Theory and experimental detection of transition radiation from a charged filament

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We present a theoretical analysis of the electromagnetic radiation (transition scattering) emitted when a permittivity wave is incident upon a stationary charged source. We report the first experimental observation and measurement of certain characteristics of the transition scattering by a charged filament of a permittivity wave generated by a high-power laser pulse in a nonlinear medium.

1. INTRODUCTION

In a paper published in 1973,[1] Ginzburg and Tsytovich examined so-called transition scattering, the electromagnetic radiation emitted when the electric field due to a permittivity wave is incident upon a charged source (see also Ref. 2). The permittivity wave can result from the propagation of high-power electromagnetic or acoustic pulses in the medium.

One of the most interesting aspects of this phenomenon is the feasibility of radiation from stationary charges or other sources of a constant electric field. The energy that is radiated away ultimately comes from the source of the permittivity wave.

Despite widespread interest in transition scattering and its importance as an investigative tool, the first experimental observation of the phenomenon did not appear until recently (see Ref. 3). In the present paper, we describe experimental observations of the transition scattering of a permittivity wave produced by a high-power laser pulse passing through an isotropic nonlinear medium. The stationary electric-field source in these experiments is a long charged filament, which gives rise to a number of interesting features in the resulting scattered radiation.[4]

In Sec. 2 we discuss the theory of transition scattering from a charged filament, and in Sec. 3 we describe the experimental procedure and results.

2. TRANSITION RADIATION FROM FIXED SOURCES OF ELECTRIC FIELDS

Suppose that a stationary charged source of constant electric field with charge density $\rho(\mathbf{r})$ resides in a medium with dielectric constant ε_0, and that a permittivity wave is incident upon that source, where the time and space dependence of the wave is given by

$$\varepsilon=\varepsilon_0+\Delta\varepsilon\cos(\mathbf{k}_0\mathbf{r}-\omega_0 t),\quad \text{where } \Delta\varepsilon\ll\varepsilon_0. \tag{1}$$

Note that $\Delta\varepsilon\ll\varepsilon_0$ holds for all cases in which an actual permittivity wave is generated. The incidence of that wave upon the source should give rise to electromagnetic radiation (transition scattering). It is extremely convenient in calculating the characteristics of that radiation to use perturbation theory, which makes it simple to calculate the spectral and angular distribution of the radiation in weakly inhomogeneous and nonsteady media.[5] Our previous results[5] show that the spectral distribution of the energy emitted by an electric-field source in a medium carrying a permittivity wave of the form

$$\varepsilon=\varepsilon_0+\varepsilon_1(\mathbf{r},t) \tag{2}$$

is given by

$$W_{\mathbf{k},\lambda}d^3k=\frac{(2\pi)^4\omega^2}{4\varepsilon_0}\left|\int d\mathbf{k}_1 d\omega_1 e_i^{\lambda}E_i^q(\mathbf{k}_1,\omega_1)\times\varepsilon_1(\mathbf{k}-\mathbf{k}_1,\omega-\omega_1)\right|^2 d^3k, \tag{3}$$

where $E^q(\mathbf{k},\omega)$ is the Fourier transform of the source's electric field amplitude in the "unperturbed" medium, $\varepsilon_1(\mathbf{k},\omega)$ is the Fourier transform of the variable part of the dielectric constant, ω is the frequency of the radiation, $\mathbf{k}$ is the wave vector of the emitted wave, and $\mathbf{e}^{\lambda}$ is its polarization vector.

We thus need to find the Fourier components of the source electric field and of the variable part of the dielectric constant. To find the former, given a charge density $\rho(\mathbf{r})$, we use the Poisson equation for the scalar potential φ:

$$\Delta\varphi=-4\pi\rho(\mathbf{r})/\varepsilon_0(0), \tag{4}$$

where $\varepsilon_0(0)$ is the static value of the dielectric constant. We expand φ and ρ in Fourier integrals of the form

$$\varphi=\int d\mathbf{k}_1 d\omega_1\varphi(\mathbf{k}_1,\omega_1)\exp[i(\mathbf{k}_1\mathbf{r}-\omega_1 t)] \tag{5}$$

