Structure functions of polarized protons and neutrons and the Gerasimov–Drell–Hearn and Bjorken sum rules

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The value of power corrections to the integrals of the polarization structure functions of proton and neutron $\int g_{1_{p,n}}(x)dx$ measured by the EMC, SMC and E142 groups, is determined based on a model which accounts for higher twist terms, has the correct asymptotic behavior at large Q^2, and satisfies the Gerasimov–Drell–Hearn sum rule at $Q^2=0$. The contribution of resonances up to $W=1.8$ GeV at $Q^2=0$ is taken into account based on the analysis of electroproduction data. It is shown that when taking into account these higher twist terms, the experimental data agree with the Bjorken sum rule, and the fraction of the proton spin projection carried by quarks, is consistent with the natural estimate of $\sim 50\%$.

Recent measurements of the deep inelastic polarized muon scattering on polarized deuterium made by the SMC group at CERN[1] and in the SLAC E142[2] experiment using polarized electrons and polarized ^{3}He allowed determination of the polarized neutron structure function $g_{1n}(x)$. The analogous proton structure function $g_{1p}(x)$ was measured previously by the EMC[3] and SLAC[4] groups. Thereby, it became possible to test the important Bjorken sum rule,[5]

$$\Gamma_p(Q^2)-\Gamma_n(Q^2)\equiv\int_0^1 dx[g_{1p}(x,Q^2)-g_{1n}(x,Q^2)]=\frac{g_A}{6}\left(1-\frac{\alpha_s(Q^2)}{\pi}\right), \qquad (1)$$

where Q^2 is the four-momentum transfer from the scattered lepton to the target, and g_A is the axial constant of β-decay. The relation (1) is written taking into account the first-order perturbative correction in QCD.[6] The higher-order corrections are also known.[7]

Let us first analyze the experimental data in the scenario advocated by Ellis and Karliner,[8] where all known perturbative QCD corrections[6,7] are accounted for and nonperturbative twist 4 terms are taken from the calculations of Ref. 9.

The SMC[1] and E142[2] data are the following. The SMC measurements correspond to the averaged value $\bar{Q}^2=4.6$ GeV^2. For scattering on polarized deuterons it was found that

$$\Gamma_d=0.023\pm0.020\pm0.015, \qquad (2)$$

where Γ_d is related to Γ_p, Γ_n by

$$\Gamma_p+\Gamma_n=2\Gamma_d(1-1.5\omega_D)^{-1}, \qquad (3)$$

and $\omega=0.058$ takes into account the D-wave admixture in the deuteron. From (2) and (3), it follows that

$$\Gamma_p+\Gamma_n=0.050\pm0.044\pm0.033 \quad \bar{Q}^2=4.6\ \text{GeV}^2. \qquad (4)$$

The EMC[3] experiment at $\bar{Q}^2=10.7$ GeV^2 found

$$\Gamma_p=0.126\pm0.010\pm0.015, \quad \bar{Q}^2=10.7\ \text{GeV}^2. \qquad (5)$$

In order to determine Γ_n from (4), (5), the data must be taken at one common value of $Q^2=Q_0^2$. As Q_0^2 we take the mean value of Q^2 in the EMC experiment $Q_0^2=10.7$ GeV^2, where higher twist corrections are smaller. In the calculation of $\Gamma_p+\Gamma_n$ at Q_0^2, we account for the perturbative QCD corrections up to α_s^3 in the nonsinglet part of $\Gamma_p+\Gamma_n$,[7] the α_s corrections in the singlet part of $\Gamma_p+\Gamma_n$ (see Eq. (21) below, where the value of Σ, $\Sigma=0.22$, was taken from Ref. 8) as well as the twist-4 correction.[9] We get

$$\frac{(\Gamma_p+\Gamma_n)|_{Q_0^2}}{(\Gamma_p+\Gamma_n)|_{4.6\ \text{GeV}^2}}=1.06.$$

