The electromagnetic structure of the vacuum in two-dimensional Yang–Mills theory with a Chern–Simons mass

V. V. Skalozub, S. A. Vilenskiĭ, and A. Yu. Zaslavskiĭ

Dnepropetrovsk State University, 320625 Dnepropetrovsk, Ukraine

(Submitted 5 August 1993)
Zh. Eksp. Teor. Fiz. **105**, 1098–1107 (May 1994)

We find the structure of the vacuum in the two-dimensional Yang–Mills theory of a topological mass corresponding to the Abrikosov lattice. In contrast to superconductivity, the solution is determined in the class of stationary (rather than static) functions for the charged components of the field and describes spatially periodic electric and magnetic fields. Such a structure is energetically preferable to a homogeneous magnetic field and can emerge spontaneously. Both the magnetic flux quantization in the fundamental cell of the lattice and the Hall effect are inherent in it. We also show that the conductivity tensor depends on the value of the topological mass and the magnetic field strength.

1. Studies of non-Abelian gauge theories have shown that the spectrum of gluons in an external magnetic field contains a tachyon mode,[1,2] which leads to instability in the perturbation-theory vacuum. This raises the question of finding a stable vacuum state. In relation to gauge theories with spontaneous symmetry breaking this problem was solved in Refs. 3, 4, and 5: it was demonstrated that the evolution of the instability leads to the appearance of a periodic structure formed by vortex strings, of the type of the Abrikosov lattice in type II superconductors.[6]

The vacuum state of two-dimensional SU(6) gluodynamics with a Chern-Simons mass in an external constant and homogeneous chromomagnetic field was studied in Ref. 7. There it was found that in this case the gluon spectrum contains a tachyon mode too, but that the presence of a Chern-Simons mass m results in a threshold in the appearance of this mode, similar to the way that the W-boson mass leads to this threshold in electroweak theory.[3]

Our goal here is to investigate the possibility of generating a periodic vacuum structure in two-dimensional SU(2) gluodynamics with a Chern–Simons mass. We find that such a structure does indeed emerge and is due exclusively to the presence of a Chern–Simons mass, in contrast to the electroweak theory, where it emerges owing to the evolution of the instability caused by the presence of a tachyon mode. For $m=0$ the structure does not occur. A peculiar feature of the periodic configuration in the two-dimensional space in an external magnetic field is that static and stationary solutions are physically of equal status. The reason is the absence of a longitudinal component in the momentum. The vacuum structure manifests itself even in magnetic fields H lower than the threshold of the tachyon mode and has an energy lower than that of the homogeneous field. The gluon fields W form a lattice of the triangular type. We calculate the normalization conditions and the currents I_μ of the gluon fields, as well as the energy and the electric and magnetic fields generated in the system. We also study the magnetic flux quantization in the fundamental cell of the lattice. All these quantities are expressed in terms of the square of the gluon condensate, $\overline{|W|^2}$. The conductivity tensor is antisymmetric, $\sigma_{ij}=\varepsilon_{ij}\sigma$, and, in view of the topological quantization of mass m (see Ref. 9), assumes a number of discrete values. We find that there is a field strength $H=H_c$ at which the energy of the resulting configuration vanishes; in fields weaker than H_c the energy becomes negative. Hence the periodic structure may occur even in the absence of an external field. Thus, this model constitutes a nontrivial example of spontaneous Lorentz-invariance breaking emerging at the classical level. As noted earlier, this is due to the presence of the mass m.

2. Let us consider the Lagrangian of a two-dimensional SU(2) gluodynamics with the Chern–Simons term:

$$\mathscr{L}=-\tfrac{1}{4}G^a_{\mu\nu}G^a_{\mu\nu}+\tfrac{1}{4}m\varepsilon_{\alpha\beta\gamma}(G^a_{\alpha\beta}A^a_\gamma-\tfrac{1}{3}g\varepsilon_{abc}A^a_\alpha A^b_\beta A^c_\gamma). \tag{1}$$

