Hole tunneling through a heterobarrier

V. Ya. Aleshkin and N. A. Bekin

Institute of Applied Physics, Russian Academy of Sciences, 603600 N. Novgorod, Russia
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We investigate theoretically within the Luttinger model the tunneling of holes through a single heterobarrier in the GaAs–$Al_xGa_{1-x}As$–GaAs structure. In view of the complexity of the valence band, hole tunneling is a multichannel process, that is, the holes can go from the light subband to the heavy one and vice versa. We find that in the event of new channels opening, the dependence of the hole tunneling coefficients on the energy and the momentum parallel to the heterojunction exhibits a behavior inherent in that of the scattering cross section near a reaction threshold. We also find that the energy dependence on the tunneling probability of heavy holes is nonmonotonic and exhibits sharp peaks. Finally, we demonstrate that by applying voltage to the heterobarrier a population inversion of the hole subbands in the collector can be created.

1. INTRODUCTION

This paper is devoted to a theoretical study of processes of holes tunneling through single heterobarriers in materials of the structure of diamond and zinc blende.

In contrast to electron tunneling, hole tunneling has yet to be thoroughly studied. Fairly recently Esaki, Mendez, Ricco, and Wang[1,2] observed in experiments the resonant tunneling of holes in a two-barrier structure, while Wessel and Altarelli[3] and Chao and Chuang[4] performed the relevant calculations. However, hole tunneling through a single barrier exhibits peculiar features. As we shortly show, the tunneling of heavy holes through a single heterobarrier formed by two heterojunctions resembles resonant tunneling. For instance, the energy dependence of the transmission coefficient displays sharp peaks. This is due to hole buildup in intermediate states, similar to electron buildup in quasistationary states in resonant tunneling.[5] Here we interpret intermediate states as those that fall off exponentially far from the barrier, similar to the interpretation given in Ref. 6 when threshold phenomena are considered.

In view of the complexity of the valence band, hole tunneling is a multichannel process, that is, in the course of tunneling the holes go from the light subband to the heavy one and vice versa. It appears that when channels open or close, the tunneling coefficients exhibit singularities similar to those that elastic scattering cross sections exhibit near a reaction threshold.[7]

When voltage is applied to the heterobarrier, the most intensive tunneling processes are from the heavy subband to the light one and from the light subband to the light one. Hence, light holes mostly find themselves behind the barrier. Below we show that by applying voltage to the barrier a population inversion in light- and heavy-hole subbands can be created.

Note that Suris[8] examined hole tunneling through rectangular heterobarriers in Kane's isotropic model.

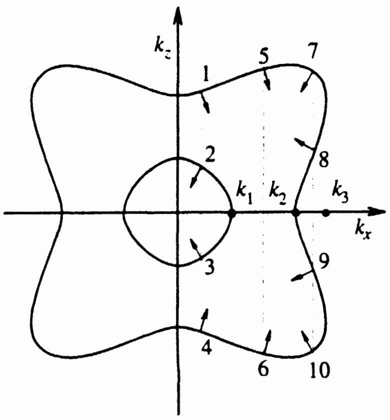

FIG. 1. Section of the constant-energy surfaces of the light and heavy holes by the plane $k_y=0$. The arrows depict the velocity vectors.

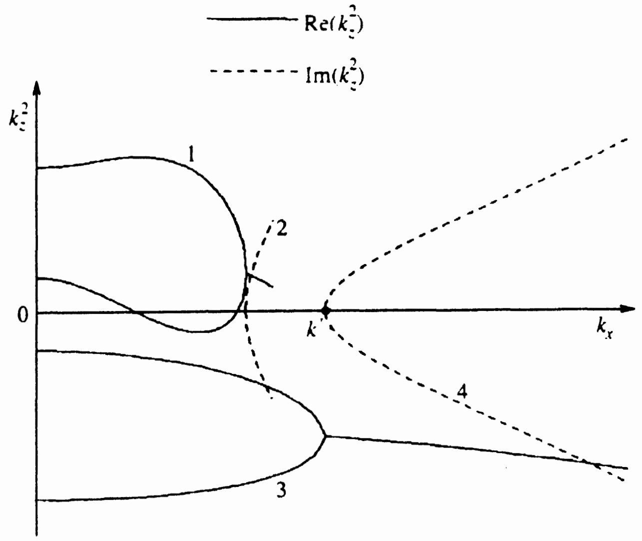

FIG. 2. The $\mathbf{k}_{\parallel}$-dependence of k_z^2 at a fixed energy. Curves *1* and *2* correspond to a hole moving outside of the barrier, and curves *3* and *4* inside the barrier.

