Volt–ampere characteristic of a tunneling junction with a thin layer of a normal metal on a dielectric barrier

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A thin layer of a normal metal on the surface of a dielectric separating two superconductors gives rise to a hysteresis in the volt–ampere characteristic in the above-threshold region $eV>\Delta_1+\Delta_2$. One feature of this hysteresis mechanism is a relatively wide voltage range over which the differential resistance is negative.

1. INTRODUCTION

The volt–ampere characteristic of a tunneling junction is usually studied by the tunneling Hamiltonian method.[1] When the junction voltage is the sum of the gaps of the single-particle excitation spectra of the two superconductors, a jump in the volt–ampere characteristic occurs.[2,3] The magnitude of the jump depends on the temperature but remains finite up to the superconducting transition temperature. The jump is due to the square root singularity in the density of states as predicted in the BCS model.

In real superconductors, there are always factors acting to smooth the square root singularity and thus to wash out the jump. One such factor is due to paramagnetic impurities. The jump smearing region in this case is of order $\Delta(\tau_S\Delta)^{-2/3}$, where τ_S is the electron spin-flip lifetime. The differential resistance in this case is always nonnegative. A more interesting situation occurs when the surface of a dielectric separating two superconductors has a thin layer (or droplets) of a normal metal. It is obvious that in this case the jump in the volt–ampere characteristic will also be washed out. However, unlike superconductors with paramagnetic impurities, the volt–ampere characteristic of such a junction is not monotonic, and a rather wide voltage range exists over which the differential resistance is negative.[4]

Below we will find the volt–ampere characteristic for a junction having a thin normal-metal interlayer on the dielectric layer separating two superconductors, in the important voltage range $|eV-\Delta_1-\Delta_2|\ll\Delta_{1,2}$.

2. DENSITY OF STATES IN A SUPERCONDUCTOR WITH A THIN NORMAL-METAL INTERLAYER ON THE SURFACE

Consider a bulk superconductor having a normal-metal layer of thickness d deposited on its plane surface.

Assume that in both the superconductor and the normal metal, the mean free path of the electron, l, is much less than the correlation length. The retarded Green's functions α and β in this case satisfy the equations[5]

$$-D\frac{\partial}{\partial r}\left(\alpha\frac{\partial\beta}{\partial r}-\beta\frac{\partial\alpha}{\partial r}\right)+2i(\alpha\Delta-\beta\varepsilon)=0,\quad \alpha^2-\beta^2=1 \tag{1}$$

and the boundary conditions on the superconductor–normal metal interface[6]

$$\beta_+=\beta_-,\quad \left(\nu D\frac{\partial\beta}{\partial n}\right)_+=\left(\nu D\frac{\partial\beta}{\partial n}\right)_-, \tag{2}$$

where $\nu=mp_0/2\pi^2$ is the density of states at the Fermi surface, and $D=vl_{tr}/3$ is the diffusion coefficient.

Note that for the system (1) to be fully applicable, it is also necessary that $l\ll d$ in the normal metal layer. This restriction is not crucial, however.

In the presence of a thin normal-metal layer, the volt–ampere characteristic of the tunneling junction can vary significantly only in the threshold region $|eV-\Delta_1-\Delta_2|\ll\Delta_{1,2}$. To calculate the one-particle tunneling current in this voltage range, we will need the Green's function α for energies ε close to Δ. In this energy range, the Green's functions α and β are large

$$|\alpha|,|\beta|\gg 1 \tag{3}$$

and the system (1) can be reduced to a single equation

$$-D\frac{\partial}{\partial r}\left(\frac{1}{\beta}\frac{\partial\beta}{\partial r}\right)+2i\Delta\left(\beta+\frac{1}{2\beta}\right)-2i\varepsilon\beta=0. \tag{4}$$

For the plane geometry considered, Eq. (4) can be solved exactly, and its solution has the form

$$\beta=\begin{cases} c/\cos^2\left[\exp\left(-\frac{i\pi}{4}\right)c^{1/2}(\varepsilon/D_N)^{1/2}Z\right]; & 0\leqslant z\leqslant d\\ \left(\frac{\Delta}{2(\varepsilon-\Delta)}\right)^{1/2}\left(\frac{Y(z)+1}{Y(z)-1}\right)^2; & d\leqslant z,\end{cases} \tag{5}$$

where the function $Y(z)$ is given by

$$Y(z)=\exp\left\{\exp\left(-\frac{i\pi}{4}\right)\right.$$

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$$\times\left(\frac{2^{3/2}[\Delta(\varepsilon-\Delta)]^{1/2}}{D_s}\right)^{1/2}(z+z_0)\Bigg\}. \qquad (6)$$

