Nonstationary electron density perturbation in a weakly collisional plasma

A. V. Maksimov and V. P. Silin

Lebedev Physical Institute, Russian Academy of Sciences, 117924 Moscow, Russia
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A nonstationary solution of the electronic kinetic equation for a weakly collisional fully ionized plasma is obtained within an approach leading to an asymptotic expansion in fractional negative powers of a large Knudsen parameter. The change in electron density due to the nonlocal effect of a nonuniform high-frequency field, to a potential electric field, and to the velocity of the ion motion is found.

1. In experiments involving the interaction of a high-power laser pulse with a solid target, it has long been found that under conditions generally viewed as collision-free for a plasma, instead of the limiting (Knudsen regime) value of electron heat flow density, one and sometimes two orders of magnitude lower heat flow density is realized.[1,2,3] The first explanations of this situation involved the assumption of ion-sound turbulence (see, e.g., Ref. 4). However, conditions for the existence of turbulence are not always realizable and so the disagreement between the kinetic theory and experiment persisted.

Numerical experiments using the Landau collision integral indicated the possibility of a several-fold reduction of electron heat flow from the Knudsen limit.[5,6] It should be noted that this numerical prediction also had no physical explanation for quite a long time.

Simultaneously with developments in numerical modeling, an attempt was made to formulate some analytical ideas concerning the description of weak-collision heat transfer. In this attempt, as in the Knudsen collisionless limit, transport was treated nonlocally, and the particular model used relied on the extrapolation, via the Padé approximation, of the strong-collisional transport results.[7,8] Underlying the model was a result of a Hilbert–Chapman–Enskog-based theory corresponding to the fact that in the strong-collisional limit, higher-order corrections correspond to expanding in even powers of the Knudsen number, this latter being the ratio of the electron mean free path to the spatial inhomogeneity scale of the electron distribution. It should be emphasized that in a fully ionized plasma there are two electron mean free paths involved, namely, the electron–ion one, l_{ei}, and the electron–electron one, l_{ee}. As shown in Ref. 9, for a plasma with a high degree of ionization ($Z \gg 1$), this leads to peculiar effects associated with the change in the velocity-symmetric part of the electron distribution function.

When speaking of the expansions involved in the Hilbert–Chapman–Enskog method used in the theory of a strongly collisional plasma, one may relate them to Maclaurin series in positive integral powers of the Knudsen number. In contrast, in the theory of highly rarefied gases, the occurrence of expansions in negative integral powers of the Knudsen number may be connected with Laurent series. As opposed to this conventional view, it may be argued that in the current kinetic theory of a plasma, a qualitatively new situation has arisen. To begin with, the development of numerical experiment[10,11,12] cast doubt on the Padé approximation approach of Refs. 7 and 8, and provided evidence to show that a new—efficient—Padé approximation corresponds to those asymptotic Knudsen number expansions of Refs. 7 and 8 which are not in even powers. However, the publications based on the numerical solution of the Boltzmann kinetic equation with the Landau collision integral exhibit an obvious divergence in the numerical experiment results presented. Thus, to Refs. 10, 11, and 12 there correspond expansions, respectively, in integer (including odd) powers; in powers of $(4/3)n$, where n is integer; and in powers of $1.44n$. All these undoubtedly interesting results are also a clear indication that an urgent need has arisen for the development of a new analytical approach with the potential to reveal a qualitatively new asymptotic region in the description of collision phenomena in plasma. Such an approach should be qualitatively different both from the Hilbert–Chapman–Enskog method and from that involved in the conventional theory of weakly rarefied gases.

Today it is already possible to call Ref. 13 the origin of the new analytic approach required. This work, concerned with the theory of laser radiation filamentation in a plasma, yields a new, stationary, analytic solution to the kinetic equation with the Landau collision integral, indicative of the existence of an expansion in terms of the fractional negative powers of a large Knudsen parameter $(Kn)^{-(2l/7)}$; where, for example, in writing the Fourier transform of the nonlocal thermal conductivity it turns out that $l=5$ (Ref. 13):

$$\varkappa_{\rm eff}=\varkappa_{SH}[1+(21,1k\lambda_e)^{10/7}]^{-1}. \tag{1.1}$$

Here $\lambda_e=2^{1/2}l_{ei}Z/3\pi^{1/2}(Z+1)^{1/2}$, Z is the degree of ionization, $l_{ei}=3m_e^2v_{Te}^4/4(2\pi)^{1/2}e^2e_i^2n_i\Lambda$ is the electron mean free path for collisions with ions, k is the "wave vector," and $\varkappa_{SH}$ the Spitzer–Härm thermal conductivity coefficient. Reference 14 shows that Eq. (1.1) leads to a nonlocal kernel in the expression for the heat flow density,

$$q(r)=-\int dr'K(r-r')(\partial\delta T_e(r')/\partial r'), \tag{1.2}$$

the three-dimensional form of the kernel being

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$$K(r)=\frac{0.07\gamma_{ei}\varkappa_B n_e f_{ei}^{4/7}}{Z^{5/7}r^{11/7}}. \tag{1.3}$$

Finally, in Ref. 15 it is shown how the relation (1.1) leads, in one dimension, to a heat transfer restriction coefficient of the form

$$q=fn_e v_{Te}\varkappa_B T_e. \tag{1.4}$$

It should be emphasized here that the results of Refs. 13, 14, and 15 are based on the stationary solution of the electron kinetic equation for the particular problem concerned with the influence exerted on a plasma by a high-frequency heating radiation. The present work takes one further step in the development of a nonlocal theory of a weakly collisional plasma. Specifically, we will obtain a nonstationary solution of the electron kinetic equation for conditions under which a fractional asymptotic expansion for the electron density perturbation is realized. As in Ref. 13, a high-frequency electromagnetic field is assumed to act on the plasma. However, unlike Ref. 13, we also consistently take into account the presence of a potential electric field $\delta\varphi$ as well as the fact that the ions move with a finite average velocity $\mathbf{u}_i$ relative to the electrons. The resulting solution of the electron kinetic equation will be employed to derive the electron density perturbation. The present work does not discuss possible applications of this theoretical result. However, some applications of this kind are obvious in the light of the use the fractional asymptotic expansion results find in Refs. 13 and 16, these being the pioneering efforts to utilize the analytic theory of nonlocal transport in the analysis of the parametric instabilities in a weakly collisional plasma.

