Surface acoustic waves caused by a magnetoelectric interaction in piezomagnetic crystals

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It has been shown that the magnetoelectric effect can provide for the existence of a new branch of surface magnetoelectroelastic waves in a piezomagnetic material adjoining a superconductor or a superdiamagnetic material, i.e., media in which the magnetic induction is equal to zero, without mechanical contact. It has been shown in the case of piezomagnetic materials of specific classes of magnetic symmetry that these waves are deeply penetrating quasibulk modes, whose penetration depth is determined by a parameter that is inversely proportional to the product of the squares of two small parameters, viz., the magnetoelectric coefficient and the piezomagnetic coefficient. When the magnetoelectric interaction is "turned off," these surface waves either disappear or transform into bulk waves. The sector for the existence of this new branch of surface acoustic waves has been found. If, however, the medium adjacent to the piezomagnetic material is a metal or an insulator, more localized surface magnetoelastic waves of the Bleustein–Gulyaev type, whose properties are determined mainly by the piezomagnetic—rather than the magnetoelectric—interaction appear instead of magnetoelectroelastic waves.

1. INTRODUCTION

In nonpiezoactive crystals, the existence of bulk elastic waves satisfying the condition of a free surface is possible for a few specific orientations of the surface and directions of propagation.[1,2] Such solutions are unstable, and small perturbations of the geometry of the problem result in their disappearance or localization and conversion into quasibulk surface waves.[3-5] Similar instability is also displayed in the "activation" of physical "perturbations" in the properties of a medium, such as electroelastic or magnetoelastic interactions. The surface waves of the Bleustein–Gulyaev type[6-9] in piezoelectric and piezomagnetic materials are a result specifically of the latter instability, and transform into bulk waves when the piezo effect is "turned off."

One more example of such physical perturbations is provided by the so-called magnetoelectric effect, which exists in crystals with a magnetic structure[9,10] and directly establishes an interrelationship between the magnetic and electric fields. It can influence the acoustic properties of crystals, i.e., the structure and dynamics of elastic fields, only with the aid of intermediaries, i.e., magnetoelastic and electroelastic interactions.[11-13] Owing to its relative weakness, the magnetoelectric interaction only slightly modifies the properties of surface electro- and magnetoacoustic waves. However, as we can see, under certain conditions neither magnetoelastic nor electroelastic interactions in themselves result in localization of the bulk-wave solution, and only consideration of the magnetoelectric effect generates a surface wave and determines its properties.

The present work focuses on a treatment of just such a situation, which arises in a piezomagnetic material adjoining a superconductor or a superdiamagnetic material without mechanical contact.

2. EQUATIONS OF MOTION AND BOUNDARY CONDITIONS

In piezomagnetic media exhibiting a magnetoelectric interaction, the mechanical stresses $\hat{\sigma}$, the magnetic induction **B**, and the electric displacement **D** are related to the strain $\hat{u}$, the magnetic field **H**, and the electric field **E** by the following expressions:[9,10]

$$\hat{\sigma}=\hat{c}\hat{u}-\hat{m}\mathbf{H}, \quad \mathbf{B}=\hat{\mu}\mathbf{H}+\hat{m}\hat{u}+\hat{\nu}\mathbf{E}, \quad \mathbf{D}=\hat{\varepsilon}\mathbf{E}+\mathbf{H}\hat{\nu}. \qquad (1)$$

Here $\hat{c}$ denotes the elastic constants; $\hat{\mu}$ is the magnetic permeability; $\hat{\varepsilon}$ is the dielectric constant; $\hat{m}$ denotes the piezomagnetic moduli, and $\hat{\nu}$ is a tensor that describes the magnetoelectric effect.

The elastic waves in such media are determined by the system of Maxwell's equations and the dynamic elasticity:

$$\operatorname{div}\mathbf{B}=0, \quad \operatorname{div}\mathbf{D}=0, \quad \rho\ddot{\mathbf{u}}=\operatorname{div}\hat{\sigma}, \qquad (2)$$

where **u** is the displacement vector in the wave and ρ is the density of the crystal.

