Ion line narrowing in a dense plasma

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The range of applicability of the quantum (with respect to the internal degrees of freedom) kinetic equation for ions in an equilibrium plasma is found. The Dicke absorption line narrowing is calculated in the limiting cases of long and short wavelengths compared to the ion mean free path. The line shape and the autocorrelation function are obtained. In the intermediate case, a simple interpolation formula for the line center intensity is proposed, which agrees with numerical results. The inclusion of Chandrasekhar's monotonically decreasing velocity dependence of the collision frequency is shown to increase the Dicke effect by 60% in the short-wavelength limit while reducing it by 40% in the long-wavelength limit.

1. INTRODUCTION

The observation of Doppler broadened spectral lines has traditionally been a means for measuring gas or plasma temperatures. If one can neglect collisions and the finite lifetime of excited states, the shape of the spectral line depends on the velocity distribution of the particles. The line contour $I(\Omega)$ is known to be given by the Fourier transform of the correlation function $\Phi(t)$ (Wiener–Khinchin theorem)

$$I(\Omega)=\frac{1}{\pi}\operatorname{Re}\int_0^\infty \Phi(t)e^{i\Omega t}dt, \tag{1}$$

where Ω is the mismatch of the field relative to the Bohr transition frequency. In turn, the autocorrelation function is found by averaging over the ensemble of atoms,

$$\Phi(t)=\langle e^{ik\Delta x(t)}\rangle. \tag{2}$$

Here the x axis is directed along the wave vector $\mathbf{k}$. If the motion is free ($\Delta x=vt$) and the distribution over velocities $\mathbf{v}$ is Maxwellian with a thermal velocity v_T, then

$$\Phi(t)=\exp(-k^2v_T^2t^2/4), \tag{3}$$

and from (1) a Doppler line shape with a characteristic width kv_T is obtained.

In a sufficiently dense plasma, the Doppler diagnostics may lead to erroneous results, however. In elastic collisions there is no slip in the phase of the optical electron, so the wave trains from one and the same atom will interfere as before. At the same time, if the atom mean free path is less than the radiation wavelength, then there is not enough time between collisions for the Doppler line shift to form for each one of the atoms. In the short mean free path limit the atom is localized and does not in fact change its coordinates. As a result, the autocorrelation function (2) tends to a constant, and the line width to zero.[1]

The effect of line narrowing in the model of weak collisions, when the change in velocity of a Brownian particle in every single collision is small and the collision frequency ν does not depend on $\mathbf{v}$, has been investigated by Rautian and Sobel'man.[2] In this case the average variation of the coordinate x vanishes and the variance is found from

$$\langle\Delta x^2(t)\rangle=\frac{v_T^2}{2\nu^2}(\nu t-1+e^{-\nu t}). \tag{4}$$

Before the first collision ($\nu t\ll 1$) the motion differs little from the free one, so that the variance is quadratic in time, $\langle\Delta x^2\rangle=(v_Tt/2)^2$. After several collisions ($\nu t\gg 1$) the motion of the Brownian particle becomes a random walk, and the variance grows according to the diffusion law $\langle\Delta x^2\rangle=2D_xt$, $D_x=v_T^2/2\nu$. The coherence time, during which a substantial reduction of the correlation function

$$\Phi(t)=e^{-k^2\langle\Delta x^2\rangle}, \tag{5}$$

takes place, depends on the radiation wavelength and is given by

$$\tau=\begin{cases}1/kv_T, & \nu\ll kv_T;\\ \nu/k^2v_T^2, & \nu\gg kv_T.\end{cases} \tag{6}$$

For short-wavelength radiation, when $\nu\ll kv_T$, the time τ is determined by the Doppler shift of the frequency of a single atom, and the function $\Phi(t)$ is close to (3). But if the wavelength exceeds the mean free path, $\nu\gg kv_T$, the time τ is controlled by diffusion, $\Phi(t)=\exp(-D_xk^2t/2)$. Therefore the spectral width, $\delta\Omega\sim 1/\tau\sim k^2v_T^2/\nu$, becomes much less than Doppler. Since $\Phi(0)=1$, the area under the line contour (1) does not depend on the parameters of the problem, so the width of the line can also be assessed from the value of $I(0)$. A more slowly decreasing $\Phi(t)$, taking collisions into account via Eq. (6), leads to a greater $I(0)$ and hence to a narrower line.

Similar line narrowing results from other constraints on the free translational motion of a particle that do not disrupt the phase, such as walls at a distance $L<1/k$ in hydrogen maser frequency standards; a transverse magnetic field, which splits the ion line into a set of cyclotron resonances if the ion Larmor radius is sufficiently small, $\rho_L<1/k$ (see Ref. 3); or the restoring force in an electrostatic trap.[4]

When the scattering of a radiating ion in a plasma is by buffer charged particles, small velocity changes dominate.

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The effective collision frequency falls off sharply for $v > v_T$ ($\nu \propto 1/v^3$) and therefore the weak collision model can only be employed for approximate plasma calculations.[1)]

The purpose of the present work is to calculate the Dicke narrowing in a plasma and to investigate the influence of the $\nu(v)$ dependence on this narrowing. Section 2 is devoted to the derivation of the quantum kinetic equation for the spectroscopic density matrix in the case of a Coulomb interaction between the particles. A brief derivation of the kinetic equation is given in Ref. 6, but the range of applicability of the equation is not discussed in sufficient detail there. In Sec. 3, we calculate the narrowing from perturbation theory for $\nu \ll kv_t$, when the Dicke effect is small; and an expression for the function $\Phi(t)$, which may prove useful for comparison with experiment, is obtained.[2)] A more complicated limiting case of a high-density plasma is analyzed in Sec. 4. The complexity of the calculation has to do with the decreasing nature of the $\nu(v)$ dependence. Even when slow ions are highly collisional, the fast ions, because of the strong dependence of the Rutherford cross section on the velocity, are as before in the rare collision regime. In Sec. 5. the results of the numerical line narrowing calculation are presented and an interpolation formula valid for $\nu \sim kv_T$ is obtained.

The limiting cases of Secs. 3 and 4 have been given in a recent Note,[9] where for the sake of brevity we restricted ourselves to an exposition of the calculation scheme used. In the present study we discuss in detail a somewhat different approach to high-density plasma calculations, one that makes it possible to increase the accuracy of the previous result obtained by the collision frequency renormalization method. What is new is the analysis in Sec. 5 of an intermediate case presumably realized in experimentally measuring the line width of an x-ray laser[10] on multiple selenium ions. In such a system, plasma is obtained by exploding foil in the focus of a high-power solid state laser. The hypothesis of appreciable collisional narrowing in such a plasma has been introduced in Refs. 11 and 12, in which, for estimation purposes, weak collision formulas are employed. While calculating the line narrowing using the Landau integral is more accurate than for weak collisions, application to the selenium plasma should be made with great caution because a high-density laser plasma is, according to estimates, a weakly nonideal and strongly turbulent plasma.

