The soliton of the Korteweg–de Vries equation: A "device" for separating an additive mixture of a determinate signal and Gaussian noise

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This paper introduces a class of nonlinear distributed filters (NDFs), which are nonlinear distributed dynamical systems, for exact separation of an additive mixture of an arbitrary determinate signal and Gaussian noise. The principal property of NDFs is that the determinate and random components of the mixture manifest themselves, in the physical sense, in dramatically different ways. The random component is responsible for the diffusive transport of a certain substance in the space of generalized variables, while the determinate component is responsible for convective transport of the same substance. One-soliton solutions of the stochastic Korteweg–de Vries (KdV) equation are taken as examples of NDF that allow for experimental verification. It is shown that many-soliton solutions of the KdV equation can also be employed as NDFs.

I. INTRODUCTION

The problem of extracting useful information contained in signals at the input ports of measuring devices, receivers in communication lines, and the like is highly important in science and technology. One reason why its solution is greatly complicated is that in addition to useful information the receiving devices take in signals containing uncertainties of some kind, that is, random signals or noise. As a result there emerge many problems related to the decoding of signals with randomly varying parameters. The literature devoted to solving such problems is vast (see, e.g., Refs. 1–4 and citations given there). Since *a priori* the probabilistic properties of signals are unknown, their fixation and subsequent separation is done via statistical hypotheses, which with a certain probability (depending on the choice of hypotheses) make it possible to evaluate the nature of the signals and the useful information contained therein. The modern approach to solving the problem incorporates estimation theory[5,6] and digital methods of signal processing based on spectral representation of the signals.[6] The problem of an additive mixture of a nonrandom signal $g(t)$ and Gaussian noise $\alpha(t)$ occupies a special place among problems of this type. Its special status is due to the widespread nature of the conditions in which noise with Gaussian statistics forms (in view of the central limit theorem familiar from mathematics) and also to the existing practice of signal formation. The author believes that the literature contains no exact methods of separating an additive mixture of an arbitrary determinate signal and arbitrary Gaussian noise.

The present study shows that it is possible in principle to separate exactly such a mixture using generally accepted statistical approaches. Here nonlinear distributed systems are suggested as filtering elements, one example being solitary nonlinear waves, or solitons. This is illustrated by the example of one-soliton solutions of the stochastically perturbed Korteweg–de Vries (KdV) equation. It must be noted, however, that a similar possibility exists if we employ many-soliton solutions of the KdV equation, as well as solutions (including soliton solutions) of other nonlinear partial differential equations.

Anticipating the conclusion we note that nonlinear dynamical systems with distributed parameters can be used as a nonlinear distributed filter (NDF) in two directions. In the first the filter is represented by a certain mathematical construction and is implemented in the form of a software package or a specialized processor. As shown in the present paper, one- or N-solutions of the stochastic KdV equation[1)] can serve as the mathematical construction. In the second case the filter is realized in the form of a physical device with a unit modeling the dynamics described by the KdV equation. What is important is that this unit can be constructed from nonlinear dispersion transmission lines, as shown by Lonngren in Ref. 7. The range of questions discussed in the present paper is limited primarily to the first case.

2. FORMULATION OF THE PROBLEM AND THE GENERAL EQUATION FOR THE MEAN[2)]

Wadati[9] used the inverse scattering method to show that the exact nonaveraged solution of the stochastic KdV equation

$$\frac{\partial u}{\partial t}+6u\frac{\partial u}{\partial x}+\frac{\partial^3 u}{\partial x^3}=\beta(t) \tag{1}$$

has the form

$$u(x,t)=v(t)-2k^2\,\mathrm{sech}^2\left[k(x-x_0)-4k^3t+6k\int_0^t v(\tau)d\tau\right], \tag{2}$$

where $\beta(t)$ is the Gaussian noise,

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$$v(t)=\int_0^t \beta(\tau)d\tau,$$