From (4) and (5) it follows that

$$\Gamma_p+\Gamma_n=0.053\pm0.046\pm0.035, \quad Q^2=Q_0^2$$

$$\Gamma_n=-0.073\pm0.04\pm0.04 \qquad (6)$$

and

$$\text{EMC,SMC}:\Gamma_p-\Gamma_n=0.202\pm0.045\pm0.045 \qquad (7)$$

The value (7) agrees with the Bjorken sum rule (1)

$$\Gamma_p-\Gamma_n=0.186\pm0.003, \quad Q^2=Q_0^2 \qquad (8)$$

(at $\alpha_s=0.25$, $\Lambda_{\text{QCD}}=200$ MeV), though the agreement is not completely convincing because of large errors. [In Eq. (8) we also took into account the corrections $\sim\alpha_s^2$ and α_s^3[7] and twist-4 terms according to Ref. 9. The error represents an estimate of the uncertainty in these corrections.]

The E142 group made their measurements at $Q^2=2$ GeV^2. Since the spins of the two protons in ^{3}He are compensated, the polarized ^{3}He scattering up to small correc-

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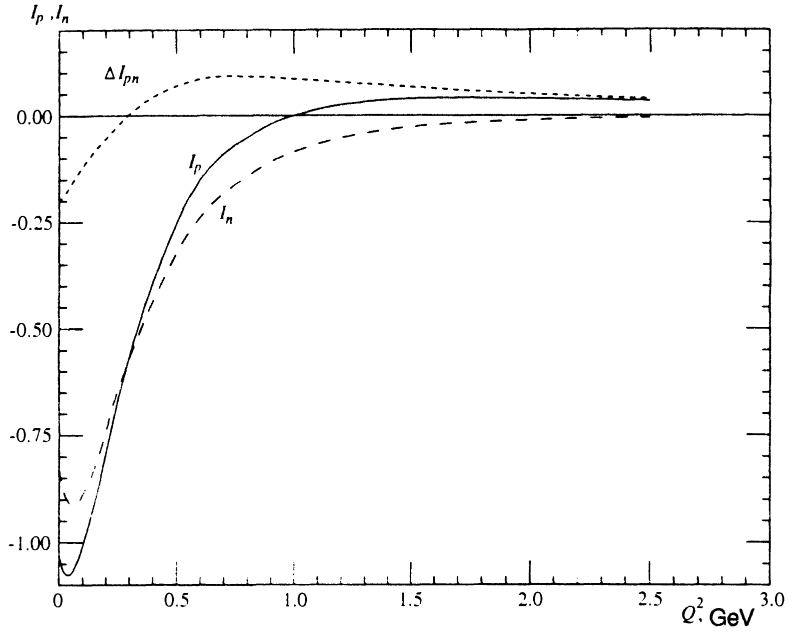

FIG. 1. The contributions of resonances up to masses $W=1.8$ GeV on the right-hand side of Eq. (14); the indices p, n, p–n refer to the cases of the deep inelastic scattering on proton, neutron and proton–neutron difference.

tions (taken into account in experiment) correspond to the scattering on a polarized neutron. In the E142 experiment it was found that:

$$\int_{0.03}^{0.6} dx g_{1n}(x) dx = -0.019 \pm 0.006 \pm 0.006. \qquad (9)$$

Extrapolating to small and large x, the E142 group determined

$$\Gamma_n = -0.022 \pm 0.011, \quad \bar{Q}^2 = 2 \text{ GeV}^2. \qquad (10)$$

[To determine Γ_n from the E142 data, Ellis and Karliner[8] used a different parametrization of $F_2(x,Q^2)$ and $R(x,Q^2)$ than was used in Ref. 2, as well as a different extrapolation at small $F_2(x,Q^2)$ and $R(x,Q^2)$ enter when finding $g_1(x)$ from the experimentally measured asymmetry. In doing so, instead of (10) they obtained $\Gamma_n = -0.028 \pm 0.006 \pm 0.009$]. Transferring the data of the E142 group to the common value Q_0^2 in the same way as in the case of the SMC data, we have

$$\frac{\Gamma_n(Q_0^2)}{\Gamma_n(Q^2 = 2 \text{ GeV}^2)} = 1.085, \quad \Gamma_n(Q_0^2) = -0.024 \pm 0.012,$$

and

$$\text{EMC, E142:} \Gamma_p - \Gamma_n = 0.150 \pm 0.021, \quad Q^2 = Q_0^2 \qquad (11)$$

in comparison with the theoretical value of the Bjorken sum rule (8). The theoretical and experimental values of the Bjorken sum rule differ significantly.