We next consider a semi-infinite piezomagnetic material adjoining a superconductor (3a), a superdiamagnetic material (3b), a metal (3c), or an insulator (3d) without mechanical contact. The boundary conditions on the interface with an internal normal **n** are, respectively,

$$\hat{\sigma}\mathbf{n}=0, \quad \mathbf{Bn}=0, \quad \mathbf{H}_t=\tilde{\mathbf{H}}_t, \quad \mathbf{Dn}=\tilde{\mathbf{D}}\mathbf{n}, \quad \mathbf{E}_t=0, \qquad (3a)$$

$$\hat{\sigma}\mathbf{n}=0, \quad \mathbf{Bn}=0, \quad \mathbf{H}_t=\tilde{\mathbf{H}}_t, \quad \mathbf{Dn}=\tilde{\mathbf{D}}\mathbf{n}, \quad \mathbf{E}_t=\tilde{\mathbf{E}}_t, \qquad (3b)$$

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$$\hat{\sigma}\mathbf{n}=0, \quad \mathbf{Bn}=\tilde{\mathbf{B}}\mathbf{n}, \quad \mathbf{H}_t=\tilde{\mathbf{H}}_t, \quad \mathbf{Dn}=\tilde{\mathbf{D}}\mathbf{n}, \quad \mathbf{E}_t=0, \tag{3c}$$

$$\hat{\sigma}\mathbf{n}=0, \quad \mathbf{Bn}=\tilde{\mathbf{B}}\mathbf{n}, \quad \mathbf{H}_t=\tilde{\mathbf{H}}_t, \quad \mathbf{Dn}=\tilde{\mathbf{D}}\mathbf{n}, \quad \mathbf{E}_t=\tilde{\mathbf{E}}_t. \tag{3d}$$

Here the letters with a tilde denote physical quantities referring to the medium adjoining the piezomagnetic material, and the subscript t denotes the tangential component of the respective vector.

Below we are interested in magnetic media of three magnetic symmetry classes:

$$\infty m', \quad 6m'm', \quad 4m'm'. \tag{4}$$

Besides the elastic constants, we then need the following magnetic and electric characteristics:

$$\mu_1=\mu_2=\mu, \quad \varepsilon_1=\varepsilon_2=\varepsilon,$$
$$m_{15}=m_{24}, \quad \nu_1=\nu_2\equiv\gamma \tag{5}$$

(the z coordinate axis is parallel to the principal symmetry axis).

3. SURFACE MAGNETOELECTROELASTIC WAVES ON INTERFACES WITH A SUPERCONDUCTOR AND A SUPERDIAMAGNETIC MATERIAL

Let us consider a semi-infinite magnetic medium from any of the classes (4) with a principal symmetry axis belonging to the surface. We select the xy plane, which is orthogonal to the principal symmetry axis, as the sagittal plane, the x axis being oriented in the direction of propagation of the wave and the y axis being parallel to the internal normal $\mathbf{n}$. We assume that boundary condition (3a) is realized on the surface, i.e., that there is a magnetoelectric contact with the superconductor leaving the surface free. This may be a thin superconducting coating on a surface, a thin gap between a piezomagnetic material and a superconductor, and in the case of shear (in the plane of the surface) waves considered below, a sliding contact between the two media.

Omitting the simple mathematical manipulations, we present the solution of interest for a surface magnetoelectroelastic wave. The displacement field in such a wave $\mathbf{u}(\mathbf{r},t)\|\mathbf{z}$ is given by

$$u(x,y,t)=A\ \exp^{-pky}\exp^{ik(x-vt)}, \tag{6}$$

where

$$p=\bar{\gamma}^2m^2, \quad \bar{\gamma}^2=\gamma^2/\varepsilon\mu, \quad m^2=m_{15}^2/\bar{c}\bar{\mu},$$
$$\bar{c}=c_{44}+m_{15}^2/\bar{\mu}, \quad \bar{\mu}=\mu(1-\bar{\gamma}^2), \tag{7}$$

A is the amplitude on the surface ($y=0$), and p is the dimensionless inverse penetration depth of the wave, which propagates with a velocity$=\omega/k$. Here

$$v^2=(1-p^2)v_t^2, \quad v_t^2=\bar{c}/\rho, \tag{8}$$

where v_t is the velocity of the bulk transverse wave in an infinite medium.