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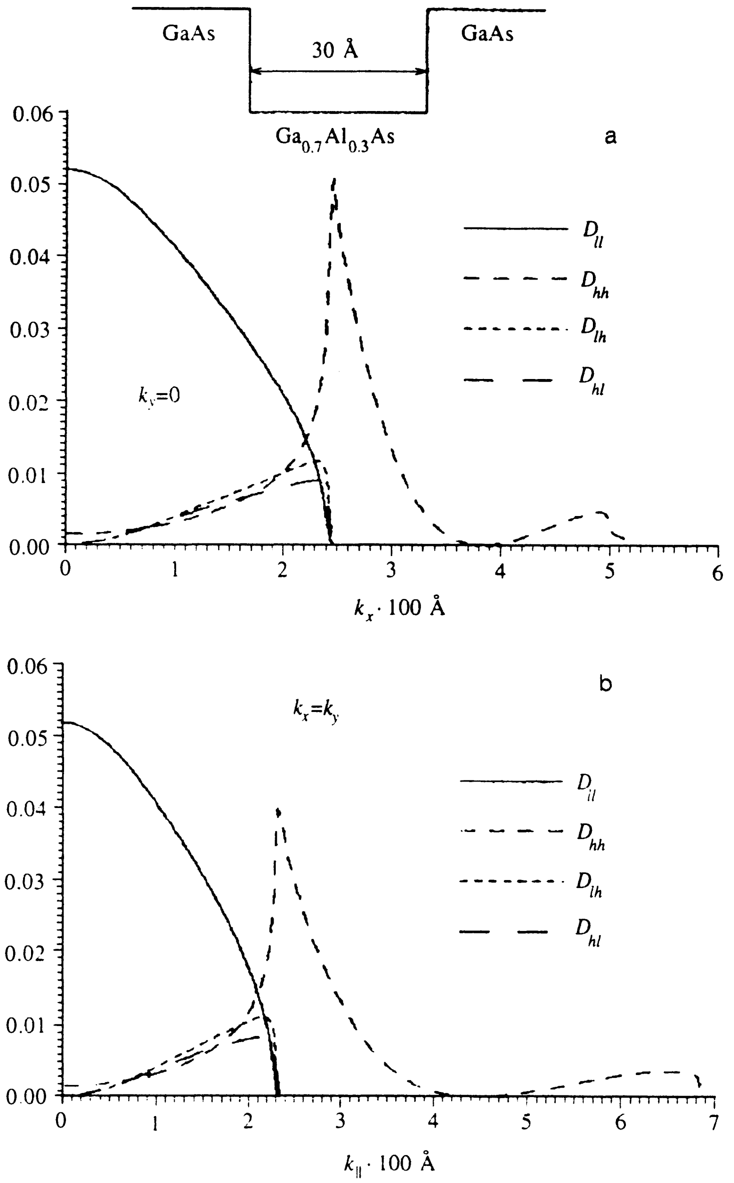

FIG. 3. Hole tunneling coefficients vs $k_{\parallel}$ for the symmetric structure depicted in the inset at (a) $k_y=0$ and (b) $k_x=k_y$. The hole energy is 25 meV.

2. GENERAL PROPERTIES OF HOLE TUNNELING

The Hamiltonian commonly used is describing the movement of holes in homogeneous semiconductors with a structure of diamond and zinc blende[9] becomes non-Hermitian when used for inhomogeneous structures, since the constants A, B, and D determining the spectrum become coordinate-dependent. If A, B, and D depend only on the z coordinate, the simplest Hermitian Hamiltonian has the form

$$\mathscr{H}=\frac{\hbar^2}{2m_0}\begin{pmatrix} F & H & I & 0 \\ H^* & G & 0 & I \\ I^* & 0 & G & -H \\ 0 & I^* & -H^* & F \end{pmatrix}, \qquad (1)$$

$$F=(A+\tfrac{1}{2}B)\hat{k}_{\parallel}^2+\hat{k}_z(A-B)\hat{k}_z+E_v(z),$$

$$H=-\tfrac{1}{2}i(D\hat{k}_z+\hat{k}_zD)\hat{k}_-=-\tfrac{1}{2}i\{D\hat{k}_z\}\hat{k}_-,$$

$$G=(A-\tfrac{1}{2}B)\hat{k}_{\parallel}^2+\hat{k}_z(A+B)\hat{k}_z+E_v(z),$$

$$I=\frac{\sqrt{3}}{2}B(\hat{k}_x^2-\hat{k}_y^2)-iD\hat{k}_x\hat{k}_y,$$

where $E_v(z)$ is the top of the valence band, $\hat{k}_-=\hat{k}_x-i\hat{k}_y$, $\hat{k}_{\parallel}^2=\hat{k}_x^2+\hat{k}_y^2$, $\hat{k}_j=-i(\partial/\partial r_j)$, and m_0 is the free-electron mass.

A hole whose motion is described by the Hamiltonian (1) has four constants of motion: the energy, the two wave-vector components perpendicular to the z axis, and the mirror parity.[10] The last constant of motion has a simple physical meaning for holes with an isotropic dispersion law: namely, parity under reflection in the plane in which the z axis and vector $\mathbf{k}_{\parallel}$ lie.

In the representation of mirror-even (+) and mirror-odd (−) wave functions, the Hamiltonian (1) has the form of a block matrix:

$$\mathscr{H}=\begin{pmatrix} H_+ & 0 \\ 0 & H_- \end{pmatrix}, \qquad (2a)$$

where

$$H_{\pm}=\frac{\hbar^2}{2m_0}\Bigg\{A\hat{k}_{\parallel}^2+\hat{k}_zA\hat{k}_z+E_v(z)$$

$$+(\tfrac{1}{2}B\hat{k}_{\parallel}-\hat{k}_zB\hat{k}_z)\sigma_z\frac{\hbar^2}{2m_0}\Bigg\{+\frac{1}{2}\{Dk_z\}[(k_x\sigma_y-k_y\sigma_x)$$

$$\pm(\sigma_xk_x+\sigma_yk_y)\sqrt{k_{\parallel}^2(3B^2\cos^2 2\theta+D^2\sin^2 2\theta)}\;]\Bigg\},\tag{2b}$$

σ_x, σ_y, and σ_z are the Pauli matrices, and θ is the angle between vector $\mathbf{k}_{\parallel}$ and the x axis. In accordance with Eqs. (2), below we consider independent mirror-even and mirror-odd holes, whose motion is described by the 2-by-2 Hamiltonians ($H+$ and H_-, respectively).

Let us now examine the restrictions imposed on the tunneling processes by the laws of conservation of the energy and the two momentum components mentioned earlier. For simplicity we start with the case where the A, B, D, and E_v are the same on both sides of the barrier. Then the z components of the initial and final states of a hole are located at the intersection of the straight line $\mathbf{k}_{\parallel}=\text{const}$ and the constant-energy surfaces of the light, $E_l(k_x,k_y,k_z)=E$, and heavy, $E_h(k_x,k_y,k_z)=E$, holes:

$$E_h=E_v+\frac{\hbar^2}{2m_0}[Ak^2+\sqrt{B^2k^4+C^2(k_x^2k_y^2+k_x^2k_z^2+k_y^2k_z^2)}\,],\tag{3a}$$

$$E_l=E_v+\frac{\hbar^2}{2m_0}[Ak^2-\sqrt{B^2k^4+C^2(k_x^2k_y^2+k_x^2k_z^2+k_y^2k_z^2)}\,],\tag{3b}$$

where $C^2=D^2-3B^2$, and the values of A, B, D, and E_v are those outside the barrier.

