Two-photon Bragg resonance and interference of atoms in separated light fields

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The two-photon Bragg resonance appearing when a beam of atoms with angular momenta of the working levels $j_a=0$ and $j_b=1$ is scattered in the field of a linearly polarized standing wave and the possibilities of using it in atom interferometry have been investigated. The specific role of spontaneous relaxation in shaping the spatial coherence in an initially incoherent atomic beam has been ascertained. The density modulation depth and the narrow resonance structure in the momentum distribution of particles scattered in separated light fields have been found.

1. INTRODUCTION

The interferometer scheme proposed several years ago in Ref. 1, which was based on forces of resonant light pressure and demonstrated the possibility of observing the interference of de Broglie waves associated with the quantum character of the translational motion of heavy neutral particles, has recently been realized experimentally in numerous variations.[2,3] Atomic interference is now also of great interest as a fundamental physical phenomenon and as a new and extremely sensitive tool for precision physical measurements.[4–6] The characteristic energy scale of the quantum effects observed when resonant atoms interact with a light field is the recoil energy $\varepsilon_r=k^2/2M$ associated with the absorption or emission of a photon. The smallness of this quantity, which takes on a value of 10^{-10} eV for optical transitions in sodium-type atoms, is responsible for the high accuracy of interferometric measurements.

Dubetskiĭ *et al.*[7,8] considered an atom interferometer, in which two-photon Bragg resonance served as the physical mechanism for splitting and mixing coherent atomic waves.[9] Such a primarily quantum situation is realized when atoms interact with the field of a weak standing wave for long times ($\varepsilon_r\tau\gg 1$). One distinguishing feature of Bragg resonance is its high selectivity with respect to the transverse momenta of the particles, under which scattering is effective only near certain (Bragg) angles of incidence of the atoms in the field.

In ordinary atomic beams with appreciable angular divergence, the scattered atomic waves overlap, and interference is manifested as spatial density modulation with a period equal to half of the optical wavelength. The influence of the large phase differences due to the Doppler shift and recoil, which rapidly destroys the interference pattern, can be eliminated in the case of scattering in two spatially separated light beams under echo conditions[1,7,8] (see Fig. 1). The first field splits the coherent atomic waves, and after diffraction in the field of the second beam, they mix again. The echo condition corresponds to compensation of the phase differences associated with the Doppler shift and recoil along the trajectories shown in Fig. 1. This is actually an atomic analog of an optical interferometer. Incidentally, the echo effect and interference of particles was recently discussed in Ref. 10. As was mentioned in Refs. 7 and 8, the realization of this scheme under the conditions of a coherent interaction runs into definite difficulties.

Spontaneous relaxation plays a major role under the conditions of a prolonged interaction of atoms with a field. As was shown in Refs. 11 and 12, relaxation is manifested in two forms in Bragg scattering. The diffractive (coherent) scattering, which now occurs in an effective complex potential, varies due to the finite width of the upper working level. This turned out to be the most significant factor for the appearance of spatial coherence in an initially incoherent atomic beam with a symmetric distribution of transverse momenta as a result of an interaction with a standing wave. In addition, incoherent scattering appears due to the recoil effect. In the simplest two-level scheme this situation results in strong suppression of the density modulation. An hypothesis was advanced in Ref. 12 that the situation with Bragg resonance in a self-bleaching transition is optimal. The present work focused on a detailed analysis of such a process in the case of an atom with a ground-state angular momentum $j_a=1$ and an excited-state angular momentum $j_b=0$ in a linearly polarized standing wave, as well as its application in atom interferometry.

2. ORIGINAL EQUATIONS

Let an atomic beam propagating in y be scattered by a linearly polarized (in z) standing light wave

$$E_z(\mathbf{r}t)=E(y)\cos(kx)\exp[-i(\Delta+\omega_{ab})t] \quad (1)$$

with a small detuning Δ from the frequency of the resonant atomic transition ω_{ab}. The function $E(y)$ differs appreciably from zero in a region with a characteristic width a and describes the distribution of the field in a cross section of the light beam.

In the stationary formulation of the scattering problem the behavior of an atom is described by the density matrix

$$\rho_{\alpha\beta}(x_1,x_2;y=v_yt,p_y)=\rho_{\alpha\beta}(x_1x_2t)F(p_y),$$
$$\alpha,\beta=(a,\mu),(b,m), \quad (2)$$

in which "a" refers to the lower (ground) state with an angular momentum $j_a=1$ and projections $\mu=0,\ \pm 1$, and

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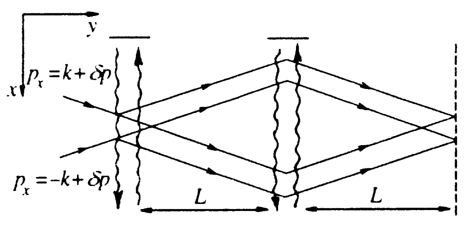

FIG. 1. Atom interferometer. Two-photon Bragg scattering of an atomic beam in spatially separated light fields. Complete cancellation of the phase shifts associated with the Doppler effect and recoil occurs along the trajectories shown.

"b" denotes the upper working level with an angular momentum $j_b=0$ and a projection $m=0$ (Fig. 2). The Wigner representation was used for the variables associated with longitudinal motion. The characteristic longitudinal momenta of ordinary hot beams significantly exceed the momentum of the resonant photon k, so that their variation during the interaction may be neglected, and it may be assumed that the longitudinal trajectory of the particles is linear and that $F(p_y)$ coincides with the distribution in the incident atomic beam. Thus, the density matrix $\rho(12)\equiv\rho(x_1x_2t)$ describes the one-dimensional transverse (parallel to the x axis) scattering of the particles caused by the recoil effect in induced and spontaneous transitions and satisfies the following system of equations (see, for example, Ref. 9):

$$i\left[\frac{\partial}{\partial t}\rho^{aa}_{\mu\mu'}(12)-\gamma_{\mu\mu'}(1-2)\rho^{bb}(12)\right]=(\hat{T}_1-\hat{T}_2)\rho^{aa}_{\mu\mu'}(12)-V^*_\mu(1)\rho^{ba}_{\mu'}(12)+\rho^{ab}_\mu(12)V_{\mu'}(2), \tag{3}$$

$$i\left(\frac{\partial}{\partial t}+\gamma\right)\rho^{bb}(12)=(\hat{T}_1-\hat{T}_2)\rho^{bb}(12)-V_\mu(1)\rho^{ab}_\mu(12)+\rho^{ba}_\mu(12)V^*_{\mu'}(2), \tag{4}$$

$$i\left(\frac{\partial}{\partial t}+\nu\right)\rho^{ba}_\mu(12)=(\hat{T}_1-\hat{T}_2)\rho^{ba}_\mu(12)-V_{\mu'}(1)\rho^{aa}_{\mu'\mu}(12)+\rho^{bb}(12)V_\mu(2),$$
$$\rho^{ab}_\mu(12)=[\rho^{ba}_\mu(12)]^*. \tag{5}$$