(the equation for ρ is the same as for φ). Substituting (5) into (4), we obtain an expression for $\varphi(\mathbf{k}_1,\omega_1)$. Since $\mathbf{E}=-\text{grad}\varphi$, we can easily obtain an expression for $\mathbf{E}(\mathbf{k}_1,\omega_1)$ if we multiply $\varphi(\mathbf{k}_1,\omega_1)$ by $-i\mathbf{k}_1$:

$$\mathbf{E}^q(\mathbf{k}_1,\omega_1)=-[4\pi i\delta(\omega_1)/\varepsilon_0(0)k_1^2]\rho(\mathbf{k}_1)\mathbf{k}_1, \tag{6}$$

where

$$\rho(\mathbf{k}_1)=(2\pi)^{-3}\int\rho(\mathbf{r})\exp(-i\mathbf{k}_1\mathbf{r})d\mathbf{r}$$

is the spatial Fourier transform of the charge density.

The Fourier transform of the variable part of the dielectric constant (1) can easily be expressed in terms of the δ function:

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$$\varepsilon_1(\mathbf{k}_1,\omega_1)=[\Delta\varepsilon/2)(\delta(\omega_1-\omega_0)\delta(\mathbf{k}_1-\mathbf{k}_0)+\delta(\omega_1+\omega_0)\delta(\mathbf{k}_1+\mathbf{k}_0)]. \qquad (7)$$

Substitution of (6) and (7) into the general equation (3) yields expressions containing the factors $\delta(\omega+\omega_0)$ and $\delta(\omega-\omega_0)$. Terms with $\delta(\omega+\omega_0)$ vanish, however. The reason is that ω and ω_0 in the argument of the δ function are real frequencies (ω_0 is the frequency of the permittivity wave, while ω is the frequency of the radiated electromagnetic wave), which are naturally positive (in contrast to ω_1, the Fourier expansion variable, which can also be negative). The δ function argument $\omega+\omega_0$ is thus always positive, and the δ function itself vanishes. Substituting (6) and (7) into (3), we easily obtain the general expression for the emitted energy. This expression diverges, however, by virtue of the square of the δ function. The reason is obvious: the transition scattering process lasts an infinitely long time, and an infinite amount of energy is thereby emitted. We therefore need to specify the energy emitted per unit time, i.e., the power. Representing one of the δ functions in the form $T/2\pi$, where T is a long time interval, we obtain for the angular and spectral power distribution of the scattered radiation

$$I_{\mathbf{k}\lambda}d^3k=\frac{(\Delta\varepsilon)^2\omega^2(2\pi)^6|\rho(\mathbf{k}-\mathbf{k}_1)|^2}{8\pi\varepsilon_0(\omega)\varepsilon_0^2(0)(k-k_0)^4}(\mathbf{e}^\lambda\mathbf{k}_0)\delta(\omega-\omega_0)d^3k. \qquad (8)$$

The radiation is therefore monochromatic at the frequency ω_0 of the permittivity wave, and the polarization vector lies in the plane formed by the vectors $\mathbf{k}_0$ and $\mathbf{k}$.

We now introduce a spherical coordinate system in which $\mathbf{k}_0$ is aligned with the polar axis. The angular power distribution for transition scattering then takes the form

$$I_{\theta,\varphi}\,d\theta d\varphi=\frac{(\Delta\varepsilon)^2\omega_0^2(2\pi)^5|\rho(\mathbf{k}-\mathbf{k}_0)|^2k_0^2k^2\sin^3\theta d\theta d\varphi}{4\sqrt{\varepsilon_0(\omega_0)}\varepsilon_0^2(0)c(k^2+k_0^2-2kk_0\cos\theta)^2}, \qquad (9)$$

where θ and φ are polar and azimuthal angles.

Note now that if we choose a stationary point charge q as the electric-field source, Eq. (9) immediately yields the results obtained in Ref. 1. The Fourier component of the charge density then becomes

$$\rho(k_1)=q/(2\pi)^3.$$

Inserting this expression into the general equation (9) gives the result obtained in Ref. 1.

We next use Eq. (9) to calculate the characteristics of transition scattering, considering first an infinitely long charged filament of infinitesimal thickness. Assume that a permittivity wave of the form given by (1) propagates along the z axis in some medium, with the filament oriented at an angle β to that axis (Fig. 1). The x axis (from which the azimuthal angle φ is measured in the spherical coordinate system) is so chosen that the filament lies in the xz plane. The permittivity wave should then give rise to scattered radiation from the filament.