2. For a description of the low frequency perturbations of the electron distribution function caused by an electromagnetic field with an electric component

$$\mathscr{E}=\tfrac{1}{2}\ Ee^{-i\omega_0 t}+\text{c.c.} \tag{2.1}$$

the (slowly changing in time) electron distribution function will be represented in the form

$$f=f_m+\delta f, \tag{2.2}$$

where f_m is the Maxwell distribution function, δf is a small perturbation, and ω_0 the frequency of the rapidly varying high-frequency field. Suppose that δf, the quadratic combination $E_iE_j^*+E_i^*E_j$ of the amplitudes of the hf field, and the perturbation $\delta\varphi$ of the slowly varying (in time) electric field, all vary with the time and coordinates as

$$\exp(i\mathbf{kr}-i\omega t), \tag{2.3}$$

where the relatively low frequency ω characterizes the low-frequency perturbation. Then, in accordance with Ref. 9, we can write for δf the following linear equation:

$$-i(\omega-\mathbf{kv})\delta f+i\frac{e\delta\varphi\mathbf{kv}}{\varkappa_B T_e}f_m-J_{ei}[\delta f]-J_{ee}[\delta f]$$
$$=\frac{e^2}{4m_e^2\omega_0^2v_{Te}^2}\left\{-i\mathbf{kv}\left(2-\frac{v^2}{3v_{Te}^2}\right)f_m\left|\mathbf{E}\right|^2\right.$$
$$-(2\pi)^{1/2}\nu_{ei}v_{Te}^3|\mathbf{E}|^2\frac{\partial}{\partial v_s}\left(\frac{v_s}{v^3}f_m\right)+\left(E_i^*E_j\right.$$
$$\left.+E_iE_j^*-\frac{2}{3}\delta_{ij}|\mathbf{E}|^2\right)\left(v_iv_j-\frac{1}{3}v^2\delta_{ij}\right)\left(\frac{i\mathbf{kv}}{2v_{Te}^2}\right.$$
$$\left.\left.+\frac{3\pi^{1/2}\nu_{ei}v_{Te}^3}{2^{1/2}v^5}\left(3+\frac{v^2}{v_{Te}^2}\right)\right)\right\}. \tag{2.4}$$

Here $v_{Te}=(\varkappa_BT_e/m_e)^{1/2}$ is the electron thermal velocity, $\nu_{ei}=2^{5/2}\pi^{1/2}3^{-1}e^2e_i^2n_i\Lambda m_e^{-2}v_{Te}^{-3}$ is the electron–electron collision frequency, and the electron–electron and electron–ion collision integrals are of the form

$$J_{ei}[\delta f]=\nu(v)\left\{\frac{\partial}{\partial v_r}\left[v^2\delta_{rs}-v_rv_s\right]\frac{\partial\delta f}{\partial v_s}+\frac{2f_m(\mathbf{u}_i\mathbf{v})}{v_{Te}^2}\right\}, \tag{2.5}$$

$$J_{ee}[\delta f]=\frac{3(2\pi)^{1/2}\nu_{ei}v_{Te}^3}{4Zn_e}\frac{\partial}{\partial v_r}$$
$$\times\left\{\int d\mathbf{v}'\frac{(\mathbf{v}-\mathbf{v}')^2\delta_{rs}-(\mathbf{v}-\mathbf{v}')_r(\mathbf{v}-\mathbf{v}')_s}{|v-v'|^3}\right.$$
$$\times\left(\frac{\partial}{\partial v_s}-\frac{\partial}{\partial v_s'}\right)[f_m(v)\delta f(\mathbf{v}')$$
$$\left.+f_m(v')\delta f(\mathbf{v})]\right\}. \tag{2.6}$$

Here $\nu(v)=3\pi^{1/2}2^{-3/2}\nu_{ei}v_{Te}^3v^{-3}$, the ion distribution function is represented in the form $f_i=f_{mi}+\delta f_i$ (where f_{mi} is the Maxwell distribution function and δf_i a small correction), and $u_i\exp(i\mathbf{kr}-i\omega t)=n_i^{-1}\int d\mathbf{v}_i\mathbf{v}_i\delta f_i$ is the average ion velocity. For our purposes one can neglect the transfer of energy in electron–ion collisions and employ formula (2.5).

Now let us represent the small perturbation δf of the distribution function as a sum of terms independent of the collision frequency ν_{ei} and of the perturbation frequency ω, plus an additional term δf_c:

$$\delta f=f_m\left[\left(\frac{v^2}{3v_{Te}^2}-2\right)I-\frac{e\delta\varphi}{\varkappa_BT_e}\right]+\frac{1}{2}OZf_m+\frac{\mathbf{u}_i\mathbf{v}}{v_{Te}^2}f_m+\delta f_c, \tag{2.7}$$

where we have introduced the notation

$$I=\frac{e^2|\mathbf{E}|^2}{4m_e^2\omega_0^2v_{Te}^2},$$

$$OZ=\frac{e^2(v_iv_j-(1/3)v^2\delta_{ij})}{4m_e^2\omega_0^2v_{Te}^4}\left(E_iE_j^*+E_i^*E_j-\tfrac{2}{3}\delta_{ij}|\mathbf{E}|^2\right). \tag{2.7'}$$

Since we are interested in the case of a high degree of ionization Z, when $Z=|e_i/e|\gg 1$, the electron–ion collision integral will be assumed to be much larger than its electron–electron counterpart when considering the angular relaxation of the electron velocity. Therefore in what follows we neglect the influence of electron–electron collisions on the angular anisotropy of the electron distribution function. Then from Eq. (2.4) we have, after substituting (2.7),

$$-i(\omega-\mathbf{kv})\delta f_c-i(\omega-\mathbf{kv})(\mathbf{u}_i\mathbf{v})v_{Te}^{-2}f_m-J_{ei}[\delta f_c]-J_{ee}[\delta f_c]$$
$$=\nu_{ei}I\left[-(2\pi)^{1/2}v_{Te}^3\frac{\partial}{\partial v_s}\left(\frac{v_s}{v^3}f_m\right)+\frac{i\omega}{\nu_{ei}}\left(\frac{v^2}{3v_{Te}^2}-2\right)f_m\right]-\frac{i\omega e\delta\varphi f_m}{\varkappa_BT_e}+\nu_{ei}OZ\left[\left(3-\frac{v^2}{2v_{Te}^2}\right)\frac{3\pi^{1/2}v_{Te}^5}{2^{1/2}v^5}+\frac{i\omega}{2\nu_{ei}}\right]f_m. \quad (2.8)$$

