Vacuum massive neutrino pair production by variable electromagnetic fields

V. V. Skobelev

Moscow State Open Pedagogical Institute, 109004 Moscow, Russia
(Submitted 18 February 1994; resubmitted 23 April 1994)
Zh. Eksp. Teor. Fiz. **106**, 385–394 (August 1994)

A first attempt is made to examine the process of vacuum production of massive neutrino pairs by a variable electromagnetic field, $F \to \nu_i \bar{\nu}_j$, allowing for mixing effects. The derived general relations are illustrated by a calculation of the probability of neutrino synchrotron radiation produced by classical charges and in collisions of Coulomb centers. The cases of Dirac and Majorana neutrinos are discussed. Fudamentally new ideas about estimating the upper limit of the neutrino mass are suggested.

1. INTRODUCTION

The effects of polarization of the charged-particle vacuum in field models are crucial in determining the validity of such models if they are confirmed in experiments. A classic example is the indirect experimental verification of the most important corollaries of QED: the light-on-light scattering in experiments in measuring the Delbrück effect[1,2] and photon splitting on a nucleus.[3,4] In the diagram technique these processes are represented by a four-leg diagram with an electron loop. Another nontrivial nonlinear QED effect, the three- or two-photon radiation generated by variable fields, was described in Ref. 5 in an invariant formulation. As for specific applications, vacuum corrections to synchrotron radiation were examined in Refs. 6 and 7, and those to the radiation generated by a classical charge in unbounded motion in the field of a fixed Coulomb center in Ref. 8, provided that the effect is caused by a variable electromagnetic field $F(k)$ of an accelerating charge for $k^2 \geqslant 0$.

It would be interesting to carry out the same program, examining the vacuum production of a massive Dirac-neutrino pair by a variable field, accounted for in the first Born approximation; generally, in models with mixing, the massive neutrinos ν_i are related to the states ν_l participating in electroweak interactions through a unitary transformation.

Bilenky and Petkov[9] showed that if considerations of hermiticity and CP-invariance are taken into account, then the effective low-energy Lagrangian of the $(\nu_i \nu j F)$-interaction can be written as

$$L = \sum_{i,j} \bar{\Psi}_i [(a_{ij} + b_{ij}\gamma^5)\sigma^{\alpha\beta}] \Psi_j F_{\alpha\beta}, \tag{1}$$

where the coefficients a_{ij} and b_{ij} are real numbers with the following symmetry properties: $a_{ij} = a_{ji}$ and $b_{ij} = -b_{ji}$. In the standard model and the one-loop approximation the form of these coefficients are fixed by the following relation:

$$\begin{Bmatrix} a_{ij} \\ b_{ij} \end{Bmatrix} = \mp \frac{3eG}{64\sqrt{2}\pi^2}(m_j \pm m_i) \sum_l U_{il}^* U_{jl} \left[\left(\frac{M_l}{m_W} \right)^2 - 2 \right], \tag{2}$$

where m_i and m_j are neutrino masses, M_l are the masses of charged leptons, and U is the mixing matrix in the lepton sector. For simplicity in what follows we employ the notation $i \to 1$, $j \to 2$, $a_{ij} \to a$, and $b_{ij} \to b$ and consider the production process $F \to \nu_1 \bar{\nu}_2$.

Using simple transformations, we arrive at the following expression for the total probability:

$$W = \frac{2}{(2\pi)^5} \int d^4k \, F_{\alpha\beta}(k) F^{\beta\alpha}(-k) A(k^2), \tag{3}$$

with

$$A(k^2) = \frac{1}{k^2}\sqrt{k^4 - 2k^2(m_1^2 + m_2^2) + (m_1^2 - m_2^2)^2} \times \left\{ \frac{(a^2+b^2)}{6k^2}[k^4 + k^2(m_1^2 + m_2^2) - 2(m_1^2 - m_2^2)^2] + m_1 m_2 (a^2 - b^2) \right\}. \tag{3a}$$