Here A^a_μ is the potential of the Yang–Mills field; $G^a_{\mu\nu}=\partial_\mu A^a_\nu-\partial_\nu A^a_\mu+g\varepsilon_{abc}A^b_\mu A^c_\nu$ $D^{ab}_\mu=\delta^{ab}\partial_\mu+g\varepsilon^{abc}A^c_\mu$ g is the coupling constant; m is the Chern–Simons mass; $\mu=0,1,2$; and $a=1,2,3$. In what follows it is convenient to shift to a "charge" (oblique) basis by introducing the notation

$$W^\pm_\mu=\frac{1}{\sqrt{2}}(A^1_\mu\pm iA^2_\mu),\quad A_\mu=A^3_\mu.$$

In terms of the fields $W^\pm_\mu$ and A_μ the Lagrangian assumes the form

$$\begin{aligned}\mathscr{L}=&-P^*_\mu W^+_\nu P^\mu W_{-\nu}+P^*_\mu W^+_\nu P^\nu W^{-\mu}-igW^+_\mu W^-_\nu F^{\mu\nu}\\&-\tfrac{1}{2}g^2[(W^+_\mu W^{-\mu})^2\\&-W^+_\mu W^{+\mu}W^-_\nu W^{-\nu}]-\tfrac{1}{2}F_{\mu\nu}F^{\mu\nu}\\&-im\varepsilon^{\mu\nu\lambda}W^+_\mu P_\nu W^-_\lambda+\tfrac{1}{2}m\varepsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda,\end{aligned} \tag{2}$$

where $F_\mu=\partial_\mu A_\nu-\partial_\nu A_\mu$, and the kinetic momentum $P_\mu=i\partial_\mu+g(\bar{A}_\mu+\hat{A}_\mu)$ takes into account the external field $\bar{A}_\mu$ and the field generated in the system $g\hat{A}_\mu$. We choose the external-field potential in the form $\bar{A}_\mu=\delta_{\mu2}Hx_1$. The equations of motion that follow from the Lagrangian (2) are:

 1063-7761/94/050588-05$10.00

$$P_\alpha^2 W^{-\mu}-P^\mu P_\alpha W^{-\alpha}+2igF^{\mu\alpha}W_\alpha^-+im\varepsilon^{\mu\alpha\beta}P_\alpha W_\beta^-$$

$$+g^2[W^{-\mu}W_\alpha^+W^{-\alpha}-W^{+\mu}W_\alpha^-W^{-\alpha}]=0, \quad (3)$$

$$\partial_\alpha F^{\alpha\beta}=I^\beta, \quad (4)$$

$$I^\beta=g[(W_\alpha^-P^{*\beta}W^{+\alpha}+W_\alpha^+P^\beta W^{-\alpha})$$

$$-2(W_\alpha^-P^{*\alpha}W^{+\beta}+W_\alpha^+P^\alpha W^{-\beta})$$

$$+(W_\beta^-P^{*\alpha}W^{+\alpha}+W_\beta^+P^\alpha W^{-\alpha})]$$

$$-m\varepsilon^{\beta\mu\nu}(\partial_\mu A_\nu-igW_\mu^-W_\nu^+). \quad (5)$$

Below we follow the method developed in the classical paper of Abrikosov,[6] describing the properties of type II superconductors in an external magnetic field whose strength is close to critical.

Let us consider the linearized version of Eq. (3):

$$\bar{P}_\alpha^2 W^{-\mu}-\bar{P}^\mu\bar{P}_\alpha W^{-\alpha}+2ig\bar{F}^{\mu\alpha}W_\alpha^-+im\varepsilon^{\mu\alpha\beta}\bar{P}_\alpha W_\beta^-=0, \quad (6)$$

where $\bar{P}_\mu=i\partial_\mu+g\bar{A}_\mu$, and $\bar{F}_{\mu\nu}=\partial_\mu\bar{A}_\nu-\partial_\nu\bar{A}_{mu}$. Applying the operator $\bar{P}_\mu$ to this operator and allowing for the commutation relation $[\bar{P}_\mu,\bar{P}_\nu]=ig\bar{F}_{\mu\nu}$, we arrive at the following corollary of the linearized equation of motion:

$$W_0^-=0. \quad (7)$$

It is easy to see that there are no static solutions in Eq. (6) with allowance made for condition (7). The reason is that, as shown below, not only a magnetic field but also an electric field develops in the system. Let us broaden the class of solutions considered and seek the solution to Eq. (6) in the form