2. QUANTUM KINETIC EQUATION IN THE CASE OF THE COULOMB INTERACTION

There are three types of quantum effects recognizable in the problem of the interaction of light with plasma ions: the quantum fluctuations of the field; the quantization of the translational motion, particularly with inclusion of the exchange interaction; and transitions between the internal states of the ion. In plasma spectroscopy, as in the case of atomic or molecular gases,[13] it is usually this last effect which is of importance. Therefore in what follows we describe the derivation of a quantum kinetic equation for the spectroscopic density matrix, which is a matrix for the internal ion states and the Wigner function for its translational states. The field will be considered classical, and the spontaneous decay of the states will be accounted for by introducing relaxation constants.

Like its classical prototype, the quantum kinetic equation is derived by the method of Bogolyubov.[14] For a short-range interaction potential that is weak compared to the kinetic energy of structureless Bose particles, the corresponding kinetic equation was first developed by Bogolyubov and Gurov.[15] The equation for the long-range Coulomb potential was found by Klimontovich and Temko[16] and solved by Silin.[17]

A collision term for the classical kinetic equation for a plasma, which generalizes the Landau integral[18] to the multiparticle interaction, was found by Balescu,[19] Lenard,[20] and Guernsey[21] simultaneously. However, as shown by Kogan,[22] the inclusion of the effect of many perturbing particles on the probe particle does not alter the binary character of the scattering, i.e., on the average multiple scattering imitates pair scattering. This equivalence is a consequence of the overlap of the ranges of applicability of the pair and multiple scattering concepts.

Let us derive a collision integral for the quantum kinetic equation for ions with internal states by considering transitions between the levels while neglecting exchange. The condition for nondegeneracy is fulfilled in gas-discharge, thermonuclear, and cosmic plasma. To zeroth order in the plasma parameter $1/N_d$ ($N_d = 4\pi N r_d^3/3 \gg 1$ is the Debye number, r_d the Debye scale), the equation for the one-particle density matrix reduces to the quantum Vlasov equation

$$i\hbar \frac{\partial \hat{\rho}_a}{\partial t} = [\hat{H}_a, \hat{\rho}_a], \quad \hat{H}_a = \hat{H}_a^0 + \hat{H}_a^1 + \mathrm{Tr}_b W_{ab} \hat{\rho}_b, \tag{7}$$

where the $\hat{\rho}_{a,b}$ are the number density operators of a- or b-type particles, a is the probe ion, b refers to the buffer ions and electrons, $\hat{H}_a^0$ is the internal Hamiltonian of the a-type particle, and $\hat{H}_a^1$ the operator for the energy of the ion in an external field. This last may be represented as a sum of the operator $\hat{V}$, the resonance interaction with the light wave field inducing transitions between the internal states, and the operator $\hat{U}$ of the ion energy in the external static (or quasistatic) electric and magnetic fields, which does not affect the internal degrees of freedom but changes the ion's translational velocity. The interaction of the probe ion with buffer charged particles enters as the trace of the product of the Coulomb interaction energy W_{ab} and the number density of the field particles (the mean field).

Changing to the energy representation for the internal variables and to a classical picture for the translational motion by making use of the Wigner representation allows one to replace the commutator of the density matrix and the quasistatic field by the Poisson bracket. As a result one obtains an equation, with a self-consistent field $\widetilde{U} = U + \mathrm{Tr}_b(W_{ab}\hat{\rho}_b)$, for the spectroscopic density matrix $\hat{\rho} = \hat{\rho}_a$:

$$\frac{\partial \hat{\rho}}{\partial t} + \{\widetilde{U}, \hat{\rho}\} = -i[\hat{V}, \hat{\rho}], \tag{8}$$

where {.,.} is a Poisson bracket. If the perturbing particles are ground-state ions, then $\hat{\rho}_b$ goes over into the classical distribution function f_b, and the trace operation, into a phase space integration and a summation over the perturbing species. Let us consider the case of a nonrelativistic plasma with no external fields, i.e., a Hamiltonian accounting for the interaction of the probe particle a with the plasma microfield, $\tilde{U}_a = p_a^2/2m + \langle W_{ab}\rangle_b$, in which the particle interaction potential obeys the Coulomb law

$$W_{ab} = \frac{Z_a Z_b e^2}{|r_a - r_b|}. \tag{9}$$

Here $Z_a e$ and $Z_b e$ are the charges on the probe ion in question and the perturbing ion.

In order to obtain the matrix density kinetic equation, let us take an average of Eq. (8) over fluctuations following, for example, the technique discussed in detail in Refs. 23, 23, and 25 for the classical equation, and in Ref. 26 for the (translationally) quantum mechanical equation for the Wigner function. These procedures can only be applied when characteristic fluctuation times are much less then other characteristic times involved.

Let us present estimates for the characteristic times for various processes in a gas-discharge plasma with an electron concentration of $N_e = 10^{14}$ cm^{-3} and electron and ion temperatures of $T_e \sim 5$ eV and $T_i \sim 1$ eV, respectively. The characteristic fluctuation time over which the average is taken is of the order of the reciprocal of the electron plasma frequency $\tau_f \simeq \omega_{pe}^{-1} = (4\pi e^2 N_e/m_e)^{-1/2} \simeq 1$ ps. This interval is much less than either the time of the change in the ion momentum, $\tau_p \simeq \nu^{-1} \simeq (N\sigma v_T)^{-1} \simeq 1$ μs or the time of interaction of an excited ion state with the light.

In fact $|V|$, the characteristic magnitude of the probe ion–light interaction matrix element is of the order of the Rabi oscillation frequency $\Omega_R = \sqrt{\Omega^2 + 4|G|^2}$ (*for* $\Omega \ll kv_T$). Here $G = \mathbf{E}\mathbf{d}/2\hbar$ is the frequency of the induced transitions due to the atom–light interaction at the exact resonance; $\mathbf{E}$ is the amplitude of the light field; and $\mathbf{d}$ the matrix element of the field-induced dipole moment. At an exact optical-wavelength resonance in the field of a relatively high-power (but not maximally so) cw laser with a flux density of 3×10^2 W/cm^2 we have $|V| \sim 10^9$ s^{-1}, hence the induced transition time $\tau_t \sim 1$ nsec. If the frequency mismatch becomes comparable to the Doppler width $(\Omega \sim kv_T)$, then for an ion mass of 40 a.u. in a gas-discharge plasma we will obtain $\tau_t \sim 0.1$ nsec, which is also much longer than the characteristic electron fluctuation time in a plasma.

In the plasma of a laser with neon-like selenium ions ($Z_a = 24$) (Ref. 10), one achieves ion and electron densities of $N_i \sim 2\times 10^{19}$ cm^{-3} and $N_e \sim 5\times 10^{20}$ cm^{-3} (about six orders of magnitude higher than in a gas discharge), and temperatures of $T_i \sim 400$ eV and $T_e \sim 900$ eV. Nevertheless, the time scales for momentum changes, spontaneous radiation ($\tau_p \sim \tau_t \sim 1$ ps), and Doppler dephasing ($1/kv_T \sim 0.1$ ps) remain substantially greater than the characteristic plasma fluctuation times $\tau_f = \omega_{pe}^{-1} \sim 1$ fs.

