of impurity atoms with steady light. These impurities were either introduced specially (Si,Ca,Ge,Ga) or coming into the specimen uncontrollably during growth (Pb,B,Ba). The most fully studied were specimens of yttrium iron garnet doped with silicon. The effect of unpolarized light on the cubic anisotropy, the induction of uniaxial anisotropy either under the action of polarized light or as a result of photomagnetic annealing, local reorientation of the magnetization in domains under local illumination, and rearrangement of the whole domain structure under the action of polarized light were observed.[6–10] The phenomenological model used to explain the polarization-dependent photomagnetic effects in $Y_3Fe_5O_{12}$:Si assumes photoinduced $Fe^{2+} \leftrightarrow Fe^{3+}$ transitions among four orientationally nonequivalent octahedral-type sites, which differ in the direction of the local trigonal axes. The photoinduced formation of a Fe^{2+} ion at one of these sites leads to the formation of a uniaxially anisotropic center with an "easy magnetization axis" directed along one of the four [111] crystallographic axes. The concentration of centers with a fixed direction of the "easy magnetization axis" depends both on the angle between the polarization vector of the light incident upon the specimen and the axis of the center, and on the angle between that axis and the direction of the local magnetization. However, existing phenomenological models only allow some of the experimental results to be explained qualitatively for $Y_3Fe_5O_{12}$ specimens with various types of doping.

The difficulties in choosing a phenomenological model of the photomagnetic effects for specimens with different types of doping are associated with the fact that for steady illumination of specimens, the experimenter is concerned with the ultimate state of the specimen, while photomagnetic effects proceed in several stages; initial excitation (formation) of anisotropic centers, followed by partial recombination of the centers and their orientational redistribution as a result of photomagnetic or just magnetic annealing. Gridnev *et al.*[11] studied the influence of radiation from a pulsed laser on the longitudinal magnetization of $Y_3Fe_5O_{12}$:Si. They showed that their observed bipolar signal in the pick-up coil was connected with changes in the magnetic properties of the specimen due to an internal photoeffect, i.e., the signal was not of a thermal nature. A similar pulse method can prove extremely useful for elucidating the nature of the photomagnetic effects. Investigation of photoinduced anisotropy using pulsed light sources makes it possible to study the properties of rapidly relaxing photoinduced centers and the formation of photoinduced centers at the instant of the action of the light pulse, without the extraneous effects of magnetic after-effect and relaxation.

In the present work a new magnetooptical spectroscopic method of studying the properties of photoinduced centers is proposed, which enables the contribution of specific optical transitions to the magnetocrystalline anisotropy of magnetic dielectrics to be determined.

2. EXPERIMENTAL METHOD

The method for studying magnetocrystalline anisotropy in the present work is based on measuring the com-

 1063-7761/94/030341-05$10.00

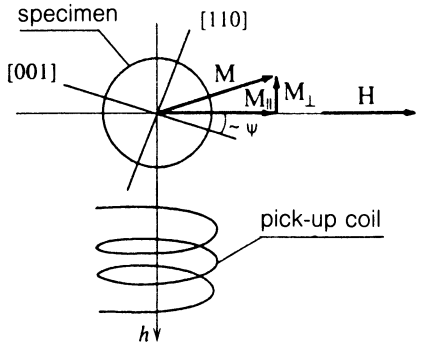

FIG. 1. Mutual orientation of specimen, pick-up coil and magnetic field.

ponent of the magnetization vector **M** of the specimen perpendicular to the external field **H** for different orientations of **H** relative to the crystallographic axes of the ferromagnet.

The dielectric $Y_3Fe_5O_{12}$ is a cubic ferromagnet, the axes of easy magnetization of which are directed along the [111] crystallographic axes. It can be shown that for different orientations of the external magnetic field **H** relative to the crystallographic directions in the (110) crystal plane, the component of magnetization perpendicular to **H**, for $K_1/MH \ll 1$, is

$$M_\perp = K_1(2\sin 2\psi + 3\sin 4\psi)/H, \qquad (1)$$

where ψ is the angle between **H** and a [001] axis, and K_1 is the first constant of the ferromagnet's cubic anisotropy. Thus, if the specimen is magnetized to saturation (i.e., $M = M_s$, the saturation magnetization) and the magnitude of M_s is constant, then the variation in the component $M_\perp$ for fixed external field is related to the variation in anisotropy, i.e., to the constant K_1. $M_\perp$ decreases with increasing K_1, and increases with decreasing K_1 (for $K_1 < 0$ as in $Y_3Fe_5O_{12}$).