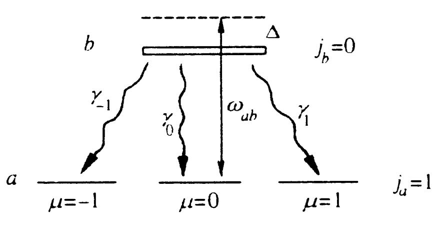

FIG. 2. Scheme of levels.

Here the derivative with respect to time is taken along the longitudinal trajectory of the particle ($\partial/\partial t=v_y\partial/\partial y$), $\nu=\Delta+i\gamma/2$, γ is the total width of the upper level associated with spontaneous decay to the ground state, and $\hat{T}_{1,2}=(-1/2M)\partial^2/\partial x^2_{1,2}$ are the operators of the transverse kinetic energy of the inversion center of an atom with a mass M. The interaction with a standing wave was written in the resonance approximation and has the form

$$V_\mu[1(2)]=V[1(2)]\delta_{\mu 0},\quad V[1(2)]=V(t)\cos kx_{1,2}, \tag{6}$$

$$V(t)=d^z_{00}E(y=v_yt),$$

where $d^i_{0\mu}=\langle j_b=0|\hat{d}_i|j_a=1;\mu\rangle$ is the matrix element of the longitudinal transition.

The relaxation matrix $\gamma_{\mu\mu'}(1-2)\equiv\gamma_{\mu\mu'}(x_1-x_2)$ describes the transverse recoil of an atom upon spontaneous transitions to sublevels of the ground state. Its Fourier components are clearly nonzero only in the interval $|q|\leqslant k$:

$$\gamma_{\mu\mu'}(x)=k^2d^i_{\mu 0}d^j_{0\mu'}\int_{-k}^{k}dq\kappa_{ij}(q)\exp(iqx). \tag{7}$$

The tensor $\kappa_{ij}(q)$ is proportional to the photon polarization density matrix $\delta_{ij}-n_in_j$ (where $\mathbf{n}$ is the direction of the momentum of the quantum) after averaging over all directions of emission in the (y,z) plane, which is equivalent to taking into account recoil only parallel to the x axis. Therefore, κ_{ij} is a diagonal matrix

$$\kappa_{xx}=1-q^2/k^2,\quad \kappa_{yy}=\kappa_{zz}=\tfrac{1}{2}(1+q^2/k^2),\quad |q|\leqslant k \tag{8}$$

with axial symmetry.

As in Refs. 7, 8, and 12, we consider a primarily quantum regime for the scattering of atoms by the field of a weak standing wave acting for a long time $\tau=a/v_y$

$$\frac{1}{\tau},\ \frac{|V|^2}{|\nu|}\ll\varepsilon_r=\frac{k^2}{2M}\ll\gamma, \tag{9}$$

under which the conditions of two-photon Bragg resonance are realized.

The transition $j_a=1\to j_b=0$ is self-bleaching. In a linearly polarized wave spontaneous relaxation results in optical pumping of the ground-state sublevels with $\mu=\pm1$, which clearly do not interact with the external field. Therefore, the number of spontaneous transitions N^1_s during the interaction must not be great:

$$N^1_s=2\gamma_1\tau w\leqslant 1. \tag{10}$$

Here $w\sim|V/\nu|\ll 1$ is the population of the upper level, and γ_1 and γ_{-1} are, respectively, partial widths associated with radiative decays to the sublevels with $\mu=\pm1$. The partial width associated with the spontaneous transition to the $\mu=0$ sublevel is denoted by γ_0, so that the total width of the upper level is $\gamma=\gamma_1+\gamma_{-1}+\gamma_0$. Although $\gamma_1=\gamma_{-1}=\gamma_0=\gamma/3$ in the specific situation under consideration, to keep the picture clear we shall retain the individual notations for the respective partial widths below.

When conditions (9) are satisfied, Eqs. (4) and (5) are easily solved, as was done in Ref. 12. First, thanks to the inequality $\gamma\tau\gg 1$, the quasistationary approximation can be used in Eqs. (4) and (5), and the derivative with respect to time may be neglected. Second, the only particles which are effectively scattered in such weak fields have a transverse kinetic energy of the order of the recoil energy: $\varepsilon(p_x)=p_x^2/2M\sim\varepsilon_r\ll\gamma$. The latter inequality is adequately satisfied for atoms with strong transitions. This permits the omission of $\hat{T}_{1,2}$ in Eqs. (4) and (5). Finally, taking into account the population of the upper level from perturbation theory, we obtain

$$\rho_\mu^{ba}\simeq-\frac{V(1)}{\nu}\rho_{0\mu}^{aa},\quad \rho_\mu^{ab}\simeq-\frac{V(2)}{\nu^*}\rho_{\mu0}^{aa},$$

$$\rho^{bb}\simeq\frac{V(1)V(2)}{|\nu|^2}\rho_{00}^{aa}. \tag{11}$$

Substituting these expressions into (3), we obtain an equation for the atomic density matrix $\rho_{\mu\mu'}^{aa}(12)\equiv\rho_{\mu\mu'}(12)$ in the ground state

$$i\frac{\partial}{\partial t}\rho_{\mu\mu'}(12)$$

$$=(\hat{T}_1-\hat{T}_2)\rho_{\mu\mu'}(12)+\frac{V^2(1)}{\nu}\delta_{\mu0}\rho_{0\mu'}(12)$$

$$-\frac{V^2(2)}{\nu^*}\rho_{\mu0}(12)\delta_{0\mu'}+\frac{V(1)V(2)}{|\nu|^2}\rho_{00}(12). \tag{12}$$

With no loss of generality, the field $V(t)$ was assumed to be real in expressions (11) and (12).