Because the characteristic times of photon emission and absorption processes are far in excess of plasma fluctuation times, Eq. (8) may be treated in the continuous medium approximation, i.e., by considering that plasma particle interactions reduce to instantaneous (relative to the optical times) screening. In this case the derivation of the quantum equation is analogous to the derivation of the classical one. As for the resulting equation itself,[6] it differs from the classical one by having on its right-hand side a commutator describing transitions between the internal states of the particle:

$$\left(\frac{\partial}{\partial t} + \mathbf{v}\frac{\partial}{\partial \mathbf{r}}\right)\hat{\rho} + \mathrm{div}_{\mathbf{v}}\hat{\mathbf{q}} = -i[\hat{V}, \hat{\rho}]; \tag{10}$$

$$\hat{q}_a = \sum_b \frac{(Z_a Z_b e^2)^2}{m_a} \int d\mathbf{v}_b U_{\alpha\beta}(\mathbf{v},\mathbf{v}_b)\left(\frac{1}{m_b}\frac{\partial f_b(\mathbf{v}_b)}{\partial v_{b\beta}} - \frac{f_b(\mathbf{v}_b)}{m_a}\frac{\partial}{\partial v_\beta}\right)\hat{\rho}, \tag{11}$$

$$U_{\alpha\beta}(\mathbf{v},\mathbf{v}_b) = \int d\omega d\mathbf{k} \frac{k_\alpha k_\beta}{k^4 |\varepsilon(\omega,\mathbf{k})|^2} \times \delta(\omega - \mathbf{k}\mathbf{v})\delta(\omega - \mathbf{k}\mathbf{v}_b). \tag{12}$$

Here $U_{\alpha\beta}$ is the Lenard–Balescu kernel and $f_b(\mathbf{v}_b)$ is the buffer particle velocity distribution.

Only in sufficiently strong fields, when $\Omega_R \gtrsim 10^{12}$ sec^{-1}, or under the action of picosecond pulses, when the pulse duration is comparable to plasma oscillation frequencies, is it impossible to average over the microfield fluctuations assuming all other processes in the system to be slow. In this case the applicability of the kinetic equation (10) is violated.

For an isothermal plasma the Lenard–Balescu kernel reduces to the Landau kernel[3)]

$$U_{\alpha\beta} = 2\pi L \frac{u^2 \delta_{\alpha\beta} - u_\alpha u_\beta}{u^3}. \tag{13}$$

Here $\mathbf{u} = \mathbf{v}_a - \mathbf{v}_b$ is the relative velocity and the Coulomb logarithm L arises from the cutoff of the integral at large $k \simeq N^{1/3}$, corresponding to small impact parameters for which the continuous medium assumption for the plasma breaks down. The collision operator $\hat{S} = -\mathrm{div}_{\mathbf{v}}\hat{\mathbf{q}}$ on the left-hand side of (10) is the divergence of the ion flux $\hat{\mathbf{q}}$ in velocity space and may be represented as a sum of two terms describing damping and diffusion,

$$\hat{q}_a = \frac{F_\alpha}{m_a}\hat{\rho} - D_{\alpha\beta}\frac{\partial \hat{\rho}}{\partial v_\beta}. \tag{14}$$

The Maxwell distribution renders the ion flux zero. Because of its divergence form the operator conserves the number of particles.

Simple estimates show that for $T_e \sim T_i$, the sum over the species b in the expression (11) is dominated by scattering by ions.[28] Scattering by electrons is small in terms of the parameter v_{Ti}/v_{Te} because only a small fraction $(\sim v_{Ti}/v_{Te})$ of electrons whose velocity components along the wave vector are small $[\mathbf{k}\mathbf{v}_e = \mathbf{k}\mathbf{v} - kv_{Ti}$, Eq. (12)] can take part. The electron contribution to the collision term might only show up in an extremely nonisothermal plasma.

Thus, for an electrically neutral plasma with equal ion and electron concentrations, the quantum (for the inner degrees of freedom) kinetic equation has the same Balescu–Guernsey–Lenard type collision integral as the classical equation. The integral has proven to be the same in the equations for the populations and for the off-diagonal density matrix elements. Since the main contribution to the effective cross section comes from small-angle scattering in the impact parameter range $N^{-1/3} \ll b \ll r_d$, where both pair and multiple scattering concepts are applicable, there are two ways to interpret this fact. From the pair collision viewpoint, it is explained by the equality of the Coulomb scattering amplitudes (whose cross section depends on the total charge alone) for different internal quantum states of the atom. From the point of view of multiple scattering, it results from the acceleration of the ion as a whole in the chaotic plasma microfield. The momentum of an ion changes adiabatically slowly compared with the Bohr frequencies of transitions between levels, so that the acceleration has no effect on the state of the internal Hamiltonian.

3. SHORT-WAVELENGTH APPROXIMATION

Intra-atomic transitions will be considered in the two-level approximation, in which case the matrix element for the dipole interaction with the field of a traveling monochromatic wave is described by the function

$$V_{mn} = -G\exp(i\mathbf{kr} - i\Omega t), \quad G = \frac{\mathbf{Ed}_{mn}}{2h}. \tag{14'}$$

Let us complete the kinetic equation (10) with terms accounting for radiation and collision relaxation, and for the excitation of the levels usually produced by electron impact from the ground or a metastable state. Denoting the relaxation constants of the levels by Γ_i, and that of polarization by Γ, for the off-diagonal matrix element $\rho_{mn}(\mathbf{v},\mathbf{r},t) = \rho(\mathbf{v})\exp(i\mathbf{kr} - i\Omega t)$ and the diagonal matrix element ρ_{jj} we obtain the following stationary homogeneous equations:

$$(\Gamma - i\Omega + i\mathbf{kv})\rho - S[\rho] = -iG\Delta N, \quad \Delta N = \rho_{mm} - \rho_{nn}, \tag{15}$$

$$\Gamma_m\rho_{mm} - S[\rho_{mm}] = \lambda_m(\mathbf{v}) - 2\,\mathrm{Re}(iG^*\rho),$$

$$\Gamma n\rho nn - S[\rho nn] = \lambda n(\mathbf{v}) + 2\,\mathrm{Re}(iG^*\rho). \tag{16}$$