In the present work the effect of pulsed light on the magnitude of the perpendicular component $M_\perp$ of the magnetization of $Y_3Fe_5O_{12}$ specimens was studied for various orientations of the external magnetic field **H** relative to the crystallographic [001] direction in the (110) crystal plane (the recording system was first described by Veselago *et al.*,[12] the optical system by Veselago *et al.*).[13]

An optical parametric oscillator was used as the light source, radiating 15–20 ns pulses in the 0.8–1.5 μm wavelength range. The mean energy density of the light incident upon the specimen did not exceed 10^{-2} J/cm^2. The repetition frequency of the light pulses was 12.5 Hz. The light beam from the oscillator was normally incident upon the specimen surface. The specimens were disks with a (110) basal plane 2 mm in diameter, and were 0.25–0.35 mm thick. The external magnetic field was applied in the plane of the disk. The sensor for recording the change in the value of $M_\perp$ under the action of a light pulse was a ten turn pick-up coil positioned close to the specimen. The axis of the coil was oriented perpendicular to the external magnetic field and, accordingly, parallel to the axis h (Fig. 1). A positive signal in the coil corresponded to a photoinduced change in $M_\perp$ in the direction of the h axis. The magnitude of the emf taken from the measuring coil was recorded using an integrating phase-sensitive pulse detector. The value of the photoinduced change in magnetization, proportional to the area of the emf pulse, was evaluated using the following expression (the shape of the pulse was approximated as triangular):

$$\Delta M = (4\pi Sk)^{-1}\int_0^\tau \left(\frac{d\Psi}{dt}\right)dt$$

$$\approx (4\pi Sk)^{-1}\left(\frac{d\Psi}{dt}\right)_{\max}\frac{\tau}{2}$$

$$= -10^4 e\tau(8\pi nSk)^{-1}\ \text{G},$$

where $k = \min(1, 1/\alpha d)$ is the light-filling factor, d is the specimen thickness, α is the absorption coefficient, e the pulse amplitude on the pick-up coil, $d\Psi/dt$ the rate of change of magnetic flux, n the number of turns in the coil, τ the emf pulse duration and S the cross-sectional area of the disk.

3. RESULTS AND DISCUSSION

We studied four types of $Y_3Fe_5O_{12}$ specimens: I) a specially undoped specimen grown from the unadulterated melt; II) a $Y_3Fe_5O_{12}$:(Pb) specimen, specially undoped, grown from a solution in a PbO–PbF_2 flux; III) a $Y_3Fe_5O_{12}$:(Si) specimen doped with silicon, grown from a solution in a PbO–PbF_2 flux; IV) a $Y_3Fe_5O_{12}$:(B–Ba) specimen specially undoped, grown from solution in a BaO–B_2O_3. The pure $Y_3Fe_5O_{12}$ specimen was taken as a basis on which photomagnetic effects were not observed under steady illumination. Si and Pb doped specimens are traditionally used in studying photomagnetic effects while the $Y_3Fe_5O_{12}$:(B–Ba) specimen is a new, little-studied material with record values for the amplitudes of photomagnetic effects over a wide temperature range. All specimens were studied beforehand in a torque magnetometer with continuous illumination (the results were presented by Doroshenko *et al.*[9]).