$$W_\mu^-(x,y,t)=W_\mu^-(x,y)e^{-i\omega t}. \quad (8)$$

Going over to the Fourier transforms

$$W_\mu^-(x,y)=\int dk2\pi\, W_\mu^-(x,k)e^{iky}, \quad (9)$$

we can write Eq. (6) as

$$(i\omega\partial_x-im[(gHx-k)]W_1^-+(-m\partial_x+\omega$$

$$\times(gHx-k))W_2^-=0,(-\omega^2+(gHx-k)^2)$$

$$\times W_1^-+(-i(gHx-k)\partial_x$$

$$-im\omega+igH)W_2^-=0,(-i(gHx-k)\partial_x$$

$$\times+im\omega-2igH)W_1^-+(-\partial_x^2-\omega^2)W_2^-=0. \quad (10)$$

The solution of this system has the form

$$W_\mu^-=\frac{C}{\sqrt{2}}\begin{pmatrix}0\\1\\i\end{pmatrix}_\mu \exp\left[-\frac{gH}{2}\left(x-\frac{k}{gH}\right)^2+iky-i\omega t,\right. \quad (11)$$

where C is a constant; ω satisfies the equation

$$\omega^2-m\omega+gH=0. \quad (12)$$

The vector

$$b_\mu=(2)^{-1/2}\begin{pmatrix}0\\1\\i\end{pmatrix}_\mu$$

is the eigenvector of the spin operator with the eigenvalue $+1$. Note that the equation obtained satisfies the condition $\bar{P}_\mu W^{-\mu}=0$. Equation (12) yields $\omega^\pm=m/2 \pm \sqrt{m^2/4-gH}$. Hence, stationary solutions exist in fields H lower than $H_0=m^2/4g$.

3. To find the vacuum structure in fields H somewhat lower than H_0, we follow Abrikosov's reasoning[6] by allowing for degeneracy in k and introduce the linear combination of the wave functions (11) with different momenta:

$$W_L(x,y,t)=e^{-i\omega t}\sum_n C_n\exp\left[inky-\frac{gH_0}{2}\left(x-\frac{nk}{gH_0}\right)\right], \quad (13)$$

where the C_n are constants. To obtain a lattice structure, we subject the C_n to the following periodicity condition:

$$C_{n+\nu}=C_n,$$

where ν is a fixed integer. Then

$$W_L\left(x+\frac{\nu k}{gH_0},y,t\right)=e^{-ik\nu y}W_L(x,y,t). \quad (14)$$

As is known (see Ref. 6), the properties of the solutions are almost entirely independent of the specific choice of the C_n and k, which determine the shape of the lattice. In what follows we drop the subscript L on W_L. The nonlinearity of the system lifts the degeneracy in k and allows for normalizing the function (13).

Let us now calculate the currents (5). To this end we write (13) as $W(x,y,t)=|W(x,y)|\exp[i\Theta(x,y)-i\omega t]$. Substituting this into (5) and allowing for the explicit form of the vector b_μ, we arrive at the following expressions for the current components:

$$I_0=-2g\omega|W|^2-mF_{12}+mg|W|^2,$$

$$I_1=\partial_2(g|W|^2+m\hat{A}_0), \quad (15)$$

$$I_2=-\partial_1(g|W|^2+m\hat{A}_0).$$

Here the terms proportional to $\hat{A}_\mu|W|^2$ are discarded owing to their smallness compared with $|W|^2$. Substituting the current (15) into the Maxwell equations (4), we find the relationship between the field $\hat{F}_{12}$ generated in the system and the W field:

$$\hat{F}_{12}=g|W|^2+m\hat{A}_0. \tag{16}$$

Thus, the potential A_0 contributes to the value of the magnetic field, and this occurs owing to the presence of the mass m. If we allow for (16), the zeroth component of Eq. (4) becomes

$$(\partial_1^2+\partial_2^2-m^2)\hat{A}_0=2g\omega|W|^2+mH. \tag{17}$$

As is known, the presence of a Chern–Simons mass generates a constant charge in the plane.[9] The term mH on the right-hand side of Eq. (17) is precisely this constant charge density, on top of which an electromagnetic field develops in the system. The term contributes nothing to the electric-field component lying in the plane and has no effect on the formation of the vacuum structure; it can be dropped in what follows. Then the solution for $\hat{A}_0$ is