Figure 1 shows that for $k_{\parallel}<k_1$ the straight line $k\|=\text{const}$ intersects the constant-energy surfaces of the light and heavy ions twice each (in Fig. 1 $k_y=0$, but the pattern is the same for all directions of $\mathbf{k}\|$). At two intersection points (*3* in the light subband and *4* in the heavy) the hole states have positive velocity components along the z axis. In the event of tunneling of holes with such $\mathbf{k}\|$, the incident holes and the holes that have passed through the barrier can be in either state *3* or state *4*. In other words, as a result of tunneling a hole either remains in the same subband (transitions of the *3*→ *3* or *4*→ *4* type) or transfers to the other subband (transitions of the *3*→ *4* or *4*→ *3* type). If we denote the ratio of the transmitted flux of holes of species j to the incident flux of holes of species i by D_{ij}, the states *1* and *4* by the letter h, and the states *2* and *3* by the letter l, for such vectors $\mathbf{k}\|$ the coefficients D_{ll}, Dhh, D_{lh}, and D_{hl} are finite, and, hence, so are the reflection coefficients R_{ll}, R_{hh}, R_{lh}, and R_{hl}.

For $k_1<k\|<k_2$ the straight line $\mathbf{k}\|=\text{const}$ intersects (twice) only the heavy-hole constant-energy surface. In this case outside the barrier there is only one state with a positive velocity. Hence only tunneling transitions from the heavy subband to the heavy subband are possible, with only D_{hh} and R_{hh} finite (we denote the states *5*, *6*, *7*, and *10* by the letter'h', too).

For $k_2<k\|<k_3$ the straight line $\mathbf{k}\|=\text{const}$ intersects the heavy-hole constant-energy surface four times. Two states, *8* and *10*, have a positive velocity. We denote the states *8* and *9* by the letter H. Then in this range the finite coefficients are D_{HH}, Dhh, D_{Hh}, D_{hH}, R_{HH}, R_{hh}, R_{Hh}, and R_{hH}. Note that the given range exists only if the heavy-hole constant-energy surface has concavities, that is, if there exists a cone of negative transverse masses.[11]

Below we give the expressions for k_1, k_2, and k_3:

$$k_1=\left(\frac{\varepsilon}{A-\sqrt{B^2+0.25C^2\sin^2 2\theta}}\right)^{1/2},\tag{4a}$$

$$k_2=\left(\frac{\varepsilon}{A+\sqrt{B^2+0.25C^2\sin^2 2\theta}}\right)^{1/2},\tag{4b}$$

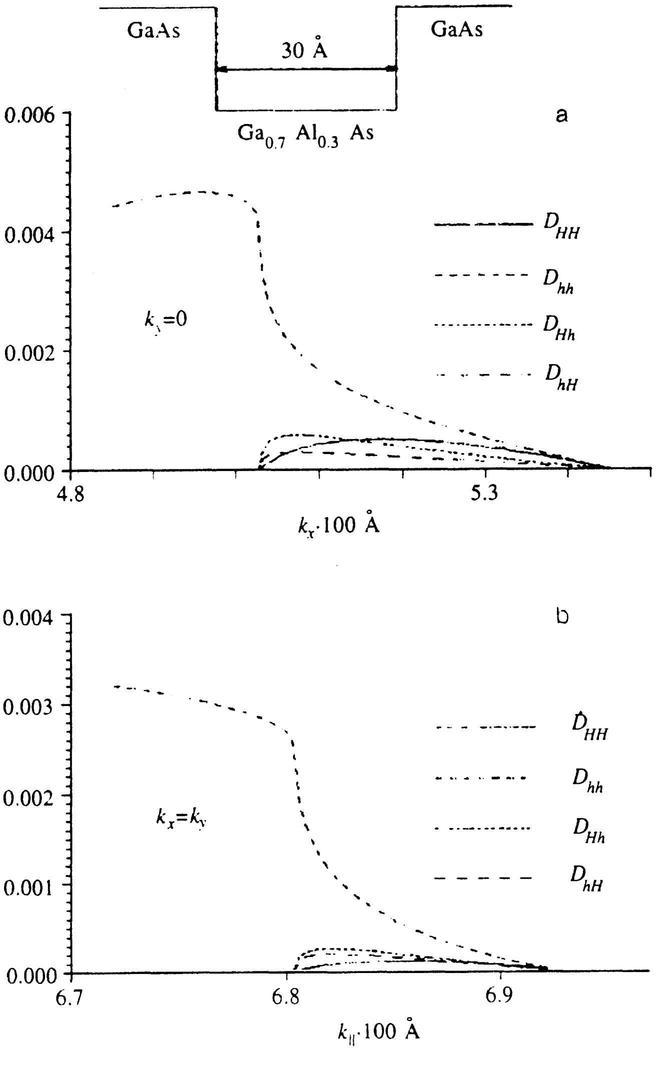

FIG. 4. The parts of Figs 3a and 3b corresponding to large $k\|$ on a greater scale.

$$k_3=\left(-\frac{AC^2\varepsilon+\sqrt{C^2\varepsilon^3(A^2-B^2)[c^2+4B^2(1-\sin^2\theta\cos^2\theta)]}}{2C^2[\frac{1}{4}C^2-(A^2-B^2)(1-\sin^2\theta\cos^2\theta)]}\right)^{1/2}, \tag{4c}$$

where $\varepsilon=2m_0(E-E_v)/\hbar^2$.