Therefore, the question is whether the nonperturbative QCD corrections, i.e., the high twist terms, determined by Balitsky, Braun and Kolesnichenko[9] that were used in the calculation of Eqs. (8), (11) are correct. We have serious doubts that the results of Ref. 9 are reliable (see discussion below). For this reason we propose another method of determination of higher twist terms.

To take into account the nonperturbative Q^2 dependence we use in this paper the idea, conjectured in Refs. 10 and 11, of a connection of Γ_p, Γ_n at large Q^2 with the Gerasimov–Drell–Hearn (GDH) sum rule, which holds at $Q^2=0$.

Following Refs. 10 and 11, we introduce the functions

$$I_{p,n}(Q^2) = \int_{Q^2/2m_{p,n}}^{\infty} \frac{d\nu}{\nu} G_{1p,n}(\nu,Q^2) \equiv \frac{2m_{p,n}^2}{Q^2} \Gamma_{p,n}(Q^2). \qquad (12)$$

At large ν and Q^2, the quantity νG_1 is related to g_1 by

$$\frac{\nu}{m_p} G_1(x,Q^2) \approx g_1(x,Q^2), \quad \nu \to \infty,$$

$$Q^2 \to \infty, \quad x = Q^2/2m_p\nu = \text{const}.$$

Therefore, at large Q^2, the $\Gamma_{p,n}(Q^2)$ defined in (12) coincide with the ones introduced in Eq. (1). At $Q^2=0$, the $I_{p,n}(0)$ satisfy the GDH sum rules,

$$I_{p,n}(0) = -\frac{1}{4} \varkappa_{p,n}^2, \qquad (13)$$

where κ_p, κ_n are the anomalous magnetic moments of the proton and neutron.

The authors of Ref. 10 proposed a vector dominance based model which described $I_{p,n}(Q^2)$ throughout the whole Q^2 region. At large $Q^2, I_{p,n}(Q^2)$ has the asymptotic form (12), and at $Q^2=0$ it satisfies the GDH sum rules. In Ref. 11 it was shown that in its original form the model is not satisfactory, since at small Q^2, the contribution of baryonic resonances to the integral (12) are important. These should be taken into account separately. We adopt here the model of Ref. 11 with this refinement and write

$$I_{p,n}(Q^2) = I_{p,n}^{res}(Q^2) + I'_{p,n}(Q^2). \qquad (14)$$

Here $I'_{p,n}$ is defined by[11]

$$I'_{p,n}(Q^2) = 2m_p^2 \Gamma_{p,n}^{as} \left[\frac{1}{Q^2+\mu^2} - \frac{c_{p,n}\mu^2}{(Q^2+\mu^2)^2} \right], \qquad (15)$$

$$c_{p,n}=1+\frac{1}{2}\frac{\mu^2}{m_p^2}\frac{1}{\Gamma^{as}_{p,n}}\left[\frac{1}{4}\varkappa^2_{p,n}+I^{res}_{p,n}(0)\right], \qquad (16)$$

where μ^2 is the vector (ρ,ω) meson mass, μ^2 0.6 GeV^2. In (15) and (16), the $\Gamma^{as}_{p,n}$ are the integrals defined in Eq. (1) or (12) at $Q^2=Q_0^2$ with higher-order twist terms excluded.

The contribution of baryonic resonances $I^{res}_{p,n}$ with masses up to $W=1.8$ GeV is known from the analysis of pion electroproduction experiments,[12,13] and is presented in Fig. 1. Knowing $I^{res}_{p,n}$, we can find $c_{p,n}$, and thereby determine all the parameters of the model. ($\Gamma^{as}_{p,n}$ can be found from the EMC and E142 experimental data taking into account the $1/Q^2$ power corrections obtained in our model. We use the E142 data because their accuracy is better than that of SMC.) Using the resonance contributions from Fig. 1, $I^{res}_p(0)=-1.028$ and $I^{res}_n(0)=-0.829$, we find:

$$c_p=0.43\pm0.10, \quad c_n=0.0^{+0.3}_{-1.2}. \qquad (17)$$

The value of c_n is defined with a large error which is due to the uncertainty in Γ_n in the E142 experiment: according to Eq. (15), Γ^{as}_n strongly affects the value of c_n. The uncertainty in $I^{res}_n(0)$ was also taken into account in (17). Substituting c_p into (15), we find that in the EMC experiment at $\bar{Q}^2=10.7$ GeV^2 the power correction comprises 8%. Thus, the experimentally measured EMC value of Γ_p corresponds (after excluding the power corrections) to

$$\Gamma^{as}_p=0.137\pm0.018. \qquad (18)$$

Analogously, making use of the E142 data for the neutron (10), the value of c_n (17), and perturbative QCD corrections, we find

$$\Gamma^{as}_n=-0.029\pm0.015, \qquad (19)$$

i.e., the power correction is 23%. The error in Eq. (19) includes the uncertainty due to the error in c_n.

The power corrections, 8% in the EMC experiment and 23% in the E142 experiment, are beyond the accuracy limit of these experiments and cannot be excluded by existing data. The power corrections for proton yield 30% at $Q^2=2$ GeV^2, while at $\bar{Q}^2=4.6$ GeV^2 the power corrections for the proton and neutron amount to 16% and 12%.

Thus, after excluding the power corrections, the Bjorken sum rule takes the form

$$\text{EMC, E142, } Q^2=Q_0^2, \quad \Gamma_p-\Gamma_n=0.166\pm0.024 \qquad (20)$$

and differs from the theoretical value by less than one standard deviation. If, instead of using directly the E142 data we use the Ellis and Karliner result,[8] the disagreement between the experimental and theoretical value of Bjorken's sum rule will be even smaller.

Let us now determine which values of the part of the proton and neutron spin projection carried by quarks and gluons correspond to the values of $\Gamma_{p,n}$ (18) and (19). Taking into account the first QCD correction[6] we have

$$\Gamma^{as}_{p,n}=\frac{1}{12}\left\{\left(1-\frac{\alpha_s}{\pi}\right)\left[\pm g_A+\frac{1}{3}a_8\right]+\frac{4}{3}\left[1-C_f\frac{\alpha_s}{\pi}\right]\Sigma\right\}-\frac{N_f}{18\pi}\alpha_s\Delta g, \qquad (21)$$

where

$$a_8=\Delta u+\Delta d-2\Delta s=3F-D, \quad \Sigma=\Delta u+\Delta d+\Delta s. \qquad (22)$$

Δu, Δd, Δs, Δg are the parts of the proton spin projection carried by u, d, s quarks and gluons, respectively, $g_A=1.257$ is the axial constant of β decay, which is related to $\Delta u,\Delta d$ by

$$g_A=\Delta u-\Delta d, \qquad (23)$$

F and D are the β-decay constants in the baryon octet, N_f is the flavor number, $C_f=(33-8N_f)/(33-2N_f)=1/3$ at $N_f=3$. We take[14]

$$3F-D=0.59\pm0.02 \qquad (24)$$

and the portion of the nucleon spin carried by gluons to be $\Delta g\approx0.5$ at $Q^2=Q_0^2$. This value will be confirmed in what follows. From (18) and (20)–(23), one can readily find Σ, Δu, Δd, Δs which correspond to the EMC experiment ($Q^2=Q_0^2=10.7$ GeV^2):

$$\Sigma=0.30\pm0.17, \quad \Delta u=0.83\pm0.06, \qquad (25)$$
$$\Delta=-0.43\pm0.06, \quad \Delta s=-0.10\pm0.06.$$

From (19) and (20)–(23), we find the values of the same quantities following from the E142 data recalculated at $Q^2=Q_0^2$:

$$\Sigma=0.53\pm0.14, \quad \Delta u=0.905\pm0.05, \qquad (26)$$
$$\Delta d=-0.355\pm0.05, \quad \Delta s=-0.02\pm0.05.$$

As is seen from a comparison of (25) and (26), after accounting for the power corrections, the EMC and E142 results agree with each other within the overlapping errors of the two experiments. The value of the part of the nucleon spin carried by all quarks agrees with the intuitively expected value $\Sigma\approx0.5$ (by analogy with the nucleon momentum fraction carried by quarks). If in accordance with the quark model we now neglect orbital momentum, the gluon share goes to $\Delta g=0.5$, the value we have adopted above. The spin fraction carried by s quarks agrees with $\Delta s\approx-0.05$, which is consistent with naive expectations.