An equivalent representation in terms of the currents generating the field F can be obtained via the substitution

$$F_{\alpha\beta}(k) F^{\beta\alpha}(-k) \to \frac{32\pi^2}{k^2}[-j_\alpha(k) j^\alpha(-k)], \tag{4}$$

which is a corollary of the Maxwell equations. Of course, Eq. (3) remains true in the absence of mixing, in which case we must put $m_1 = m_2$, $b = 0$, and $2a = \mu$, where μ is the anomalous magnetic moment of the massive Dirac neutrino. [10] (We note in passing that this moment can also be obtained from Eq. (2) for the diagonal transitions $\mu_{ii} = 2a_{ii}$ combined with the fact that U is unitary.) The integration range is specified by the obvious condition $k^2 \geqslant (m_1 + m_2)^2$, and when the contact approximation of the form (1) and (2) is employed, the value of $k_{0\,\text{eff}}$ must be much smaller than the mass of the lightest charged particle of the model's vacuum (the electron mass). In the applications considered below this condition is met

in view of the rapid decrease in $F(k)$ for $k_0 \gtrsim M_e$. The variable electromagnetic field $F(k)$ can form, for instance, as a result of the accelerated motion of charges (protons and nuclei), whose structure is not taken into account in the present approach, nor is radiation recoil. Below we will see that these restrictions agree with those mentioned earlier.

 1063-7761/94/080215-05$10.00

Note that the vacuum mechanism of neutrino pair production by classical charges in the present setting is predominant, in contrast to the photon generation by the "charge→ F→ 3γ" scheme, which in any case is negligible compared to the ordinary radiation emitted by accelerating charges.

2. THE SYNCHROTRON MECHANISM OF VACUUM MASSIVE NEUTRINO PRODUCTION

Imagine a classical charge Q moving in an ultrarelativistic manner along a circle of radius R with a frequency ω. The Fourier transforms of the potentials have the form

$$A_0(k)=\frac{2(2\pi)^2Q}{k^2\omega}J_\nu R|\mathbf{k}|\sin\theta,$$

$$\begin{Bmatrix}A_1(k)\\A_2(k)\end{Bmatrix}=\frac{2(2\pi)^2QR}{k^2}\times\left\{\begin{pmatrix}\cos\varphi\\\sin\varphi\end{pmatrix}\frac{\nu}{|\mathbf{k}|R\sin\theta}J_\nu+i\begin{pmatrix}\sin\varphi\\-\cos\varphi\end{pmatrix}J'_\nu\right\}, \quad (5)$$

where $\nu=k_0/\omega\gg1$, θ and φ are the angles specifying the position of vector **k** in a spherical system of coordinates, and the z\ axis is perpendicular to the plane of motion. Substituting the potentials in the form (5) into the general expression (3) and doing trivial integration with respect to φ, we arrive at the following expression for the spectral–angular (over the total neutrino momentum) distribution of the pair-production rate (probability per unit time) normalized to unity:

$$W_t=\frac{4Q^2\omega^3}{\pi}\int\int_{(\Gamma)}\frac{d\nu\,d\bar\nu\bar\nu^2}{\nu^3(1-\bar\nu^2)^3}\times\sqrt{\nu^4(1-\bar\nu^2)^2-2\nu^2(1-\bar\nu^2)(\tilde m_1^2+\tilde m_2^2)+(\tilde m_1^2-\tilde m_2^2)^2}$$
$$\times\{\tfrac{1}{3}(a^2+b^2)[\nu^4(1-\bar\nu^2)^2+\nu^2(1-\bar\nu^2)(\tilde m_1^2+\tilde m_2^2)-2(\tilde m_1^2-\tilde m_2^2)^2]+2\tilde m_1\tilde m_2(a^2-b^2)\nu^2(1-\bar\nu^2)\}$$
$$\times\int_{-1}^{1}d\cos\theta\left[\left(\frac{1}{\bar\nu^2\sin^2\theta}-1\right)J_\nu^2+v^2J_\nu'^2\right], \quad (6)$$

with

$$\bar\nu=\frac{|\mathbf{k}|}{k_0},\quad \arg J_\nu=v\nu\bar\nu\sin\theta,\quad \Gamma=\{\nu^2(1-\bar\nu^2)\geq\tilde m^2\},$$
$$\tilde m_{1,2}=\frac{m_{1,2}}{\omega},\quad \tilde m=\tilde m_1+\tilde m_2. \quad (6a)$$

Analyzing Eq. (6) by analytical methods is generally impossible, in view of which we will discuss the asymptotic behavior of (6) in relation to parameter $\tilde m$.