A solution to this equation will be sought assuming the condition

$$kv\ll|\nu(v)+i\omega|, \quad (2.9)$$

which, as shown in Ref. 13, may be fulfilled for perturbation wavelengths $\lambda=k^{-1}$ less than the electron mean free path l_{ei} if the characteristic electron velocities dominating the plasma perturbation are much less than the thermal one. Let $\delta f_c=\delta f_0+\delta f_a$, where $\delta f_0=\langle\delta f_c\rangle$ is the angle-averaged—and hence isotropic—part of the perturbation δf_c and $\delta f_a=\delta f_c-\langle\delta f_c\rangle$ is the anisotropic part of the perturbation δf_c. Then averaging (2.8) over the angles yields

$$-i\omega\delta f_0+i\langle\mathbf{kv}\delta f_a\rangle-J_{ee}[\delta f_0]+i\mathbf{ku}_i\frac{v^2}{3v_{Te}^2}f_m$$
$$=\nu_{ei}I\left\{-(2\pi)^{1/2}v_{Te}^3\frac{\partial}{\partial v_s}\left(\frac{v_sf_m}{v^3}\right)+\frac{i\omega}{\nu_{ei}}\left(\frac{v^2}{3v_{Te}^2}-2\right)f_m\right\}-\frac{i\omega e\delta\varphi}{\varkappa_BT_e}f_m. \quad (2.10)$$

Subtracting this from (2.8) and neglecting the combination $i\mathbf{kv}\delta f_a-i\langle\mathbf{kv}\delta f_a\rangle$ compared with $J_{ei}[\delta f_a]+i\omega\delta f_a$ [corresponding to the condition (2.9)] and also neglecting $J_{ee}[\delta f_a]$ compared with $J_{ei}[\delta f_a]$ [corresponding to $Z\gg 1$], we obtain the second equation:

$$-i\omega\delta f_a+i\mathbf{kv}\delta f_0$$
$$=J_{ei}[\delta f_a]+\frac{i\omega\mathbf{u}_i\mathbf{v}}{v_{Te}^2}f_m+\nu_{ei}OZ\left[\left(3-\frac{v^2}{2v_{Te}^2}\right)\times\frac{3\pi^{1/2}v_{Te}^5}{2^{1/2}v^5}+\frac{i\omega}{2\nu_{ei}}\right]f_m-ik_\alpha u_{i\beta}\left(v_\alpha v_\beta-\frac{1}{3}v^2\delta_{\alpha\beta}\right)\frac{f_m}{v_{Te}^2}. \quad (2.11)$$

This last equation has the following explicit exact solution:

$$\delta f_a=-i(\mathbf{kv}\delta f_0-\omega\mathbf{u}_i\mathbf{v}v_{Te}^{-2}f_m)[2\nu(v)-i\omega]^{-1}+\{OZ([(3v_{Te}^2/v^2)-(1/2)]2\nu(v)+(i\omega/2))-ik_\alpha u_{i\beta}v_{Te}^{-2}(v_\alpha v_\beta-(1/3)v^2\delta_{\alpha\beta})\}[6\nu(v)-i\omega]^{-1}. \quad (2.12)$$

Thus the problem of determining the electron distribution perturbation is reduced to the solution of an equation for the isotropic function δf_0,

$$\left\{-i\omega+\frac{k^2v^2}{3[-i\omega+2\nu(v)]}\right\}\delta f_0-J_{ee}[\delta f_0]$$
$$=I\left\{-(2\pi)^{1/2}\nu_{ei}v_{Te}^3\frac{\partial}{\partial v_s}\left(\frac{v_sf_m}{v^3}\right)+i\omega\left(\frac{v^2}{3v_{Te}^2}-2\right)f_m\right\}-\frac{i\omega e\delta\varphi}{\varkappa_BT_e}f_m+\frac{\omega\mathbf{ku}_iv^2f_m}{3v_{Te}^2[-i\omega+2\nu(v)]}-\frac{i\mathbf{ku}_iv^2f_m}{3v_{Te}^2}-\frac{4k^2v^4If_m}{45v_{Te}^2[-i\omega+2\nu(v)][-i\omega+6\nu(v)]}\times\left[\nu(v)\left(\frac{6v_{Te}^2}{v^2}-1\right)+i\omega\right]. \quad (2.13)$$

In the following we take into account that $(\partial/\partial\mathbf{v})(\mathbf{v}v^{-3})=4\pi\delta(\mathbf{v})$, where $\delta(\mathbf{v})$ is the delta function.

3. In this section the perturbation of the electron density will be determined, which requires the solution of Eq. (2.13) for δf_0, the symmetric correction to the distribution function. We will consider the low-frequency limit, assuming

$$\omega\ll\nu(v), \quad (3.1)$$

whereupon the condition (2.9) takes the form

$$kv\ll\nu(v). \quad (3.2)$$

For $kv_{Te}\gg\nu_{ei}$, this inequality can only be realized when the perturbed nonequilibrium electron distribution is dominated by electrons whose velocities are much less than thermal. With this indication of the importance of low electron velocities—and so recognizing the hope for obtaining solutions for sufficiently large values of k [see condition (3.2)]—it should be noted that care must also be taken to track the contribution from *large* velocities. In fact, it is only when this contribution is negligible that we may hope to construct a satisfactory theory.

In view of inequality (3.1), for the function $F(x)$ defined by

$$\delta f_0(v)=\frac{9\pi^{1/2}}{8k^2l_{ei}^2}f_m(v)F\left(\frac{v^2}{2v_{Te}^2}\right)\equiv\frac{Z}{2N}f_mF(x), \quad (3.3)$$

where $l_{ei}=v_{Te}\nu_{ei}^{-1}$ and

$$N=\frac{4}{9\pi^{1/2}}Zk^2l_{ei}^2, \quad (3.4)$$

the following integro-differential equation can be written according to (2.13):

$$\frac{1}{N}L[F(x)]-x^3F(x)\left[1-\frac{i\omega Z}{2\nu_{ei}Nx^{5/2}}\right]$$
$$=\left[\pi^{1/2}\left[-1+\delta(x)+\frac{2i\omega x^{1/2}}{\pi^{1/2}\nu_{ei}}\left(1-\frac{x}{3}\right)\right]\right.$$
$$\left.+\frac{i\omega e\delta\varphi}{\nu_{ei}\varkappa_BT_e}x^{1/2}+\frac{2i\mathbf{ku}_i}{3\nu_{ei}}x^{3/2}.\right. \qquad (3.5)$$