Here $\lambda_j(\mathbf{v})$ is the excitation function of the jth level. For weak fields ($|G^2| \ll \Gamma\Gamma_j$), one can neglect saturation and consider only the polarization equation (15), with the population density $\Delta N = \lambda_m(\mathbf{v})/\Gamma_m - \lambda_n(\mathbf{v})/\Gamma_n$, $\lambda_j(\mathbf{v}) = Q_j W(\mathbf{v})$ on the right-hand side. The central problem of plasma spectroscopy reduces to the evaluation of an off-diagonal element of the density matrix ρ, the integral of the element over velocities being expressed in terms of the observable absorbed power per unit volume as

$$\mathscr{P}(\Omega) = -2\hbar\omega \int d^3v\,\mathrm{Re}(iV^*_{mn}\rho_{mn})$$
$$= -2\pi\hbar\omega|G|^2 N_{mn} I(\Omega), \tag{16'}$$

$$N_{mn} = \frac{Q_m}{\Gamma_m} - \frac{Q_n}{\Gamma_n}.$$

As shown in Sec. 2, in the Coulomb interaction case the collision integral in Eq. (10) may be written in the form of a differential operator

$$S[\rho_{ij}] = \frac{\partial}{\partial v_\alpha}\left(-\frac{F_\alpha}{m}\rho_{ij} + D_{\alpha\beta}\frac{\partial\rho_{ij}}{\partial v_\beta}\right). \tag{17}$$

If the velocity distribution of the buffer particles and excitation function λ_j has, or differs little from, a Maxwellian form, it is possible to obtain explicit expressions[25] for the dynamic damping and diffusion tensor corresponding to the Landau integral (13). Setting $T_{ia} = T_{ib} = T$, $m = m_{ia} = m_{ib}$ in the following analysis, from Eqs. (11), (13), and (14) we obtain

$$F_\alpha = -\nu m v_T \xi_\alpha \Phi_l(\xi),$$
$$D_{\alpha\beta} = \frac{\nu v_T^2}{2}\left[\Phi_l(\xi)\frac{\xi_\alpha\xi_\beta}{\xi^2} + \Phi_{tr}(\xi)\left(\delta_{\alpha\beta} - \frac{\xi_\alpha\xi_\beta}{\xi^2}\right)\right], \tag{18}$$

where $\xi_\alpha = v_\alpha/v_T$ is the α-component of the dimensionless velocity,

$$\nu = \frac{16\sqrt{\pi}LN_i(Z_a e^2)^2}{3m^2 v_T^3} \tag{19}$$

is the effective transport frequency of ion–ion collisions, and N_i is the ion concentration. If the plasma contains ions of different degrees of ionization Z_b, then the frequency ν involves the effective charge $N_i = \Sigma_b Z_b^2 N_b$. The functions

$$\Phi_l(\xi) = 3\int_0^1 \lambda^2 e^{-\lambda^2\xi^2} d\lambda,$$
$$\Phi_{tr}(\xi) = \frac{3}{2}\int_0^1 (1-\lambda^2)e^{-\lambda^2\xi^2} d\lambda \tag{20}$$

are expressed in terms of the Chandrasekhar function $g(\xi) = [\mathrm{erf}(\xi) - \xi\,\mathrm{erf}'(\xi)]/2\xi^2$ and are related by

$$\frac{1}{2\xi}\frac{d\Phi_l}{d\xi} + \Phi_l = \frac{\Phi_{tr} - \Phi_l}{\xi^2}, \quad \xi\frac{d\Phi_l}{d\xi} + 3\Phi_l = 3\exp(-\xi^2). \tag{21}$$

At high velocities, the damping force and the longitudinal (with respect to the velocity) diffusion become unimportant and a dominant role is played by the transverse (with respect to angles in velocity space) diffusion. In a coordinate system with the z axis along the probe ion velocity, the diffusion tensor (18) is diagonal, and for $\xi \ll 1$ it is also isotropic.

Let us investigate the change in the shape of the spectrum due to ion–ion scattering. The collision integral (17) is most conveniently written using a spherical coordinate system with the z axis along the wave vector $\mathbf{k}$:

$$S[\rho] = \nu\mathscr{L}\rho,$$

$$\mathscr{L} = \frac{1}{2}\left[\frac{\Phi_{tr}}{\xi^2}\left(\frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\sin\vartheta\frac{\partial}{\partial\vartheta} + \frac{1}{\sin^2\vartheta}\frac{\partial^2}{\partial\varphi^2}\right)\right.$$

$$+\frac{1}{\xi^2}\frac{\partial}{\partial\xi}\xi^2\Phi_l\left(\frac{\partial}{\partial\xi}+2\xi\right)\Bigg], \tag{22}$$

where ϑ and φ are angles in velocity space.

A solution to the quantum kinetic equation (15) for an off-diagonal element of the density matrix for a two-level system may be obtained in the form of a series in powers of the parameter $\eta = \nu/kv_T \ll 1$

$$\rho(\mathbf{v}) = \sum_{n=0}^{\infty} \eta^n \left(\frac{i\mathscr{L}}{z-\xi\cos\vartheta}\right)^n \bar{\rho}, \quad z=\frac{\Omega+i\Gamma}{kv_T}, \tag{23}$$

where

$$\bar{\rho}=\frac{GN_{mn}}{kv_T(v_T\sqrt{\pi})^3}\frac{\exp(-\xi^2)}{z-\xi\cos\vartheta} \tag{24}$$

is the off-diagonal element of the density matrix in the absence of collisions. The power absorbed by the two-level system can also be represented in the form of an expansion in powers of the parameter η:

$$\mathscr{P}=\mathscr{P}_0\,\mathrm{Re}\left[i\sum_{n=0}^{\infty}\left(\frac{i\nu}{kv_T}\right)^n J_n(z)\right], \tag{25}$$

where

$$\mathscr{P}_0=-2\hbar\omega|G|^2N_{mn}\frac{\sqrt{\pi}}{kv_T},$$

$$J_n=\frac{1}{\pi^2}\int d^3\xi\left(\frac{\mathscr{L}}{z-\xi\cos\vartheta}\right)^n\frac{\exp(-\xi^2)}{z-\xi\cos\vartheta}. \tag{26}$$

The coefficient of the zeroth term in the expansion (25) can be expressed in terms of the probability integral of a complex variable,

$$iJ_0(\Omega)=w(z)=\exp(-z^2)\left(1+\frac{2i}{\sqrt{\pi}}\int_0^z\exp(t^2)dt\right), \tag{27}$$

and describes the Voigt line contour for a collisionless plasma.

For a tenuous plasma, the frequency of Coulomb collisions is small compared to the Doppler line width and we need to keep only the terms $n=0,1$ in the expansion (25). Substituting the collisional integral $\mathscr{L}$, Eq. (22), into the integral J_1 defined by Eq. (26) and integrating by parts, the coefficient of the first power of the small parameter η can be written in the form

$$-J_1=\int_0^\infty\frac{d^3\xi}{2\pi^2}\frac{\Phi_l\cos^2\vartheta+\Phi_{tr}\sin^2\vartheta}{(z-\xi\cos\vartheta)^4}\exp(-\xi^2)$$

$$=\int_0^\infty\frac{d^3\xi}{3\pi^2}\frac{\Phi_l 2\xi\cos\vartheta\exp(-\xi^2)}{(z-\xi\cos\vartheta)^3}. \tag{28}$$