The term with the transverse kinetic energy plays a significant role in Eq. (12). If $p_x=p\approx k$, $kv_x\tau\sim\varepsilon_r\tau\gg1$, i.e., during the interaction the atom traverses a distance along the standing wave greater than the spatial period of the field. Just this circumstance ensures the narrowness of the Bragg resonance and the high selectivity of the scattering with respect to the transverse momenta of the atoms. The second and third terms on the right-hand side of Eq. (12) describes the interaction of an atom in the ground state with the effective complex potential[11]

$$U(xt)=\frac{V^2(t)}{\Delta+i\gamma/2}\cos^2 kx, \tag{13}$$

which appears as a result of induced transitions between the ground-state $\mu=0$ sublevel and the excited state under the action of the external field. In a weak field the upper level acts as an intermediate state with a finite lifetime.

To complete the discussion we briefly recall the properties of a two-photon Bragg resonance.[9] A diffraction (Bragg) pattern appears in the scattering due to the fact that coherent mixing of states with momenta differing by $2k$ occurs in periodic potential (13). In a weak field satisfying condition (9) there is effective mixing only of states with the momenta $p=\pm k+\delta p$ and similar kinetic energies

$$\Omega=\varepsilon(k+\delta p)-\varepsilon(-k+\delta p)=\frac{2k\delta p}{M}\sim\frac{1}{\tau}\ll\varepsilon_r, \tag{14}$$

so that the width δp of the two-photon Bragg resonance

$$\delta p/k\sim1/\varepsilon_r\tau\ll1 \tag{15}$$

becomes smaller than the momentum of the photon when the interaction time is long. We note that Bragg scattering results in the appearance of coherence between states with momenta equal to $\pm k+\delta p$ in an initially incoherent atomic beam.

The last term on the right-hand side of Eq. (12) is associated with a process involving an induced transition of the atom to the excited state followed by spontaneous decay to one of the ground-state sublevels. Therefore, only the diagonal element $\rho_{00}(12)$ of the density matrix appears in this term. Recoil due to spontaneous emission results in incoherent mixing of states with momenta in a range of width $2k$.

It can be seen from (12) that an independent equation is observed for $\rho_{00}(12)$ and that $\rho_{00}(12)$ appears as a source in the equations for the remaining elements of the density matrix.

In the usual situation it may be assumed that the incident atomic beam lacks coherence (both for the internal states and for states of translational motion), that the sublevels are equally populated, and that the density of the particles is constant. Then the initial condition for Eq. (12) has the form

$$\rho_{\mu\mu'}(x_1x_2,t=-\infty)=F_0(x_1-x_2)\delta_{\mu\mu'}. \tag{16}$$

The Fourier component $F_0(p)$ describes the initial distribution with respect to the transverse momenta and has, for example, a Gaussian form with a characteristic width p_0, which is smaller than p_y, but significantly greater than the momentum of the photon in an ordinary atomic beam ($k\ll p_0\ll p_y$).

Equation (12) is invariant with respect to the translational transformation

$$x_{1,2}\to x_{1,2}+\pi/k, \tag{17}$$

which likewise does not alter initial condition (16). Therefore, the solution may be represented in the form of a mixed expansion in a series and a Fourier integral:[12]

$$\rho_{\mu\mu'}(x_1x_2t)=\int\frac{dp}{2\pi}\sum_s\rho_s^{\mu\mu'}(p,t)\exp[ip(x_1-x_2)$$

$$+isk(x_1+x_2)]. \tag{18}$$

We also have a useful relation between the harmonic amplitudes $\rho_s(p)$ and the Fourier components $\rho(p_1,p_2)$:

$$\rho(p+q/2,p-q/2)=2\pi\sum_s\rho_s(p)\delta(q-2sk). \tag{19}$$

EQUATIONS FOR HARMONICS

To describe the lowest-order Bragg resonance, it is sufficient to retain the harmonics with $s=0$, ±1 in expansion (18). Then, for the harmonic $\rho_s^{00}(pt)\equiv n_s(pt)$ we have the following system of equations:

$$i\frac{\partial}{\partial t}n_0(pt)=4i\,\mathrm{Im}\,U(t)n_0(pt)$$

$$+U(t)[n_1(p+k,t)+n_{-1}(p-k,t)]$$

$$-U^*(t)[n_1(p-k,t)+n_{-1}(p+k,t)]$$

$$-2i\frac{\gamma_0}{\gamma}\,\mathrm{Im}\,U(t)\int_{-k}^{k}dq\varphi_1(q)$$

$$\times[n_0(p-q-k,t)+n_0(p-q+k,t)], \qquad (20)$$

$$i\frac{\partial}{\partial t}n_1(pt)=[\varepsilon(p+k)-\varepsilon(p-k)+4i\,\mathrm{Im}\,U(t)]n_1(pt)$$

$$+U(t)n_0(p-k,t)-U^*(t)n_0(p+k,t)$$

$$-2i\frac{\gamma_0}{\gamma}\,\mathrm{Im}\,U(t)\int_{-k}^{k}dq\varphi_1(q)n_0(p-q,t), \qquad (21)$$

$$n_{-1}(pt)=n_1^*(pt), \quad U(t)=\frac{V^2(t)}{4\nu},$$

$$\varphi_1(q)=\frac{3}{4k}\kappa_{zz}(q).$$

These equations coincide with the equations which were presented in Ref. 12. The functions $n_{\pm1}(pt)$ differ appreciably from zero only in a small neighborhood of the Bragg resonance $p=\delta p\ll k$, where the energy difference $\Omega=\varepsilon(p+k)-\varepsilon(p-k)$ is small. The function $n_0(pt)$ varies significantly over a broad range of momenta. Therefore, only the zeroth harmonic was retained in the integral terms in (20) and (21), and the contribution from $n_{\pm1}(pt)$ is small owing to $\delta p/k$. The initial conditions for the harmonics have the form