Here we have taken into account the relation $\delta(\mathbf{r})=(4\pi r^2)^{-1}\delta(r)$ for the δ-function and used the notation

$$L[F(x)]=F''(x)\tfrac{3}{2}\int_0^x dyy^{1/2}e^{-y}+\left[\frac{3}{2}x^{1/2}-x^{3/2}\right]$$
$$\times\int_x^\infty dye^{-y}[F'(x)-F'(y)]$$
$$-\int_0^x dyy^{3/2}e^{-y}[F'(x)-F'(y)]. \qquad (3.6)$$

A solution to Eq. (3.5) will be sought in the approximation where

$$N\gg 1. \qquad (3.7)$$

Moreover, assuming ω to be small, we represent the solution of Eq. (3.5) in the form

$$F(x)=IF_0(x)+I(\omega/\nu_{ei})F_1(x). \qquad (3.8)$$

In the solution of Eq. (3.5) we will restrict ourselves to terms linear in ω.

The equation for $F_0(x)$ has the form

$$\frac{1}{N}L[F_0(x)]-x^3F_0(x)=\pi^{1/2}[\delta(x)-1]. \qquad (3.9)$$

In the approximation (3.7) this equation was solved in Ref. 13 and its solution has the form

$$F_0(x)=-\left(\frac{2}{7}\right)^{1/7}\Gamma\left(\frac{6}{7}\right)\frac{4N^{8/7}}{\pi^{1/2}\xi^{1/4}}\sin\frac{\pi}{7}K_{1/7}\left(\frac{4}{7}\xi^{7/4}\right)$$
$$+N^{6/7}\Psi(\xi,[\pi^{1/2}])$$
$$\equiv\tilde{F}_0(\xi), \qquad (3.10)$$

where to make things compact we have used the functional

$$\Psi(\xi,[\varphi(\xi)])=\frac{4}{7}\xi^{-1/4}\left\{I_{1/7}\left(\frac{4}{7}\xi^{7/4}\right)\right.$$
$$\times\int_\xi^\infty d\xi\xi^{-1/4}\varphi(\xi)K_{1/7}\left(\frac{4}{7}\xi^{7/4}\right)$$
$$+K_{1/7}\left(\frac{4}{7}\xi^{7/4}\right)\int_0^\xi d\xi\xi^{-1/4}\varphi(\xi)$$
$$\left.\times I_{1/7}\left(\frac{4}{7}\xi^{7/4}\right)\right\}. \qquad (3.11)$$

Here $\xi=xN^{2/7}$, $\Gamma(z)$ is the Γ function, and $I_{1/7}(z)$ and $K_{1/7}(z)$ are modified Bessel functions. In the search for the function $F_1(x)$ we represent it in the form of a sum of four terms as

$$F_1(x)=I\left[2F_{1/2}(x)-\frac{2}{3}F_{3/2}(x)\right]+\frac{e\delta\varphi}{\varkappa_BT_e}F_1(x)$$
$$+\frac{2\mathbf{ku}_i}{3\omega}F_{3/2}(x)+\frac{ZI}{2N}\delta F(x), \qquad (3.12)$$

where for the three functions involved we have from Eq. (3.5) the following equations:

$$\frac{1}{N}L[F_{1/2}(x)]-x^3F_{1/2}(x)=x^{1/2}, \qquad (3.13)$$

$$\frac{1}{N}L[F_{3/2}(x)]-x^3F_{3/2}(x)=x^{3/2}, \qquad (3.14)$$

$$\frac{1}{N}L[\delta F(x)]-x^3\delta F(x)=-x^{1/2}F_0(x). \qquad (3.15)$$

The last of these, Eq. (3.15), is a result of iteration after replacing $F(x)$ by $F_0(x)$ in the term in $i\omega$ on the left-hand side of Eq. (3.5).

The solution procedures for Eqs. (3.13) and (3.14) are given in Appendix I, and that for Eq. (3.15) in Appendix II.

Note that the term containing $\mathbf{ku}_i$ in (3.12) provides a correct expression for the ion-velocity correction to the isotropic part of the distribution, not only in the low-velocity region, see Eq. (3.2), but also in the opposite limit when $kv\gg\nu(v)$. This is shown in Appendix III. Thus the $\mathbf{ku}_i$ term in (3.12) may be used to describe the ion-velocity correction for any value of electron velocity. The solutions of Eqs. (3.13)–(3.15) enable the electron density perturbation to be evaluated:

$$\delta n_e=\int d\mathbf{v}\delta f(v)=-n_eI-n_e\frac{e\delta\varphi}{\varkappa_BT_c}+\delta n_{ec}, \qquad (3.16)$$

where the relation (2.7) has been used and where from (3.3), (3.8), (3.12), and Appendices I and II,

$$\delta n_{ec}=\int d\mathbf{v}\delta f_0(v)$$
$$=-n_e\left\{I\left(\frac{\beta_0Z}{N^{2/7}}+\frac{i\omega}{\nu_{ei}}\left[\frac{\delta\beta Z^2}{N^{4/7}}+\frac{2\beta_{1/2}Z}{N^{5/7}}-\frac{\beta_{3/2}Z}{N}\right]\right)\right.$$
$$\left.+\frac{i\omega e\delta\varphi}{\nu_{ei}\varkappa_BT_e}\frac{\beta_{1/2}Z}{N^{5/7}}+\frac{i\mathbf{ku}_i}{\nu_{ei}}\frac{\beta_{3/2}Z}{N}\right\}. \qquad (3.17)$$