2a. $\tilde m\gg1$

Here the applicability condition for representation (5), $\nu_{\rm eff}\gg1$, is satisfied by a large margin, and under the additional restrictions

$$\nu_{\rm eff}\gg(1-v^2\bar\nu^2\sin^2\theta)^{-3/2}_{\rm eff}, \quad (7a)$$

$$\bar\nu_{\rm eff},\quad (\sin\theta)_{\rm eff}\approx1 \quad (7b)$$

we can employ the "tangent approximation" for the Bessel functions:[11]

$$J_\nu\approx\frac{1}{\sqrt{2\pi\nu\varepsilon^{1/2}}}\exp\left(-\frac{\nu}{3}\sqrt{\varepsilon^3}\right),\qquad \varepsilon=1-v^2\bar\nu^2\sin^2\theta. \quad (8)$$

Assuming

$$m_1m_2\neq0,\quad a\neq0 \quad (9)$$

and performing certain transformations, we obtain

$$W_t\approx\frac{16Q^2\omega^3(2\tilde m_1\tilde m_2)^{3/2}a^2}{\pi^2\tilde m}\int_0^1\frac{d\bar\nu}{(1-\bar\nu^2)^{3/2}}\int_0^1 d\times\cos\theta\left(\frac{1-\bar\nu^2\sin^2\theta}{\sqrt\varepsilon}+\sqrt\varepsilon\right)\int_1^\infty dx\,\sqrt{x-1}\times\exp(-cx), \quad (10)$$

with

$$c=\frac{2\tilde m}{3\sqrt{1-\bar\nu}}\sqrt{\varepsilon^3}, \quad (10a)$$

where by integrating with respect to ν we introduced a new variable x via the formula

$$\nu=x\frac{\tilde m}{\sqrt{1-\bar\nu^2}} \quad (11)$$

and all powers of velocity v have been set, where possible, to unity. Analysis of Eq. (10) suggests that in the ultrarelativistic case condition (7b) is met, which together with (7a) is reduced to the following final applicability condition for the approximation used in Sec. 2a (with $\nu_{\rm eff}\sim\tilde m_\gamma$):

$$\xi=\frac{\tilde m}{\gamma^2}\gg1,\quad \gamma=\frac{W_Q}{m_Q}\gg1, \quad (12)$$

with m_Q and E_Q the mass and energy of charge Q. Then the condition for ignoring radiation recoil, $k_{0\,\rm eff}=\omega\nu_{\rm eff}\ll E_Q$, is equivalent to $(m_1+m_2)/m_Q\ll1$ and is always met.

The integral with respect to x in Eq. (10) is evaluated elementarily, and the method of the ensuing approximate integration with respect to $\bar\nu$ and $\cos\theta$ is discussed in Ref. 12. Performing all the necessary calculations, we get

$$W_t\approx\frac{16Q^2\omega^3a^2(\tilde m_1\tilde m_2)^{3/2}}{\sqrt\pi\,3^{5/4}\tilde m^{7/2}}\gamma^4\exp(-\sqrt3\,\xi). \quad (13)$$

Equations (12) and (13) suggest that the probability is exponentially small in the parameter of approximation (12), which is quite similar to the exponential dependence when one considers e^+e^- pair production in the QED framework.[13]