$$\hat{A}_0(\mathbf{r})=-\frac{g\omega}{\pi}\int d^2r'\,K_0(m|\mathbf{r}-\mathbf{r}'|)\,|W(\mathbf{r}')|^2, \tag{18}$$

where $K_0(x)$ is the modified Bessel function of the second kind, and $\mathbf{r}$ and $\mathbf{r}'$ are radius vectors in the (x,y) plane. A simpler expression for $\hat{A}_0$, which makes it possible to obtain results in explicit form, is obtained in the approximation $\Delta\hat{A}_0/\hat{A}_0\ll m^2$. As a result we have

$$\hat{A}_0(x,y)=-\frac{2g\omega}{m^2}|W(x,y)|^2. \tag{19}$$

Thus, as noted earlier, the presence of the Chern–Simons mass leads to the appearance of an electric field $E_k=\hat{F}_{k0}=\partial_k\hat{A}_0$. In final form the expressions for the magnetic and electric fields generated in the system are

$$\hat{F}_{12}=g(1-2\omega m)|W|^2,\ \hat{F}_{k0}=-2g\omega\, m^2\,\partial_k|W|^2. \tag{20}$$

4. We return to the solution (13) and normalize W with allowance for the nonlinearity of the system. Let us examine the expression for the energy of the system of fields:

$$U=\int d^2x\,T_{00},$$

$$=\int d^2x\,[i\partial_0W_\mu^+P_0W^{-\mu}-i\partial_0W_\mu^-P_0^*W^{+\mu}+m\varepsilon^{\nu0\mu}W_\nu^+\partial_0W_\mu^+-\mathscr{L}], \tag{21}$$

where $\mathscr{L}$ is the Lagrangian (2). Suppose that the fields W and A minimize the configuration's energy. Replacing W with $(1+\varepsilon)W$, $\varepsilon\ll1$, seting the energy variation equal to zero, we get

$$\begin{aligned}\delta U=2\varepsilon\int d^2x\,\{&-i(P_0^*+gA_0^{(1)})W^{+\mu}\partial_0W_\mu^-\\&+i(P_0+gA_0^{(1)})W^{-\mu}\partial_0W_\mu^++m\varepsilon^{\mu o\nu}W_\nu^-\partial_0W_\mu^-\\&-W^{+\nu}(P_\nu P_\mu+gA_\nu^{(1)}P_\mu+gP_\nu A_\mu^{(1)})W^{-\mu},\\&+W_\nu^+(P_\mu+2gA_\mu^{(1)})P^\mu W^{-\nu}+2ig(F_{\mu\nu}\\&+F_{\mu\nu}^{(1)})W^{+\mu}W^{-\nu},+im\varepsilon^{\alpha\beta\gamma}W_\alpha^+(P_\beta\\&+gA_\beta^{(1)})W_\gamma^-+g^2(W_\mu^+W^{-\mu})^2+\tfrac{1}{2}F^{\mu\nu}F_{\mu\nu}^{(1)}\\&-m\varepsilon^{\alpha\beta\gamma}A_\alpha\partial_\beta A_\gamma^{(1)}\}=0,\end{aligned} \tag{22}$$

where the terms proportional to ε^2 have been discarded. Here the A_μ field is represented in the for m $A_\mu=\bar{A}_\mu^{(0)}+A_\mu^{(1)}$ where $A_\mu^{(1)}$ results from the W field and *the deviation of* H from H_0. Integrating by parts, employing Eq. (6) the mulas (15) the current components, allowing the fact that $W^{-\mu}$ satisfies the condition $\bar{P}_\mu W^{-\mu}=0$, we reduce the expression (24) the energy to the for m

$$\delta U=2\varepsilon\int d^2x\left[\frac{\omega^3}{m}|W|^2-g|W|^2F_{12}^{(1)}+g^2\left(1-\frac{2\omega}{m}-\frac{4\omega^2}{m^2}\right)|W|^4\right]=0. \tag{23}$$