Here

$$\beta_0=-\frac{N^{-8/7}}{\pi^{1/2}}\int_0^\infty d\xi\xi^{1/2}\exp(-\xi N^{-2/7})\tilde{F}_0(\xi)$$
$$=\frac{4}{\pi}\left(\frac{2}{7}\right)^{1/7}\Gamma\left(\frac{6}{7}\right)\sin\frac{\pi}{7}\int_0^\infty d\xi\xi^{1/4}K_{1/7}\left(\frac{4}{7}\xi^{7/4}\right)=1,17;$$
$$(3.18)$$

$$\beta_{1/2}=-\frac{N^{-5/7}}{\pi^{1/2}}\int_0^\infty d\xi\xi^{1/2}\exp(-\xi N^{-2/7})\tilde{F}_{1/2}(\xi)$$

$$=\frac{8}{7\pi^{1/2}}\int_0^\infty d\xi\xi^{1/4}K_{1/7}\left(\frac{4}{7}\xi^{7/4}\right)$$

$$\times\int_0^\xi dyy^{1/4}I_{1/7}\left(\frac{4}{7}y^{7/4}\right)=0{,}56; \tag{3.19}$$

$$\beta_{3/2}=-\frac{2N^{-3/7}}{3\pi^{1/2}}\int_0^\infty d\xi\xi^{1/4}\exp(-\xi N^{-2/7})\tilde{F}_{3/2}(\xi)$$

$$=\frac{8}{21\pi^{1/2}}\int_0^\infty d\xi K_{1/7}\left(\frac{4}{7}\xi^{7/4}\right)\int_0^\xi dyI_{1/7}\left(\frac{4}{7}y^{7/4}\right)$$

$$\times[y^{5/4}\xi^{1/4}+\xi^{5/4}y^{1/4}]\exp(-\xi N^{-2/7})$$

$$=\frac{4\ln N}{21\pi^{1/2}}, \tag{3.20}$$

$$\delta\beta=-\frac{N^{-13/7}}{2\pi^{1/2}}\int_0^\infty d\xi\xi^{1/2}\exp(-\xi N^{2/7})\delta\tilde{F}(\xi)$$

$$=\frac{8}{7\pi}\left(\frac{2}{7}\right)^{1/7}\sin\frac{\pi}{7}\int_0^\infty d\xi K_{1/7}\left(\frac{4}{7}\xi^{7/4}\right)$$

$$\times\int_0^\xi dyI_{1/7}\left(\frac{4}{7}y^{7/4}\right)\left[\xi^{1/4}K_{1/7}\left(\frac{4}{7}y^{7/4}\right)\right.$$

$$\left.+y^{1/4}K_{1/7}\left(\frac{4}{7}\xi^{7/4}\right)\right]=1.9. \tag{3.21}$$

The estimation of the contribution to the integrals (3.18)–(3.21) from the region $\xi\gg\xi_0$, $N^{2/7}\gg\xi_0\gg1$, yields: a contribution $\sim N^{-2/7}\xi_0^{-3/2}\ll\beta_0\sim1$ to the integral (3.18); $\sim\xi_0^{-1}\ll\beta_{1/2}\sim1$ to (3.19); $\sim\ln(N^{2/7}/\xi_0)<\beta_{3/2}\sim\ln(N^{2/7})$ to (3.20); and $\sim\max(N^{-2/7}\xi^{-4},N^{-4/7}\xi_0^{-3/2})\ll\delta\beta\sim1$ to (3.21). We emphasize that $\exp(-\xi N^{-2/7})$ in the integrals (3.18), (3.19)–(3.21) may be replaced by unity because the main contribution is from the values $\xi\approx1$ or, equivalently from the velocities

$$v\sim v_{Te}N^{-1/7}. \tag{3.22}$$

This enables one to argue that the iteration leading to Eq. (3.15) implies the following restriction on the frequency ω

$$\omega\ll\nu_{ei}Z^{-1}N^{2/7}. \tag{3.23}$$

This also follows from (3.17) by comparison of the terms proportional to β_0 and $\delta\beta$.

For the integral on the right-hand side of Eq. (3.20), $\exp(-\xi N^{-2/7})$ cannot be replaced by unity because of its role in restricting contributions from velocities exceeding the electron thermal velocity ($v>v_{Te}$). The dominant contribution to the integral is given by the region $1\lesssim\xi\lesssim N^{2/7}$, with the consequence that $\beta_{3/2}\sim\ln(N^{2/7})$. This means, in particular, that the dependence of $u_i(t)$ on time should not be restricted by the inequality (3.23) but rather by a more stringent one,

$$\omega\ll\nu_{ei}N/Z. \tag{3.24}$$

However, one can also employ $F_{3/2}(x)$ in the region moderate x values. The proof is given in Appendix III.

Since the relation (3.1), with (3.22), reduces to the inequality $\omega\ll\nu_{ei}N^{3/7}$, it holds automatically under the condition (3.24), which may be regarded as the principal restriction our analysis imposes on the frequency ω.

As to the formula (3.17), we also note that in view of the condition (3.7) the term $\sim\beta_{3/2}I$ in it may be ignored. Finally, on account of (3.7) and especially noting that $Z\gg1$, the term $\sim I\beta_{1/2}$ may be neglected. Therefore we have

$$\delta n_e=-n_eI\left\{1+\frac{C_0Z^{5/7}}{(kl_{ei})^{4/7}}+\frac{i\omega}{\nu_{ei}}\frac{C_1Z^{10/7}}{(kl_{ei})^{8/7}}\right\}-n_e\frac{e\delta\varphi}{\varkappa_BT_e}$$
$$\times\left\{1+\frac{i\omega}{\nu_{ei}}\frac{C_2Z^{2/7}}{(kl_{ei})^{10/7}}\right\}-n_e\frac{i\mathbf{k}\mathbf{u}_i}{\nu_{ei}}\frac{C_3}{(kl_{ei})^{2}}, \tag{3.25}$$

where $C_0=(81\pi/16)^{1/7}\beta_0=1.73$ was obtained in Ref. 13 and

$$C_1=\left(\frac{81\pi}{16}\right)^{2/7}\delta\beta=4.2;$$

$$C_2=\left(\frac{81\pi}{16}\right)^{5/14}\beta_{1/2}=1.5;$$

$$C_3=\left(\frac{81\pi}{16}\right)^{1/2}\beta_{3/2}=\frac{3}{7}\ln N. \tag{3.25'}$$

To conclude this section it is appropriate to discuss the validity of the assumption (2.9) which, using (3.1), and for the velocities given by (3.22), takes the form

$$(kl_{ei})^{1/7}\gg Z^{-4/7}. \tag{3.26}$$

This inequality is satisfied if the conditions (3.7) and $Z\gg1$ are as well, which together with (3.24) are the principal conditions determining the applicability of Eq. (3.25) to the electron density perturbation.