In the cartesian coordinate system defined above, the charge density of the filament is

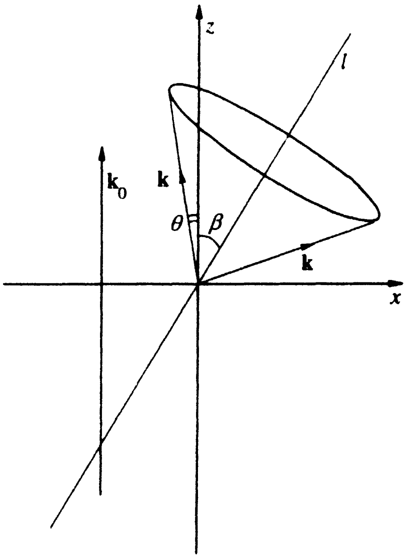

FIG. 1. Geometry of the interaction of a laser beam with a charged filament in a nonlinear medium. l: charged filament; $\mathbf{k}_0$: wave vector of the permittivity wave.

$$\rho(\mathbf{r})=\sigma\delta(y)\delta(x\cos\beta-z\sin\beta), \qquad (10)$$

where σ is the linear charge density of the filament. The spatial Fourier transform of that charge density is

$$\rho(k)=\sigma\delta(k_z+k_x\tan\beta)/(2\pi)^2\cos\beta. \qquad (11)$$

Inserting (11) into the general equation (9) yields an expression for the angular distribution of the scattered power. It is clear from (11) that this expression contains the square of a δ function, $\delta^2(k_z-k_0+k_x\tan\beta)$, which makes it diverge. Obviously, the reason for the divergence is that the filament is infinitely long. We must therefore consider instead the radiated power per unit length of the filament:

$$\frac{dI_{\theta,\varphi}}{dL}d\theta d\varphi=\frac{(\Delta\varepsilon)^2\omega_0^2\sigma^2k^2k_0^2\sin^3\theta d\theta d\varphi\delta(k_z-k_0+k_x\tan\beta)}{4\sqrt{\varepsilon_0(\omega_0)}\varepsilon_0^2(0)c(k^2+k_0^2-2kk_0\cos\theta)^2\cos\beta}, \qquad (12)$$

where $k_z=k\cos\theta$ and $k_x=k\sin\theta\cos\varphi$.

It can be seen from (12) that the filament has the remarkable property at infinity that radiation is only possible when $k_z-k_0+k_x\tan\beta=0$. Upon further consideration, this can be shown to be equivalent to requiring that the velocity of the intersection between the permittivity wave and the filament, which is equal to $\omega_0/(k_0\cos\beta)$, be greater than the phase velocity of the emitted wave, ω_0/k. This is particularly obvious for a permittivity wave that propagates along the filament ($\beta=0$). The radiation requirement is then satisfied when

$$\cos\theta=k_0/k=c/\sqrt{\varepsilon_0}\,v_\varepsilon, \qquad (13)$$

where v_ε is the phase velocity of the permittivity wave. Equation (13) is precisely the analog of the condition for Cherenkov radiation.

Omitting the details of a lengthy but straightforward integration over angles θ and φ, we quote the final expression for the power radiated per unit length of the filament:

$$\frac{dI}{dL}=\left(\frac{\Delta\varepsilon}{\varepsilon_0(0)}\right)^2\frac{\sigma^2\omega_0\pi(1+\cos^2\beta)}{4\varepsilon_0(\omega_0)(k^2/k_0^2-1)}. \quad (14)$$

If the permittivity wave is produced by the passage of a high-power laser pulse through a nonlinear medium, and the permittivity depends (in a medium with a center of inversion) on the electric field strength E of the wave as

$$\varepsilon=\varepsilon_0+\varepsilon_2E^2, \quad (15)$$

then the wave vector $\mathbf{k}_0$ and frequency ω_0 of the permittivity wave will be equal to twice the wave vector and frequency of the incident laser wave, and the phase velocity of the permittivity wave will be $c/\sqrt{\varepsilon_0(\omega_0/2)}$. The phase velocity of the radiated wave will then be $c/\sqrt{\varepsilon_0(\omega_0)}$. Thus, the filament invariably radiates in a medium with a normal dispersion law ($k>k_0$), while it only radiates when the "Cherenkov" condition (13) is satisfied in a medium with anomalous dispersion ($k<k_0$).