$$n_0(p,t=-\infty)=F_0(p), \quad n_{\pm1}(p,t=-\infty)=0. \qquad (22)$$

We also need equations for the harmonics of other elements of the density matrix. For $\rho_s^{-11}(pt)\equiv m_s(pt)$ [and similarly for $\rho_s^{-1-1}(pt)$] we obtain

$$i\frac{\partial}{\partial t}m_0(pt)=-2i\frac{\gamma_1}{\gamma}\,\mathrm{Im}\,U(t)\int_{-k}^{k}dq\varphi_2(q)$$

$$\times[n_0(p-q-k,t)+n_0(p-q+k,t)], \qquad (23)$$

$$i\frac{\partial}{\partial t}m_1(pt)=[\varepsilon(p+k)-\varepsilon(p-k)]m_1(pt)$$

$$-2i\frac{\gamma_1}{\gamma}\,\mathrm{Im}\,U(t)\int_{-k}^{k}dq\varphi_2(q)n_0(p-q,t), \qquad (24)$$

$$m_{-1}(pt)=m_1^*(pt), \quad \varphi_2(q)=\frac{3}{8k}[\kappa_{yy}(q)+\kappa_{xx}(q)].$$

We note that $\mathrm{Im}\,U(t)<0$, and Eq. (23) simply describes the effect of the optical pumping of the sublevels with $\mu=\pm1$.

Finally, the equations for $\rho_s^{-11}\equiv r_s$ differ from (23)–(24) only in that φ_2 is replaced by $\varphi_3=(3/8k)(\kappa_{yy}-\kappa_{xx})$ in the integral terms.

The initial conditions for these equations are $m_0(p,t=-\infty)=F_0(p)$, and all the other quantities are equal to zero.

3. TWO-PHOTON BRAGG RESONANCE

The problem reduces to the solution of system (20)–(21) for the harmonics of density matrix element $\rho_{00}(12)$, which are then employed as the source in the equations like (23)–(24) for the remaining elements. We follow Ref. 12, in which equations similar to (20)–(21) were analyzed in detail.

The harmonic $n_0(pt)$, which is clearly the momentum distribution of the atoms in the $\mu=0$ sublevel [see (19)], is written in the form of a sum of two terms:

$$n_0(pt)=\bar{n}_0(pt)+\delta n_0(pt). \qquad (25)$$

The first term is the "background" on which the Bragg resonance is traced. The behavior of $\bar{n}_0(pt)$ is determined by two factors: damping due to spontaneous decays to other sublevels and diffusion in momentum space due to processes involving incoherent mixing of the states with momenta p and $p\pm k+q$, where $|q|\leqslant k$ is the recoil momentum for a spontaneous transition to the $\mu=0$ sublevel. We assume that the number of such transitions during the interaction is small [in analogy to condition (10)]

$$N_s^0=\gamma_0\tau w\ll 1. \qquad (26)$$

In this case the weak-field diffusional variation of the momentum is of the order of k and scarcely alters the broad initial distribution $F_0(p)$, causing only small corrections of order $(k/p_0)^2\ll 1$. Therefore, for $\bar{n}_0(pt)$ it is sufficient to use the expression[12]

$$\bar{n}_0(pt)\simeq\exp\left[-\frac{\gamma_1}{|\gamma|^2}\int_{-\infty}^{t}V^2(t')dt'\right]F_0(p). \qquad (27)$$

The second term $\delta n_0(pt)$ in (25) describes the contribution from the Bragg resonance caused by the coherent variation of the momentum by a definite amount $2k$ (or $-2k$) and is appreciably nonzero only near $p=\pm k$. After $\bar{n}_0$ has been separated out, the set of remaining functions in Eqs. (20)–(21) $\delta n_0(\pm k+\delta p,t)$ and $n_{\pm1}(\delta p,t)$, which refer to the process of diffractive Bragg scattering per se, may be represented in the form of the matrix

$$\begin{pmatrix}\delta n_0(k+\delta p,t) & n_1(\delta p,t)\\ n_{-1}(\delta p,t) & \delta n_0(-k+\delta p,t)\end{pmatrix}$$

$$=\exp\left[-\frac{\gamma}{2|\nu|^2}\int_{-\infty}^{t}V^2(t')dt'\right]\hat{R}(t). \qquad (28)$$

Then from (20)–(21) for $\hat{R}$ we obtain the inhomogeneous equation

$$i\frac{\partial\hat{R}}{\partial t}=\hat{H}\hat{R}-\hat{R}\hat{H}^{+}+i\hat{\Gamma}(t) \qquad (29)$$

with the non-Hermitian Hamiltonian

$$\hat{H}=\frac{\Omega}{2}\sigma_3+U(t)\sigma_1, \qquad (30)$$

where $U(t)=V^2(t)/4\nu$ is the complex potential, $\Omega=2k\delta p/M$, and σ_1 and σ_3 are Pauli matrices, and with the inhomogeneous term

$$\hat{\Gamma}(t)\simeq-\frac{\gamma_1}{\gamma_0}\frac{dg}{dt}F_0\sigma_1,$$

$$g(t)=\exp\left[\frac{\gamma_0}{2|\gamma|^2}\int_{-\infty}^{t}\right]V^2(t')dt']. \qquad (31)$$

Here it was assumed that $F_0(p)$ is a fairly broad distribution and that it can be replaced by the constant F_0 with accuracy to small corrections of order $(k/p_0)^2$ in the vicinity of $p\sim k$.

We introduce the evolution operator $\hat{S}(t)$, which obeys the equation

$$i\frac{\partial\hat{S}}{\partial t}=\hat{H}\hat{S}. \qquad (32)$$

Then the general solution of Eq. (29) can be written in the form

$$\hat{R}(t)=\hat{S}(t)\hat{R}_0\hat{S}^{+}(t)+\hat{S}(t)\int_{-\infty}^{t}dt'\hat{S}^{-1}(t')\hat{\Gamma}(t')\times[\hat{S}^{+}(t')]^{-1}\hat{S}^{+}(t), \qquad (33)$$

where the first term contains the arbitrary constant matrix $\hat{R}_0$, and the second term is a particular integral of inhomogeneous equation (29).