4. To summarize, a discussion is given of the principles of a new approach to the nonstationary kinetic theory of a fully ionized plasma in the rare collision limit, when collision effects show up in the asymptotic expansion in negative fractional powers of the Knudsen parameter. A nonstationary solution of the kinetic equation is obtained. It is shown that for nonstationary phenomena, the electron density perturbation is controlled by variations in the symmetric part of the electron distribution function for electron velocities less than thermal. The peculiar new properties of the electron distribution obtained are due to the effect of the electron–electron collisions. One should emphasize the qualitative difference of our asymptotic approach from any one of the previous analytical approaches to the kinetic theory of a collisional plasma. Note that the formula (3.25) offers a possibility of a consistent theoretical description of the influence of electron collision effects on the collective excitations in a weakly collisional plasma and on the properties of parametric instabilities.

APPENDIX I

In this Appendix the solution to Eqs. (3.1) and (3.14) is obtained. We first note that owing to the condition (3.7), the asymptotic dependences for $x \gg 1$ follow from Eqs. (3.13) and (3.14) as

F1/2(x)=1x3 {−x1/2+C1/2N },

$$F_{3/2}(x) = \frac{1}{x^3}\left\{-x^{3/2} + \frac{C_{3/2}}{N}\right\}; \qquad \text{(AI.1)}$$

where

$$C_{1/2} = \int_0^\infty dy y^{3/2} e^{-y} F'_{1/2}(y),$$

$$C_{3/2} = \int_0^\infty dy y^{3/2} e^{-y} F'_{3/2}(y). \qquad \text{(AI.2)}$$

Now consider Eqs. (3.13) and (3.14) in the opposite limit $x \ll 1$. Neglect the integral terms in (3.13) and (3.14) assuming that in the region $x \ll 1$ the following inequalities hold:

$$\frac{1}{N}\left|\int_0^\infty dy y^{3/2} e^{-y} F'_{1/2}(y)\right| \ll A_{1/2}, \qquad \text{(AI.3)}$$

$$\frac{1}{N}\left|x^{1/2}\int_x^\infty dy e^{-y} F'_{1/2}(y)\right| \ll A_{1/2}, \qquad \text{(AI.4)}$$

$$\frac{1}{N}\left|\int_0^\infty dy y^{3/2} e^{-y} F'_{3/2}(y)\right| \ll A_{3/2}, \qquad \text{(AI.5)}$$

$$\frac{1}{N}\left|x^{1/2}\int_x^\infty dy e^{-y} F'_{3/2}(y)\right| \ll A_{3/2}, \qquad \text{(AI.6)}$$

where

$$A_{1/2} = \max\left\{\left|x^3 F_{1/2}(x)\right|; \left|\frac{1}{N} x^{1/2} F'_{1/2}(x)\right|\right\}, \qquad \text{(AI.7)}$$

$$A_{3/2} = \max\left\{\left|x^3 F_{3/2}(x)\right|; \left|\frac{1}{N} x^{1/2} F'_{3/2}(x)\right|\right\}. \qquad \text{(AI.8)}$$

Then for $x \ll 1$, Eqs. (3.13) and (3.14) take the following respective forms:

$$N^{-1}(x^{3/2} F'_{1/2}(x))' - x^3 F_{1/2}(x) = x^{1/2}, \qquad \text{(AI.9)}$$

$$N^{-1}(x^{3/2} F'_{3/2}(x))' - x^3 F_{3/2}(x) = x^{3/2}. \qquad \text{(AI.10)}$$

The solutions of these equations which are regular at infinity and zero can be written in the form

$$F_{1/2}(x) \equiv \tilde{F}_{1/2}(\xi) = -N^{5/7}\Psi(\xi,[\xi^{1/2}]), \qquad \text{(AI.11)}$$

$$F_{3/2}(x) \equiv \tilde{F}_{3/2}(\xi) = -N^{3/7}\Psi(\xi,[\xi^{3/2}]), \qquad \text{(AI.12)}$$

where the functional $\Psi(\xi,[\varphi])$ is given by (3.11), and $\xi = xN^{2/7}$. In the region $\xi \gg 1$ the asymptotic forms of (AI.11) and (AI.12) are identical to the corresponding expressions (AI.1) with $C_{1/2}$ and $C_{3/2}$ neglected. Expressions (AI.11) and (AI.12), when substituted into (AI.2), give

$$C_{1/2} = N^{2/7}\int_0^\infty d\xi \xi^{3/2} d\Psi(\xi,[\xi^{1/2}])/d\xi \sim N^{2/7}, \qquad \text{(AI.12}')$$

$$C_{3/2} = \int_0^\infty d\xi \xi^{3/2} d\Psi(\xi,[\xi^{3/2}])/d\xi \sim N^0. \qquad \text{(AI.12}'')$$

This means that for $x \gg 1$, $C_{1/2}$ and $C_{3/2}$ can be neglected in formulas (AI.1). Since for this very reason the asymptotic $N \gg 1$ forms of the solutions (AI.11) and (AI.12) are identical with the asymptotics of (AI.1), it follows that these solutions are applicable to any x value.

One needs to check the validity of the conditions (AI.3)–(AI.6), however. To carry out this check, for $x \ll 1$ we shall start with the region $N^{2/7} \gg \xi \gg 1$ in which, according to (AI.11) and (AI.12), we have

$$F_{1/2}(x) = -x^{-5/2}, \quad F_{3/2}(x) = -x^{-3/2}. \qquad \text{(AI.13)}$$

These expressions give the respective forms

$$A_{1/2} = x^{1/2}, \quad A_{3/2} = x^{3/2}. \qquad \text{(AI.14)}$$

Equations (AI.13) yield for the left-hand sides of the inequalities (AI.3) and (AI.5) the estimates $N^{-5/7}$ and $N^{-1}\ln N$, respectively. Comparison of these estimates with expressions (AI.14) enables one to infer the validity of the inequalities (AI.3) and (AI.5) in the region $N^{2/7} \gg \xi \gg 1$. Also satisfied in this region are the inequalities (AI.4) and (AI.6), because Eqs. (AI.13) yield for the left-hand sides of them the respective estimates $N^{-1}x^{-2}$ and $N^{-1}x^{-1}$. We turn next to the region $\xi \ll 1$, in which by (AI.10) and (AI.11) we have

$$F_{1/2}(x) = -N^{5/7}[\alpha_{1/2} - \tfrac{2}{3} x N^{2/7}], \qquad \text{(AI.15)}$$

$$F_{3/2}(x) = -N^{3/7}[\alpha_{3/2} - \tfrac{1}{5}(xN^{2/7})^2], \qquad \text{(AI.16)}$$

where

$$\alpha_{1/2} = \frac{2(2/7)^{8/7}}{\Gamma(8/7)}\int_0^\infty dy y^{1/4} K_{1/7}\left(\frac{4}{7} y^{7/4}\right) = 2\left(\frac{2}{7}\right)^{3/7}\Gamma\left(\frac{3}{7}\right)\Gamma\left(\frac{2}{7}\right)\left[\Gamma\left(\frac{1}{7}\right)\right]^{-1},$$

$$\alpha_{3/2} = \frac{2(2/7)^{8/7}}{\Gamma(8/7)}\int_0^\infty dy y^{5/4} K_{1/7}\left(\frac{4}{7} y^{7/4}\right) = 2\left(\frac{7}{2}\right)^{1/7}\Gamma\left(\frac{5}{7}\right)\Gamma\left(\frac{4}{7}\right)\left[\Gamma\left(\frac{1}{7}\right)\right]^{-1}. \qquad \text{(AI.16}')$$

