Current–voltage characteristic of an inhomogeneous tunnelingjunction in the above-threshold region $eV>\Delta_1+\Delta_2$

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(Submitted 16 March 1994)
Zh. Eksp. Teor. Fiz. **105**, 1814–1820 (June 1994)

We show that a region of negative differential resistance can appear on the current–voltage characteristic of a tunneling junction manufactured from inhomogeneous superconductors. What is important is that with such a hysteresis mechanism the right end of the region of negative differential resistance always proves to be displaced to the right of the threshold by a distance considerably greater than the "step" on the current–voltage characteristic.

1. INTRODUCTION

At low temperatures the current–voltage characteristic of a tunneling junction[1,2] can differ considerably from that given by an idealized model based on the tunneling Hamiltonian in the second-order perturbation theory in the barrier penetrability and on the BCS model.[3] Describing the experimental data requires allowing both for higher-order terms in the perturbation-theory expansion in powers of barrier penetrability and for the various depairing mechanisms, which smear out the square-root singularity in the density of states of single-particle excitations. Moreover, inhomogeneities, always present in superconductors, must be taken into account.[4]

Two-particle tunneling causes steps to appear on the current–voltage characteristic of the junction at voltages V that obey the condition $eV=\Delta_{1,2}$, where Δ_1 and Δ_2 are the order parameters in the first and second superconductors, respectively.[5,6] Allowing for the depairing mechanisms in the self-consistent field approximation makes the width $E_{1,2}$ of the gap in the single-particle excitation spectrum less than $\Delta_{1,2}$ and smears out all singularities on the current–voltage characteristic. Allowing for inhomogeneities also reduces the density of states at energies $\varepsilon_{1,2}$ less than $E_{1,2}$ and changes the form of the density of states in the energy range $\varepsilon_{1,2}>E_{1,2}$ (see Ref. 4). Although these corrections are small, they nevertheless lead to new phenomena and are therefore considered below.

2. THE CURRENT-VOLTAGE CHARACTERISTIC IN THE REGION ABOVE THE BARRIER

At low temperatures ($T\ll\Delta_{1,2}$), the current–voltage characteristic of a tunneling junction is specified by the following formula:

$$-eR_Nj=\int_0^{eV}d\omega\,\mathrm{Re}\,\alpha_1(\omega)\,\mathrm{Re}\,\alpha_2(eV-\omega),\tag{1}$$

where R_N is the junction resistance in the normal state, and $\alpha_{1,2}$ are the retarded Green's functions integrated with respect to the energy variable.[3] In the self-consistent field approximation and in inhomogeneous superconductors, α near the single-particle excitation threshold satisfies a cubic equation.[4] To study the above-barrier region we must refine this equation so that it contains correction terms quadratic in the inhomogeneities, terms emerging from the low-momentum region.[4]

Following Ref. 4, we arrive at the following system of equations for the Green's functions α and β averaged over the inhomogeneity distribution:

$$\alpha^2-\beta^2=1-\frac{M_0\Delta^2\alpha^2}{8\pi}\exp\left(\frac{i\pi}{4}\right)\left(\frac{6}{vl_{\mathrm{tr}}}\right)^{3/2}(\omega\alpha-\Delta\beta)^{-1/2},$$
$$\alpha\Delta-\omega\beta-i\alpha\beta\Gamma+\frac{M_0\Delta^2\alpha\beta}{4\pi}\left(\frac{6}{vl_{\mathrm{tr}}}\right)^{3/2}\times\exp\left(\frac{i\pi}{4}\right)(\omega\alpha-\Delta\beta)^{1/2}=0.\tag{2}$$

Here Δ is the average value of the order parameter in the superconductor, v is the electron velocity on the Fermi surface, and l_{tr} is the electron transit length (free path length).

For a weakly inhomogeneous superconductor we can write the order parameter $\Delta(r)$ as

$$\Delta(r)=\Delta+\Delta_1(r),\tag{3}$$

with $\langle\Delta_1(r)\rangle=0$.