k the parameter of the spectral problem, and x_0 the position of the soliton initially, $t=0$. Wadati[9] studied the evolution of the soliton's mean envelope $\langle u(x,t)\rangle$ under Gaussian white noise $\beta(t)$ with $\langle\beta(t)\rangle=0$ and the correlation function $\langle\beta(t+\tau)\beta(t)\rangle=2D\delta(\tau)$. Here $\langle...\rangle$ stands for statistical averaging over the ensemble of realizations of the process $\beta(t)$. In Refs. 10 and 11, on the basis of the structure of one-soliton solutions, for instance (2), the general problem of calculating mean values of the form $\langle\Phi(z+w(t))\rangle$ was formulated and solved, with $\Phi(z)$ a nonrandom function of variable z,

$$w(t)=\int_0^t \alpha(\tau)d\tau$$

a random process of a Gaussian process $\alpha(t)$. The variables z and $w(t)$ are assumed independent. It was found that for arbitrary Gaussian perturbations $\alpha(t)$ [it is assumed that $\langle\alpha(t)\rangle=0$ and $\langle\alpha(t)\alpha(\tau)\rangle=K(t,\tau)$], the mean $\langle\Phi\rangle$ characterizes a "diffusion process" in the space of variables t and z:

$$\frac{\partial\langle\Phi\rangle}{\partial t}=D(t)\frac{\partial^2\langle\Phi\rangle}{\partial z^2}, \tag{3}$$

where the diffusion coefficient $D(t)$ depends on "time" t and is given by the following relation:

$$D(t)=\int_0^t \langle\alpha(t)\alpha(\tau)\rangle d\tau=\int_0^t K(t,\tau)d\tau. \tag{4}$$

Here, obviously, $\langle\Phi\rangle|_{t=0}=\Phi(z)$. The variable t is assumed to be the time variable, although one must bear in mind that in some problems a spatial coordinate may act as the variable t. Of the process $\alpha(t)$ is time independent, $K(t,\tau)=K(|t-\tau|)$ and

$$D(t)=\int_0^t K(\tau)d\tau.$$

Now suppose that the right-hand side of the KdV equation is subjected to a perturbation $\xi(t)=f(t)+\beta(t)$, with $f(t)$ an arbitrary nonrandom function of time, that is, $\xi(t)$ is a combination of a determinate signal and additive Gaussian noise, acting as background. The structure of the nonaveraged solution of the KdV equation does not change under such interference. The one thing to keep in mind is that now

$$v(t)=\int_0^t \xi(\tau)d\tau,$$

so that the argument of $\mathrm{sech}^2(...)$ acquires an additional nonrandom term that is the double integral of the determinate term $f(t)$ in the signal $\xi(t)$:

$$-\int_0^t d\tau\int_0^\tau f(\tau_1)d\tau_1.$$

Physically, the model describes the dynamics of a soliton in a force field containing a regular term $f(t)$ in addition to the random component $\beta(t)$.

Let us generalize the results of Refs. 10 and 11. Suppose that $\eta(t)=g(t)+\alpha(t)$ is an additive mixture of a determinate function $g(t)$ and a random Gaussian process $\alpha(t)$ with the characteristics

$$\langle\alpha(t)\rangle=0,\quad \langle\alpha(t)\alpha(\tau)\rangle=K(t,\tau).$$

Within the general formulation, the mixture $\eta(t)$ corresponds to the problem of calculating the statistical mean in the Gaussian measure of the functional

$$\Phi(z,t)=\Phi\left(z+\int_0^t \eta(\tau)d\tau\right)$$
$$\equiv\Phi\left(z+\int_0^t g(\tau)d\tau+\int_0^t \alpha(\tau)d\tau\right).$$