We can summarize the principal properties of transition scattering by a long charged filament, first of all, by pointing out that it is monochromatic and takes place at the frequency of the permittivity wave (when the permittivity is given by (15), the filament radiates at twice the frequency of the incident laser wave). The radiation is polarized (in the plane defined by vectors $\mathbf{k}$ and $\mathbf{k}_0$) and directional (all radiated wave vectors lie on the generatrices of a right circular cone whose axis coincides with both the z axis and the filament itself). The radiative intensity is not uniformly distributed about the cone, however, since with the exception of the special case $\beta=0$, the problem is not cylindrically symmetric about the axis identified with the filament. The rate at which photons are emitted is independent of ω.

In actual transition scattering experiments, the filament cannot be assumed to be of infinitesimal thickness: its diameter always far exceeds the wavelengths of the incident and scattered waves. The power scattered by a filament of finite thickness should obviously decrease with increasing filament radius R, the physics here being that the static field strength decreases with increasing R.

Calculations based on (9) for a filament of finite radius R show that all features of transition scattering described above are preserved, except that (for large enough R) the radiated power is reduced by a factor proportional to $1/kR$ that is especially simple when $\beta\ll 1$:

$$\frac{dI_R}{dL}=\frac{dI}{dL}\frac{1}{2\pi R\sqrt{k^2-k_0^2}}, \quad (16)$$

where dI/dL is given by (14).

3. EXPERIMENTAL SETUP AND PROCEDURE

To produce the permittivity wave in the medium we employed a $YAlO_3$:Nd^{3+} picosecond laser ($\lambda=1.06$ μm) consisting of an optical cavity, a system to isolate a single pulse, and two amplifiers. We used a spatial filter to improve beam quality between the amplifiers and the pulse isolator. The master oscillator used a traveling-wave ring resonator to improve mode synchronization. The resonator was passively Q-switched by a solution of No. 2972y polymethine dye in a pumped cell. The laser output energy E was approximately 0.1 J per pulse, the duration was $\tau\approx 30$ ps, and the beam diameter was approximately 5 mm.

In this experiment, the system consisting of the medium and the charged filament was modeled as a cylindrical capacitor filled with a nonlinear fluid. The inner plate was a taut conductor of diameter $d\approx 100$ μm; the capacitor length was $l=65$ mm, and its outer diameter was $D\approx 60$ mm. The capacitor faces were made of optical glass and served as the input windows for the laser beam. Using such a system, it was possible to produce electric fields of approximately 10^6 V/m near the filament. The linear charge density in Eq. (14) is given by

$$\sigma=2\pi\varphi\varepsilon_0\varepsilon(0)/\ln(R_2/R_1), \quad (17)$$

where φ is the potential at the capacitor plate, R_1 and R_2 are the radii of the inner and outer capacitor plates, ε_0 is the permittivity of the vacuum, and $\varepsilon(0)$ is the static permittivity of the medium. At $\varphi=11$ kV, the linear charge density is $\sigma\approx 2.6\times 10^{-7}$ C/m. A high-voltage pulse lasting $\tau\approx 50$ ns was applied to the cell, which contained nitrobenzol, at the same time as the pulse of light. The optical spectrum emerging from the cell was observed spectrographically and recorded on photographic film. Energy measurements were made with an FÉU-30 photomultiplier calibrated at the second harmonic of the neodymium laser.

In this experiment we investigated the angular distribution of transition scattering, making use of a set of apertures of various diameters to cover the photomultiplier, which then enabled us to check the validity of the Cherenkov condition on the angular distribution of the scattered radiation [see Eq. (13)]. In the present setup, $\theta=12.6°$.