Let us finally compare the value $\Gamma_p+\Gamma_n$ at $\bar{Q}^2=4.6$ GeV^2 (4) measured by SMC with the theoretical expectation in our model. The power corrections by which Γ_p, Γ_n deviate at $Q^2=4.6$ GeV^2 from their values Γ^{as}_p, Γ^{as}_n at Q^2 are, correspondingly 16% and 12%. Taking them into account, using Eq. (21), the values $3F-D$ given by (24) and $\Sigma=\Delta g=0.5$, we obtain at $\bar{Q}^2=4.6$ GeV^2

$$(\Gamma_p+\Gamma_n)_{\text{theor}}=0.10 \qquad (27)$$

in comparison with the experimental value (4). Again, the difference is less than one standard deviation.

The values of the power corrections that follow from our model agree with the value found in Ref. 15, where the coefficient of the higher twist correction was considered a free parameter, chosen by the requirement that the EMC, SMC and E142 results for Σ be consistent with each other. However, our values of power corrections are several times larger than the twist-4 contribution calculated in Ref. 9.

When comparing these two approaches one should take into consideration the following. The authors of Ref. 9 calculated the contributions of twist-4 terms to Γ_{p+n} and Γ_{p-n}. In finding the vacuum expectation values induced by an external axial field—which profoundly influence the final answer—the authors of Ref. 9 have really taken the octet field instead of the singlet one and used the dominance of massless goldstones (π or η), which is incorrect for the singlet field case. For this reason there are serious doubts about the reliability of Ref. 9 calculations for the Γ_{p+n} case. For the octet case (the Bjorken sum rule), the situation is better, in principle. But, unfortunately, in this case the main contribution to the sum rule used by the authors comes from the highest-order term included in the operator product expansion (OPE)—the term $\sim m_0^2\langle\bar{\psi}\psi\rangle^2$ of dimension 8. It is possible that in this case the contribution of terms left out of the OPE is substantial, or even that the OPE series is divergent at characteristic values of the Borel parameter $M^2=1$ GeV^2 used in the sum rule. On the physical side of the sum rule, the contribution of the continuum comprises 80%, and the nucleon pole term from which Γ_{p-n} was obtained gives only 20%. This also spoils the accuracy of the calculations in Ref. 9. Finally, the result of Ref. 9 (unlike the other results obtained by the QCD sum rule) depends on the ultraviolet cutoff. This circumstance introduces an uncontrollable uncertainty into the calculation. We therefore do not believe that the twist-4 results obtained in Ref. 9 are justifiable. The crucial factor for checking our approach would be a precision study of the Q^2 dependence in polarized deep-inelastic $e(\mu)$ nucleon scattering.

In a recent preprint, Ji and Unrau[16] attempted to construct a model of the Q^2 dependence of $\Gamma(Q^2)$ that satisfies the GDH sum rule at $Q^2=0$. They found high twist corrections to the EMC, SMC and E142 experiments much smaller than in our model. We disagree with Ji and Unrau's considerations at low Q^2. The main ingredient of their approach is the dominant role played by elastic contributions (nucleon pole in the Compton amplitude) at low Q^2. However, as follows from the original derivation[17] (see also Ref. 18), the polarized Compton amplitude taking account of crossing terms, which appears on the left-hand side of GDH sum rule, has no nucleon pole term at $Q^2=0$ and is a constant. For this reason, at $Q^2=0$ the elastic term is absent from the right-hand side, only excited states contribute and the GDH sum rule is most definitely nontrivial. At $Q^2>0$, the pole term on the left-hand side is completely cancelled by the pole term on the right-hand side, and as a consequence the elastic contribution can be omitted in constructing a model describing the Q^2 dependence of $\Gamma(Q^2)$. It must also be mentioned that elastic contributions are not measured in any experiment on deep inelastic scattering.

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