These results can easily be generalized to incorporate the case where the positions of the top of the valence band, E_v, to the right and left of the barrier differ. Here k_1, k_2, and k_3 are different on the right and left of the barrier, so to distinguish the two values we use the superscripts R and L. To determine the possible values of the first and second subscripts in D_{ij}, we must find the ranges of $k\|$ within which the incident holes and the holes that have passed the barrier fall with respect to the values of k_1, k_2, and k_3. For instance, if an incident hole "fits" into the range $k_2^L<k\|<k_3^L$ and a hole that has passed the barrier into the range $k\|<k_1^R$, then D_{hl}, d_{hh}, D_{Hl}, and D_{Hh} are finite.

Now we give the expression for the z component of the flux, which can be obtained in the usual manner[7] by employing (2b):

$$j_z=\frac{\hbar}{2m_0}[A(\langle\psi|\hat{k}_z\psi\rangle+\langle\hat{k}_z\psi|\psi\rangle)+B(\langle\hat{k}_z\psi|\sigma_z\psi\rangle+\langle\psi|\hat{k}_z\sigma_z|\psi\rangle)+D\langle\psi|\sigma_yk_x-k_y\sigma_x|\psi\rangle], \tag{5}$$

where $|\psi\rangle$ is a two-component spinor, the wave function of a hole.

Note that for a state that is a linear combination of light and heavy holes or states h and H with the same energies and $k\|$, the component jz is the sum of the z components of the fluxes of the respective constituents, and an interference term is absent (both j_x and j_y have such terms). This can easily be understood if we take into account the law of flux conservation ($\operatorname{div}\mathbf{j}=0$) and the law of variation of the interference term in space: $\sim\cos([k_{zh}-k_{zl}]z-\xi)$, where ξ is a phase factor, and k_{zl} and k_{zh} the z components of the wave vectors of light and heavy holes.

3. TUNNELING THROUGH RECTANGULAR BARRIERS

Let us consider hole tunneling through a rectangular barrier. Suppose that the heteroboundaries lie in the planes $z=0$ and $z=a$, so that the range $0<z<a$ constitutes a barrier for the holes (see the inset in Fig. 3a). The wave function outside the barrier can be represented as a plane-wave expansion:

$$\psi=\sum_{j=1}^{4}b_j\psi_j=\sum_{j=1}^{4}c_j\exp(-ik_{zj}z)\psi_j, \tag{6}$$

$$\psi_j=|\psi_j\rangle=\frac{1}{\sqrt{V_j}}\begin{vmatrix}\varphi_j\\ \chi_j\end{vmatrix}\exp(i\mathbf{k}\cdot\mathbf{r}+ik_{zj}z), \tag{7}$$

$$\varphi_j=\frac{k_-}{N}(iDk_{zj}-0.5\lambda k_\|\sqrt{3B^2\cos^2 2\theta+D^2\sin^2 2\theta}), \tag{8a}$$

$$\chi_j=\frac{1}{N}\left[\left(A+\frac{B}{2}\right)\mathbf{k}_\|^2+(A-B)k_{zj}-E-E_v\right], \tag{8b}$$

where $N=\sqrt{|\varphi|^2+|\chi|^2}$, $V_j=|\langle\psi_j|\hat{V}_z|\psi_j\rangle|$ is the magnitude of the mean value of the velocity component along the z axis, and $\lambda=1$ for mirror-even holes, and $\lambda=-1$ for mirror-odd holes. These normalization condition corresponds to $j_z=1$. Below we give the formulas for the four values of k_{zj}:

$$k_z=\mp\sqrt{\frac{R\mp\operatorname{sgn}(\varepsilon)\sqrt{R^2-(A^2-B^2)[(\varepsilon-A\mathbf{k}_\|^2)^2-B^2\mathbf{k}_\|^4-C^2\mathbf{k}_\|^4\sin^2\theta\cos^2\theta]}}{A^2-B^2}},$$

where $R=A\varepsilon-(A^2-B^2-\frac{1}{2}C^2)\mathbf{k}_\|^2$. For k_{z1} let us select the "+" sign in front of both radical signs. Then in the range $k\|<k_1$, k_{z1} corresponds to a light hole moving to the left (state *2* in Fig. 1), while in the range $k_2<k\|<k_3$, k_{z1} corresponds to an H-hole moving to the right (state *8* in Fig. 1). For k_{z3} we select the "−" sign in front of the inner radical sign and "+" in front of the outer. Then k_{z3} corresponds to heavy holes moving to the left (states *1*, *5*, and *7* in Fig. 1). We put $k_{z2}=-k_{z1}$ and $k_{z4}=-k_{z3}$. Note that in $k_1<k_\|<k_2$ the imaginary part of k_{z1} is positive.

Expansion (6) is valid above the barrier, too. In this region, however, k_z is purely imaginary if $C=0$, that is, the holes obey an isotropic dispersion law. For $C\neq0$ the $\mathbf{k}\|$-dependence of k_z^2 is depicted in Fig. 2. The reader can see that for $C\neq0$ and $k\|>k'$ above the barrier, k_z acquires a positive real part.[4]

The matching conditions for the wave functions are obtained by integrating the Schrödinger equation with the Hamiltonian (2) near a heteroboundary. If the heteroboundary lies in plane $z=0$, the boundary conditions for the spinor components ψ_β (here β is the spinor index) and their derivatives can be written as

