Tunneling transmissivity of a disordered superlattice with scatterers in the potential barriers

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The transfer-matrix method is used to calculate the tunneling transmission coefficient of disordered superlattices with deep-lying impurity centers inside the potential barriers. The disorder arises because the widths of he quantum wells assume random values along the superlattice chain. It is established that (a) the maximum value of the transmissivity T of an electron through the lattice increases by several orders of magnitude as compared to the value in the absence of impurities, and (b) an energy range forms in which $T(E)\approx 1$, where E is the electron energy.

It is well known that the presence of deep-lying impurities in potential barriers can have a strong effect on the tunneling properties of a semiconductor structure (see, e.g., Ref. 1 and literature cited there and also Refs. 2–6). For instance, in order to control the energy spectrum, Beltram and Capasso[2] suggested introducing "impurity planes of deep-lying levels" into the potential barriers of a semiconductor superlattice; the eigenstates corresponding to these impurities are highly localized only along an axis perpendicular to the barrier–well interface. The method of creating such deep-lying levels is well known.[7] Beltram and Capasso[2] considered a periodic superlattice. The calculations were done using the Kronig–Penney model, and the potential of the impurity centers was taken in the form of a delta function. The authors of Ref. 2 showed that introducing impurities produces very strong broadening and displacement of the energy minibands. Similar results were reported in Refs. 3–5 for other tunnel-resonance structures; the transfer-matrix method was used in Refs. 3 and 4. At the same time it is important to know the effect of deep-lying impurities on the tunneling characteristics of disordered superlattices, which have been intensively studied lately. In this paper we calculate the tunneling spectra (the dependence of the electron transmissivity through a superlattice on the electron energy) of a disordered superlattice whose potential barriers contain deep-lying impurity centers. We find that these differ dramatically from the spectra of superlattices without scatterers. As in Refs. 8 and 9, disorder arises because the quantum-well widths assume different values along the superlattice chain.

Let us consider a superlattice consisting of a finite sequence of one-dimensional rectangular potential barriers of the same height V. Suppose that a flux of electrons with energy E is traveling from left to right along the x\ axis, with the effective mass of the charge carriers assumed independent of x. Suppose that each barrier contains only one impurity plane. The potential of the scattering centers is modeled by a delta function: $U(x)=\Omega\delta(x-x_j)$, where Ω is the strength of the scatterer and x_j is its position. The electronic states inside the barriers are described by the equation

$$\left(\frac{d^2}{dx^2}-\kappa^2\right)\psi(x)=\beta\delta(x-x_j)\psi(x), \tag{1}$$

where $\kappa^2=2m(V-E)$, $\beta=2m\Omega$, and $\hbar=e=m_0=1$. The solution of Eq. (1) and of the equation describing the states of electrons in the quantum wells is of the form

$$\psi(x)=Ae^{ikx}+Be^{-ikx}, \tag{2}$$

with $k_b=-i\kappa$ in the barriers. We also assume that the values of the wave numbers with the same energy in different quantum wells are the same ($k_w=\sqrt{2mE}$), and the same is true for the barriers independently from the wells. We assume that the value of A for the incident wave (in the region in front of the superlattice) is equal to unity, and that of B in the region back of the last barrier is zero, which corresponds to the absence of a reflected wave back of the superlattice. We seek the solution to the system of equations for the coefficients of the wave functions by employing the transfer-matrix method. The matrix transferring the solution across a barrier–well interface is[10]

$$R_n=\frac{1}{2k_n}\begin{pmatrix}(k_n+k_{n+1})\exp\{i(-k_n+k_{n+1})x_n\} & \\ (k_n-k_{n+1})\exp\{i(k_n+k_{n+1}x_n)\} & \\ \times\begin{matrix}(k_n-k_{n+1})\exp\{i(-k_n-k_{n+1})x_n\}\\ (k_n+k_{n+1})\exp\{i(k_n-k_{n+1})x_n\}\end{matrix}\end{pmatrix}, \tag{3}$$

where the subscript n refers the solution to a certain well (barrier), for n even we have $k_n=k_b$, for n odd $k_n=k_w$. The matrix transferring the solution across the impurity centers is[10]

$$M_j=\frac{1}{2\kappa}\begin{pmatrix}2\kappa-\beta & -\beta\exp\{-2\kappa x_j\}\\ \beta\exp\{2\kappa x_j\} & 2\kappa+\beta\end{pmatrix}. \tag{4}$$

The transmission coefficient of an electron across the superlattice is[10]

$$T(E)=\left|\left(\prod_{n-1}^{r}R_n'\right)_{11}\right|^{-2}, \tag{5}$$

where the vertical bars designate an absolute value, r is the number of barrier–well interfaces (including the first and last interfaces), for n even we have $R_n'=R_{2s}$, and for n odd $R_n'=R_{2s-1}M_s$, with $s=1,2,3,....$

We calculate the transmission coefficient $T(E)$ via (5) for the silicon–silicon-carbide structure, for which