The magnetic and electric components of the wave are given by

$$\mathbf{H}=-\text{grad}\ \Phi_H, \quad \Phi_H=\frac{m_{15}}{\bar{\mu}}Ae^{ik(x-vt)}(e^{-pky}-\bar{\gamma}^2e^{-ky}),$$

$$\mathbf{E}=-\text{grad}\ \Phi_E, \quad \Phi_E=\frac{\gamma}{\varepsilon}\frac{m_{15}}{\bar{\mu}}Ae^{ik(x-vt)}(e^{-ky}-e^{-pky}). \tag{9}$$

The field of mechanical stresses in the wave, as well as the magnetic induction and the electric displacement, are assigned by plugging expressions (6) and (9) into relations (1). The smallness of p in (7), which is of order $\bar{\gamma}^2m^2$, means that the surface wave under consideration is a deeply penetrating (quasibulk) wave. We note that the degree of localization of Bleustein–Gulyaev waves at the surface is higher, since the value of p is not so small for them: $p\sim m^2$ (see below). It is significant that in the absence of a magnetoelectric interaction, $p=0$ in expressions (6)–(8), i.e., localization disappears, and the surface wave transforms into a bulk wave propagating along the boundary.

We now turn to boundary problem (3b), in which the piezomagnetic medium under consideration adjoins a superdiamagnetic material without mechanical contact. It is not difficult to see that the shear wave is also described in this case by Eqs. (6)–(8) after the following replacement:

$$p\rightarrow p/(\bar{\varepsilon}+1), \quad \bar{\varepsilon}=\varepsilon(1-\bar{\gamma}^2) \tag{10}$$

(for simplicity, the dielectric constant of the superdiamagnetic material is assumed to be equal to unity). Accordingly, the following replacement should be made in Eqs. (9):

$$e^{-ky}\rightarrow\frac{\mu\varepsilon}{\bar{\varepsilon}+1}e^{-ky}. \tag{11}$$

In addition, while in the preceding case the entire wave field was concentrated in the piezomagnetic material, now the electric field "leaks" into the adjoining medium:

$$\tilde{\mathbf{E}}|_{y<0}=-\text{grad}\ \tilde{\Phi}_E, \quad \tilde{\Phi}_E=-\frac{\gamma m_{15}\varepsilon}{\bar{\varepsilon}+1}Ae^{ky}e^{ik(x-vt)}. \tag{12}$$

A completely analogous physical picture emerges in piezoelectric media of symmetry classes (4) with a magnetoelectric effect when their dielectric constant is much greater than that of the adjoining medium. Here it can be assumed that $\mathbf{Dn}\rightarrow0$ on the boundary, although $\mathbf{H}_t\neq0$, and all the magnetic parameters in the derived relations like (6) and (10)–(12) can be replaced by the corresponding electrical parameters and vice versa.

4. SURFACE MAGNETOELASTIC WAVES ON A BOUNDARY WITH A METAL OR AN INSULATOR

A qualitatively different picture emerges if the medium adjoining the piezomagnetic material is a metal or an insulator [boundary problems (3c) and (3d)]. The magnetoelectric interaction plays a secondary role in these cases, and the main cause of localization of the wave is piezomagnetism. The expression for the inverse penetration depth of the wave now has the form

$$p=\frac{m^2}{\mu+1}(1+\gamma^2\eta), \tag{13}$$

$$\eta=\begin{cases}1/\varepsilon & (14)\\ (\varepsilon+\mu+1)/\varepsilon[(\varepsilon+1)(\mu+1)-\gamma^2]. & (15)\end{cases}$$

Expression (14) corresponds to a boundary with a metal (its magnetic permeability is assumed to be equal to unity), and expression (15) corresponds to a boundary with an insulator (for simplicity, both the dielectric constant and the magnetic permeability are set equal to unity). Here, in fact, we are dealing with surface magnetoelastic waves of the Bleustein–Gulyaev type, which have been modified only by the influence of the magnetoelectric effect. When the latter is absent, i.e., when $\gamma=0$, the cases covered by Eqs. (13)–(15) are identical, and a "classical" wave of the Bleustein–Gulyaev type appears in the piezomagnetic material.