We represent $\hat{S}(t)$ in the form

$$\hat{S}(t)=\exp\left(-i\frac{\Omega t}{2}\sigma_3\right)\hat{B}(t), \qquad (34)$$

where the exponential function describes free evolution, and the matrix $\hat{B}(t)$ for the Bragg scattering per se[12]

$$\hat{B}(t)=\begin{pmatrix}\alpha(t) & \bar{\beta}(t)\\ \beta(t) & \bar{\alpha}(t)\end{pmatrix},\quad \hat{B}(-\infty)=1, \qquad (35)$$

is built up from the spinor $u(t)=\binom{\alpha(t)}{\beta(t)}$, which obeys the equation

$$i\frac{\partial u}{\partial t}=\exp(i\Omega t\sigma_3)U(t)\sigma_1 u,\quad u(-\infty)=\begin{pmatrix}1\\0\end{pmatrix}. \qquad (36)$$

The equation for the "conjugate" functions $\bar{\alpha}(t)$ and $\bar{\beta}(t)$ are obtained from (36) by means of the replacement $\Omega\to-\Omega$, i.e.,

$$\bar{\alpha}(t;\Omega)=\alpha(t;-\Omega),\quad \bar{\beta}(t;\Omega)=\beta(t;-\Omega). \qquad (37)$$

After the interaction with a field ends as $t\to\infty$, the functions $\alpha(t)$ and $\beta(t)$ take the constant values $\alpha(\infty)\equiv\alpha$ and $\beta(\infty)\equiv\beta$, which specify the complete Bragg scattering matrix $\hat{B}(\infty)\equiv\hat{B}$.

Equations (28) and (33)–(35) solve the problem of Bragg scattering in general form, expressing $\delta n_0(\pm k+\delta p,t)$ and $n_{\pm 1}(\delta p,t)$ in terms of the solutions $\alpha(t)$ and $\beta(t)$ of Eq. (36) and the initial matrix $\hat{R}_0$. Of course, the functions $\alpha(t)$ and $\beta(t)$ for a field with a smooth envelope $V^2(t)$ of arbitrary form can be found only to some approximation, for example by perturbation theory. The explicit forms of $\alpha(t)$ and $\beta(t)$ for the envelope $V^2(t)=V_0^2/\cosh(t/\tau)$, which allows a known exact solution of Eq. (36), were written down in Ref. 12.

The main features of Bragg scattering can, however, be analyzed on the basis of the properties of Eq. (36) without using the explicit form of the solution. Initial condition (22) means that $\hat{R}_0=\hat{R}(-\infty)=0$, and $\hat{R}$ is determined only by the inhomogeneous solution [the second term on the right-hand side of Eq. (33)]. The integration of the equations like (23)–(24) for other elements of the density matrix is elementary. Here it is sufficient to leave only $\bar{n}_0(pt)$ as defined in Eq. (27) in the integral terms in Eqs. (23)–(24).

4. SPATIAL DENSITY MODULATION AND MOMENTUM DISTRIBUTION

The density of atoms (in the ground state in the $\mu=0$ sublevel) at the point $(x,\ y=v_y t)$ is specified by the diagonal element $\rho_{00}(xxt)$. The zeroth harmonic [see (18)] clearly gives a contribution which is not dependent on x, and the $n_{\pm 1}$ harmonics produce spatial oscillations of the density with a period π/k. These oscillations are the result of the interference of coherent atomic waves appearing as a result of diffraction of the particles in the standing light wave.

Using expressions (28) and (33)–(35) and initial condition (22), we obtain the harmonic $n_1(\delta p,y)$ after an interaction with a field (when $t\gg\tau$, i.e., $y\gg a$)

$$n_1(\delta p,y)=\frac{2\gamma_1}{\gamma}F_0\exp(-N_s-i\Omega y/v_y)[(1+\bar{C})\alpha\beta^*+(1+C)\bar{\beta}\bar{\alpha}^*-D\alpha\bar{\alpha}^*-D^*\bar{\beta}\beta^*]. \qquad (38)$$

Here N_s is the total number of spontaneous transitions; α, β, and the "conjugates" are specified by the constant Bragg scattering matrix;

$$C=\int_{-\infty}^{\infty}dt(|\alpha|^2+|\beta|^2)\frac{dg}{dt},\quad \bar{C}(\Omega)=C(-\Omega),$$

$$D=\int_{-\infty}^{\infty}dt(\bar{\alpha}\beta^*+\bar{\beta}\alpha^*)\frac{dg}{dt}. \qquad (39)$$

It can be seen from expression (38) that n_1 vanishes and there is no spatial density modulation in the absence of spontaneous relaxation.

As was noted in Refs. 11 and 12, a major role is played by the finite width of the excited state in effective complex potential (13), which specifies Bragg scattering matrix (35). It follows from Eq. (36) for the real potential $U(t)$ that the replacement $\Omega\to-\Omega$ is equivalent to the operation of complex conjugation. Therefore, bearing the initial conditions in mind, from relations (37) we obtain

$$\bar{\alpha}(t)=\alpha^*(t),\quad \bar{\beta}(t)=-\beta^*(t), \qquad (40)$$

and $\bar{B}(t)$ becomes a unitary matrix. In this case the expression in square brackets in Eq. (38) vanishes, since $C=\bar{C}$, the first two terms simply cancel one another, and $D=0$.

The fact that n_1 as defined in (38) turned out to be proportional to the partial width γ_1 is due to expression (31) for the inhomogeneous term, which does not take into account the difference between $F_0(p)$ and the constant at small momenta. It can be shown that if these corrections are taken into account, n_1 does not vanish when $\gamma_1=0$. In this sense it may be stated that the density modulation is caused by violation of the unitary character of the Bragg scattering matrix.

If we formally set $\gamma_0=0$, we have $dg/dt=0$ and $C=D=0$. In this sense Eq. (38) is simplified significantly and coincides with the expression obtained in Ref. 12, i.e.,

$$n_1(\delta p)\sim e^{-N_s}(\alpha\beta^*+\bar{\beta}\bar{\alpha}^*). \tag{41}$$

This result has a visualizable physical interpretation. The first term $\alpha\beta^*$ is the contribution to the density modulation from the particles with an initial transverse momentum $k+\delta p$, and the second term $\bar{\beta}\bar{\alpha}^*$ is the contribution from the particles with a momentum $-k+\delta p$. There are no other contributions, since there are no spontaneous transitions to the $\mu=0$ sublevel (the partial width $\gamma_0=0$). In a real potential, these contributions exactly cancel.