The correlation function of the random quantity $\Delta_1(\mathbf{r})$ can be written as

$$\langle\Delta_1(r)\Delta(r_1)\rangle=\Delta^2\int\frac{d^3\mathbf{k}}{(2\pi)^3}M_k\exp\{i\mathbf{k}(\mathbf{r}-\mathbf{r}_1)\}.\tag{4}$$

The function M_k depends on the type of inhomogeneities and their distribution.[4] According to Ref. 4, the parameters M_0 and Γ in Eq. (2) are

$$M_0=M_{k=0},\quad\Gamma/\Delta=\frac{3\Delta}{\pi^2vl_{\mathrm{tr}}}\int_0^\infty d\mathbf{k}H_k.\tag{5}$$

 1063-7761/94/060982-04$10.00

In deriving Eq. (2) we have assumed that the electron mean free path in a superconductor is short:

$$l_{tr} \ll v/\Delta . \tag{6}$$

Let us find the current–voltage characteristic of the junction in the above-barrier region,

$$\Delta_{1,2} \gg eV-\Delta_1-\Delta_2 \gg \Delta(\Gamma/\Delta)^{2/3}. \tag{7}$$

As shown below, it is in this region that inhomogeneities can give rise to an essentially new phenomenon, a region of negative differential resistance. To obtain the current–voltage characteristic in the region defined in (7) we need only to solve the system of equations (2) using perturbation-theory techniques. As a result of simple calculations we get

$$\alpha=\frac{\omega}{(\omega^2-\Delta^2)^{1/2}}[1+\alpha^{(1)}(\omega)], \tag{8}$$

$$\alpha^{(1)}=-\frac{i\Gamma}{\Delta}\left(\frac{\Delta^2}{\omega^2-\Delta^2}\right)^{3/2}\frac{3M_0\Delta^4\exp(i\pi/4)}{16\pi(\omega^2-\Delta^2)^{5/4}}\left(\frac{6}{vl_{tr}}\right)^{3/2}.$$

Combining (1) and (8), we get

$$-eR_Nj=\frac{\pi\sqrt{\Delta_1\Delta_2}}{2}+\frac{3\pi}{16}(eV-\Delta_1-\Delta_2)\frac{\Delta_1+\Delta_2}{\sqrt{\Delta_1\Delta_2}}+I,$$

$$I=\frac{1}{4}\left(\int\frac{d\omega\sqrt{\Delta_1\Delta_2}}{\sqrt{\omega-\Delta_1}\sqrt{eV-\omega-\Delta_2}}\cdot\alpha_1^{(1)}(\omega)+(1\rightleftharpoons 2)\right). \tag{9}$$

Here and Δ_1 and Δ_2 are the average values of the order parameters in the first and second superconductors, respectively. The integral is evaluated along a contour that starts at the point $\omega = eV - \Delta_2 - i\delta$ and bypasses the singularities of the function α in the neighborhood of the point $\omega = \Delta_1$. The symbol $(1\leftrightarrow 2)$ indicates that we must add to the integral written explicitly the same integral with the indices 1 and 2 interchanged. Evaluating the integral in (9), we get

$$I=\frac{9\sqrt{3}\sqrt{\Delta_1\Delta_2}}{32\cdot 2^{1/4}\pi(eV-\Delta_1-\Delta_2)^{5/4}}\left[\Delta_1^{5/4}\left(\frac{\Delta_1}{(vl_{tr})_1}\right)^{3/2}M_0^{(1)}+\Delta_2^{5/4}\left(\frac{\Delta_2}{(vl_{tr})_2}\right)^{3/2}M_0^{(2)}\right]\times B\left(\frac{1}{2},-\frac{3}{4}\right)-\frac{\pi\sqrt{\Delta_1\Delta_2}(\Gamma_1\sqrt{\Delta_1}+\Gamma_2\sqrt{\Delta_2})}{8\sqrt{2}(eV-\Delta_1-\Delta_2)^{3/2}}, \tag{10}$$

where $B(x,y)$ is the Euler function. From Eqs. (9) and (10) we can easily derive the condition for the appearance of a region of negative differential resistance on the current–voltage characteristic of a tunneling junction. To do this, we introduce the dimensionless parameters $\kappa_{1,2}$, which characterize the distribution of inhomogeneities in a superconductor:

$$\kappa_{1,2}^{5/4}=\frac{48\pi}{5M_0^{(1,2)}}\left[\sqrt{\frac{2}{3}}\left(\frac{vl_{tr}}{3\Delta}\right)_{1,2}\right]^{3/2}. \tag{11}$$

If we combine this with Eq. (10) for I, we arrive at a simpler equation for the current:

$$I=\frac{3\cdot 3^{1/4}}{5\sqrt{2}}B\left(\frac{1}{2},-\frac{3}{4}\right)\sqrt{\Delta_1\Delta_2}\left\{\frac{1}{\kappa_1^{5/4}}\left(\frac{\Delta_1}{eV-\Delta_1-\Delta_2}\right)^{5/4}+\frac{1}{\kappa_2^{5/4}}\left(\frac{\Delta_2}{eV-\Delta_1-\Delta_2}\right)^{5/4}\right\}-\frac{\pi\sqrt{\Delta_1\Delta_2}(\Gamma_1\sqrt{\Delta_1}+\Gamma_2\sqrt{\Delta_2})}{8\sqrt{2}(eV-\Delta_1-\Delta_2)^{3/2}}. \tag{12}$$