To derive an equation that controls the evolution of $\langle\Phi\rangle$ we expand the function $\Phi(...)$ in a Fourier series and average the result over the Gaussian statistics of the noise $\alpha(t)$. The result is

$$\langle\Phi(z+g(t)+w(t))\rangle$$
$$=\frac{1}{2\pi}\int_{-\infty}^{\infty}\exp\left\{ipz+ipq(t)-\frac{p^2}{2}\int_0^t\int_0^t\langle\alpha(\tau_1)\alpha(\tau_2)\rangle d\tau_1 d\tau_2\right\}\Phi(p)dp,$$

where $\Phi(p)$ is the Fourier transform of $\Phi(z)$, and

$$q(t)\int_0^t g(\tau)d\tau.$$

The desired equation for $\langle\Phi\rangle$ can easily be obtained by taking the derivative of both sides of the above result with respect to t:

$$\frac{\partial\langle\Phi\rangle}{\partial t}=g(t)\frac{\partial\langle\Phi\rangle}{\partial z}+D(t)\frac{\partial^2\langle\Phi\rangle}{\partial z^2}, \tag{5}$$

where the varying diffusion coefficient $D(t)$ is still given by formula (4). Obviously, $\langle\Phi\rangle|_{t=0}=\Phi(z)$ at $t=0$. Comparison of Eqs. (3) and (5) shows that now two mechanisms are responsible for the evolution of $\langle\Phi\rangle$: (1) diffusion with the diffusion coefficient $D(t)$, and (2) convection with the transfer rate $g(t)$.

What is important here is that these are two different physical mechanisms and, as the form of Eq. (5) implies, a specific component in the combined perturbation $\eta(t)=g(t)+\alpha(t)$ is responsible for each of these mechanisms. Convection occurs owing to the action of the nonrandom component of the signal $\eta(t)$ (or a force if we are dealing with the stochastic KdV equation), and the transfer rate is simply $g(t)$.

Let us apply this result to the stochastic KdV equation (1). We assume that the right-hand side of Eq. (1) is perturbed by an additive force $\xi(t)=f(t)+\beta(t)$, with the functions $\xi(t)$ and $\eta(t)$ related as

$$\eta(t)=\int_0^t \xi(\tau)d\tau=\int_0^\tau f(\tau)d\tau+\int_0^t \beta(\tau)d\tau.$$

Thus, the "convective transfer rate" $g(t)$ of the soliton's mean envelope is characterized by the magnitude of the integral of $f(t)$. What is important is that the envelope is still entirely determined by the nonrandom component of $\xi(t)$. The noise component $\beta(t)$ of η is responsible for the diffusion of the soliton.

We have therefore established that the actions of the components of an additive mixture of Gaussian noise and a determinate signal on a nonlinear distributed system described by a dynamical variable

$$\Phi\left(z+q(t)+\int_0^t \alpha(\tau)d\tau\right)$$

are exactly separable. The random component corresponds to the diffusion transport mechanism of a certain substance $\langle\Phi\rangle$, and the determinant component controls the convective transport of $\langle\Phi\rangle$ with a rate $g(t)$. In view of what has been said, the nonlinear distributed dynamical models studied here can be considered as filters for separating additive mixtures of determinate signals with Gaussian noise. Note that the form of the components in the mixture can be quite arbitrary and that the conditions of integrability implicitly employed do not constitute severe restrictions.

Let us write the general solution to Eq. (5). In the theory of Markov processes this equation describes (for $\langle\Phi\rangle$ nonnegative and normalizable) the class of Bachelier processes.[12] We have

$$\langle\Phi\rangle=\frac{1}{\sqrt{2\pi\theta(t)}}\int_{-\infty}^{\infty}\exp\left\{-\frac{(z-q(t)-y)^2}{4\theta(t)}\right\}\Phi(y)dy, \tag{6}$$