Upon illuminating the specimens with circularly polarized light from a parametric oscillator in external magnetic fields $H > 1$ kOe, directed at an angle of 60° to the [1$\bar{1}$0] axis at a temperature of 78 K, we observed emf pulses in the pick-up coil, the sign and magnitude of which depended on the type of specimen and the incident wavelength (Fig. 2). The spectral characteristics of the photoinduced change in the perpendicular component of magnetization for all types of specimens is shown in Fig. 3. A positive sign for the signal in Figs. 2 and 3 corresponds to an increase in $|M_\perp|$. The maximum change in $M_\perp$ was observed in external magnetic fields close to the magnitude of the saturation field H_s for illumination with light polarized with **E**$\parallel$ [111]. In external fields $H > H_s$, the photoinduced change in the perpendicular component of the magnetization $\Delta M_\perp$ decreases according to a $1/H$ law.

We will now discuss the angular dependence of the photoinduced change in the perpendicular component of magnetization for each type of specimen, shown in Figs.

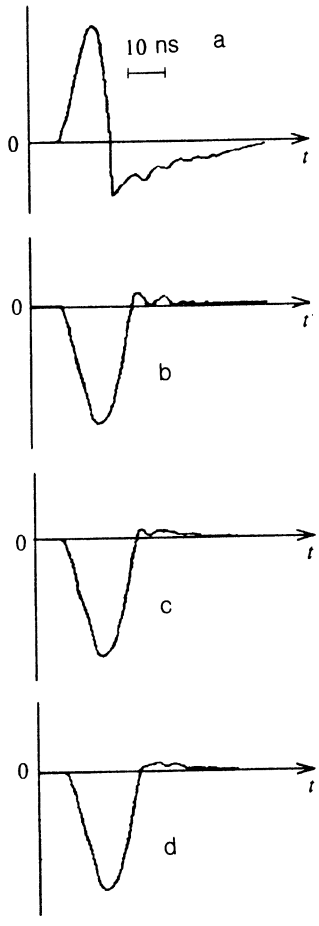

FIG. 2. Shape of pulses on the pick-up coil for $H=2$ kOe, $T=78$ K, $\psi\approx-30°$ (circularly polarized light); a) $Y_3Fe_5O_{12}$, $Y_3Fe_5O_{12}$:(Pb), $\lambda=0.97$ μm; b) $Y_3Fe_5O_{12}$:(Pb), $\lambda=1.06$ μm; c) $Y_3Fe_5O_{12}$:(Si), $\lambda=1.06$ μm; d) $Y_3Fe_5O_{12}$:(B–Ba), $\lambda=1.06$ μm;

4–7 separately, and compare them with results on the photomagnetic properties for continuous illumination.

1. $Y_3Fe_5O_{12}$ specimen

Upon illuminating the specimen with light having $\lambda<1$ μm, we observed bipolar emf pulses, which suggest comparatively short relaxation times of the magnetization to the original direction [see Fig. 2a and Fig. 3 (curve 1)]. At 78 K, the relaxation time is at most 60 ns. As the temperature increases, the relaxation time becomes appreciably shorter, while the amplitude of the effect decreases monotonically up to about 180 K. The polarization of the incident light has no influence on the angular dependence of the photoinduced change in the perpendicular component $\Delta M_\perp$ of the magnetization, which is shown in Fig. 4. The function $\Delta M_\perp(\psi)$ is satisfactorily approximated by Eq. (1) for the angular dependence of $M_\perp$, which corresponds to an increase in cubic anisotropy energy of order 10^3 erg/cm^3. The effect is observed in the vicinity of the fundamental absorption of the specimen in the range 0.85–1 μm, corresponding to the excited states of Fe^{3+} ions in octahedral positions undergoing an increase in anisotropy energy.