5. SECTOR FOR THE EXISTENCE OF THE BRANCH OF SURFACE MAGNETOELECTROELASTIC WAVES

We have thus encountered a new type of localization of elastic waves on a mechanically free interface between a piezomagnetic material and a medium exhibiting the Meissner effect. The specific features of the new surface wave stem from the fact that its localization disappears together with the magnetoelectric interaction. The stability of this type of localization against a small perturbation in the geometry of the problem or the physical constants (with lowering of the symmetry) is a fundamental question.

As an example, let us consider a small deviation in the direction of propagation of the wave through an angle φ from the symmetric direction with maintenance of the orientation of the surface and the remaining parameters [for simplicity, we restrict ourselves to an analysis of only transversely isotropic media of classes $\infty m'$ and $6m'm'$ from (4)]. Just as was done in Ref. 14, where the sectors for the existence of Bleustein–Gulyaev waves in piezoelectric materials were considered, it can be shown that all the expressions obtained above remain valid under such a perturbation,[1)] if the parameter p in them is replaced by

$$p(\varphi)=p-(a^2p_l/f)\varphi^2. \tag{16}$$

Here a is a parameter proportional to the elastic anisotropy:

$$a=\frac{c_{13}}{c_{66}}\left[\frac{c_{12}(c_{13}+c_{44})}{c_{13}(c_{11}-c_{44})}-1\right], \tag{17}$$

$$f=(1+p_{t'}^2)^2-4p_lp_{t'}, \tag{18}$$

$$p_l=\sqrt{1-\frac{c_{44}}{c_{11}}},\quad p_{t'}=\sqrt{1-\frac{c_{44}}{c_{66}}}. \tag{19}$$

The sign of the addition to p in (16) is specified by the sign of f. According to Refs. 5 and 14, $f>0$ when $\rho v_R^2<c_{44}<c_{66}<c_{11}$, and $f<0$ when $\rho v_R^2>c_{44}$, where v_R is the velocity of the Rayleigh wave, which always exists on the free surface in a transversely isotropic direction. In the former case ($f>0$), which is illustrated in Fig. 1a, the Rayleigh wave belongs to the subsonic range of velocities and is clearly maintained even when $\varphi\neq 0$. On the other hand, the magnetoelectroelastic wave exists only within the sector

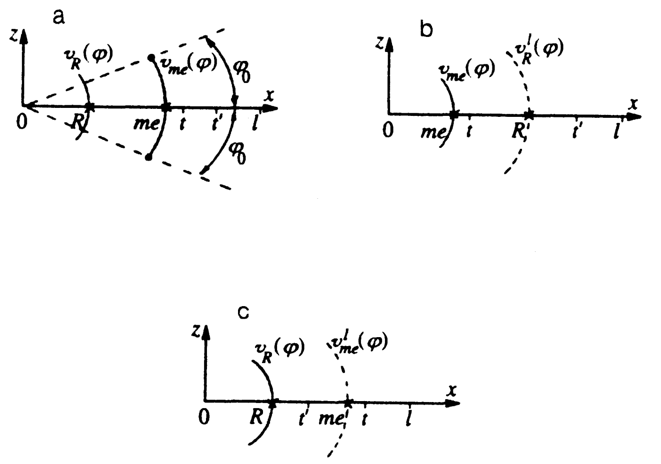

FIG. 1. Polar diagram of the dependence of the velocity of surface waves on the propagation direction in a small neighborhood of a transversely isotropic orientation. The crosses denote surface magnetoelectroelastic (me) and Rayleigh (R) waves existing in a transversely isotropic direction. The letters t, t', and l indicate the velocities of the bulk (transverse and longitudinal) waves. a) Two branches of surface waves $v_R(\varphi)$ and $v_{me}(\varphi)$ exist. The circles denote bulk waves on the boundaries of the sector for the existence of surface magnetoelectroelastic waves. b) Two branches of surface magnetoelectroelastic waves $v_{me}(\varphi)$ and a branch of pseudosurface (leaky) Rayleigh waves $v_R^l(\varphi)$ (dashed line) exist. c) A branch of surface Rayleigh waves $v_R(\varphi)$ and a branch of pseudosurface magnetoelectroelastic waves $v_{me}^l(\varphi)$ exist.