where the variable

$$\theta(t)=\int_0^t D(\tau)d\tau$$

characterizes the effective time sale of the dynamics of $\langle\Phi\rangle$. At large times the structure of the initial profile of $\Phi(z)$ is forgotten and the mean is transformed into a Gaussian packet whose width and height vary as $\sqrt{\theta(t)}$ and $1/\sqrt{\theta(t)}$, respectively. Here, as the solution (6) implies, the position of the packet's peak is fixed by the determinate component of $\eta(t)$. The resulting solution (6) shows that by studying the transformation of $\langle\Phi\rangle$ in time we can reconstruct the characteristics of the additive mixture $\eta(t)$ of interest to us, since the shape of the correlation function of the noise is directly linked to the width and height of the profile of $\langle\Phi\rangle$. To illustrate this point we examine the case where the process $\alpha(t)$ is time-independent. Then the diffusion coefficient is given by the formula

$$D(t)=\int_0^t K(\tau)d\tau.$$

At times when the Gaussian self-similarity of the profile of $\langle\Phi\rangle$ is realized, the correlation function $K(t)$ can be expressed in terms of its width $h(t)$ in the following manner:

$$K(t)=2\frac{d}{dt}\left(h(t)\frac{dh(t)}{dt}\right).$$

In practice, we often have to do with an additive mixture of Gaussian noise and a periodic signal. For instance, suppose that $g(t)=g\cos\omega t$. Then $q(t)=g\sin(\omega t)/\omega$ and, hence the position of the peak of the Gaussian packet performs oscillates harmonically with the frequency ω. When $g(t)$ is a sum of harmonics with different frequencies and amplitudes,

$$g(t)=\sum_k a_k\exp\{i\omega_k t\},$$

we can use the movement of the peak of the Gaussian packet to reconstruct the envelope of the sum of harmonics.

3. EXAMPLES OF NONLINEAR DISTRIBUTED FILTERS

Here are some examples of NDF. The first is the one-soliton solution (2) of the stochastic KdV equation (1) perturbed by a force $\xi(t)$. In this case the diffusion equation (5) controls the evolution of the difference

$$\langle u(x,t)\rangle-\int_0^t f(\tau)d\tau\equiv\langle\Phi\rangle.$$

Hence, the evolution of the mean envelope of the soliton is described by the expression

$$\langle u(x,t)\rangle=\int_0^t f(\tau)d\tau+\frac{1}{\sqrt{2\pi\theta(t)}}\times\int_{-\infty}^{\infty}\exp\left\{-\frac{[z-q(t)-y]^2}{4\theta(t)}\right\}\Phi(y)dy, \tag{7}$$

where $z=k(x-x_0)-4k^3t$, $\Phi(z)=-2k^2\operatorname{sech}^2 z$, and

$$q(t)=\int_0^t d\tau\int_0^\tau f(\tau_1)d\tau_1,$$

$$\theta(t)=\int_0^t d\tau\int_0^\tau d\tau_1\int_0^\tau d\tau_2\int_0^{\tau_1}K(\tau_2,\tau_3)d\tau_3.$$

This example is interesting because the mathematical model (1) allows for physical modeling by, for example, a nonlinear dispersive transmission line.[8] Usually such a line constitutes a chain of nonlinear sections whose elements are variable-capacity diodes or induction coils with saturable ferromagnetic cores. The random force $\beta(t)$ acting on the soliton of the KdV equation can be taken here as a random emf at the input section of the chain. If we are speaking of physically modeling the transformation of the mixture of Gaussian noise and a determinate signal via the soliton of the KdV equation by nonlinear dispersive transmission lines, an emf varying according to the law $\xi(t)$ is input to the chain.