The foregoing theory [see Eq. (14)] predicts that the scattered power will increase as the nonlinearity of the medium rises [i.e., as ε_2 in (15) increases], which is why we chose nitrobenzol as the nonlinear fluid: it has an ε_2 several times greater than other well-known media. (In our initial experiments[3] we used benzol, which is much less nonlinear than nitrobenzol, and with which we were therefore observing at the limit of our instrumental sensitivity.) Inserting the parameters of the medium into Eq. (14), and using the factor in (16) and the transition-scattered (TS) pulse duration, which is governed by the effective interaction length and the propagation velocity in the nonlinear medium ($\tau_{\rm rad}\approx 130$ ps in the present case), we can easily obtain the dependence of the scattered energy on the laser energy for a given potential difference (specifically, for 11 kV):

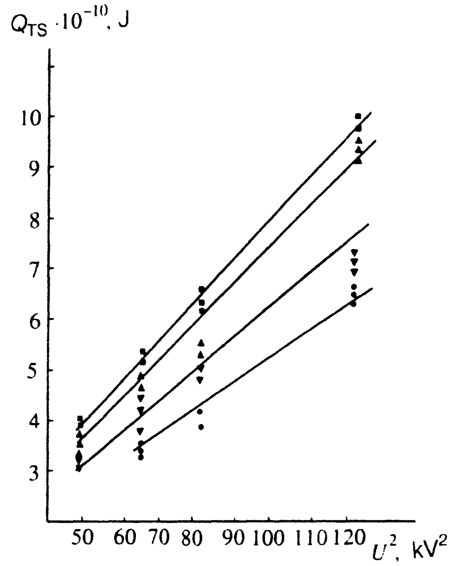

FIG. 2. Dependence of transition-scattered energy on the square of the filament potential, for $Q_L=0.0717$ J (■), $Q_L=0.0696$ J (▲), $Q_L=0.0635$ J (▼), and $Q_L=0.0584$ J (●).

$$Q_{TS}[J]=2.32\cdot 10^{-7}Q_L^2[J]. \quad (18)$$

When we take reflection and absorption losses in the optical elements of the detection system into account, (18) becomes

$$Q_{TS}[J]=0.85\cdot 10^{-7}Q_L^2[J]. \quad (19)$$

In these experiments, we studied the dependence of the scattered energy on the linear charge density of the filament and the laser pulse energy. The results have been plotted in Figs. 2 and 3.

4. DISCUSSION OF EXPERIMENTAL RESULTS. COMPARISON WITH THEORY

We can summarize our experimental results as follows.

1. Application of an electric field gives rise to a polarized component of the transition-scattered radiation at the frequency of the second harmonic, with directional properties described by (13).

2. The radiated energy is a quadratic function of the charge density on the filament. In particular, at a potential of approximately 11 kV and a laser energy of approximately 1 J, the total scattered energy is $\approx 2\cdot 10^{-9}$ J.

3. The scattered energy is proportional to the square of the laser energy producing the permittivity wave.

4. In addition to transition-scattered radiation, we also see unpolarized cooperative scattering of the laser radiation at the second harmonic,[6] which has a different angular spread than transition scattering, thereby allowing the effects to be discriminated spatially.

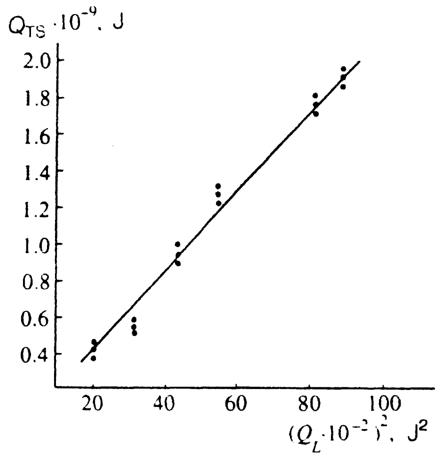

FIG. 3. Dependence of transition-scattered energy on the square of the laser energy at $U=11$ kV.

It is worth mentioning that the scattered energy is roughly twice the theoretical value suggested by Eq. (19), possibly due to self-focusing of the laser radiation, which would increase the intensity and thus the nonlinearity of the medium.

5. CONCLUSION

In this paper we have presented a theoretical analysis of the possibility that electromagnetic radiation may be generated when a permittivity wave is incident upon a stationary charged source. We experimentally investigated the transition scattering of laser radiation by a charged filament in a nonlinear medium. Our results (in particular, the dependence of the scattered energy on the filament's charge density) leave no doubt that we have confirmed the feasibility of transition scattering from an arbitrary charged source.

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