2b. $\tilde{m}\ll 1$

In view of the assumed smallness of the neutrino masses this case can also be realistic, and in the asymptotic expressions we must put $m_1=m_2=0$ in Eq. (6) and use the asymptotic representation of the Bessel functions in terms of modified Bessel functions, as is done in the classical problem of studying the spectral characteristics of ordinary synchrotron radiation.[14] Then the spectral–angular distribution of the rate of neutrino production (probability per unit time) is given by the expression

$$W_t\approx\frac{8Q^2\omega^3(a^2+b^2)}{9\pi^3}\int_0^1 dv\int_0^1 d\cos\theta\int_0^\infty d\nu\ \nu^3[\varepsilon(1-\bar{v}^2\sin^2\theta)K_{1/3}^2(\tfrac{1}{3}\nu\varepsilon^{3/2})+\varepsilon^2K_{2/3}^2(\tfrac{1}{3}\nu\varepsilon^{3/2})]. \qquad (14)$$

As in the problem of synchrotron radiation, $\nu_1\bar{\nu}_2$ pairs are emitted into a narrow cone near the rotation plane with an angle of the order of γ^{-1}, with the peak in emission at the harmonics $\nu_{\rm eff}\sim\gamma^3$. The condition for ignoring radiation recoil in this case has the form $\gamma^2(\omega/m_Q)\ll 1$ and is always met in the contact approximation employed here [see Eq. (16b)].

Performing fairly simple computations, we can write the total probability in the limit $v\to 1$ as

$$W_t\approx\frac{13}{2\sqrt{3}\,\pi}\,Q^2\omega^3(a^2+b^2)\gamma^5. \qquad (15)$$

For the form factors the contact approximation is valid if $\omega\nu_{\rm eff}\ll M_e$, which in the asymptotic situations considered here can be represented, respectively, by the following inequalities:

$$\left(\frac{m_1+m_2}{M_e}\right)\gamma\ll 1, \qquad (16a)$$

$$\left(\frac{\omega}{M}\right)\gamma^3\ll 1. \qquad (16b)$$

For reasonable values of E_Q these inequalities are nearly always valid.

The condition for the quark structure of protons (or nuclei) not to be excited, in which case the vacuum contribution to neutrino production considered here is predominant, is that the effective pair energy $k_{0\,\rm eff}$ be small compared to the inverse characteristic scale $l_{\rm nucl}^{-1}$ on which the nucleon structure manifests itself ($l_{\rm nucl}\sim 10^{-14}$ cm). Mathematically this condition can be written as $\nu_{\rm eff}\omega\ll l_{\rm nucl}^{-1}$, which is sure to be met with a large margin in the contact approximation under discussion.

3. VACUUM MASSIVE NEUTRINO PRODUCTION BY A CHARGE IN UNBOUNDED MOTION IN THE FIELD OF A COULOMB CENTER

The equation of the trajectory of a charge Q_1 in unbounded nonrelativistic motion in the field of a fixed Coulomb center Q_2 is given in Ref. 15, and the Fourier transforms of the current density ($Q_1Q_2<0$) have the form

$$j_1=i\pi Q_1 f H_{i\nu}^{(1)\prime}(i\nu\varepsilon),$$
$$j_2=i\pi Q_1 f\varepsilon^{-1}\sqrt{\varepsilon^2-1}\,H_{i\nu}^{(1)}(i\nu\varepsilon), \qquad (17)$$
$$j_3=0,$$

and j_0 is determined from the continuity equation. Here $H_p^{(1)}(x)$ is the Hankel function of the first kind, of order p, and we have introduced the following notation:

$$\nu=\frac{k_0\alpha_{12}}{\mu v_0^3},\quad \alpha_{12}=|Q_1Q_2|,$$
$$\varepsilon=\sqrt{1+\frac{\mu^2v_0^4\rho^2}{\alpha_{12}^2}},\quad f=\frac{\alpha_{12}}{\mu v_0^2}, \qquad (18)$$

where μ is the mass of charge Q, and v_0 and ρ are the velocity of Q_1 at infinity and the impact parameter. In calculating (17) we assumed that

$$k_0\alpha_{12}\ll\mu v_0^2, \qquad (19a)$$

which is strictly true if radiation recoil is ignored, or

$$k_0\ll\mu v_0^2. \qquad (19b)$$

Combining Eqs. (3), (4), and (17) and integrating with respect to the spatial components k_α and the impact parameter (after multiplication by $2\pi\rho\, d\rho$), we arrive at a general result for the differential cross section of scattering with neutrino pair production as a function of total pair energy:

$$\frac{d\sigma}{dk_0}=\frac{16\pi Q_1^2\alpha_{12}^3}{3\mu^3v_0^5}|H_{i\nu}^{(1)}(i\nu)H_{i\nu}^{(1)\prime}(i\nu)|\frac{1}{k_0k_-}\{\tfrac{1}{6}(a^2+b^2)\times[k_0^4A_1(s,u)+k_0^2(m_1^2+m_2^2)A_s(s,u)-2(m_1^2-m_2^2)^2A_3(s,u)]+(a^2-b_2)k_0^2m_1m_2A_2(s,u)\}. \qquad (20)$$

Here the $A_i(s,u)$ are defined as

$$A_1(s,u)=2us[1-\tfrac{1}{3}s(1+u)+\tfrac{1}{15}s^2(1-u+u^2)]E-[2(1-s)+\tfrac{2}{3}u(1-u)s^2+\tfrac{1}{5}u(1-3u+2u^2)s^3]F+2(1-s)(1-us)\Pi, \qquad (21a)$$

$$A_2(s,u)=-us[1+\tfrac{1}{3}s(1+u)]E+[3-s+\tfrac{1}{3}u(1-u)s^2]F-(3-us-s-us^2)\Pi+2s(1-s)(1-us)\Pi', \qquad (21b)$$

$$A_3(s,u)=s[-uE-F+(1+u)\Pi+(-1-s-us+3us^2)\Pi'+s(1-s)(1-us)\Pi''], \qquad (21c)$$

where

$$k_\pm=\sqrt{k_0^2-(m_1\pm m_2)^2},\quad s=\frac{k_+^2}{k_0^2},\quad u=\frac{k_-^2}{k_+^2},$$
$$E\equiv E(u^{-1/2}),\quad F\equiv F(u^{-1/2}),\quad \Pi\equiv\Pi(s,u^{-1/2}) \qquad (21d)$$

elliptic integrals in the notation used in Ref. 11, and the derivative of Π is taken with respect to the first argument.

The factor $N\equiv(k_0k_-)^{-1}\{\cdots\}$ in (20) assumes different forms for different values of the neutrino mass ratio:

(a) for $m_1\ll m_2$ ($m_1=0$)

$$N=\frac{(a^2+b^2)k_0^2\sqrt{s}}{4}\left[1-\frac{8s}{3}+\frac{11s^2}{5}-\frac{(1-s)^3}{2\sqrt{s}}\ln\left(\frac{1+\sqrt{s}}{1-\sqrt{s}}\right)\right]; \tag{22a}$$

(b) for $m_1, m_2 \ll k_0$ $(m_1=m_2=0)$

$$N=\tfrac{2}{15}(a^2+b^2)k_0^2; \tag{22b}$$

and (c) for $m_1=m_2=m$

$$N=\tfrac{1}{6}(a^2+b^2)(k_0^2A_1+2m^2A_2)+(a^2-b^2)m^2A_2, \tag{22c}$$

with

$$A_1=\tfrac{2}{15}(11-6s+s^2)E(\sqrt{s})-\tfrac{1}{15}(22-23s+s^2)F(\sqrt{s}),$$

$$A_2=-\tfrac{1}{3}(10+s)E(\sqrt{s})+\tfrac{1}{3}(10-4s)F(\sqrt{s}).$$

As follows from the expression for parameter ν in (18), the value of ν is determined by the product of the small quantity $k_0\alpha_{12}/\mu v_0^2$ and the large quantity v_0^{-1} in accordance with the restriction (19a) and the nonrelativistic setting of the problem in Sec. 3. Thus, within the approximation two cases are possible, $\nu\ll 1$ and $\nu\gg 1$, in which the product of the Hankel functions in (20) is expressed in terms of elementary functions as follows:

(a) for $\nu\ll 1$

$$|\cdots|\approx\frac{4}{\pi^2\nu}\ln\frac{2}{\gamma\nu}, \tag{23a}$$

and (b) $\nu\gg 1$

$$|\cdots|\approx\frac{4}{\pi\nu\sqrt{3}} \tag{23b}$$

(here $\gamma=e^C$, where C is Euler's constant). Combining this with Eqs. (22) exhausts the asymptotic representations of the general expression (20).