It may be stated that the two spatial lattices have a phase shift of π. The loss of the unitary character due to the finite lifetime of the excited state alters the phase shift, and density modulation appears.

To illustrate this point we consider expression (41) for the special case $\delta p=0$, in which Eq. (36) is easily interpreted for an arbitrary field envelope and gives

$$\alpha=\bar{\alpha}=\cos\chi,\quad \beta=\bar{\beta}=-i\sin\chi,\quad \chi=\int_{-\infty}^{\infty}U(t)dt. \tag{42}$$

Then, for n_1 and the phase difference φ between the atomic lattices corresponding to the two terms in Eq. (41), we obtain

$$n_1\sim e^{-N_s}\sinh\frac{N_s}{2},\quad \cot\frac{\varphi}{2}=\frac{\sinh(N_s/2)}{|\sin N_i|},$$

$$N_i=2\,rmRe\,\chi=\frac{\Delta}{2|\nu|^2}\int_{-\infty}^{\infty}V^2(t)dt, \tag{43}$$

$$N_s=-4\,\mathrm{Im}\,\chi=\frac{\gamma}{2|\nu|^2}\int_{-\infty}^{\infty}V^2(t)dt,$$

where N_i and N_s are the numbers of induced and spontaneous transitions during the interaction. When $N_s=0$, we see that the phase difference equals π and $n_1=0$. When $N_s\gg 1$, the value decreases exponentially. This is perfectly natural, since spontaneous relaxation strongly disrupts the coherence.

In the general case atoms with initial momenta from a range of width $2k$, rather than just the particles with momenta equal to $\pm k+\delta p$, also make a contribution to density modulation (38) due to spontaneous transitions to the $\mu=0$ sublevel when $\gamma_0\neq 0$.

The first harmonic $m_1(\delta p,t)=\rho_1^{11}=\rho_1^{-1-1}$ of the diagonal elements of the density matrix for other sublevels is obtained by elementary integration of Eq. (24) with consideration of (27), and after an interaction with a field it has the form

$$m_1(\delta p,y)=\gamma_1F_0\exp(-i\Omega y/v_y)\int_{-\infty}^{\infty}dt\frac{V^2(t)}{4|\nu|^2}\times\exp\left(i\Omega t-\frac{\gamma_1}{|\nu|^2}\int_{-\infty}^{t}V^2(t')dt'\right). \tag{44}$$

To obtain observable quantities, averaging with the initial momentum distribution $F(p_y)F_0(p)$ must be performed. We use angle brackets to denote this operation. Then the oscillating part of the particle density has the form

$$\delta n(x,y)=[\langle n_1(\delta p,y)\rangle+2\langle m_1(\delta p,y)\rangle]\exp(2ikx)+\mathrm{const}. \tag{45}$$

Due to the large phase difference $\Omega y/v_y\sim y/a$ associated with the Doppler effect and recoil, which, incidentally, are of the same order of magnitude when $p\sim k(kv\sim\varepsilon_r)$, the averaged quantities $\langle n_1\rangle$ and $\langle m_1\rangle$ become vanishingly small as soon as the atoms leave the interaction region $(y>a)$, and there is no density modulation. Dephasing caused by reversible relaxation is known to be eliminable under echo conditions when scattering takes place in two light fields.[1] We shall examine this equation in the next section. Attention should still be focused on the relationship between the contributions $\langle n_1\rangle$ and $\langle m_1\rangle$ of atoms in different sublevels to the resultant modulation depth (45). These quantities can be calculated analytically only by perturbation theory for weak fields $(\tau V^2/|\nu|\ll 1)$, in which the number of induced and spontaneous transitions is small. Restricting ourselves to the lowest order of perturbation theory in solving (36) for a field with a Gaussian envelope $[V^2(t)=V_0^2\exp(-t^2/\tau^2)]$, from (38) and (39) we obtain

$$\langle n_1(\delta p,y)\rangle=-2\langle m_1(\delta p,y)\rangle=-\frac{N_s^1}{2}\frac{k}{p_0}\frac{\exp(-y^2/a^2)}{\varepsilon_\tau\tau}, \tag{46}$$

$$N_s^1=\frac{\sqrt{\pi}}{2}\frac{V_0^2\gamma_1\tau}{|\nu|^2}\ll 1.$$

We see that in a weak field the contributions to the modulation depth from atoms in different sublevels cancel (in the linear approximation with respect to N_s^1). If the field parameter is of order unity, then, as the results of the numerical calculation presented in Fig. 3 show, complete cancellation does not occur, and $\langle n_1\rangle$ and $\langle m_1\rangle$ have positive signs.

Momentum distribution

Bragg resonance is also manifested in the distribution function with respect to the transverse momentum $n_0(p)$ for atoms in the $\mu=0$ sublevel. Against the background of the smooth distribution (27), near momenta equal to $\pm k$ there is a narrow resonance structure (with a width $\delta p\ll k$), which is described by the functions $\delta n_0(\pm k+\delta p)$. After an interaction with a field, the value of, for example,

$$\delta n_0(k+\delta p)=\exp(-N_s)R_{11}(\delta p,t\to+\infty) \tag{47}$$

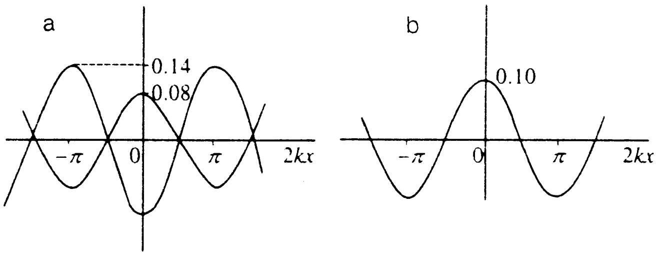

FIG. 3. Numerical calculation (for $N_s=1$, $\Delta/\gamma=1$) of spatial modulation of the atomic density: a) in the immediate vicinity of the zone of interaction ($t=0$) with a standing light wave in different states with $\mu=0$ *(1)* and $\mu=\pm 1$ *(2)*; b) under echo conditions ($t=20\tau$, $L=10a$).