Let us consider a simpler NDF also formed from the solution of the stochastic KdV equation, perturbed not additively, as in the previous example, but parametrically:

$$\frac{\partial u}{\partial t}+\alpha(t)\frac{\partial u}{\partial x}+u\frac{\partial u}{\partial x}+\Delta\frac{\partial^3 u}{\partial x^3}=0. \tag{8}$$

Now $\alpha(t)$ characterizes homogeneous perturbations of the speed of the nonlinear wave. As shown in Ref. 13 (see also Ref. 14), the exact solution to Eq. (8) has the form

$$u(x,t)=U_0 \operatorname{sech}^2\left[\sqrt{\frac{U_0}{12\Delta}}\left(x-\frac{U_0 t}{\Delta}-\int_0^t \alpha(\tau)d\tau\right)\right]. \tag{9}$$

Let us replace $\alpha(t)$ with the additive mixture $\xi(t)$. We still think of $\alpha(t)$ as Gaussian noise with a zero mean and a correlation function $K(t,\tau)$ of arbitrary form. We employ the above result (5). For Φ we taken solution (9) with the variable z in the form $z=(U_0/12\Delta)^{1/2}(x-U_0 t/\Delta)$ and

$$q(t)=\sqrt{\frac{U_0}{12\Delta}}\int_0^t g(\tau)d\tau,$$

$$\theta(t)=\frac{U_0}{12\Delta}\int_0^t d\tau\int_0^\tau d\tau_1\, K(\tau,\tau_1).$$

In contrast to the previous example, the convective transfer of $\langle u(x,t)\rangle$ occurs at a rate proportional to the amplitude of the determinate "signal" $g(t)$, or $\dot{q}(t)=\sqrt{U_0/12\Delta}f(t)$.

Thus, the movement of the peak of the mean envelope of the soliton can be used to verify the presence of a determinant component in $\xi(t)$. Here the diffusion dynamics of the soliton is determined by the noise component of the mixture.

To conclude this section we note that for an NDF used to separate additive mixtures we can take one-soliton solutions of other nonlinear stochastic equations, for instance, the sine-Gordon equation and the nonlinear Schrödinger equation.

4. MANY-SOLITON SOLUTIONS AS NDF

Above we discussed the possibility of using the one-soliton solutions of the KdV equation as nonlinear distributed filters. In the class of solutions to the KdV equation this solution is the simplest. Wadati and Akutsu[15] gave the exact nonaveraged many-soliton solutions of the KdV equation describing the evolution of N solitons in the field of a random force. For one thing, they discussed in great detail the transformation of two-soliton solutions in the field of a Gaussian random force. They also showed that, as in the case of one soliton, the action of a random force causes solitons to diffuse in such a manner that at large times the two-soliton solution is the sum of two Gaussian packets.

In this section we establish that many-soliton solutions of the KdV equation can also be used as nonlinear distributed filters for separating additive mixtures of a determinate signal and Gaussian noise. According to the results of Ref. 15 (see also Ref. 14, p. 76), a nonaveraged N-soliton solution of the KdV equation has the form

$$u(x,t)=-2\frac{d^2}{dx^2}[\ln \det C(x,t)], \tag{10}$$

where $C(x,t)$ is a matrix whose elements are

$$c_{nm}(x,t)=\delta_{nm}+\frac{\beta_n(t)}{k_n+k_m}\exp[-(k_n+k_m)x], \tag{11}$$

with

$$\beta_n(t)=\frac{b_n(t)}{i\dot{a}(ik_n)},\quad 0<k_1<k_2<\ldots<k_n,\quad n=1,2,\ldots,N, \tag{12}$$

$$b_n(t)=b_n(0)\exp\left[8k_n^3-12k_n\int_0^t v(\tau)d\tau\right],$$

$$\dot{a}(ik_n)=-\left.\frac{\partial a(k)}{\partial k}\right|_{k=ik_n}, \tag{13}$$

and

$$v(t)=\int_0^t \beta(\tau)d\tau.$$

Let us now turn to the case $N=2$. The form of solution (10) at $N=2$ readily suggests that the solution is the sum of exponentials depending on function arguments of the type