In Eqs. (5) and (6), the constants of integration c and z_0 can be found from the boundary conditions (2). Substituting expressions (5) for the Green's function β into the conditions (2), we find

$$\left(\frac{\Delta}{2(\varepsilon-\Delta)}\right)^{1/2}\left(\frac{Y(d)+1}{Y(d)-1}\right)^2 = \frac{c}{\cos^2\left[\exp\left(-\frac{i\pi}{4}\right)\cdot c^{1/2}(\varepsilon/D_N)^{1/2}d\right]};$$

$$-2v_sD_s^{1/2}\left(\frac{\Delta}{\varepsilon-\Delta}\right)^{1/2}(2\Delta(\varepsilon-\Delta))^{1/4}\frac{(Y(d)+1)Y(d)}{(Y(d)-1)^3}$$
$$=v_ND_N^{1/2}c^{3/2}\varepsilon^{1/2}\sin\left[\exp\left(-\frac{i\pi}{4}\right)c^{1/2}(\varepsilon/D_N)^{1/2}d\right]\Big/\cos^3\left[\exp\left(-\frac{i\pi}{4}\right)c^{1/2}(\varepsilon/D_N)^{1/2}d\right]. \qquad (7)$$

For convenience, we introduce two dimensionless parameters,

$$\varkappa=(\Delta/D_N)^{1/2}d, \quad \gamma=2^{1/2}\frac{v_sD_s^{1/2}}{v_ND_N^{1/2}}. \qquad (8)$$

In the case considered, the parameter $\varkappa$ is small and the parameter γ is generally of order unity. We will check below that if the parameter γ is not too large, so that

$$\varkappa^{2/3}\gamma^{1/3}\ll 1, \qquad (9)$$

$|c|^{1/2}\varkappa$ will also be small. This enables one to eliminate $Y(d)$ from (7), and thus to reduce the latter to a single third-degree equation for $c^{1/2}$. After a little manipulation we find an equation for $c^{1/2}$,

$$c^{3/2}+\frac{\gamma}{2\varkappa}\exp\left(\frac{i\pi}{4}\right)\left[c\left(\frac{2(\varepsilon-\Delta)}{\Delta}\right)^{1/2}-1\right]=0. \qquad (10)$$

The threshold value E in the one-particle excitation spectrum is determined from the condition

$$\left(\frac{\partial c}{\partial\varepsilon}\right)_{\varepsilon=E}=\infty. \qquad (11)$$

From Eqs. (10) and (11), we find

$$E=\Delta\left(1-\frac{9}{2}\left(\frac{\varkappa}{\gamma}\right)^{4/3}\right), \quad c(\varepsilon=E)=-i\left(\frac{\gamma}{\varkappa}\right)^{2/3}. \qquad (12)$$

Equation (12) leads to the aforementioned restriction (9) on the domain of applicability of Eq. (10). Equation (10) can be solved in a standard manner giving

$$c=-\frac{i}{4}\left(\frac{\gamma}{\varkappa}\right)^{2/3}\left(1-\frac{2y}{9}\right)\left[1+\frac{1}{2}(\exp(\varphi_1)+\exp(-\varphi_1))+\frac{i\sqrt{3}}{2}(\exp(\varphi_1)-\exp(-\varphi_1))\right]^2, \qquad (13)$$

where

$$\frac{\varepsilon-\Delta}{\Delta}=\left(\frac{\varkappa}{\gamma}\right)^{4/3}(y-9/2);$$

$$\exp(\psi_1)=\left\{\frac{2}{\left(1-\frac{2y}{9}\right)^{3/2}}-1+\left[\left(\frac{2}{\left(1-\frac{2y}{9}\right)^{3/2}}-1\right)^2-1\right]^{1/2}\right\}^{1/3}. \qquad (14)$$

Expression (13) is convenient for use in the region

$$0\leqslant y\leqslant 9/2. \qquad (15)$$

In the range $9/2\leqslant y$, the expression for c should be rewritten in the form

$$C=\frac{i}{4}\left(\frac{\gamma}{\varkappa}\right)^{2/3}\left(\frac{2y}{9}-1\right)\left\{1+\exp\left(\frac{i2\pi}{3}\right)\left(1+\frac{2i}{\left(\frac{2y}{9}-1\right)^{3/2}}+\left(\left(1+\frac{2i}{\left(\frac{2y}{9}-1\right)^{3/2}}\right)^2-1\right)^{1/2}\right)^{1/3}\right.$$
$$\left.+\exp\left(-\frac{i2\pi}{3}\right)\left(1+\frac{2i}{\left(\frac{2y}{9}-1\right)^{3/2}}+\left(\left(1+\frac{2i}{\left(\frac{2y}{9}-1\right)^{3/2}}\right)^2-1\right)^{1/2}\right)^{1/3}\right\}^2. \qquad (16)$$