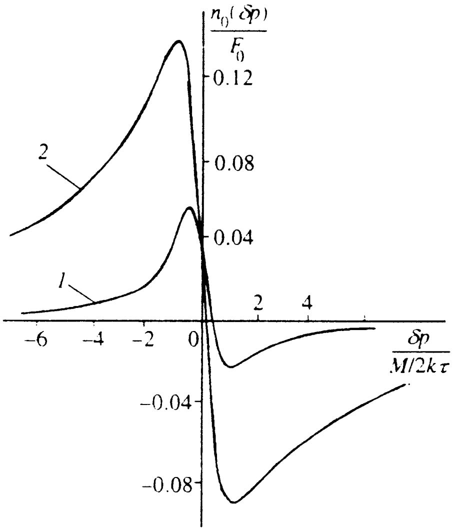

FIG. 4. Momentum distribution of particles forming a spatial lattice in the vicinity of the momentum $p=k$ [numerical calculations for $\Delta/\gamma=1$ *(1)* and $\Delta/\gamma=0$ *(2)*].

is specified by the matrix element R_{11} of the second summand in matrix (33) and has a fairly cumbersome form in the general case.

In the case of weak fields, if we restrict ourselves to the terms quadratic in U, for R_{11} we obtain

$$R_{11}=\frac{4\gamma_1}{\gamma}F_0\,\mathrm{Re}\int_{-\infty}^{\infty}dt[U(t)-U^*(t)]\times\int_0^{\infty}dt'\,U^*(t+t')\exp(-i\Omega t'). \quad (48)$$

Using a field with a model envelope of the form $V^2(t)=V_0^2\exp(-|t|/\tau)$ for simplicity, after all the integrations we arrive at the following expression for $\delta n_0(k+\delta p)$:

$$\delta n_0(k+\delta p)=F_0\frac{\gamma_1 N_s^2}{16\gamma}\frac{1-\xi^2-4\xi\Delta/\gamma}{(1+\xi^2)^2}, \quad (49)$$

where $\xi=\Omega\tau$ and $N_s=V_0^2\gamma\tau|\nu|^2\ll 1$.

Distribution (49) describes a curve that is asymmetric with respect to the point $\xi=0$ and has an area equal to zero. The characteristic width $\xi\sim 1$, i.e., $\delta p/k\sim 1/\varepsilon_r\tau\ll 1$. The amplitude of the resonance increases with increasing Δ/γ, and the asymmetry depends on the sign of the detuning. As the numerical calculations presented in Fig. 4 show, the characteristic features of the resonance curve just mentioned are also maintained in stronger fields. Averaging over the longitudinal velocity (i.e., over the time of flight $\tau=a/v_y$) with the distribution function $F(p_y)$ does not qualitatively alter the form of resonance curve (49). The resonant momentum structure in the vicinity of $p=-k$ is obtained by means of the replacement $\xi\to-\xi$.

5. ATOMIC INTERFERENCE IN SEPARATED LIGHT FIELDS

The first harmonics of the diagonal elements of the density matrix describe the spatial coherence in an initially incoherent atomic beam as a result of two-photon Bragg scattering in the field of a standing wave. This coherence essentially exists only in the region where the light field acts, and it rapidly breaks down at $y>a$ due to the phase differences associated with the Doppler frequency and the recoil energy. The coherence destroyed by reversible relaxation processes can be restored under echo conditions via successive perturbations, which create a time-reversal effect. Therefore, we consider the successive two-photon Bragg scattering of an atomic beam in two standing light waves separated in space by a distance L.

As we have already noted, the diagonal element $\rho_{00}(12)$ of the density matrix obeys an independent equation [see (12)]. Therefore, we describe a general solution scheme for this element, assuming, of course, that the light fields have identical linear polarization.

After scattering in the first wave, which is located in the vicinity of $y=0$, $\rho_{00}(12)$, which corresponds to the exponential factor in operator (34), evolves freely in the interval $a\ll y\ll L-a$, and its constant value as $t\to\infty$ can be taken for the Bragg scattering matrix.

The action of the second field is specified by the general solution of (33), in which both terms, i.e., the homogeneous and inhomogeneous solutions, must now be taken into account. We recall that the initial atomic beam, which is incident upon the first wave, was assumed to be incoherent, that $\hat{R}_0=0$, and that only the inhomogeneous solution corresponded to the interaction with the first field. The matrix $\hat{R}$ obtained as a result of the effects of the first wave and free evolution in the interval L should be taken as the initial matrix $\hat{R}_{II}^0$ for the interaction with the second field:

$$\hat{R}_{II}^0=F_0\exp\left(-i\frac{\Omega T}{2}\sigma_3\right)\hat{B}_I\hat{P}_I\hat{B}_I^+\exp\left(i\frac{\Omega T}{2}\sigma_3\right),$$
$$\hat{P}_I=\int_{-\infty}^{\infty}\hat{S}_I^{-1}\hat{\Gamma}_I(\hat{S}_I^+)^{-1}dt,\quad T=L/v_y. \quad (50)$$

Here the subscript "*I*" denotes quantities referring to the first field.

Under echo conditions at a distance $L+1(l\ll L)$ from the second wave ($y=2L+l$) we obtain

$$\hat{R}_{II}(T+l/v_y)=F_0\exp[-i(\Omega/2)(T+l/v_y)\sigma_3]\hat{B}_{II}(\hat{R}^0_{II}+\hat{P}_{II})\hat{B}^+_{II}\exp[i(\Omega/2)(T+l/v_y)\sigma_3]. \tag{51}$$

The subscript *"II"* refers to the second wave. The term associated with the inhomogeneous solution ($\sim\hat{P}_{II}$) produces coherent effects and density modulation similar to those occurring in the first wave. Therefore, after averaging over velocity, they exist only in the immediate vicinity of the second field and vanish at distances $y-L\gg a$.