Here $\Phi_l\cos^2\vartheta+\Phi_{tr}\sin^2\vartheta$ is a component of the diffusion tensor (18) along the wave vector $\mathbf{k}$, and the last equality is obtained with the aid of the identities (21). Making the substitution $1/z^3=(1/2)\int_0^\infty d\tau\tau^2e^{iz\tau}$ for $\mathrm{Im}\,z>0$ and performing the Gaussian integration over the velocities, we obtain for the collision correction to the line shape

$$-J_1(\Omega)=\int_0^\infty\frac{d\tau\tau^3}{2\sqrt{\pi}}\int_0^1\frac{d\lambda\lambda^2}{(\lambda^2+1)^{5/2}}$$

$$\times\exp\left(-\frac{\tau^2}{4(\lambda^2+1)}+i\tau z\right)$$

$$=\int_0^\infty\frac{d\tau\tau^3}{2\sqrt{\pi}}\int_0^{1/\sqrt{2}}dxx^2$$

$$\times\exp\left(-\frac{\tau^2}{4}(1-x^2)+i\tau z\right). \tag{29}$$

To first order in ν/kv_T, the autocorrelation function can be expressed in terms of the Fourier transforms, with respect to Ω, of the integrals J_0 and J_1,

$$\Phi(t)=\left(1+\frac{2\nu}{kv_T}(y-F(y))\exp(y^2)\right)\exp(-\Gamma t-2y^2), \tag{21'}$$

$$y=\frac{tkv_T}{2\sqrt{2}},$$

where $F(y)=\exp(-y^2)\int_0^y dx\exp(x^2)$ is the Dawson integral. In the limit $|z|=|i\Gamma+\Omega|/kv_T\ll 1$, the integral $J_1=-(2/\sqrt{\pi})(\sqrt{2}-\ln(\sqrt{2}+1))$, so that the inclusion of collisions somewhat increases the absorption intensity at the line center:

$$\mathscr{P}=\mathscr{P}_0\left(1+\frac{2}{\sqrt{\pi}}\frac{\tilde{\nu}}{kv_T}\right),$$

$$\tilde{\nu}=(\sqrt{2}-\ln(\sqrt{2}+1))\nu\cong 0.53\nu. \tag{30}$$

Since the area under the line contour is independent of ν, an increase in amplitude at the maximum implies a narrowing of the line.

For the weak collision model with a constant collision frequency Eq. (32), in the same limiting case $\mathscr{P}=\mathscr{P}_0(1+2\nu/3\sqrt{\pi}kv_T)$. The question naturally arises as to how to explain the increase from 0.33 to 0.53 in the coefficient for the correction. One would expect that the dropping velocity dependence of the Coulomb collisions should reduce rather than increase the coefficient. The point is that a correct comparison of the models requires that the friction forces of the probe ions be compared. For this purpose let us turn to Eq. (14) for the particle flux in velocity space. The first term, corresponding to the dynamic friction force, describes the damping of the probe particle. The second term, the diffusion flow in velocity space, arises if the probe particle distribution $f(\mathbf{v})$ has a nonzero gradient. Consider the one-dimensional case. In the weak-collision constant-diffusion-coefficient model, the flows marked by arrows in Fig. 1 compensate each other, so that the average velocity of the probe particles decreases only because of the friction force. If the diffusion coefficient $D(v)$ decreases with increasing absolute velocity, the flow toward lower velocities exceeds that directed toward higher velocities. As a result, the damping force increases. For the case of a narrow distribution function, the correction term is proportional to the derivative of the diffusion coefficient at the center of the distribution. If $dD/d|v|>0$

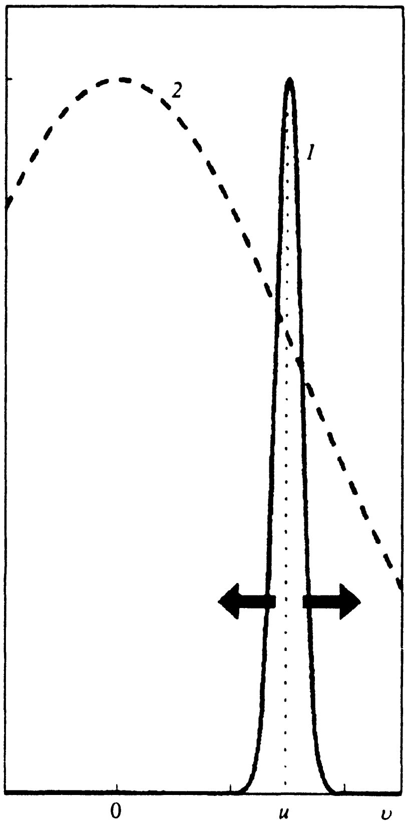

FIG. 1. One-dimensional probe-particle distribution function in velocity space (curve 1), and the diffusion coefficient (curve 2) with a decreasing velocity dependence. Arrows indicate the direction of diffusion flows.

for $v=u$, the correction decreases the friction force. In three dimensions, the total force acting on probe particles with the distribution $f(\mathbf{v})$ is

$$\int d^3v\left(F_\alpha f(\mathbf{v})-mD_{\alpha\beta}\frac{\partial f(\mathbf{v})}{\partial v_\beta}\right)$$

$$=\int d^3v\left(F_\alpha+m\frac{\partial D_{\alpha\beta}}{\partial v_\beta}\right)f(\mathbf{v}). \tag{30'}$$

The additional damping force is determined by the divergence of the diffusion tensor and vanishes for $\partial D_{\alpha\beta}/\partial v_\beta=0$. Using the identities (21), we obtain that $\partial D_{\alpha\beta}/\partial v_\beta=F_\alpha/m$, that is, the damping force on a probe particle in a plasma doubles because of the dropping dependence of the diffusion tensor on the magnitude of the velocity. The total force on particles with nonzero velocities is still less than $2F_\alpha(0)$ because of the decrease in collision frequency with increasing velocity. As a result, the coefficient $\tilde{\nu}$ for the correction to the work of the field, Eq. (30), was found to be less than $2\nu/3$ but about 60% larger than in the constant-coefficient model.

Using Eqs. (27) and (29), one can obtain an asymptotic expansion of $\mathscr{P}$ for $|z|\gg 1$:

$$\mathscr{P}=\mathscr{P}_0\frac{kv_T}{\sqrt{\pi}}\left[\frac{\Gamma}{\Omega^2}+\frac{k^2v_T^2}{2}\frac{3\Gamma+\nu/\sqrt{2}}{\Omega^4}-\frac{\Gamma^3}{\Omega^4}+\cdots\right]. \tag{31}$$

In the far wings of the curve one obtains a power-law decrease in intensity. Note that the integral J_1 increases the coefficient of z^{-4}. The intensity distribution in the wings as obtained here differs from the weak collision result of Ref. 2 by a factor of $1/\sqrt{2}$ (at collision frequency ν), which reflects the velocity dependence of the diffusion coefficient.