The region of negative differential resistance exists if

$$\frac{3\pi}{16}\frac{\Delta_1+\Delta_2}{\sqrt{\Delta_1\Delta_2}}+\left(\frac{\partial I}{\partial eV}\right)_{eV_0}\leqslant 0, \tag{13}$$

where eV_0 is the extremal point of $(\partial I/\partial eV)$, that is, the point at which

$$\left(\frac{\partial^2 I}{\partial(eV)^2}\right)_{eV_0}=0. \tag{14}$$

Combining (12) and (14), we find the position of the extremal point eV_0:

$$eV_0-\Delta_1-\Delta_2=\left\{\frac{5\pi}{18\cdot 3^{1/4}B(1/2,-3/4)}\times\frac{\Gamma_1\sqrt{\Delta_1}+\Gamma_2\sqrt{\Delta_2}}{[(\Delta_1/\kappa_1)^{5/4}+(\Delta_2/\kappa_2)^{5/4}]}\right\}^4. \tag{15}$$

Substituting this into Eq. (13), we arrive at an inequality that is the condition for the appearance of a region of negative differential resistance:

$$[\Gamma_1\Delta_1^{1/2}+\Gamma_2\Delta_2^{1/2}]\left[\frac{5\pi}{18\cdot 3^{1/4}B(1/2,-3/4)}\times\frac{\Gamma_1\Delta_1^{1/2}+\Gamma_2\Delta_2^{1/2}}{(\Delta_1/\kappa_1)^{5/4}+(\Delta_2/\kappa_2)^{5/4}}\right]^{-10}\geqslant 9\sqrt{2}\frac{\Delta_1+\Delta_2}{\Delta_1\Delta_2}. \tag{16}$$

3. THE GAP IN THE SINGLE-PARTICLE EXCITATION SPECTRUM

The Green's functions α and β in the self-consistent field approximation are given by Eqs. (2). In this approximation the single-particle excitation spectrum contains a gap of width E. When $0<\omega<E$ holds, both α and β are purely imaginary and we can assume

$$\alpha=-ix,\quad \beta=-iy,\quad \omega\alpha-\Delta\beta=i\Delta z^2. \tag{17}$$

Suppose that the parameters Δ/Γ and κ are large, that is,

$$\{\Delta/\Gamma,\kappa\}\gg 1. \tag{18}$$

In this case, near the threshold the functions X and Y satisfy the inequality

$$\{x,y\}\gg 1. \tag{19}$$

Using this inequality and Eqs. (2), we arrive at the following system of equations for the functions x and y at the threshold:

$$\gamma x^2 z^2 + \gamma^2 \frac{x^2}{8z^3} + \frac{\gamma}{4z^2} + \frac{4z}{x} - 2z^3 = 0,$$
$$\frac{1}{x} + \frac{\gamma x}{2z} + (\Gamma/\Delta)x^2 - \gamma z x^2 - z^2 = 0, \tag{20}$$

where the dimensionless parameter γ is given by the formula

$$\gamma = \frac{M_0}{4\pi}\left(\frac{6\Delta}{v l_{tr}}\right)^{3/2} = \frac{24\sqrt{2}}{5\kappa^{5/4}}\left(\frac{2}{3}\right)^{3/4}. \tag{21}$$

Equations (17), (20), and (21) determine the values of the Green's functions α and β at the threshold point E, and also the location of this point.

Let us examine the most interesting region of small values of parameter γ,

$$\gamma \ll (\Gamma/\Delta)^{5/6}. \tag{22}$$

If this condition is met, we can employ perturbation theory to solve the system of equations (20). Also, the values of parameter κ happen to lie inside the range of values for which condition (16) for the appearance of a hysteresis loop on the current–voltage characteristic is satisfied. By means of simple calculations we find

$$E = \Delta\left\{1 - \frac{3}{2}\left(\frac{\Gamma}{\Delta}\right)^{2/3} + \frac{42}{5\kappa^{5/4}}\left(\frac{2}{3}\right)^{3/4}\left(\frac{\Delta}{\Gamma}\right)^{1/6}\right\},$$
$$\alpha(E) = -i\left(\frac{\Delta}{\Gamma}\right)^{1/3}\left[1 + \frac{41\gamma}{48\sqrt{2}}\left(\frac{\Delta}{\Gamma}\right)^{5/6}\right], \tag{23}$$
$$z(E) = \sqrt{2}\left(\frac{\Gamma}{\Delta}\right)^{1/6}\left[1 - \frac{31\gamma}{192\sqrt{2}}\left(\frac{\Delta}{\Gamma}\right)^{5/6}\right].$$