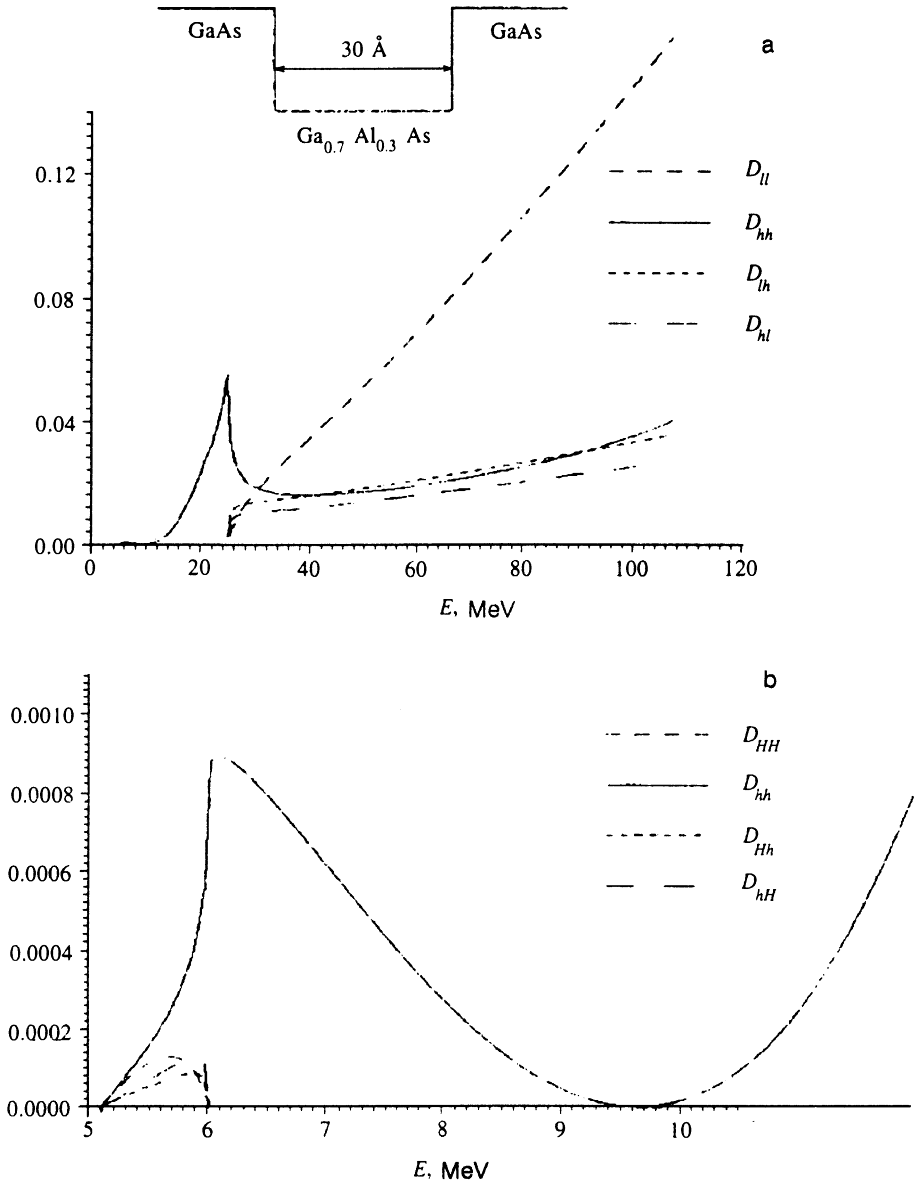

FIG. 5. Hole tunneling coefficients vs energy for the structure depicted in the inset at $k_y=0$ and $k_x=2.4\times10^6$ cm^{-1}. The low-energy part of (a) is depicted in (b) on a larger scale.

$$\left|\begin{array}{c}\psi_1(+0)\\ \frac{d}{dz}\psi_1(+0)\\ \psi_2(+0)\\ \frac{d}{dz}\psi_2(+0)\end{array}\right| = \hat{L}\left|\begin{array}{c}\psi_1(-0)\\ \frac{d}{dz}\psi_1(-0)\\ \psi_2(-0)\\ \frac{d}{dz}\psi_2(-0)\end{array}\right|, \qquad (9)$$

where $\hat{L}$ is a 4-by-4 matrix with the following finite elements:

$$L_{11}=L_{33}=1,\ L_{22}=\frac{A(-0)-B(-0)}{A(+0)-B(+0)},$$

$$L_{23}=\frac{k_+[D(+0)-D(-0)]}{2[A(+0)-B(+0)]},$$

$$L_{41}=-\frac{k_-[D(+0)-D(-0)]}{2[A(+0)+B(+0)]},$$

$$L_{44}=\frac{A(-0)+B(-0)}{A(+0)+B(+0)}.$$

The expansion coefficients c_j in (6) on the two sides of the barrier are related via a transfer matrix $\hat{T}$ (see Ref. 5):

$$c_j(-0)=T_{jn}c_n(a+0), \qquad (10)$$

$$\hat{T}=\hat{M}^{-1}(-0)\hat{L}(0)\hat{M}(+0)NM^{-1}\times(a-0)L(a)M(a+0), \qquad (11)$$

where $M_{1j}=\varphi_j$, $M_{2j}=ik_{zj}\varphi_j$, $M_{3j}=\chi_j$, $M_{4j}=ik_{zj}\chi_j$, and $N_{ln}=\exp(-ik_{zn}a)\delta_{ln}$, with δ_{ln} the 4-by-4 identity matrix. The M matrix establishes the transition from the representation (6) to the representation (9).

Let us assume that a hole is incident on the barrier from the region $z<0$. Irrespective of the type of hole incident on the barrier, behind the barrier (in the region $z>a$) there can be heavy holes moving to the right. On the other