Note that for $\nu k^2v_T^2\gg\Gamma^3$, the terms in the series (23) for $\rho(v)$ diverge for velocity values close to resonance ($\mathbf{kv}=\Omega$). Rather than proving the cancellation of terms that diverge as $\Gamma\to 0$ in each order of the expansion (23), we may re-sum this series so as to make the divergences vanish. In fact, take for a zeroth approximation, instead of $\bar{\rho}$, a solution of the kinetic equation with velocity-independent but nonzero diffusion and friction coefficients (thus approximately accounting for particle collisions). In this model, in contrast to Eq. (22), the collision integral has the simpler form

$$S[\rho]=\nu\mathscr{L}^0\rho,$$

$$\mathscr{L}^0=\frac{1}{2\xi^2}\left(\frac{1}{\sin\vartheta}\frac{\partial}{\partial\vartheta}\sin\vartheta\frac{\partial}{\partial\vartheta}+\frac{1}{\sin^2\vartheta}\frac{\partial^2}{\partial\varphi^2}+\frac{\partial}{\partial\xi}\xi^2\left(\frac{\partial}{\partial\xi}+2\xi\right)\right). \tag{32}$$

A quantum kinetic equation with such a collision term can be solved exactly to give

$$\rho^0(\xi)=\int d^3\xi'\hat{\mathscr{G}}^0(\xi,\xi')\frac{-iGN_{mn}}{kv_T(\sqrt{\pi}v_T)^3}\exp(-\xi'^2),$$

$$\hat{\mathscr{G}}^0(\xi,\xi')=\int_0^\infty\frac{d\tau e^{iz\tau}}{[(1-e^{-2\eta\tau})\pi]^{3/2}}\exp\left(-\frac{(\xi-\xi'e^{-\eta\tau})^2}{(1-e^{-2\eta\tau})}+\frac{i\mathbf{k}}{\eta k}\frac{(\xi'-\xi e^{-\eta\tau})}{(1+e^{-\eta\tau})}-\frac{(\eta\tau/2-\tanh(\eta\tau/2))}{\eta^2}\right),$$

$$\eta=\frac{\nu}{kv_T}. \tag{33}$$

If in Eq. (15) we make the substitution $\rho=\rho^0+\Delta\rho$, we obtain the equation for $\Delta\rho$, which differs from (15) only on the right-hand side,

$$(\Gamma-i\Omega+i\mathbf{kv})\Delta\rho-\nu\mathscr{L}\Delta\rho=\nu(\mathscr{L}-\mathscr{L}^0)\rho^0, \tag{34}$$

and hence $\Delta\rho$ can again be found in an approximate way. Repeating the above procedure we obtain the solution in series form:

$$\rho(\xi)=\sum_{n=0}^{\infty}\eta^n\rho^{(n)}(\xi), \tag{35}$$

$$\rho^{(n+1)}(\xi)=\int d^3\xi'\hat{\mathscr{G}}^0(\xi,\xi')(\mathscr{L}-\mathscr{L}^0)\rho^{(n)}(\xi'). \tag{36}$$

The terms in this expansion remain finite as $\Gamma\to 0$, and the corrections to the work of the field, to first order in $\eta=\nu/kv_T$, are identical to Eqs. (30) and (31).

4. LONG-WAVELENGTH APPROXIMATION

In the long wavelength limit $kv_T \ll \nu$, $\eta \gg 1$, it is convenient to expand the distribution function in terms of Legendre polynomials in a coordinate system with z axis along the vector $\mathbf{k}$. Thus

$$\rho(\xi)=\sum_{l=0}^{\infty}\left(\frac{ikv_T}{\nu}\right)^l Y_{l0}(\cos\vartheta)\times R_l(\xi)\frac{-iGN_{mn}}{\nu v_T^3}\exp\left(-\frac{\xi^2}{2}\right),$$

$$Y_{l0}(x)=\left(\frac{2l+1}{4\pi}\right)^{1/2}P_l(x). \tag{37}$$

Then Eq. (15) reduces to a system of ordinary differential equations

$$\mathscr{H}^l R_l(\xi)+\frac{\xi}{\sqrt{2l+1}}\left(\frac{lR_{l-1}(\xi)}{\sqrt{2l-1}}-\left(\frac{kv_T}{\nu}\right)^2\frac{(l+1)R_{l+1}(\xi)}{\sqrt{2l+3}}\right)=\delta_{l0}\frac{2}{\pi}\exp\left(-\frac{\xi^2}{2}\right).$$

$$\mathscr{H}^l=\frac{\Gamma-i\Omega}{\nu}-\frac{1}{2\xi^2}\left(\left(\frac{d}{d\xi}-\xi\right)\xi^2\Phi_l\left(\frac{d}{d\xi}+\xi\right)-l(l+1)\Phi_{tr}\right),\quad l=0,1,2,\dots .$$

$$R_l(\infty)=0,\quad R_l(\xi)|_{\xi\to 0}\sim\xi^l. \tag{38}$$

In this system, equations for different l are coupled by terms that are small with respect to the parameter kv_T/ν. Thus, the contribution to the equation for $R_l(\xi)$ from the term in $R_{l+1}(\xi)$ has a smallness of order kv_T/ν. To leading order, the only nonzero term is

$$R_0(\xi)=\frac{2}{\pi}\frac{\nu}{\Gamma-i\Omega}\exp\left(-\frac{\xi^2}{2}\right), \tag{38'}$$

and the velocity distribution function becomes Maxwellian. (Since the collision term is exactly zero for the Maxwell distribution, the expression $\mathscr{H}^0R_0(\xi)=((\Gamma-i\Omega)/\nu)R_0(\xi)$ acquires an additional smallness when $|\Gamma-i\Omega|\ll\nu$, so for the zeroth approximation to be applicable, Eq. (38) for $l=0$ indicates that $(kv_T/\nu)^2$ must be small not only compared to 1 but to $|\Gamma-i\Omega|/\nu$ as well.) To first order in kv_T/ν, or for $kv_T \gg \sqrt{|\Gamma-i\Omega|\nu}$, one needs to retain only the first two terms in the expansion (37), with $l=0$ and $l=1$. As a result, we are left with a system of two differential equations.

The differential equations (38) can be reduced to an infinite system of algebraic equations for the coefficients in the expansion of $R^l(\xi)$ over a basis of orthogonal functions; for these, it is convenient to take Laguerre polynomials $R_n^l(\xi)$, which are the eigenfunctions for the problem with a velocity independent collision frequency for $\mathbf{k}=0$. We have

$$R^l(\xi)=\sum_{n=0}^{\infty}\frac{1}{\pi^{3/4}}a_n^l R_n^l(\xi),$$

$$R_n^l(\xi)=A_n^l\xi^l\exp\left(-\frac{\xi^2}{2}\right)L_n^{l+1/2}(\xi^2),$$

$$A_n^l=[n!2/\Gamma(n+l+3/2)]^{1/2}, \tag{39}$$

$$L_n^{l+1/2}(x)=\frac{e^x}{n!x^{l+1/2}}\frac{d^n}{dx^n}[x^{n+l+1/2}e^{-x}].$$

The Laguerre functions satisfy the boundary conditions and, for l's equal, are orthogonal on the real half-axis,

$$\int_0^\infty \xi^2 d\xi R_n^l(\xi)R_{n'}^l(\xi)=\delta_{nn'}. \tag{39'}$$

If we multiply the lth equation in the system (38) by $R_n^l(\xi)$ and integrate with respect to ξ we obtain