We also reproduce the expression for c valid in the range of small (large) values of the argument y:

$$c=-i\left(\frac{\gamma}{\varkappa}\right)^{2/3}\times\begin{cases}1+\frac{2i}{3}y^{1/2}; & 0\leqslant y\ll 1\\ \frac{i}{(2(y-9/2))^{1/2}}\left[1-\frac{2^{1/4}\exp(-i\pi/4)}{(y-9/2)^{3/4}}\right]; & 1\ll y.\end{cases} \qquad (17)$$

Equations (14) and (16) determine the Green's function $\beta(z=0)=c$ in the energy range

$$|\varepsilon-\Delta|\ll\Delta, \qquad (18)$$

making it possible to obtain the volt–ampere characteristic of the tunneling junction over the entire hysteresis region.

3. VOLT–AMPERE CHARACTERISTIC

To second order in the barrier transparency, the zero-temperature one-particle current j across the barrier, for a nonhomogeneous superconductor, can be written in the form[7]

$$SR_N j = -\int_{E_1}^{eV-E_2} d\varepsilon \int_s \mathrm{Re}\,\alpha_1(\varepsilon)\mathrm{Re}\,\alpha_2(eV-\varepsilon)ds, \tag{19}$$

where s is the junction area and R_N is the resistivity of the junction in the normal state. For a junction with no normal metal layers, the current is equal to

$$eR_N j = -\begin{cases} 0; \quad eV < \Delta_1+\Delta_2 \\ \dfrac{\pi(\Delta_1\Delta_2)^{1/2}}{2}\left[1+\dfrac{3(\Delta_1+\Delta_2)(eV-\Delta_1-\Delta_2)}{8\Delta_1\Delta_2}\right]; \\ \quad 0 < eV-\Delta_1-\Delta_2 \ll \Delta_{1,2}. \end{cases} \tag{20}$$

The coefficient of the second (linear in $eV-\Delta_1-\Delta_2$) term in (20) depends weakly on the presence of a thin layer of normal metal. Therefore the expression (19) for the current through a junction with normal-metal layers can be written in the form

$$eR_N j = -(\Delta_1\Delta_2)^{1/2}\left\{F+\frac{3\pi}{16\Delta_1\Delta_2}(\Delta_1+\Delta_2) \times (eV-\Delta_1-\Delta_2)\right\}, \tag{21}$$

where

$$F = \frac{1}{s(\Delta_1\Delta_2)^{1/2}}\int_{E_1}^{eV-E_2} d\varepsilon \int_s \mathrm{Re}\,\beta_1(\varepsilon) \times \mathrm{Re}\,\beta_2(eV-\varepsilon)ds. \tag{22}$$

The Green's functions $\beta_{1,2}$ are determined by expressions (13) and (16). In these we have omitted small corrections leading to a slow linear growth of the current with the voltage. Equations (13), (16), (21), and (22) are also valid for nonuniform deposition of a normal-metal layer on either or both sides of the dielectric, provided the corresponding areas are large enough (i.e., much greater than the square of the correlation length). An explicit expression for the function F depends on the relative position of the normal metal layers. We present expressions for F for bilateral and unilateral deposition which actually covers the general case.

For a bilateral deposition (deposit thicknesses $d_{1,2}$)

$$F \equiv F_{12} = \begin{cases} \dfrac{\pi}{18} z_1 z_2; \quad z_{1,2} \ll 1 \\ \dfrac{\pi}{2} - \dfrac{B(-1/4,1/2)}{2^{5/4}}\left(\dfrac{1}{(z_1+9/2)^{3/4}}+\dfrac{1}{(z_2+9/2)^{3/4}}\right); \\ \quad z_{1,2} \gg 1, \end{cases} \tag{23}$$

where $B(-1/4,1/2)$ is the Euler beta function,

$$z_1 = \frac{eV-E_1-E_2}{\Delta_1}\left(\frac{\gamma_1}{\varkappa_1}\right)^{4/3}; \quad z_2 = \frac{eV-E_1E_2}{\Delta_2}\left(\frac{\gamma_2}{\varkappa_2}\right)^{4/3};$$