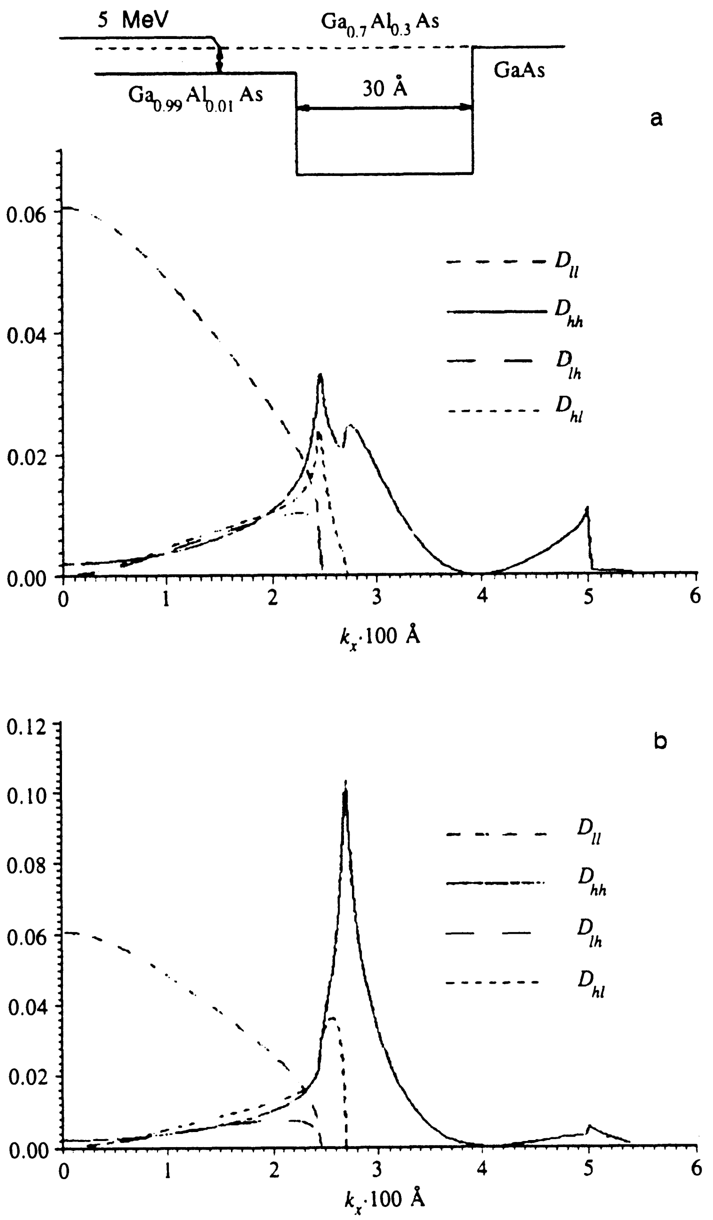

FIG. 6. Hole tunneling coefficients vs $k\|$ for the slightly asymmetric structure depicted in the inset for (a) mirror-even and (b) mirror-odd holes at $k_y=0$. The hole energy is 25 meV.

hand, the region behind the barrier can contain light holes moving to the right only if $k_1^R > k_\|$ (if $k_\| > k_2^R$, there can be H-holes moving to the right). If $k_1^R < k_\| < k_2^R$, the region behind the barrier contains light-hole states that decay as $z \to \infty$. The absence in this region of holes moving to the left (or states exponentially growing as $z \to \infty$ for $k_1^R < k_\| < k_2^R$) leads to the vanishing of two expansion coefficients $c_j(a+0)$: namely, $c_3(a+0)=0$ for all $\mathbf{k}_\|$, and $c_1(a+0)=0$ for $k_\| < k_1^R$ or $c_2(a+0)=0$ for $k\| > k_1^R$.

Let us now examine the wave functions of holes in front of the barrier. If a unit flux of light holes (heavy H-holes) is incident on the barrier, $c_2(-0)=1$ ($c_1(-0)=1$), and $c_4(-0)=0$ since there are no h-holes incident on the barrier. If a unit flux of heavy h-holes is incident on the barrier, $c_4(-0)=1$ and $c_2(-0)=0$ for $k_\| < k_1^L$, since there are no heavy H-holes incident on the barrier. But if $k_1^L < k_\| < k_2^L$, we have $c_1(-0)=0$, since within this range there can be no states growing with $z \to -\infty$. Thus, in front of the barrier one of the coefficients c_j is equal to unity and another is zero.

Bearing in mind what was said earlier, we see that Eqs. (10) and (11) lead to a system of four inhomogeneous linear equations for determining the four unknown coefficients c_j. The tunneling coefficients are expressed in terms of the $c_j(a+0)$ and the reflection coefficients in terms of the $c_j(-0)$.

Figures 3a and 3b show how D_{ij} depends on $k\|$ at $\theta=0$ and $\theta=\pi/4$, respectively, for the structure depicted in the inset. For symmetric barriers the tunneling coefficients of mirror-even holes, D_{ij}, are equal, respectively, to the coefficients D_{ji} of mirror-odd. This property is not present in asymmetric barriers.

Figures 3a and 3b clearly demonstrate the following three specific features of hole tunneling. First, D_{hh} assumes its maximum value at $k\|=k_1$ rather than at $k\|=0$, as is the case with electrons. Second, the $k\|$-derivative of D_{hh} becomes equal to $\pm\infty$ at $k_\|=k_1,k_2$. And, finally, within the range $k_\| > k_1$ the $k\|$-dependence of D_{hh} is nonmono-