$$z_j+\kappa_j\int_0^t v(\tau)d\tau,\quad j=1,2,$$

where the z_j are variables, the κ_j are constant parameters, and the label j numbers the solitons. The concrete form of these characteristics is unimportant. Having in mind the general formulation, let us consider the problem of calculating the mean of the function

$$\Phi\left(z_1+\kappa_1\int_0^t v(\tau)d\tau, z_2+\kappa_2\int_0^t v(\tau)d\tau\right),$$

where z_1 and z_2 are independent variables, κ_1 and κ_2 are parameters, and $v(t)$ is a Gaussian process with a zero mean and an arbitrary correlation function. The difference from the previous case is that the function Φ acquires a second argument. To establish the form of the equation for $\langle\Phi\rangle$, we expand Φ in a Fourier series and average the result over the Gaussian statistics of process $v(t)$. We have

$$\langle\Phi(\ldots)\rangle=\frac{1}{(2\pi)^2}\iint_{-\infty}^{\infty}\exp(ip_1z_1+ip_2z_2)G(p_1,p_2,t)\times\tilde{\Phi}(p_1,p_2)dp_1,dp_2,$$

where we have introduced the notation

$$G(p_1,p_2,t)=\exp\left[-\frac{1}{2}(\kappa_1p_1+\kappa_2p_2)^2\times\iint_0^t\langle v(\tau_1)v(\tau_2)\rangle d\tau_1 d\tau_2\right].$$

After finding the derivatives of both sides of the expression for $\langle\Phi\rangle$ and performing straightforward transformations we arrive at the desired equation, which controls the dynamics of $\langle\Phi\rangle$:

$$\frac{\partial\langle\Phi\rangle}{\partial t}=\tilde{D}(t)\left(\kappa_1\frac{\partial}{\partial z_1}+\kappa_2\frac{\partial}{\partial z_2}\right)^2\langle\Phi\rangle, \tag{14}$$

with

$$\tilde{D}(t)=\int_0^t\langle v(t)v(\tau)\rangle d\tau.$$

Equation (14) should be augmented with the initial conditions

$$\langle\Phi\rangle|_{t=0}=\Phi(z_1,z_2).$$

The above result implies that $\langle\Phi\rangle$ characterizes a diffusion process taking place now in the space of the two variables z_1 and z_2. Importantly, in this space there is cross-diffusion in addition to diffusion along the coordinates z_1 and z_2. The cross-diffusion coefficient is

$$D_{12}(t)=D_{21}(t)=\tilde{D}(t)\kappa_1,\kappa_2.$$

Since the temporal behavior of the diffusion tensor is characterized by the same functional dependence of $\tilde{D}(t)$, the two-dimensional diffusion equation (14) can easily be reduced to an equation with constant coefficients by the substitution

$$\theta(t)=\int_0^t\tilde{D}(\tau)d\tau.$$

Now let us add a nonrandom function $f(t)$ to the random process $v(t)$ in the arguments of the function Φ in the integrand. Similar computations lead to the equation for $\langle\Phi\rangle$,

$$\frac{\partial\langle\Phi\rangle}{\partial t}=f(t)\left(\kappa_1\frac{\partial}{\partial z_1}+\kappa_2\frac{\partial}{\partial z}\right)\langle\Phi\rangle$$
$$+\tilde{D}(t)\left(\kappa_1\frac{\partial}{\partial z_1}+\kappa_2\frac{\partial}{\partial z_1}\right)^2\langle\Phi\rangle, \tag{15}$$

with the same initial conditions $\langle\Phi\rangle|_{t=0}=\Phi(z_1,z_2)$. Thus, the above result implies that the components of the additive mixture of the signal $f(t)$ and Gaussian noise $v(t)$ are exactly separated in their physical manifestations. What is important here is that such separation occurs in the space of a larger number of variables. In our example the variables are z_1 and z_2. Another important aspect is that the nonrandom component $f(t)$ still implements the convective transfer of substance $\langle\Phi\rangle$, while the random component $v(t)$ implements the diffusive transfer of $\langle\Phi\rangle$. The solution to Eq. (15) shows that the temporal behavior of the extrema $\langle\Phi\rangle$ is characterized by the determinate component of the mixture. The width and height of the profile of $\langle\Phi\rangle$ is controlled by its stochastic component. Thus, the mathematical construction