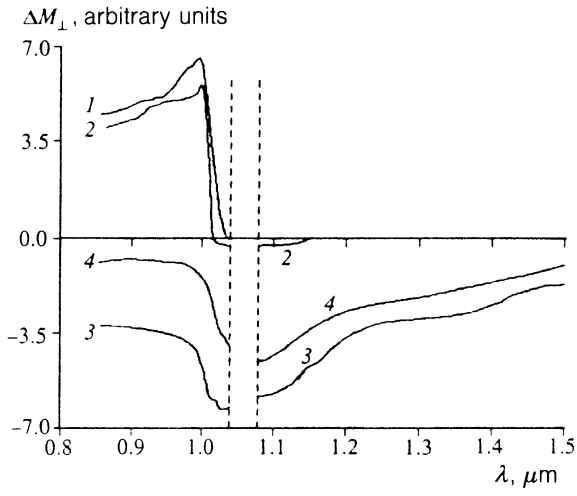

FIG. 3. Spectral dependences of photoinduced change in the perpendicular component of magnetization for $H=2$ kOe, $T=78$ K, $\psi=-30°$ (circularly polarized light): *1*—$Y_3Fe_5O_{12}$; *2*—$Y_3Fe_5O_{12}$:(Pb); *3*—$Y_3Fe_5O_{12}$:(Si); *4*—$Y_3Fe_5O_{12}$:(B–Ba). The vertical dashed lines indicate the region of degenerate action of the parametric light oscillator.

Photomagnetic effects were not observed in this specimen under steady illumination because of the short relaxation times.

2. $Y_3Fe_5O_{12}$:(Pb) specimen

In the region of the fundamental absorption $\lambda=0.8$–1.0 μm, the time and angular dependences of $\Delta M_\perp$ are completely analogous to the corresponding dependences of the previous specimen (see Figs. 2a and 4), i.e., a photoinduced increase in cubic anisotropy is observed. In the region of impurity absorption in the wavelength interval 1.0–1.15 μm, a unipolar negative pulse is observed (Figs. 2b and 3, curve *2*), the amplitude of which only reaches the maximum after a series of light pulses. No such temporal effect was subsequently observed upon varying the magnitude or direction of the external field. It can be concluded that the initial illumination of the specimen leads to the formation of photoinduced anisotropic centers before achieving the saturation concentration. Further illumination with light of wavelength in the interval 1.0–1.15 μm leads to photomagnetic effects associated with secondary

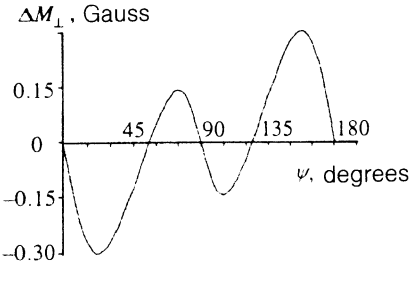

FIG. 4. Angular dependence of the photoinduced change in the perpendicular component of magnetization of $Y_3Fe_5O_{12}$ and $Y_3Fe_5O_{12}$:(Pb) for $T=78$ K, $\lambda=0.97$ μm, $H=2$ kOe, ψ is the angle between the vector **H** and the [001] direction.

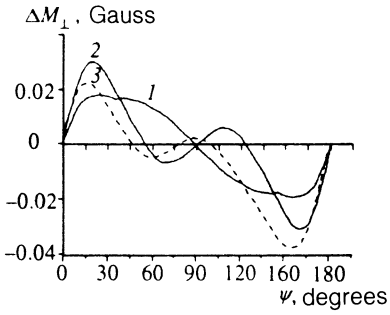

FIG. 5. Angular dependence of the photoinduced change in the perpendicular component of magnetization for $Y_3Fe_5O_{12}$:(Pb) for $T=78$ K, $\lambda=1.06$ μm, $H=2$ kOe; *1*—**E**⊥ **H**, *2*—**E**∥ **H**, *3*— **EH**$=-45°$.

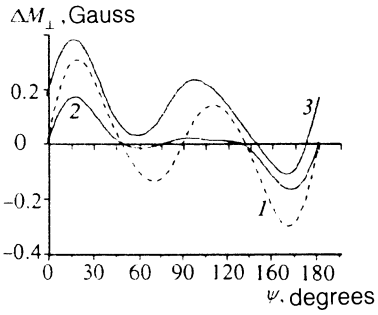

FIG. 7. Angular dependence of the photoinduced change in the perpendicular component of magnetization for $Y_3Fe_5O_{12}$:(B–Ba) for $T=78$ K, $\lambda=1.06$ μm, $H=2$ kOe; *1*—**E**⊥ **H**, *2*—**E**∥ **H**, *3*—**EH**$=45°$.

excitation of photoinduced centers. When the specimen is illuminated with light at $\lambda=1.06$ μm (the region of impurity absorption) the angular dependence of $\Delta M_\perp$ depends strongly on the polarization of the incident light (see Fig. 5). With the plane of polarization **E**⊥ **H**, uniaxial photoinduced anisotropy directed along the $[1\bar{1}0]$ axis arises. For the polarization-dependent effect at $\lambda=1.06$ μm, the value of $\Delta M_\perp$ is about 20 times less than when it is illuminated with light at $\lambda<1.0$ μm. With increasing temperature to 150 K, the effect falls monotonically to zero.