From expressions (50) and (51) we obtain the first harmonic of the density-matrix element ρ_{00} at a distance $L+l$ from the second wave

$$n_1(\delta p, y=2L+l)=\frac{2\gamma_1}{\gamma}F_0\exp(-N_s)\times\mathrm{Tr}\left\{\exp[-i\Omega(T+l/v_y)\sigma_3]\hat{B}_{II}\exp\left(-i\frac{\Omega T}{2}\sigma_3\right)\hat{B}_I\hat{P}_I\hat{B}^+_I\exp\left(i\frac{\Omega T}{2}\sigma_3\right)\hat{B}^+_{II}\sigma\right\}. \tag{52}$$

Here N_s is the total number of spontaneous transitions during the interaction with the two fields, and the matrix $\sigma=\sigma_1-i\sigma_2$ plays role of a projection operator. Bearing in mind the subsequent averaging over velocity, in calculating Tr $\{\cdots\}$ we should retain only the terms not containing the large phase $\Omega T=\Omega L/v_y$. These terms correspond to the phase trajectories of an atom presented in Fig. 1. As a result, the density modulation depth has the form

$$\langle n_1\rangle=\frac{\gamma_1 k}{2\gamma}F_0\left\langle\frac{\exp(-N_s)}{\varepsilon_\tau\tau}\int_{-\infty}^{\infty}d\xi\exp(-il\xi/a)\times[(1+\bar{C})\alpha\beta^*+(1+C)\bar{\beta}\bar{\alpha}^*-D\alpha\bar{\alpha}^*-D\bar{\beta}\beta^*]_I(\beta^*\bar{\beta})_{II}\right\rangle. \tag{53}$$

Here $\tau=a/v_y$, integration with respect to transverse momentum has been written in explicit form ($\xi=\Omega T$), averaging over longitudinal momentum is denoted by angle brackets, and subscripts *"I"* and *"II"* refer to the first and second fields.

It follows from Eq. (36) that in fields with a smooth envelope, the waves β and $\bar{\beta}$ decrease exponentially as a function of the parameter $\Omega\tau=\xi$ at $\xi\gg1$. Therefore, values $\xi\sim1$ contribute to the integral over ξ in Eq. (53). This means that the atomic lattice has a dimension of the order of the width of the light beam ($l\sim a$) in space. If the field parameter $V^2\tau/|\nu|\sim1$ for each of the waves, the integral in (53) is of order unity, and the modulation depth is equal in order of magnitude simply to the relative number of particles which effectively participate in two-photon Bragg scattering

$$\langle n_1\rangle\sim\frac{\delta p}{p_0}\sim\frac{k}{p_0\varepsilon_\tau\tau}. \tag{54}$$

In an atomic beam with hot velocities v_y and angular divergence $\theta\sim p_0/Mv_y\sim10^{-3}$–$10^{-2}$, the modulation depth may reach a value of the order of 10^{-3}–10^{-2} of the mean particle density in the beam.

An exact analytical calculation of $\langle n_1\rangle$ can be performed for weak fields. For two identical fields with a Gaussian envelope, after all the integrations in lower-order perturbation theory we obtain

$$\langle n_1\rangle=-\frac{\pi}{\sqrt{3}}\frac{\gamma_1}{\gamma}\frac{N_s\exp(-N_s)}{kv_0\tau}\left(\frac{V_0^2\tau}{4|\nu|}\right)^2\exp(-l^2/3a^2), \tag{55}$$

$$v_0=p_0/M,\quad N_s=\frac{\sqrt{\pi}}{2}\frac{V_0^2\gamma\tau}{|\nu|^2}\ll1.$$

Although the number of spontaneous decays N_s is small, to visualize the situation we retain the exponential factor $\exp(-N_s)$.

The other sublevels (with $\mu=\pm1$) do not contribute to the resultant modulation depth, since they do not display the echo effect. In fact, "time reversal" requires a repeated coherent perturbation. In Eq. (24) there is only an incoherent source, and there are no induced transitions here to bring about cancellation of large phases upon repeated perturbation. Thus, the echo effect for spatial coherence presents itself in the following manner. The signal appearing immediately after the first perturbation, i.e., the spatial density modulation, is given by the first harmonics for all three sublevels [see (45)]. At this point the contributions $\langle n_1\rangle$ and $2\langle m_1\rangle$ partially cancel [and in weak fields the cancellation is essentially complete, see (46)]. After the second perturbation, only the signal associated with $\langle n_1\rangle$ is reproduced under echo conditions. Since no cancellation occurs here, the resultant density modulation signal may be greater than the original signal, in principle. Such amplification of the signal is especially evident in weak fields. The narrow Bragg resonance structure in the momentum distribution of the atoms, which is shown in Fig. 4, is also reproduced under echo conditions.

6. CONCLUSIONS

We considered two-photon Bragg resonance and the possibility of its utilization as a mechanism for splitting and mixing coherent atomic waves in atom interferometry. The $1\leftrightarrow0$ transition, which is self-bleaching in a linearly polarized field, was employed as the working atomic transition. The characteristic features of the Bragg scattering process, primarily the special role played by spontaneous relaxation in the formation of coherence in an initially incoherent atomic beam and the resultant echo signal, were ascertained in this concrete example.

The results obtained may be generalized to other angular momenta, such as those of the levels in transitions of the $1/2\leftrightarrow1/2$ type, which are encountered in alkali-metal atoms. A situation similar to the one considered can be realized when a standing wave with circular polarization acts on atoms.

We stress that departure from the simplest two-level scheme, i.e., consideration of the degeneracy of the levels and spontaneous relaxation, expands the possibilities for controlling interference effects. For example, the depth of the spatial density modulation under echo conditions can be manipulated by varying the angle between the directions of the polarizations of the separated light waves. This is due to the fact that atoms in states representing the superposition of states with different projections of the angular momentum are subjected to the coherent influences in the second wave and that disappearance of the echo signal can be achieved at a definite angle.

Finally, we note the Bragg resonance structure in the momentum distribution (Fig. 4). The width of the resonance is determined by the magnitude of the momentum, which corresponds to an energy far smaller than the recoil energy. The possibility of obtaining such an ultranarrow distribution was pointed out in Refs. 7, 8, 11, and 12. The form of this narrow Bragg resonance was obtained in the present work.

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