Bearing in mind that F12(1)=H−H0 + g(1− 2ω/m)|W|2, *we arrive at the following normalization condition*:

$$\left[\frac{\omega^3}{m}+g(H_0-H)\right]\overline{|W|^2}=\frac{4g^2\omega^2}{m^2}\overline{|W|^4}, \tag{24}$$

where by $\overline{|W|^2}$ and $\overline{|W|^4}$ we have denoted the quantities averaged over the volume V occupied by the field. The normalization condition written in this way plays an important role since it is independent of the form of the function W, that is, independent of the nature of the structure emerging in the vacuum. Note that at $m=0$ it follows from Eq. (24) with allowance for (12) that $\overline{|W|^4}=0$, that is, no gluon condensate is present. The normalization condition obtained differs substantially from those that emerge in the superconductivity theory[6] and the electroweak theory.[3] For one thing, the value of the gluon condensate $|W|^2$ does not vanish at $H=0$ if we select $\omega=\omega^+$ The calculation of the energy density conducted below shows that the resulting solution W is the one corresponding to $\omega=\omega^+$, the one having the minimum energy. Hence, in what follows we give the results for ω^+. As noted earlier, the lattice is selected by fixing the momentum k and the number ν, which makes it possible to calculate explicitly the value of the ratio

$$\beta=\frac{\overline{|W|^4}}{(\overline{|W|^2})^2}. \tag{25}$$

As is well known (see Ref. 10), the values $\nu=2$ and $\beta=1.16$ correspond to a triangular lattice and the values $\nu=1$ and $\beta=1.18$ to a square lattice. This data is sufficient for calculating the macroscopic characteristics of the system.

The magnetic induction B is the average value of the microscopic field. Allowing for (20), we can write

$$B=H+g\left(1-\frac{2\omega}{m}\right)\overline{|W|^2}. \tag{26}$$

Using the normalization condition and the definition of β yields

$$B=H+\frac{m^2}{4g\omega^2\beta}\left(1-\frac{2\omega}{m}\right)\left(\frac{\omega^3}{m}+g(H_0-H)\right). \tag{27}$$

We can easily see that, depending on the choice of solution of the equation for ω, Eq. (12), the gluon condensate either screens the external magnetic field (at $\omega=\omega^+$) or antiscreens it (at $\omega=\omega^-$). The energy density of the configuration obtained is calculated by employing Eqs. (21) and (3)–(5):

$$\varepsilon=\frac{H^2}{2}+\frac{\omega^3}{m}\overline{|W|^2}-\frac{6g^2\omega^2}{m^2}\overline{|W|^4}, \tag{28}$$

with $\varepsilon=U/V$. Combining the normalization condition (24) and the definition (25) of β, we reduce the above expression to

$$\varepsilon=\frac{H^2}{2}-\frac{1}{8g^2\omega^2\beta}[\omega^3+3mg(H_0-H)]\ [\omega^3+mg(H_0-H)]. \tag{29}$$

To determine which lattice is realized we must know ε as a function of β with a fixed B. However, Eqs. (27) and (29) do not make it possible to express the energy density in terms of B explicitly. Numerical calculation shows that the energy drops as β decreases. Hence it is the triangular lattice described in Ref. 10 that occurs.

Let us examine ε as a function of the external field H. Numerical calculations show that for any H satisfying the conditions $0\leqslant H\leqslant H_0$ the energy of the configuration increases monotonically, and its value is always lower than the energy of the homogeneous magnetic field. Specifically, $\varepsilon=\frac{1}{2}H_0^2(1-1/4\beta)$ at $H=H_0$ and $\varepsilon=-35H_0^2/8\beta$ at $H=0$. There exists a critical field value H_c at which the system's energy density vanishes; at $\omega=\omega^+$ this value is $H_c\approx 0.912H_0$. Thus, even without an external field there is a periodic structure, and this is due to the Chern–Simons mass.

5. Let us examine the magnetic flux quantization in the fundamental cell of the lattice. Using the representation $W^\mu(x,y,t)=b^\mu|W(x,y)|\exp[i\Theta(x,y)-i\omega t]$ and allowing for the explicit form of vector b^μ, we write the expression for the spatial components of the current I^k as

$$I^k=-m\varepsilon^{ki}\partial_iA_0+2g^2T^k_{ilm}W^{+l}W^{-m}\left(A^i-\frac{1}{g}\partial^i\Theta\right), \tag{30}$$

where we have introduced the notation $T^k_{ilm} = g_{lm}\delta^k_i - 2g_{im}\delta^k_l$, and ε_{ik} is the totally antisymmetric tensor. We use this expression to express the potential A^i as

$$A^i=\frac{1}{g}\partial^i\Theta+(I^k+m\varepsilon^{ki}\partial_iA_0)(2g^2T^k_{ilm}W^{+l}W^{-m})^{-1}. \tag{31}$$