Upon continuous illumination, in $Y_3Fe_5O_{12}$:(Pb) photoinduced variations in the dynamic magnetic susceptibility and in the coercive force are observed in the temperature range 78–150 K, as is photoinduced uniaxial anisotropy along the $[1\bar{1}0]$ axis, but not photoinduced variations in cubic anisotropy.[9]

3. $Y_3Fe_5O_{12}$:Si specimen

Over the whole spectral range studied, approximately the same unipolar pulses of negative polarity (see Fig. 2c) were observed for the $Y_3Fe_5O_{12}$:Si specimen. The amplitude of the pulses, independent of the initial illumination, fell monotonically to zero with increasing temperature to 270 K. The $\Delta M_\perp$ (ψ) dependence is sensitive to the polarization of the incident light (see Fig. 6). the contribution of cubic magnetic anisotropy (the variation in the an isotropy constant by an amount of the order of 3×10^3 erg/cm^3) and uniaxial anisotropy along the polarization direction of the light can be extracted from the angular dependence of $\Delta M_\perp$.

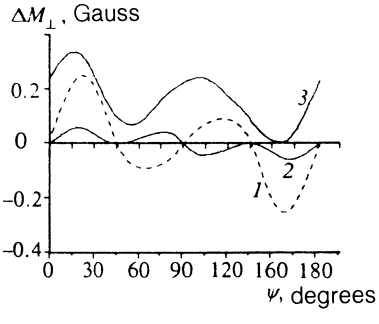

FIG. 6. Angular dependence of the photoinduced change in the perpendicular component of magnetization for $Y_3Fe_5O_{12}$:Si for $T=78$ K, $\lambda=1.06$ μm, $H=2$ kOe; *1*—**E**⊥ **H**, *2*—**E**∥ **H**, *3*—**EH**$=45°$.

The experimental results for $Y_3Fe_5O_{12}$:Si agree satisfactorily with results of experiments with continuous illumination.[9] The only difference is in the magnitude of the effects, which is an order of magnitude greater for the pulsed method.

4. $Y_3Fe_5O_{12}$:(B–Ba) specimen

The temporal and angular dependences of $\Delta M_\perp$ for the $Y_3Fe_5O_{12}$:(B–Ba) specimen at $\lambda=1.06$ μm are similar to the corresponding dependences for $Y_3Fe_5O_{12}$:Si (Figs. 2d and 6), i.e., for both specimens, characteristic features include large relaxation times (compared with the light pulse duration) for the direction of magnetization toward background position, photoinduced cubic anisotropy with an "axis of easy magnetization" along [100] equivalent directions, and the appearance of uniaxial anisotropy along the polarization direction of linearly polarized light.

Unlike $Y_3Fe_5O_{12}$:Si, however, temporal effects are observed in $Y_3Fe_5O_{12}$:(B–Ba) specimens: the maximum values of $\Delta M_\perp$ are reached after preliminary illumination of the specimen for 2–3 minutes. After preliminary illumination, temporal effects are no longer observed when the external magnetic field or specimen orientation are changed. We thus conclude that the change in the perpendicular component of magnetization $\Delta M_\perp$ is associated with secondary excitation of photoinduced centers. The amplitude of $\Delta M_\perp$ falls monotonically to zero as the temperature increases to 190 K.