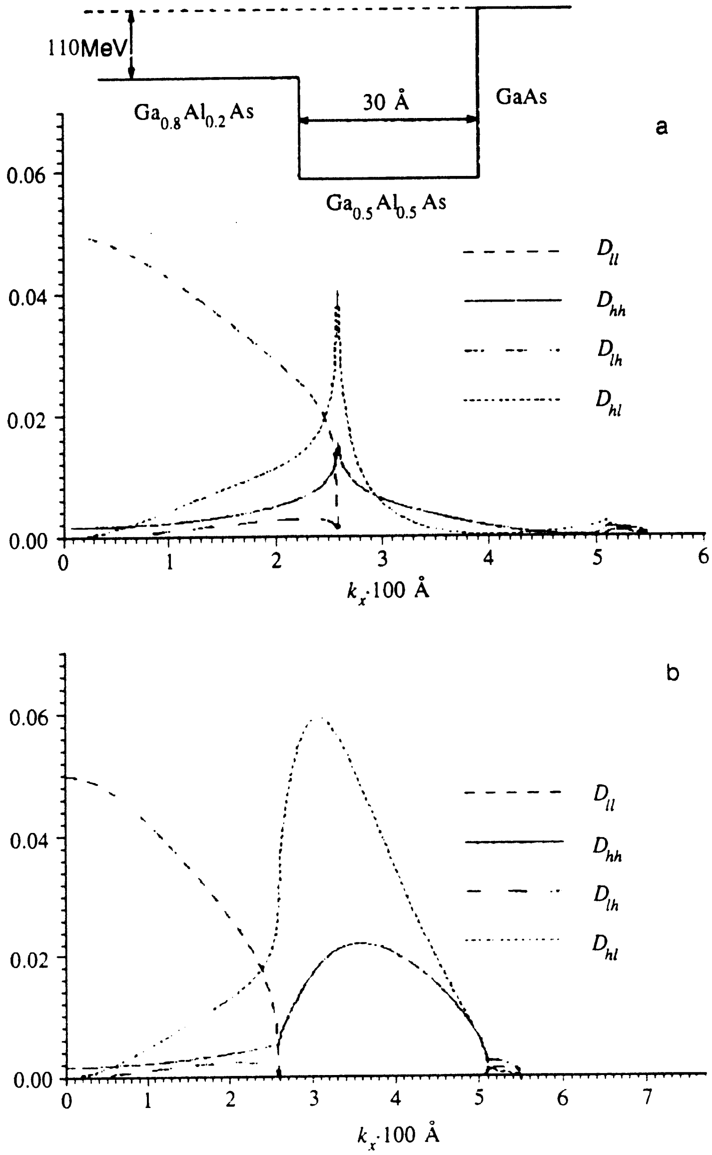

FIG. 7. Hole tunneling coefficients vs $k\|$ for the highly asymmetric structure depicted in the inset for (a) mirror-even and (b) mirror-odd holes at $k_y=0$. The hole energy is 25 meV.

tonic (D_{hh} has a peak in the range $k_1 < k_{\|} < k_2$ and a second peak in the neighborhood of $k_{\|}=k_2$).

The reason why D_{hh} increases with $k\|$ is that there are two tunneling channels under the barrier; one corresponds to light holes, and the other to heavy holes. Clearly, the "light" channel is more transparent since in it the damping of the wave function in the barrier is greater. At $k_{\|}=0$ the heteroboundaries do not mix the states of light and heavy holes (the $H_{\pm}$ are diagonal), and therefore the "light" channel is closed for heavy holes. As $k\|$ increases, the "light" channel becomes more and more open for heavy holes, and D_{hh} increases.

For $k_2 > k\| > k_1$ only the motion of heavy h-holes is infinite, while the states of light holes and of heavy H-holes decay exponentially far from a heterobarrier (an intermediate state). Strictly speaking, dividing holes in an intermediate state into heavy and light is purely nominal because the values of k_z for both lie on the same loop of the $k_z(\varepsilon,\mathbf{k}\|)$ dependence (see Fig. 2). The cloud of light holes, whose spatial distribution is proportional to $\exp\{2\,\mathrm{Im}(k_{z1})z\}$ to the left of the barrier and to $\exp\{-2\,\mathrm{Im}(k_{z1})z\}$ to the right, spreads over ever growing distances as $k\|$ approaches k_1 from above, since $\mathrm{Im}\,(k_{z1})=0$ at $k\|=k_1$. Such "inflation" leads to hole buildup in intermediate states in the neighborhood of a barrier, similar to electron buildup in a quasistationary state in resonant tunneling.[5] The greater fraction of holes in an intermediate state lies to the left of the heterobarrier, that is, on the side of the incident flux. The holes in an intermediate state can leave this state and transform into heavy holes. Here the reflected flux of heavy holes interferes with the flux from the intermediate state. When interference is switched on ($k\|=k_1$) or of ($k\|=k_2$), the $\mathbf{k}\|$-dependence of D_{hh} changes drastically, as a result of which D_{hh} exhibits peaks at $k\|=k_1$ and in the neighborhood of $k\|=k_2$. Note that for holes obeying an isotropic dispersion law ($C=0$), when there are no H-states, $D_{hh}(k\|)$ decreases monotonically for $k\| > k_1$.

Since tunneling in the $k_2 < k\| < k_3$ range is weak, for the structure depicted in the inset in Fig. 3a the behavior of the tunneling coefficients are shown in Figs. 4a and 4b on a magnified scale.