In particular, Eqs. (AI.15) and (AI.16) yield

$$A_{1/2} = \tfrac{2}{3} x^{1/2}, \quad A_{3/2} = \tfrac{2}{5} x^{3/2}. \qquad \text{(AI.17)}$$

Since (AI.15) gives for the left-hand side of (AI.3) the form $(4/15)x^{5/2}$, whereas for the left-hand side of (AI.5) the expression (AI.16) gives $(4/35)x^{7/2}$, comparison with (AI.17) shows these two inequalities to hold. Further, by estimating from (AI.15) the left-hand side of inequality (AI.4) we obtain $x^{1/2}N^{-2/7}$. Since $N \gg 1$, inequality (AI.4) is realized. Therefore Eq. (AI.15) gives the solution to Eq. (3.13) for the entire range of x values under the assumption (3.7).

It remains to consider inequality (AI.6) in the region $\xi \ll 1$ in which, from (AI.16), the left-hand side of this inequality turns out to be of order $x^{1/2}N^{-4/7}$. Comparison of this estimate with $A_{3/2}$ as defined by (AI.17) shows that inequality (AI.16) may fail (if at all) only for $\xi \lesssim N^{-2/7} \ll 1$. However the contribution that $F_{3/2}(x)$, in this region of its argument, makes to the electron density perturbation is small compared to that from larger ξ values.

APPENDIX II

We present here the basic steps in the approximate method used to solve Eq. (3.15). Despite the analogy with the discussion in Appendix I, there arises here a certain complication, due to the special behavior of the right-hand side of (3.15) for $x \ll N^{-2/7} \ll 1$, when, from Eqs. (3.10) and (3.11),

$$F_0(x) \cong -\frac{2\pi^{1/2}N}{x^{1/2}} = -\frac{2\pi^{1/2}N^{8/7}}{\xi^{1/2}}. \qquad \text{(AII.1)}$$

In the opposite limit of moderately small x values, when

$$x \gg N^{-2/7} \qquad \text{(AII.2)}$$

we have from (3.10) and (3.11) the asymptotic form

$$F_0(x) = \frac{\pi^{1/2}}{x^3}. \qquad \text{(AII.3)}$$

By using (AII.3) one readily obtains, from Eq. (3.15) for $x \gg 1$, the asymptotic formula

$$\delta F(x) = \frac{1}{x^3}\left\{\frac{\pi^{1/2}}{x^{5/2}} + \frac{\delta\tilde{C}}{N}\right\}, \qquad \text{(AII.4)}$$

where

$$\delta\tilde{C} = \int_0^\infty dy y^{3/2} e^{-y} \delta F'(y). \qquad \text{(AII.5)}$$

In accordance with the approach of Ref. 13 and Appendix I, we next consider Eq. (3.15) for $x \ll 1$. To simplify this equation, assume the following inequalities to hold:

$$\frac{1}{N}\left|\int_x^\infty dy y^{3/2} e^{-y} \delta F'(y)\right| \ll A, \qquad \text{(AII.6)}$$

$$\frac{1}{N}\left|x^{1/2}\int_x^\infty dy e^{-y} \delta F'(y)\right| \ll A, \qquad \text{(AII.7)}$$

where

$$A = \max\{x^3|\delta F(x)|; N^{-1}x^{1/2}|\delta F'(x)|\}. \qquad \text{(AII.8)}$$

Then for $x \ll 1$, Eq. (3.15) can be reduced to the differential equation

$$\frac{1}{N}[x^{1/2}\delta F'(x)]' - x^3 \delta F = -\left[x^{1/2}F_0(x) + \frac{\delta\tilde{C}}{N}\right]. \qquad \text{(AII.9)}$$

The solution to this equation which satisfies the boundedness conditions at both infinity and zero can be written in the form

$$\delta F(x) = \delta\tilde{F}(\xi) = N^{6/7}\Psi\left(\xi, \left[(\xi N^{-2/7})^{1/2}\tilde{F}_0(\xi) + \frac{\delta\tilde{C}}{N}\right]\right), \qquad \text{(AII.10)}$$

where the functional $\Psi(\xi,[\varphi(\xi)])$ is defined by (3.11).

The asymptotic behavior of the solution (AII.10) for $\xi \gg 1$ turns out to be consistent with Eq. (AII.4). This implies that (AII.10) gives the solution of Eq. (3.15) for any values of the argument x provided the condition (3.7) and, as well, the conditions (AII.6) and (AII.7) hold. We shall later show that the last two conditions are fulfilled if so is inequality (3.7).

However, before proceeding to the proof, let us estimate the quantity $\delta\tilde{C}$. Substitution of (AII.10) into (AII.5) gives

$$\begin{aligned}\delta\tilde{C}\Bigl\{1 - N^{-4/7}\int_0^\infty & d\xi\xi^{3/2}\exp(-\xi N^{-2/7}) \\ &\times (d\Psi(3,[1])/d\xi)\Bigr\} \\ = N^{2/7}\int_0^\infty & d\xi\xi^{3/2}\exp(-\xi N^{-2/7}) \\ &\times (d\Psi(\xi,[\xi^{1/2}\tilde{F}_0(\xi)])/d\xi).\end{aligned} \qquad \text{(AII.11)}$$

In view of the rapid decrease of the functionals Ψ in the region (AII.2), even for $x \ll 1$, it is possible in the integrals in (AII.11) to replace the exponentials by unity. This allows one to write

$$\delta\tilde{C} = N^{2/7}\int_0^\infty d\xi\xi^{3/2}(d\Psi(\xi,[\xi^{1/2}\tilde{F}_0(\xi)])/d\xi). \qquad \text{(AII.12)}$$

From this, making use of (3.10), there at once follows the estimate $\delta\tilde{C} \sim N^{10/7}$. Therefore in the asymptotic formula (AII.4) the term $\delta\tilde{C}/N$ is only important for $x \gg N^{-6/35}$, or equivalently for $\xi \gg N^{4/35} > 1$.