Under steady illumination, only a photoinduced increase in cubic magnetocrystalline anisotropy by 10^3 erg/cm^3 is observed in $Y_3Fe_5O_{12}$:(B–Ba), independent of the polarization of the light, as well as uniaxial anisotropy resulting from photomagnetic annealing along [111] axes.[9]

The pulse method of studying the photoinduced change in the perpendicular component of magnetization makes it possible to obtain the angular dependence of $\Delta M_\perp$ to a relative accuracy sufficient to verify the model representations. We now discuss the phenomenological model of photoinduced anisotropic centers, traditionally

used to interpret photomagnetic effects in $Y_3Fe_5O_{12}$:Si (Ref. 14), to analyze the angular dependence of $\Delta M_\perp$. We propose that at the moment the illumination is turned on, photoinduced uniaxial anisotropic centers appear with local axes directed along the equivalent [111] directions. The probability of the formation of an anisotropic center with a specific axis depends, then, on the direction of the magnetization and the polarization of the light. Such suggestions are fully justified, since in our experiments the maximum values of $\Delta M_\perp$ are reached for an orientation of the plane of polarization along the equivalent [111] directions. We can then write for the free energy per $\mathscr{E}$ unit specimen volume

$$\mathscr{E}=K_1(m_x^2m_y^2+m_x^2m_z^2+m_y^2m_z^2)+K_2(m_x^2m_y^2m_z^2)$$
$$-\mathbf{mH}+\sum_{i=1}^{4}N_0\varepsilon_i(\mathbf{m})W_i(\mathbf{m},\mathbf{e}), \qquad (2)$$

where K_1 and K_2 are the cubic anisotropy constants, $\mathbf{H}$ the magnetic field, $\mathbf{M}$ the magnetization, $\mathbf{m}=\mathbf{M}/M$, N_0 the concentration of photoinduced centers, and $W_i(\mathbf{m},\mathbf{e})$ the probability of formation of a center with the ith local axis (it depends on the direction of magnetization $\mathbf{m}$ and the polarization of the light $\mathbf{e}$ relative to the local axis of the center); $\varepsilon_1=\varepsilon_0\sin^2\beta_1$ is the anisotropy energy of the center with a local ith axis. We shall assume[14] that

$$W_i(\mathbf{m},\mathbf{e})=C(1+A\cos^2\gamma_i)(1+B\cos^2\beta_i),$$

where β_i is the angle between the ith local axis and the vector $\mathbf{m}$, γ_i is the angle between the ith local axis and $\mathbf{e}$; A and B are dimensionless constants, $-1<(A,B)<1$, and C is a normalization constant.

Minimizing the free energy (Eq. 2), we can determine the equilibrium orientation of the magnetization $\mathbf{m}$ for $\varepsilon_0=0$ and the orientation of the magnetization $\mathbf{m}$ in the presence of illumination for $\varepsilon_0\neq0$. The magnitude of $\Delta M_\perp$ can be calculated from the relation

$$\Delta M_\perp(\psi)=0.5\sqrt{2}\mathbf{M}[-(\tilde{m}_x-m_x)\cos\psi+(\tilde{m}_y-m_y)$$
$$\times\cos\psi+\sqrt{2}(\tilde{m}_z-m_z)\sin\psi]$$

and compared with the experimentally observed angular dependence of $\Delta M_\perp$.

Very good agreement between theory and experiment can be obtained for all three $\Delta M_\perp$ dependences corresponding to different light polarizations for the $Y_3Fe_5O_{12}$:(B–Ba) specimen with the following values of the adjustable parameters: $K_1=-0.1$ kOe, $K_2=0$, $N_0\varepsilon_0=-0.01$ kOe, $A=-0.3$, $B=0.6$. The probability of formation of a center with a specific local axis for $A<0$ will be a maximum for orthogonal orientation of the light polarization and the induced local axis of the center. A negative value of the contribution of photoinduced centers to the anisotropy energy ($N_0\varepsilon_0<0$) suggests that under the action of light, centers are formed with the an isotropy of the {111} easy plane. Knowing the value of the absorption coefficient $\alpha=80$ cm^{-1} for $h\nu_0=1.17$ eV and the energy density of the incident light $I\approx0.011$ J/cm^2, we can calculate the anisotropy energy of a single center,

$$\varepsilon_0=(N_0\varepsilon_0)Mh\nu_0/I\alpha\approx2\times(10^{15}-10^{14})\text{erg}.$$