$$\left\langle\Phi\left(z_1+\kappa_1\int_0^t v(\tau)d\tau,z_2+\kappa_2\int_0^t v(\tau)d\tau\right)\right\rangle$$

can also be interpreted as an NDF for exact separation of an additive mixture of a determinate signal $f(t)$ and random Gaussian noise $\alpha(t)$. In the case of the two-soliton solution of the KdV equation with the right-hand side in the form of a mixture of noise and signal, one can easily verify that the dynamics of the soliton peaks is controlled by the signal and their widths and heights by the noise.

Similarly, the N-soliton solution of the KdV equation can be shown to exactly separate the additive mixture $f(t)+\beta(t)$. Here, however, an NDF corresponds to a transfer process in a space of N variables, where convective transfer is implemented by the determinate signal f and diffusive transfer by the random component in the mixture.

5. CONCLUSION

Our study has shown that a nonlinear distributed filter, either a specific physical system or a mathematical construction, serves as an effective means for separating an additive mixture of Gaussian noise and a determinate signal. What is important is that in the expanded space of variables t and z the random and determinate components of the mixture are exactly separated and manifest themselves differently in the physical sense. The random component of the mixture is responsible for the diffusive transport of a certain "substance" $\langle\Phi\rangle$, and the determinate component for the convective transport of the same substance. Here the diffusion coefficient is directly related to the shape of the correlation function of the noise, while the convective transfer rate is directly related to the determinate component.

Note that from the viewpoint of studying the statistical properties of Gaussian noise, the result (5) at $f(t)\equiv 0$ has an independent status, since it enables the self-correlation function of Gaussian noise to be recovered. The results possess still another important informative property: namely, the deviation of the observed evolution of $\langle\Phi\rangle$ from (6) means that the noise component $\alpha(t)$ is not a Gaussian process, and quantitative characteristics of the deviation can be specified. This fact also has independent value.

In this paper we have discussed the case of an additive mixture of Gaussian noise and a determinate signal. Reasoning along similar lines, we could have examined the problem of separating an additive mixture with Poisson noise $\alpha(t)$. In this connection we point to an important fact that emerges when noise with arbitrary statistics is described: generally, the determinate component is only responsible for the convective transport of the mean $\langle\Phi\rangle$, irrespective of the statistics of the noise.

Note that in practical application (in the form of a mathematical construction) of the results obtained here there emerge a number of extremely delicate questions concerning the ergodic properties of the process $\alpha(t)$. The reason is that the above results are exact in the statistical sense when the operation of statistical averaging over the ensemble of realizations of $\alpha(t)$ is defined. In practice, however, the averaging is over characteristic time scales. In view of this it would be proper to assume that the results can be used to test the ergodicity of random processes.

One more fact is worth noting. When we construct an NDF as a physical device based on a real physical process, information is needed about the conditions in which this process is described, say, by the KdV equation (or another equation), and, therefore, has the necessary structure of the functional dependence of the solution on the stochastic variable. In relation to the KdV studied in this paper, this question has been discussed in the literature quite fully. For instance, as applied to the description of stress dynamics in nonlinear dispersive transmission lines, the conditions of applicability of the KdV equation were formulated in Ref. 7, together with the results of experimental studies corroborating the soliton nature of the propagation of a perturbation along the chain.

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[1)]Actually, this statement is more general because any exact solution of a partial differential equation can serve as the mathematical construction of an NDF, provided that it possesses the necessary functional dependence.

[2)]The results of this section were announced in Ref. 8.

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