It is impossible to determine the total cross section for $Q_1Q_2<0$ (unlike charges) in the approach considered here because of the low-energy nature of the approximation specified by (19a) and (19b).

For $Q_1Q_2>0$ (like charges) we must multiple Eq. (20) by $e^{-2\pi\nu}$ and, in any case, the total cross section is finite. If an additional requirement $\alpha_{12}\gg v_0$ is met and if the motion is nonrelativistic, $v_0\ll 1$, the total cross section can be found from the formula

$$\sigma(Q_1Q_2>0)=\int_{m_1+m_2}^{\infty}\frac{d\sigma(Q_1Q_2<0)}{dk_0}\exp(-2\pi\nu)\,dk_0, \tag{24}$$

since both conditions (19a) and (19b) are met because of the exponential convergence of the integral at the upper limit. It was found impossible to calculate the integral in (24) because of the complicated structure of the integrand.

As in Sec. 2, a restriction of the "structural type" agrees with the low-energy approximation, which in turn is reduced to considering values of k_0 much smaller than M_e in analyzing the differential cross

section (20) and to the inequality

$$\mu v_0^2\ll\frac{\alpha_{12}}{v_0}M_e$$

in calculating the total cross section (24).

4. DISCUSSION AND ANALYSIS OF RESULTS

Let us first analyze the possibility in principle of observing neutrino production by employing the existing proton devices. The approximate parameters of the Fermilab ring are:[1)] $E_p\sim 10^3$ GeV, magnetic induction $B\sim 4.4\times 10^4$ G, and proton flux $\sim 10^{31}$ cm^{-2} s^{-1} (the parameters of the DESY and CERN machines have the same order of magnitude). The characteristic parameter $\tilde{m}$ (6a) and the approximation parameter ξ (12) for ultrarelativistic motion of a charge in a magnetic field can be written as

$$\tilde{m}=3.84\times 10^{-3}\left(\frac{A}{Z}\right)\left(\frac{m_\nu}{\text{eV}}\right)\left(\frac{E_Q}{\text{GeV}}\right)\left(\frac{B_0}{B}\right), \tag{25a}$$

$$\xi=3.34\times 10^{-3}\left(\frac{A^3}{Z}\right)\left(\frac{m_\nu}{\text{eV}}\right)\left(\frac{\text{GeV}}{E_Q}\right)\left(\frac{B_0}{B}\right), \tag{25b}$$

where

$$A=\frac{m_Q}{m_p},\quad Z=\left|\frac{Q}{e_0}\right|,\quad m_\nu=m_1+m_2,$$

$$B_0=\frac{M_e^2}{e_0}=4.41\times 10^{13}\text{ G}.$$

For proton beams with these parameters we obtain

$$\tilde{m}\sim 4\times 10^9\left(\frac{m_\nu}{\text{eV}}\right), \tag{26a}$$

$$\xi\sim 3\times 10^3\left(\frac{m_\nu}{\text{eV}}\right). \tag{26b}$$

The probable range of neutrino masses is 10–0.1 eV, and in this case the conditions for the applicability of the approximation $\tilde{m},\xi\gg 1$ in Sec. 2a are met, including condition (16a). However, in view of the exponential smallness of (13) in the parameter ξ, the probability

is negligible irrespective of the value of the structural constant a in specific variants of electroweak models. As (12) shows, the acceptable value $\xi\sim 1$ is attained by increasing the beam energy and the magnetic induction by a factor of 100 simultaneously. In the planned next-generation machines this situation could be realized.