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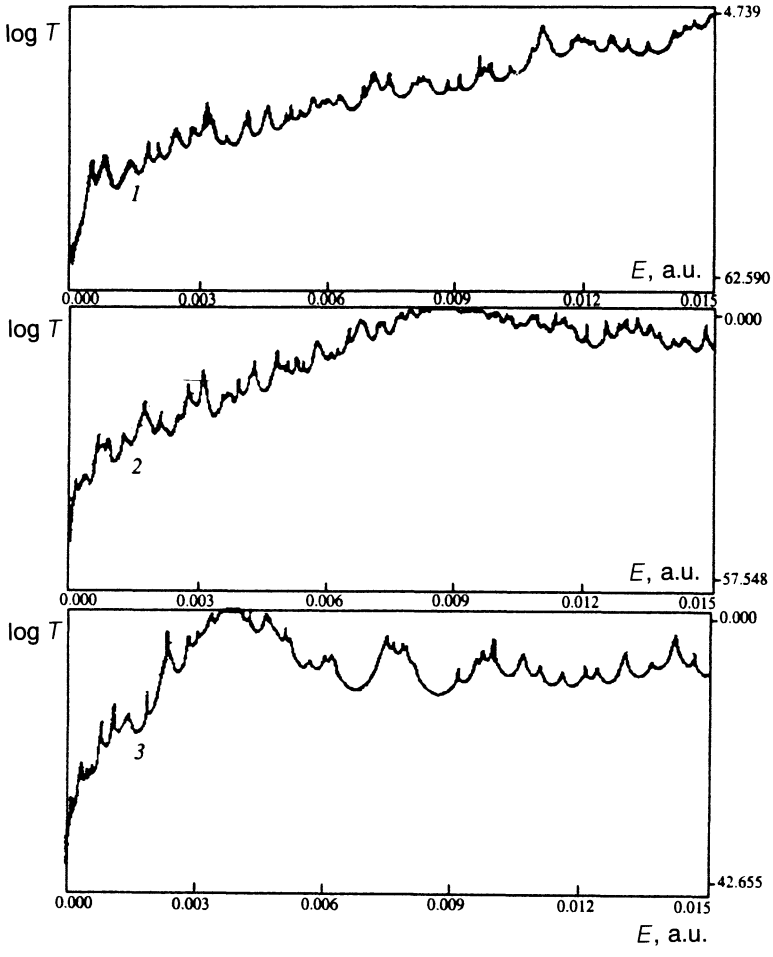

FIG. 1. The dependence of the transmission coefficient T on the electron energy E for a disordered superlattice based on the silicon–silicon-carbide structure with scatterers inside the potential barriers. The scatterer strengths for curves *1*, *2*, and *3* are, respectively, $\beta_1=0$, $\beta_2=-0.1$ a.u., and $\beta_3=-0.13$ a.u. The values of the other parameters are: $V=0.4$ eV, $l=20$, $b=60$ a.e., and $m=0.2m_0$. The scatterers are in the middle of the barriers.

$V=0.4$ eV and $m=0.2m_0$ (see Ref. 11). Calculations show that the transmission coefficient T of the superlattice strongly depends on such parameters as the scatterer strength Ω (or β), the positions occupied by the scatterers inside the barriers, and the number of lattice periods. Here only the results of calculations of $T(E)$ as a function of β are given; the typical behavior found in such calculations is shown in Fig. 1. To encompass the values of the transmission coefficient in the entire range from 0 to V, we plot log $T(E)$ along the vertical axis from the minimum value to the maximum on a uniform scale (the minimum and maximum values of log $T(E)$ are specified in the right part of the figure). The barrier thickness is $b=60$ a.u., the number of superlattice periods $l=20$, and the quantum-well width distribution is assumed Gaussian with a large spread, sufficient to make the width distribution random for all practical purposes. The main features of the resulting spectra described below were reproduced in each of the 300 realizations considered of the random set of well widths. The values of the scatterer strengths for curves *1*, *2*, and *3* are, respectively, $\beta_1=0$ (absence of scatterers), $\beta_2=-0.1$ a.u. and $\beta_3=-0.13$ a.u. Comparison of these curves shows that the tunneling spectra of a disordered superlattice with deep-lying impurity centers in the barriers differ considerably from the spectra of a lattice without scatterers. More precisely, (1) the maximum value of $T(E)$ increases by several orders of magnitude in comparison to the case where there are no scatterers in the superlattice, and (2) an energy interval develops in which $T(E)\approx 1$. The position of this interval on the horizontal (energy) axis depends on the value of $|\beta|$: as $|\beta|$ grows, the interval moves toward smaller values of E (see curves *2* and *3*). Note that the energies at which $T(E)\approx 1$ are in the vicinity of E_m, which corresponds to a resonant level in an isolated (single) barrier, and the maximum transmission coefficient is attained at $E=E_m$.

To estimate the accuracy with which the obtained curves are reproduced, we note, first, that the maximum values $T(E_m)$ of the transmission coefficient for 300 realizations considered (for given values of the parameters) are greater than 0.999 and differ only in the fourth decimal place. Second, the limits of variation of the energy interval ΔE within which $T(E)>0.5$ in 300 realizations considered are

$$0.58\times 10^{-3}\text{ a.u.}\leqslant \Delta E\leqslant 1.25\times 10^{-3}\text{ a.u.}\,.$$

In conclusion we note that there are energies at which the transmission coefficient is close to unity for a broad range of values of the parameters b, l, and β of the problem.

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