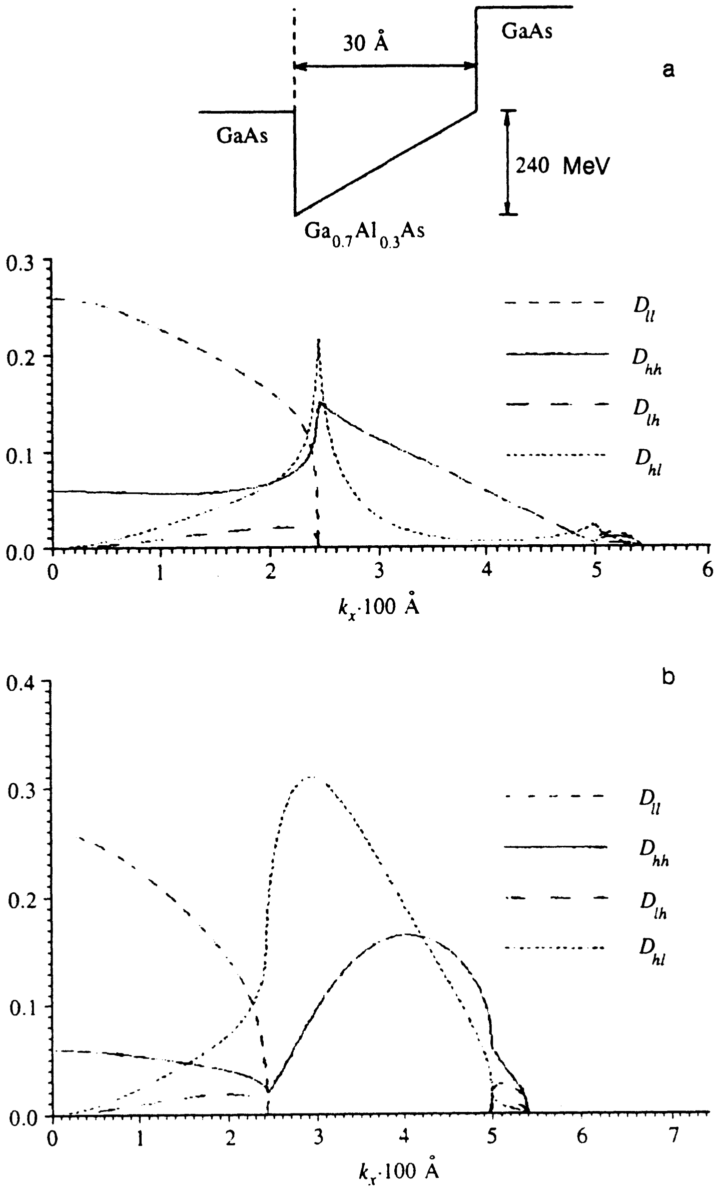

FIG. 8. Hole tunneling coefficients vs $k\|$ for (a) mirror-even and (b) mirror-odd holes for a structure to which voltage is applied. The hole energy is 25 meV, and $k_y=0$.

The presence of peaks in $D_{hh}(\mathbf{k}_\|)$ leads to two peaks in the energy dependence of the transmission at constant $\mathbf{k}_\|$ (Fig. 5). The peak at low energy appears at $k\|=k_2$. The second peak corresponds to $k\|=k_1$.

Reasoning in the same manner as Landau and Lifshitz[7] did in connection with the energy dependence on the elastic scattering cross section near the reaction threshold, we can show that in the neighborhood of $k_{1,2}$,

$$D_{hh}(k_\|)\approx|\sqrt{D_{hh}(k_{1,2})}+P_{1,2}\sqrt{k_\|-k_{1,2}}|^2,$$

where $P_{1,2}$ are constants. Clearly, the k‖-derivative of $D_{hh}(k_\|)$ has branch points of order 2 at $k_\|=k_{1,2}$ and, if the imaginary part of $P_{1,2}$ is finite, experiences a discontinuity at these points.

Figures 6a, 6b, 7a, and 7b demonstrate the behavior of the tunneling coefficients for (a) mirror-even and (b) mirror-odd holes in their passage through asymmetric heterobarriers, depicted in the insets. Clearly, here mirror-odd holes tunnel better than mirror-even. Moreover, in contrast to the case of symmetric barriers, the off-diagonal coefficients D_{lh} and D_{hl} vanish at different values of $k_\|$ owing to the different values of k_1, k_2, and k_3 to the right and to the left of the heterobarrier. This also explains the increase in the number of singularities in the transmission coefficients and in the derivatives of the coefficients, since the hole buildup in intermediate states to the right and to the left of the barrier now occurs at different values of $k_\|$. Fig. 7 also shows that if the barrier is highly asymmetric, the areas under the Dll vs $k\|$ and D_{hl} vs $k\|$ curves are considerably greater than those under the D_{hh} vs $k\|$ and D_{lh} vs $k\|$ curves, that is, mostly light holes find themselves behind the barrier.

HOLE TUNNELING THROUGH A HETEROBARRIER IN AN ELECTRIC FIELD

The above method of finding the tunneling coefficients can be applied to heterobarriers in an electric field. However, now the matrix linking ψ_i and $d\psi_i/dz$ in the planes $z=+0$ and $z=a-0$, equal to the matrix product

$\hat{M}(+0)NM^{-1}(a-0)$ in Eq. (11) for a rectangular barrier, must be found by numerically solving the Schrödinger equation with the Hamiltonian (2b).

Figures 8a and 8b depict the behavior of the transmission coefficients for the structure in the inset in Fig. 8a. Qualitatively, the behavior of D_{ij} is similar to that depicted in Fig. 7. Note that if the voltage drop on the barrier is 5 mV, the transmission coefficients for all practical purposes coincide with the respective coefficients depicted in Fig. 6.

Application of voltage to the heterobarrier can produce population inversion between the subbands of light and heavy ions within a certain range of momenta. The reason is not that primarily light holes find themselves behind the barrier; rather, the heavy holes that have passed through the barrier have a wave vector differing from that of light holes. Moreover, when the applied voltage is high, a situation may emerge in which the momentum of the light holes that have passed the barrier is much higher than the characteristic momentum of equilibrium holes in the right junction (the collector). Thus, in the momentum-space region corresponding to the light holes that have passed through the barrier there may be no heavy holes, that is, population inversion is established. This could lead to generation of radiation at the respective frequency.

A hole that has undergone scattering after tunneling contributes nothing to generation of radiation. Hence, for the sake of an estimate we can assume that the ratio of the luminescence power to the scattered power is approximately equal to the ratio of the frequency of spontaneous photon emission to the scattering rate. The frequency (rate) of spontaneous photon emission via a hole in the transition from the light subband to the heavy subband can be written as

$$W=\frac{e^2(E_l-E_h)^2B\sqrt{\kappa}}{2c^3\hbar^2m_0}, \qquad (12)$$

where c is the speed of light, and κ is the high-frequency dielectric constant of a narrow-gap semiconductor. If $|E_l-E_h|=0.2$ eV, for GaAs we have $W\sim10^7$ Hz. The frequency of spontaneous emission of an optical photon is roughly 10^{13} Hz. Thus, no more than one millionth of the power liberated in the structure will go into light in spontaneous transitions of holes from the light subband to the heavy subband.

In conclusion we note that in examining hole tunneling we ignored the charge of the intermediate-state holes, which leads to a change in barrier shape. It is a well-known fact that in resonant tunneling the buildup of electrons in a quasistationary state leads to a hysteresis loop in the current–voltage characteristic.[12] Similarly, hole buildup in intermediate states can lead to a hysteresis loop in the current vs voltage dependence.

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