Integrating along the contour on which $I^k+m\varepsilon^{ki}\partial_iA_0=0$, we obtain

$$\Phi=\oint_r\mathbf{A}\cdot\mathbf{r}=\frac{1}{g}\oint_r(\nabla\Theta)\cdot d\mathbf{r}=\frac{2\pi}{g}n, \tag{32}$$

where $n=\pm1,\pm2,\ldots$. This suggests that the flux quantum Φ_0 is $2\pi/g$. Applying this reasoning to the fundamental cell of the lattice and taking into account the translational invariance of the cell, we arrive at the following result:

$$\oint_\Gamma(I^k+m\varepsilon^{ki}\partial_iA_0)(2g^2T^k_{ilm}W^{+l}W^{-m})^{-1}dx^i=0. \tag{33}$$

The integration is carried out along the cell boundary. We have also allowed for the fact that, as Eqs. (15) and (19) imply, the streamlines coincide with lines on which $|W|^2$ is constant. Thus, the magnetic flux quantum is equal to Φ_0, just as in the electroweak theory.[3]

6. The relation connecting the electric field strength and the current vector is specified in the standard manner: $I_i=\sigma_{ij}E_j$, where σ_{ij} is the conductivity tensor. Comparing this with Eqs. (15) and (20), we obtain the explicit form of σ_{ij}:

$$\sigma_{ij}=\varepsilon_{ij}m\left(1-\frac{m}{2\omega}\right)\equiv\varepsilon_{ij}\sigma. \tag{34}$$

Clearly, $\mathbf{I}$ is perpendicular to $\mathbf{E}$, and the conductivity σ, due to Eq. (12), is a function of the external fields, as in the case of the Hall effect. Formula (34) acquires the simplest form in the limit of $H\to 0$, that is, $\sigma=\frac{1}{2}m$. As is known, the gauge invariance of the Yang–Mills theory with the Chern–Simons term is not broken if m assumes discrete values[9]

$$m=\frac{g^2}{4\pi}n,\quad n \text{ an integer}. \tag{35}$$

Thus, we arrive at the condition topological quantization of conductivity:

$$\sigma=\frac{g^2}{8\pi}n. \tag{36}$$

Comparison of this result with the expression for the Hall conductivity $\sigma_{\mathrm{H}}=(g^2/2\pi)n_{\mathrm{H}}$ (see Ref. 8) yields

$$n_{\mathrm{H}}=\frac{n}{4}. \tag{37}$$

Of course, the analogy with conductivity quantization is superficial and the numbers n and n_{H} in Eqs. (35) and (37) have different physical meanings. The number n in Eq. (35) is a macroscopic parameter that determines the size of the topological mass, which, therefore, is a number specific to the model. On the other hand, n_{H} in the Hall effect determines the number of conductivity quanta for a given state,

7. Before we discuss the results a remark is in order. The properties of the ground state in non-Abelian gauge theories is one of the fundamental questions of quantum field theory. Savvidy[11] discovered the phenomenon of spontaneous magnetization of the vacuum of gluodynamics in three-dimensional space. Moreover, in Refs. 1 and 2 it was demonstrated that this vacuum state is unstable because the gluon spectrum contains a tachyon mode. Attempts to find a stable vacuum state due to the tachyon evolution have led to the construction in the electroweak

theory of a periodic vacuum structure in the form of the Abrikosov lattice.[3,4,5] The lattice was found to develop in an external field H somewhat more strongly than the unstable-mode threshold. The lattice is not the ground state of the model in weak fields, where there is no tachyon. Spontaneous magnetization of the vacuum of two-dimensional SU(2) gluodynamics with a Chern–Simons mass was studied in Ref. 7. There it was discovered that the gluon spectrum also has a tachyon mode, with the value of mass m determining the threshold for this mode.