Interestingly, in the above reasoning of Sec. 2a the neutrinos are ultrarelativistic, $(k_{0\nu}/m_\nu)_{\text{eff}}\sim E_Q/m_Q\gg 1$; nevertheless, the probability (13) depends on their mass in an exponential manner. The very fact that neutrino production is detected sets the upper limit on the value of mass, irrespective of the threshold behavior (up to now all laboratory experiments have been based on this fact). In this sense a process of the form $e\to e\nu\bar{\nu}$ in a magnetic field, whose probability within a proper range of values of E_e and B also contains the factor $\exp[-\sqrt{3}(m_\nu/\omega)(M_e/E_e)^2]$, would be more appropriate for establishing the upper limit on the neutrino mass. The reason is the low perturbation-theory order, not related to vacuum polarization, and the corresponding properties of the electron storage rings. This question, however, merits separate study because of the basic difference in computational procedure and does not constitute the aim of the present study.

These prospects for estimating the upper limit on the neutrino mass may improve when megagauss targets are used in scattering experiments involving powerful proton and electron beams.

Note that in fact the results obtained in Secs. 2 and 3 can also be applied to Majorana neutrinos. To be precise, it follows from CP- and T-invariance that $a_{ij}^{(M)}=0$, while $b_{ij}^{(M)}$ (equal to $-b_{ji}^{(M)}$) is nonzero only if the neutrinos have the same CP-parity. Thus, in the absence of mixing or when the CP-parities are different the probability of the $F\rightarrow\nu_i^{(M)}\nu_j^{(M)}$ process is identically zero, and in any case it is suppressed in the approximation discussed in Sec. 2a because the structural constant $a^{(M)}$ vanishes. Obviously, the latter circumstance offers a chance to establish the nature of neutrino in the discussed experiments.

As for vacuum neutrino production as a result of collisions of Coulomb centers discussed in Sec. 3, cautious estimates yield, in addition to other mechanisms, a contribution of nonrelativistic proton–proton collisions to neutrino production of collapsed objects such as neutron stars and white dwarfs (in which the electron component is degenerate). This statement, however, is valid only for a nonstandard "upper" value $\mu_\nu\sim 10^{-10}\mu_B$, which in itself is far from obvious.

[1)]This information was most kindly supplied to the author by A. V. Borisov.

[1] M. Delbrück, Z. Phys. **84**, 144 (1933).
[2] H. E. Jackson and K. J. Wetzel, Phys. Rev. Lett. **22**, 1008 (1969).
[3] M. Bolsterly, Phys. Rev. **94**, 282 (1962).
[4] G. Jarlskog *et al.*, Phys. Rev. D **8**, 3813 (1973).
[5] D. Galtsov and V. Skobelev, Phys. Lett. B **36**, 238 (1971).
[6] V. V. Skobelev, Phys. Lett. A **48**, 229 (1974).
[7] V. V. Skobelev, Opt. Spektrosk. **39**, 1160 (1975) [Opt. Spectrosc. (USSR) **39**, 667 (1975)].
[8] V. V. Skobelev, Opt. Spektrosk. **42**, 614 (1977) [Opt. Spectrosc. (USSR) **42**, 351 (1977)].
[9] S. M. Bilenky and S. T. Petkov, p **59**, 671 (1987).
[10] K. Fujikawa and R. E. Shrock, Phys. Rev. Lett. **45**, 963 (1980).
[11] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series and Products*, 4th ed., Nauka, Moscow, 1971, p. 919, 977.
[12] V. V. Skobelev, Izv. Vyssh. Uchebn. Zaved. **18**, 101 (1975).
[13] V. I. Ritus, Nucl. Phys. B **44**, 236 (1972).
[14] A. A. Sokolov, I. M. Ternov, and V. G. Bagrov, in *Synchrotron Radiation*, Nauka, Moscow, 1976, p. 18 (in Russian).
[15] L. D. Landau and E. M. Lifshitz, *The Classical Theory of Fields*, 4th ed., Nauka, Moscow, 1967, p. 241.

Translated by Eugene Yankovsky

This article was translated in Russia. It is reproduced here the way it was submitted by the translator, except for stylistic changes by the Translation Editor.