$$B(-1/4,1/2) = -\frac{4\sqrt{\pi}\Gamma(3/4)}{\Gamma(1/4)}. \tag{24}$$

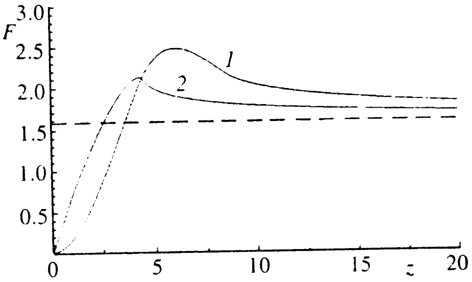

FIG. 1. Function F (volt–ampere characteristic) for a tunneling junction made of two identical superconductors, for bilateral (curve *1*) and unilateral (curve *2*) coating of the dielectric by a normal metal layer of thickness d. Dash-dot horizontal line is the asymptote ($\pi/2$) of the function F for $Z \to \infty$.

An expression for F for $z_{1,2} \gg 1$ has been obtained in Ref. 4.

For a dielectric coated only on the side facing the first (second) superconductor, Eqs. (12) and (22) yield

$$F \equiv F_{1(2)} = \begin{cases} \dfrac{\pi}{3\sqrt{2}} z_{1(2)}; \quad z_{1(2)} \ll 1 \\ \dfrac{\pi}{2} - \dfrac{B(-1/4,1/2)}{2^{5/4}} \dfrac{1}{(z_{1(2)}+9/2)^{3/4}}; \quad z_{1(2)} \gg 1. \end{cases} \tag{25}$$

Figure 1 presents the values of the function F for a tunneling junction made of two identical superconductors, for a bilateral (curve *1*) and unilateral (curve *2*) coating on the dielectric by a normal-metal layer of thickness d.

Now suppose that on the side toward the first superconductor, the normal metal area is s_1; on the side toward the second, it is s_2; on both sides, s_{12}. Then the expression for the junction current j can be written in the form

$$eR_N j = -(\Delta_1\Delta_2)^{1/2}\left\{F_1 \cdot \frac{s_1-s_{12}}{s} + F_2\frac{s_2-s_{12}}{s} + F_{12}\frac{s_{12}}{s} + \frac{\pi}{2}\left(1-\frac{s_1+s_2-s_{12}}{s}\right)\theta(eV-\Delta_1-\Delta_2) + \frac{3\pi}{16}\frac{(\Delta_1+\Delta_2)}{\Delta_1\Delta_2}(eV-\Delta_1-\Delta_2) \times \theta(eV-\Delta_1-\Delta_2)\right\}, \tag{26}$$

where the functions F_1, F_2, and F_{12} are given by (22), (23), and (25).

The boundaries of the negative differential resistance region are found from the condition

$$\partial j/\partial eV = 0. \tag{27}$$

For a symmetrical junction (both sides of the dielectric coated with a normal-metal layer), from Fig. 1 and Eqs. (21) and (25) we find the region of negative differential resistance to be

$$6.168 < z < -9/2 + \left[-\frac{2^{3/4}}{\pi} \cdot B(-1/4,1/2)(\gamma/\varkappa)^{4/3} \right]^{4/7}. \quad (28)$$

From (28) it follows that the negative differential resistance region is quite wide, and [owing to the large parameter $(\gamma/\varkappa)^{16/21}$] is larger than the region of smearing of the step in the volt–ampere characteristic.[4]

4. CONCLUSION

We have studied a model of a superconducting tunneling junction in which the surface of the dielectric interface is coated, partially or completely, by a thin film of normal metal. In experimental samples, such a film may result from the particular technique used in tunneling junction preparation.[2] The film need not necessarily be made of a normal metal: it is enough if the superconducting transition temperature of the film material is considerably lower than that of the electrode material.

Within the model employed, the square-root singularity in the density of states is washed out and there always exists a voltage range in which the differential resistance is negative. In the present model this region turns out to be quite wide and—because of the large parameter $(\xi/d)^{16/21}$, where ξ is the correlation length—it is larger than the smearing region of the jump in the volt–ampere characteristic.

In real samples, there always exist other mechanisms which also lead to the washing out of the density of states singularity, such as paramagnetic impurities, inelastic scattering of electrons, etc. Some of these mechanisms—for example, paramagnetic impurities—lead to a monotonic volt–ampere characteristic. As a result of the competition among the various mechanisms involved, the volt–ampere characteristic may or may not possess a region with negative differential resistance.

Also note that in the present work we have restricted our discussion to the low-temperature case $T \ll \Delta$. This last circumstance is not in fact a critical one, and a finite-temperature analysis does not lead to any new physical phenomena.

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