$$\sum_{n'=0}^{\infty}H_{nn'}^l a_{n'}^l-\left(\frac{l+1}{\eta^2}\frac{\sqrt{n+l+3/2}a_n^{l+1}+\sqrt{n}a_{n-1}^{l+1}}{\sqrt{(2l+1)(2l+3)}}-l\frac{\sqrt{n+1+1/2}a_n^{l-1}+\sqrt{n+1}a_{n+1}^{l-1}}{\sqrt{(2l+1)(2l-1)}}\right)=\delta_{n0}\delta_{l0}, \tag{40}$$

where the matrix element

$$H_{nn'}^l=\int_0^\infty \xi^2 d\xi R_n^l(\xi)\mathscr{H}^l R_{n'}^l(\xi) \tag{41}$$

can be expressed in terms of Γ functions of a half-integer argument. The absorbed power depends only on the coefficient a_0^0, namely,

$$\mathscr{P}(\Omega)=\mathscr{P}_0\frac{kv_T}{v\sqrt{\pi}}\operatorname{Re}a_0^0 \tag{42}$$

and we can find this coefficient in the limit $kv_T/\nu=1/\eta\ll 1$, to terms of order $1/\eta^2$. Since for the matrix elements we have $H_{n0}^0=H_{0n}^0=\delta_{n0}(\Gamma-i\Omega)/\nu$, the equation for a_0^0 involves the coefficient a_0^1 only with the factor $1/\eta^2$,

$$\frac{\Gamma-i\Omega}{\nu}a_0^0-\frac{1}{\eta^2}\sqrt{\frac{1}{2}}a_0^1=1. \tag{43}$$

Hence this coefficient can be obtained to zeroth order in $1/\eta$. In this case the system of equations (40) divides into independent subsystems with different l. All coefficients a_n^0 except a_0^0 are small in $1/\eta^2$, and for a_n^1 one obtains the equation

$$\sum_{n'=0}^{\infty}H_{nn'}^1 a_{n'}^1=\delta_{n0}\sqrt{\frac{1}{2}}a_0^0. \tag{44}$$

From this,

$$a_0^1=\lim_{M\to\infty}\frac{(H_{00}^1)_M^*}{\det\mathscr{H}_M^1}a_0^0, \tag{45}$$

where M is the number of terms retained in the series (39), $\mathscr{H}_M^1$ is the matrix that results in this case from (41), and $(H_{00}^1)_M^*$ is the algebraic complement to the element H_{00}^1 of the matrix $\mathscr{H}_M^1$. Substituting (45) into (43) we find

$$a_0^0=\frac{\nu}{\Gamma-i\Omega+\dfrac{(kv_T)^2}{2\lambda}},\quad \lambda=\nu\lim_{M\to\infty}\frac{\det\mathscr{H}_M^1}{(H_{00}^1)_M^*}. \tag{46}$$

TABLE I.

M	0	1	2	3	4	5	6	7
$\bar{\nu}/\nu$	0.7071	0.5992	0.5942	0.5942	0.5942	0.5942	0.5942	0.5942

Because of the rapid decrease of H^l_{mn} with $|n-m|$, one can calculate λ by approximating $R^{0,1}(\xi)$ by the first M terms of the series (39). If only the term with $n=0$ is kept,

$$H^0_{00}=\frac{\Gamma-i\Omega}{\nu},\quad \lambda=\nu H^1_{00}=\frac{\nu}{\sqrt{2}}+\Gamma-i\Omega,$$

$$\mathscr{P}(\Omega)=\mathscr{P}_0\frac{kv_T}{\sqrt{\pi}}\,\mathrm{Re}\left(\Gamma-i\Omega+\frac{(kv_T)^2}{2\left(\frac{\nu}{\sqrt{2}}+\Gamma-i\Omega\right)}\right)^{-1}. \quad (47)$$

At the line center $|\Gamma-i\Omega|\ll\nu$, the coefficient $\lambda\to\bar{\nu}=\nu/\sqrt{2}$ plays the role of the effective collision frequency. The value $\bar{\nu}=\nu/\sqrt{2}$ for the effective collision frequency was obtained by the frequency renormalization method in Ref. 9. This means that the renormalization of frequency is, in this problem, equivalent to retaining the leading term in the Laguerre polynomial expansion.

Retaining two terms in the expansion (39)

$$\lambda=\nu\left(H^1_{00}-\frac{(H^1_{01})^2}{H^1_{11}}\right)$$

$$=\frac{\nu}{\sqrt{2}}+\Gamma-i\Omega-\frac{9\nu^2}{(59\sqrt{2}\nu+80(\Gamma-i\Omega))},$$

$$\mathscr{P}(\Omega)=\mathscr{P}_0\frac{kv_T}{\sqrt{\pi}}\,\mathrm{Re}\left[\Gamma-i\Omega+\frac{(kv_T)^2}{2}\left(\frac{\nu}{\sqrt{2}}+\Gamma-i\Omega-\frac{9\nu^2}{59\sqrt{2}\nu+80(\Gamma-i\Omega)}\right)^{-1}\right]^{-1}. \quad (48)$$

At the line center $|\Gamma-i\Omega|\ll\nu$, the coefficient $\lambda\to\bar{\nu}=0.599\nu$. If the number M of terms retained in the expansion (39) is increased, the value of ν will decrease. However, starting with $M=2$, it remains virtually unchanged as illustrated in Table I. As is seen, at $M=1$ the value of $\bar{\nu}$ differs from its limiting value by as little as one percent.[4)] Thus, the formula (48) provides the contour of the spectral line in the limit $\nu\gg kv_T$. This contour is depicted in Fig. 2 for two values of the parameter ν/kv_T. The asymptotic behavior of the absorbed power at the line center is, in the limit $1/\eta\ll1$, $\Gamma\ll\nu$, of the form

$$\mathscr{P}=\mathscr{P}_0\frac{kv_T}{\sqrt{\pi}}\frac{1}{\Gamma+(kv_T)^2/2\bar{\nu}},\quad \bar{\nu}\cong0.594\nu. \quad (49)$$

In the far wing of the line ($|\Gamma-i\Omega|\gg1$), to terms of order $\nu/|\Gamma-i\Omega|$ the expression (46) takes the form

$$\lambda=\nu H^1_{00}+O(\nu/|\Gamma-i\Omega|)$$

$$=\Gamma-i\Omega+\frac{\nu}{\sqrt{2}}+O(\nu/|\Gamma-i\Omega|). \quad (49')$$

Substituting this into (42) and expanding in $1/\Omega$, we obtain for the work of the field the expression (31).