4. THE RANGE OF VOLTAGES BELOW THE THRESHOLD

Fluctuations in the inhomogeneity distribution also lead to the appearance of a density of states in the energy region[4]

$$\Delta - E \ll E - \omega \ll \Delta. \tag{24}$$

The density of states here is determined by the same parameters Γ/Δ and κ as in the above-barrier region. We write it in the form[7]

$$\operatorname{Re}\alpha(\omega) = f_1 = 2.48\left(\frac{3}{2}\right)^{3/8}\left(\frac{5}{48\kappa\pi}^{1/2}\right)\left(\frac{\Delta}{\Gamma}\right)^{2/3}$$
$$\times\left[\left(\kappa\frac{(E-\omega)}{\Delta}\right)^2 + A\right]^{9/16}$$
$$\times\left[-\left(\left(\kappa\frac{E-\omega}{\Delta}\right)^2 + A\right)^{5/8}\right] 0 < E-\omega, \tag{25}$$

with $0 < E-\omega$. Here the dimensionless parameter A is of order unity and must be considered an adjustable parameter, so that Eq. (25) can give the best approximation for the density of states in the entire energy range $0 < E-\omega \ll \Delta$.

Equations (2) and (25) make it possible to derive a good interpolation formula for the density of states, suitable for all values of ω and exact for energies far from the threshold:[7]

$$\operatorname{Re}\alpha(\omega) = \begin{cases} f_1(\omega), & \omega < E, \\ f_1(\omega) + f_2(\omega), & \omega > E, \end{cases} \tag{26}$$

where $f_1(\omega)$ is an even function of the argument $E-\omega$, and for $E-\omega>0$ is given by Eq. (25). The function $f_2(\omega)$ is the real part of the function $\alpha(\omega)$ defined in Eqs. (2).

Comparison of the experimental data with the current–voltage characteristic determined via Eqs. (1), (2), (25), and (26) allows an accurate determination of the parameters Γ/Δ, κ, and A of a superconductor.[7]

Below we give an expression for the current, valid for voltages obeying the inequality

$$\Delta_1 + \Delta_2 - (E_1 + E_2) \ll (E_1 + E_2) - eV. \tag{27}$$

Combining Eqs. (1) and (25), for a symmetric junction we get

$$-eR_N j = \frac{0.877\Delta}{\kappa^2}\left(\frac{\Delta}{\Gamma}\right)^{4/3}\left[\left(\kappa\frac{E - eV/2}{\Delta}\right)^2 + A\right]^{21/16}$$
$$\times\exp\left[-2\left(\left(\kappa\frac{E - eV/2}{\Delta}\right)^2 + A\right)^{5/8}\right]. \tag{28}$$

Comparison of the experimental data with the formula (28) allows an accurate determination of the parameter κ.

5. CONCLUSION

For inhomogeneous superconductors the density of states is determined with good accuracy by the self-consistent field approximation. The single-particle excitation spectrum acquires a gap of width E in this approximation, with E smaller than the order parameter Δ. In weakly inhomogeneous superconductors the shift $\delta_{1,2} = \Delta_{1,2} - E_{1,2}$ of the edge of the spectrum is small.

Fluctuations in the distribution of inhomogeneities also result in the appearance of a finite density of states for energies lower than E.

The current–voltage characteristic of a tunneling junction near the threshold $eV \approx E_1 + E_2$ makes it possible to determine the parameters Γ/Δ and κ characterizing the magnitude and distribution of inhomogeneities. The density of states and the current–voltage characteristic of the tunneling junction depend on various combinations of Γ/Δ and κ. As a result the current–voltage characteristic of the junction in the above-barrier region $eV - E_1 - E_2 > \delta_1 + \delta_2$ can be found via perturbation-theory techniques. This suffices to clarify the question of the appearance of a section with negative differential resistance and of its size. If the appearance of a section with negative differential resistance is due to inhomogeneities in the superconductors, the right edge of this section is shifted to the right by an amount exceeding the shift $\delta_{1,2}$ of the edge of the spectrum by a factor on the order of the large parameter. When the section with negative differential resistance appears this parameter is of the order of $\kappa^{10/27}$.

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Translated by Eugene Yankovsky

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