We now proceed to discuss the validity of inequalities (AII.6) and (AII.7). Consider first the region $N^{2/7} \gg \xi \gg 1$ in which, by (AII.4),

$$A = \frac{\pi^{1/2}}{x^{5/7}} + \frac{\delta\tilde{C}}{N}. \qquad \text{(AII.13)}$$

In determining the left-hand sides of the inequalities (AII.5) and (AII.6) in the region $N^{2/7} \gg \xi \gg 1$, one can again employ Eq. (AII.4). Then, for example, the left-hand side of inequality (AII.6) takes the form

$$\frac{1}{N^{4/7}\xi^{3/2}}\left[\frac{2\delta\tilde{C}}{N} + \frac{11\pi^{1/2}}{8x^{5/2}}\right]. \qquad \text{(AII.13')}$$

Comparison of this expression with (AII.13) shows that in the region $N^{2/7} \gg \xi \gg 1$ inequality (AII.6) is satisfied. Similarly, the left-hand side of inequality (AII.7) for $N^{2/7} \gg \xi \gg 1$ has the form

$$\left[\frac{\delta\tilde{C}}{N} + \frac{\pi^{1/2}}{x^{5/2}}\right]\frac{1}{N^{2/7}(xN^{2/7})^{5/2}}. \qquad \text{(AII.13'')}$$

Comparison of this expression with (AII.13) shows that in the region $N^{2/7} \gg \xi \gg 1$ inequality (AII.7) is realized. In the opposite limit $\xi \ll 1$, Eq. (AII.10) yields the following asymptotic behavior:

$$\delta F(x) = N^{13/7}[4\pi^{1/2}(xN^{2/7})^{1/2} - \alpha], \qquad \text{(AII.14)}$$

where

$$\frac{\alpha}{4\pi^{1/2}} = \left(\frac{2}{7}\right)^{9/7} \frac{2\Gamma\left(\frac{6}{7}\right)}{\pi\Gamma\left(\frac{8}{7}\right)} \sin\frac{\pi}{7} \int_0^\infty dy \left[K_{1/7}\left(\frac{4}{7}y^{7/4}\right)\right]^2$$

$$= \left(\frac{2}{7}\right)^{5/2} \frac{\Gamma\left(\frac{3}{7}\right)\left[\Gamma\left(\frac{2}{7}\right)\right]^2}{\Gamma\left(\frac{1}{7}\right)\Gamma\left(\frac{4}{7}\right)}. \qquad \text{(AII.15)}$$

In the case of small values $\xi \ll 1$ we have

$$A = 2\pi^{1/2}N. \qquad \text{(AII.16)}$$

A particularly simple form takes in this case inequality (AII.7), which can be written

$$\frac{1}{N} x^{1/2} |\delta F(x)| \sim (xN^{2/7})^{1/2} N^{5/7} \alpha \ll 1.$$

In view of the expression (AII.16), we have thus shown that inequality (AII.7) is satisfied. Finally, the left-hand side of inequality (AII.6), dominated by the integration over small values of x, has the form $\pi^{1/2}N^{3/7}$. Comparison of this last expression with (AII.16) shows inequality (AII.6) to hold in the region $\xi \ll 1$. A somewhat closer look at the above expressions involved in the inequalities (AII.6) and (AII.7) shows that these inequalities also hold for $\xi \sim 1$. Thus we have shown in this Appendix that Eq. (AII.10) represents the solution of the Eq. (3.15) when inequality (3.7) holds.

APPENDIX III

Remembering that we must compare the contributions to the nonequilibrium electron distribution made by electrons with low and high velocities, in this Appendix we will show that the term in $\mathbf{ku}_i$ in (3.12), which is determined by Eq. (3.14), not only provides a correct contribution for the region of low velocities, Eq. (3.2) but for the high-velocity region as well $kv > \nu(v)$.

From the initial kinetic equation (2.4) with the collision integral (2.5), it is easy to obtain, in the limit of low frequencies ω [cf. Eq. (3.1)], the following expression for the ion-velocity correction $\delta f_{0(i)}$ to the isotropic part of the distribution function [for large velocities $kv > \nu(v)$]:

$$\delta f_{0(i)} = -\frac{2i\nu(v)\mathbf{ku}_i}{v_{Te}^2 k^2} f_m = -\frac{3i\pi^{1/2}\nu_{ei}\mathbf{ku}_i}{4k^2 v_{Te}^2 x^{3/2}} f_m. \qquad \text{(AIII.1)}$$

Let us compare this result with that given by the approach we used in Sec. III for the case $kv < \nu(v)$. We note above all that the inequality $kv > \nu(v)$ can be rewritten in the form $x^2 > (kl_{ei})^{-1}$ or $\xi^2 > z^{1/2}N^{1/4} \gg 1$, i.e., the high-velocity region $kv > \nu(v)$ corresponds to the region of values $\xi \gg 1$. In the range $\xi \gg 1$, the solution for the function $F_{3/2}(x)$ [which function corresponds, by (3.12), to the ion velocity contribution to the distribution function] has a solution of the form (see Appendix I)

$$F_{3/2}(x) = -x^{-3/2}. \qquad \text{(AIII.2)}$$

The ion velocity correction to the isotropic part of the distribution, $\delta f_{0(i)}$, can be expressed in terms of $F_{3/2}(x)$ in the following manner in accordance with Eqs. (3.3), (3.8), and (3.12):

$$\delta f_{0(i)} = \frac{3i\pi^{1/2}\mathbf{ku}_i}{4\nu_{ei}k^2 l_{ei}^2} F_{3/2}(x) \qquad \text{(AIII.3)}$$

and hence from (AIII.2) is, for $kv > \nu(v)$, identically equal to the expression (AIII.1) obtained from the solution of the initial kinetic equation.

Thus the ion-velocity correction $\delta f_{0(i)}$ to the isotropic part of the distribution function is determined by Eq. (AIII.3) for both the regions $kv < \nu(v)$ and $kv > \nu(v)$, that is, for any value of electron velocity.

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