5. INTERMEDIATE CASE

The interaction with the electromagnetic field violates the isotropy of the distribution of ions in velocity space. If a particle has a nonzero velocity component along the electromagnetic wave vector, it acquires an additional phase of $\Delta\phi=\mathbf{k}\mathbf{v}_T$. Collisions change ion velocities in a random way, thus partially restoring the isotropy. For $\nu\sim kv_T$, in the low-velocity range $|v|\leqslant v_T$, the anisotropic portion of the distribution function is of the order of the isotropic portion, which suggests a rapid convergence of the Legendre polynomial expansion. As for the particles with large velocities $|v|\gg v_T$, for which the collision frequency decreases with increasing phase, their contribution to the work of the field is small because of the exponential decrease in the distribution function at high velocities.

The dependence of the work of the field at the line center on the ratio ν/kv_T has been determined numerically by using the system of equations obtained from (40) by truncating after finite n and l values. This was solved by the Gauss elimination method to give the value of a^0_0. Obtained in this way, a^0_0 represents the solution of the infinite system (40) only if it tends to some limit as n and l increase. It turned out that for $\nu/kv_T\geqslant0.2$, the coefficient a^0_0 ceases to

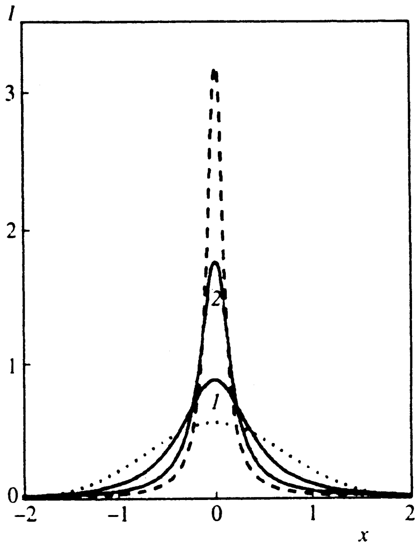

FIG. 2. Line contour $I(\Omega)$ in the frequent collision limit. $x=\Omega/kv_T$ denotes the Doppler-width normalized detuning from the resonance, $\Omega=\omega-\omega_{mn}$. Broken lines represent the narrow Lorentz absorption contour for a single particle (—) and the wide contour of the Doppler broadened line (.....) for $kv_T=10\Gamma$. Solid lines represent the contours given by Eq. (48) for $\nu=3kv_T$ (curve *1*) and $\nu=10kv_T$ (curve *2*).

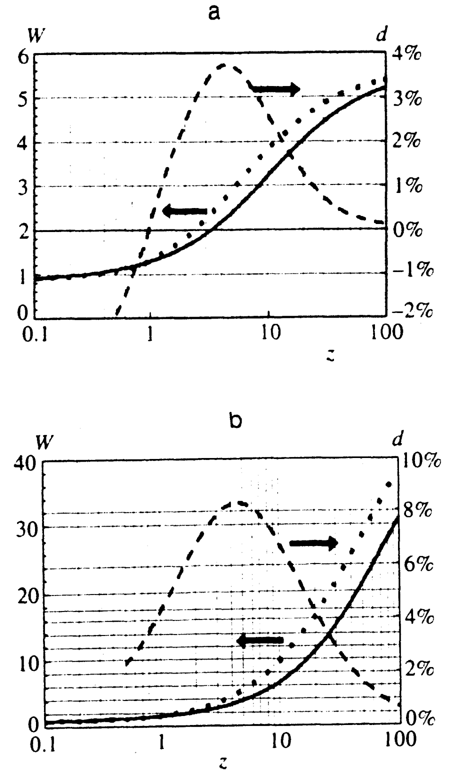

FIG. 3. The dependence of the normalized absorbed power $W=\mathscr{P}/\mathscr{P}_0$ at the line center ($\Omega=0$) upon the ratio of the collision frequency to the Doppler line width, $z=\nu/kv_T$; $\mathscr{P}$ being determined by Eq. (26). (—): numerical calculation; (-----): relative error d in the interpolation formula (50); (....): weak collision model. a) $kv_T=10\Gamma$; b) $kv_T=100\Gamma$.

change for $n\geqslant 4$, $l\geqslant 6$. Results are shown in Fig. 3 (solid line). They agree well with those obtained analytically for the limiting cases of long and short wavelengths.

The asymptotic expressions (30) and (49) for the absorbed power at the line center have the same dependence on ν, kv_T, and Γ as in the weak collision model,[2] but the coefficients are different.[5)] A comparison of the models is clearly illustrated by Fig. 3, which shows results for the weak collision model (points) with a velocity independent collision frequency. As is seen from the figure, the falling off of the collision frequency with increasing velocity reduces the effect by 40% compared to that obtained in the limit $\nu \gg kv_T \gg \sqrt{\Gamma\nu}$. In the short-wavelength limit the effect increases by 60% [see Eq. (30)].

We conclude by reproducing a simple interpolation formula for the absorbed power at the line center,

$$\mathscr{P}=\mathscr{P}_0\frac{kv_T}{\sqrt{\pi}}\frac{1}{\Gamma+\dfrac{(kv_T)^2}{2\bar{\nu}+\sqrt{\pi}kv_T}},\quad \bar{\nu}=0.59\nu, \tag{50}$$

which goes over into the asymptotic form (49) when $kv_T\ll\nu$. In the limiting case $kv_T\gg\nu$, this formula differs from the asymptotic expression only in the coefficient of the small correction, namely, $\bar{\nu}/\nu=0.59$ instead of 0.53 as in Eq. (30):

$$\mathscr{P}=\begin{cases}\mathscr{P}_0\dfrac{kv_T}{\sqrt{\pi}}\dfrac{1}{\Gamma+\dfrac{(kv_T)^2}{2\bar{\nu}}} & \text{for } kv_T\ll\nu,\\[2ex] \mathscr{P}_0\left(1+\dfrac{2\bar{\nu}}{\sqrt{\pi}kv_T}\right) & \text{for } kv_T\gg\nu.\end{cases} \tag{50'}$$

Also shown in Fig. 3 is the error in the interpolation formula (dashed line). It is seen that even in the intermediate case $kv_T\sim\nu$ the error does not exceed 10%, showing the interpolation formula to be adequate for any interrelation between the parameters ν and kv_T.

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[1)] In Ref. 5 it is shown that for a mixture of atomic or molecular gases, the Fokker–Planck equation with a constant collision frequency also has a limited region of applicability. The model can only be employed for an extremely heavy excited particle, with an atomic weight $\gtrsim 200$, even with hydrogen as a buffer gas.

[2)] A number of recent experiments involving the detection of the Dicke effect in atomic[7] and molecular[8] spectra did include the measurement of the correlation function.

[3)] In the case of a strongly nonisothermal plasma it follows from Eq. (12) that there appears an additional term in the kernel (13), which reflects dynamic polarization effects in the plasma. The spectroscopic implications of this term were considered in Ref. 27.

[4)] The sharp increase in accuracy due to the use of two Laguerre polynomials, and the slow reduction in error despite a further increase of number M, have been noticed by workers on the theory of transport in a plasma.[28]

[5)] In Ref. 29 it is shown that independent of the particular form of the collision integral, the absorbed power expression has the form of a Lorentz line contour $(kv_T)^2/\mu$, where μ^{-1} is the sum of the inverse eigenvalues of the collision operator.

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