Satisfactory agreement with experiment for the $Y_3Fe_5O_{12}$:(Pb) specimen can be obtained for $A=0.33$ and $B=0.15$, i.e., the probability of formation of a center is a maximum for collinear orientation of the plane of polarization relative to the center's local axis. The angular dependence of $\Delta M_\perp$ for $Y_3Fe_5O_{12}$:Si cannot be represented satisfactorily by a single choice of the constants A and B. Curves closest to experiment are obtained for $A=-0.52$ and $B=0.39$.

Doroshenko *et al.*[9] took account for both octahedral and tetrahedral centers to describe the effects of magnetic annealing in $Y_3Fe_5O_{12}$:Si. We carried out similar calculations taking account of tetrahedral centers, but we did not obtain appreciable improvements in describing the experimental curves. We consider that the unsatisfactory agreement between theory and experiment for $Y_3Fe_5O_{12}$:Si is connected with either the difficulty of a correct choice in our model of the initial magnetic state of the specimen after a series of light pulses, due to the long relaxation times of the photoinduced centers, or with the fact that the phenomenological model is not applicable to $Y_3Fe_5O_{12}$:Si.

We note in conclusion that the pulse method for studying the influence of light on magnetic anisotropy can be applied in spectral investigations of any magnetic dielectrics with excited state lifetimes greater than 2×10^{-8} s.

[1] U. Enz and H. van der Heide, Solid State Commun. **6**, 347 (1968).
[2] R. W. Teale and D. W. Temple, Phys. Rev. Lett. **19**, 904 (1967).
[3] K. Hisatake and K. Hirasaki, Phys. Status Solidi A **27**, K61 (1975).
[4] J. F. Dillon, E. M. Gyorgy, and J. P. Remeika, Phys. Rev. Lett. **22**, 643 (1969).
[5] V. F. Kovalenko and É. L. Nagaev, Usp. Fiz. Nauk **148**, 561 (1986). [Sov. Phys. Usp. **29**, 297 (1986)].
[6] V. F. Kovalenko, E. S. Kolezhuk, and P. S. Kuts, Zh. Eksp. Teor. Fiz. **81**, 1399 (1981) [Sov. Phys. JETP **54**, 742 (1981)].
[7] V. F. Kovalenko, P. S. Kuts, and V. P. Sokhatskiĭ, Fiz. Tverd. Tela **24**, 145 (1982) [Sov. Phys. Solid State **24**, 80 (1982)].
[8] P. J. Flanders, C. D. Graham, J. F. Dillon, E. M. Gyorgy, and J. P. Remeika, J. Appl. Phys. **42**, 1443 (1971).
[9] R. A. Doroshenko, M. S. Setchenkov, and I. V. Vladimirov, Fiz. Tverd. Tela **34**, 377b (1992) [Sov. Phys. Solid State **34**, 202 91992)].
[10] R. A. Doroshenko, M. S. Setchenkov, and I. V. Vladimirov, Fiz. Tverd. Tela **30**, 2834 (1988) [Sov. Phys. Solid State **30**, 1633 (1988)].
[11] V. N. Gridnev, R. V. Pisarev, S. I. Shablaev, and M. G. Khalmuratov, Fiz. Tverd. Tela **30**, 3399 (1988) [Sov. Phys. Solid State **30**, 1951 (1988)].
[12] V. G. Veselago, A. A. Minakov, and S. G. Rudov, Zh. Eksp. Teor. Fiz. **87**, 629 (1984). [Sov. Phys. JETP **60**, 360 (1984)].
[13] V. G. Veselago, G. I. Vinogradova, A. A. Garmonov, S. G. Rudov, É. G. Zhukov, I. Kurbanklychev, and V. A. Levshin, Zh. Eksp. Teor. Fiz. **97**, 559 (1990) [Sov. Phys. JETP **70**, 311 (1990)].
[14] J. F. B. Hawkes and R. W. Teale, J. Phys. C **5**, 481 (1972).

Translated by R. Berman