Here we have shown that the presence of a Chern–Simons mass makes possible the generation of a periodic vacuum structure. We found that the ground state of two-dimensional SU(2) gluodynamics constitutes a triangular lattice formed by gluon fields with a period of order m^{-1}. It appears that the tachyon instability threshold H_0 is not reached, and the structure can develop even in the absence of an external magnetic field. The decisive factor causing a lattice structure to develop spontaneously in a nonlinear system of fields is the presence of a topological mass. A periodic configuration has an energy that lies below the Savvidy level. Hence, a spontaneously magnetized vacuum state is not realized in the given model. Partitioning the plane into cells with a characteristic size of the order of m^{-1} leads to a situation in which the tachyon mode, in view of the absence of a longitudinal momentum component developing in the entire space, plays no important role, in contrast to the case in the electroweak theory. The solution obtained, we believe, constitutes the first nontrivial example of a solution of field-theory equations that has the meaning of a vacuum breaking Lorentz invariance.

Another feature of our model is the generation in the system of a periodic electric field. Note that the potential A_μ is time-independent, so that the field strength $\mathbf{E}$ is determined solely by the gradient zeroth component of the potential. Thus, we have actually found a new type of lattice formed by electric and magnetic fields. This constitutes the main difference from the case of a lattice in a type II superconductor, where there is only a magnetic field. As noted in the Introduction, in a two-dimensional world there can be no momentum component "along the field," and static and stationary field configurations are therefore physically equivalent. In this case, in view of the fact that the presence of a Chern–Simons mass leads to mixing of the electric and magnetic components of the tensor $F_{\mu\nu}$, a purely magnetic structure cannot develop in the system. It is primarily for this reason that the solutions forming the lattice are nonstatic.

Our result agrees with that obtained by Teh,[12] who found an exact solution for the classical Yang–Mills equations with a Chern–Simons mass in the form of a solitary vortex of electric and magnetic fields. Of these stationary solutions of the equations of motion only one is realized, the solution that has the minimum energy in the vicinity of H_0. The gluon condensate partially screens the external field. The condensate amplitude and, as a consequence, the observed physical quantities are time-independent. As Eq. (34) implies, the diagonal components of the conductivity tensor vanish, so that the system is nondissipative.

The condition $\Delta A_0/A \ll m^2$ used in calculating A_0 can be interpreted as the zeroth approximation to the nonlocal interaction (18). The comparison of the results of Refs. 3 and 5 done in Ref. 5 suggests that allowing for nonlocal corrections has no effect on the behavior of the system and changes the final result only slightly (by approximately 10%). Formation in the two-dimensional model of a Hall effect is caused by a topological mass and, hence, closely linked to the gauge invariance of the theory.

The two-dimensional quantum field theory is a possible candidate for describing layer systems. Hence, our results can find application in solid-state physics if it is found that collective effects in two-dimensional electron systems can be described by the non-Abelian model studied in this paper.

[1] V. V. Skalozub, Yadernaya Fiz. **28**, 228 (1978) [Sov J. Nucl. Phys. **28**, 113 (1978)].
[2] N. K. Nielsen and P. Olesen, Nucl. Phys. B **144**, 376 (1978).
[3] V. V. Skalozub, Yadernaya Fiz. **43**, 1045 (1986) [Sov J. Nucl. Phys. **43**, 665 (1986)]; **45**, 1708 (1987) [**45**, 1058 (1987)].
[4] J. Ambjørn and P. Olesen, Nucl. Phys. B **315**, 315 (1989).
[5] S. W. MacDowell and O. Törnkvist, Phys. Rev. D **45**, 3833 (1992).
[6] A. A. Abrikosov, Zh. Eksp. Teor. Fiz. **32**, 1442 (1952).
[7] V. V. Skalozub, S. A. Vilenskiffi, and A. Yu. Zaslavskiffi, Yadernaya Fiz. **56** (5), 218 (1993) [Sov. J. Nucl. Phys. **56**, 687 (1993)].
[8] *The Quantum Hall Effect*, edited by R. E. Prange and S. M. Girvin, 2nd ed., Springer, New York, 1990.
[9] S. Deser, R. Jackiw, and S. Templeton, Ann. Phys. (N.Y.) **140**, 372 (1982).
[10] W. H. Kleiner, L. M. Roth, and S. H. Autler, Phys. Rev. **133A**, 1226 (1964).
[11] G. K. Savvidy, Phys. Lett. B **71**, 133 (1977).
[12] R. Teh, J. Phys. G **16**, 175 (1990).

Translated by Eugene Yankovsky

This article was translated in Russia. It is reproduced here the way it was submitted by the translator, except for stylistic changes by the Translation Editor.