$$|\varphi|<\varphi_0\equiv\frac{1}{|a|}\sqrt{\frac{fp}{p_l}}. \tag{20}$$

Within this sector $p(\varphi)>0$ characterizes the degree of localization of the surface wave; on the boundaries of the sector $p(\pm\varphi_0)=0$, localization disappears, and outside sector (20), i.e., when $|\varphi|>\varphi_0$, $p(\varphi)<0$ corresponds to a nonphysical growing solution inside the crystal. At a fixed value of φ, a decrease in γ constricts the sector, so that at some moment the direction of φ changes from being an internal direction with respect to the sector to being an external direction, and the corresponding wave transforms from a surface wave into a nonphysical wave. In other words, in the case under consideration ($f>0$), sector (20) contains a complete branch of solutions for surface magnetoelectroelastic waves, which is governed entirely by the magnetoelectric interaction and disappears (together with the spectrum) as $\gamma\to 0$.

In the latter case ($f<0$), the velocity of the Rayleigh wave falls in the transonic range (Fig. 1b). When the symmetric orientation is perturbed, such a wave must transform into a pseudosurface (leaky, hence the index l in the notation for the velocity) wave,[14,15] i.e., it acquires a small-amplitude bulk partial wave that drains energy from the surface, and, accordingly, an imaginary addition to the phase velocity ($v_R\to v_R^l - iv_l'$) that ensures decay of the amplitude as the wave propagates over the surface. On the other hand, the second wave remains subsonic, and its degree of localization $p(\varphi)$ [see (16)] only increases in response to a deviation. Physically, we are dealing with a known branch of quasibulk waves[5] that has been modified by the magnetoelectric interaction. Of course, this modification is significant only as long as the second term in (16) does not excessively exceed the first.

Thus, in both cases a new branch of solutions for surface waves is realized in a small neighborhood of the symmetric orientation ($\varphi=0$). Their localization is completely determined by the magnetoelectric effect when $f>0$, and is largely determined by this effect when $f<0$.

Finally, if $c_{66}<c_{44}<c_{11}$, a situation corresponding to Fig. 1c is realized, in which the velocity of the magnetoelectroelastic wave v_{me} in a transversely isotropic direction falls in the transonic range, and a deviation from the symmetric orientation generates the corresponding leaky branch.

We close with one final remark. For simplicity, we restricted ourselves above to consideration of piezomagnetic materials of symmetry classes (4), which have a symmetric magnetoelectric interaction tensor $\hat{\nu}$ with zero off-diagonal components. Considerable interest has recently been evoked by media with a toroidal moment characterized by a tensor $\hat{\nu}$ with a nonzero antisymmetric part. If we consider crystals from the $\infty 2'$ and $62'2'$ symmetry classes, for which $\nu_{11}=\nu_{22}=0$ and $\nu_{21}=-\nu_{12}=q$, as an example, the entire analysis can be performed in analogy to the preceding discussion. It then turns out that localization does not appear ($p=0$) on the boundary with a superconductor [boundary conditions (3a)] in the transversely isotropic direction. An ordinary quasibulk wave[5,14] exists in the vicinity of this direction under the condition $f<0$ [see the second term in (16)]. On a boundary with a superdiamagnetic material [boundary conditions (3b)] "antilocalization" ($p<0$) appears:

$$p=-\frac{\bar{q}^2 m^2 \varepsilon}{\bar{\varepsilon}+1}, \tag{21}$$

where

$$\bar{q}^2=\frac{q^2}{\varepsilon\mu},\quad m^2=\frac{m_{15}^2}{\bar{c}\mu},\quad \bar{c}=c_{44}+\frac{m_{15}^2}{\mu},\quad \bar{\varepsilon}=\varepsilon(1-\bar{q}^2).$$

As before, Eq. (16) holds in the vicinity of the symmetric direction. When $f>0$, "antilocalization" is clearly maintained over the entire neighborhood [$p(\varphi)<0$], and when $f<0$, the second term in (16) becomes larger than the first outside of sector (20) and localization appears despite the magnetoelectric interaction, not because of it.

Thus, at least in the case just considered, the antisymmetric part of $\hat{\nu}$ opposes localization more than it promotes it.

6. CONCLUSIONS

As we have seen, the quasibulk transverse surface wave considered is not the only solution for the geometry under consideration. In particular, another fundamental solution corresponding to an ordinary Rayleigh wave slightly modified by the piezomagnetic and magnetoelectric interactions always exists simultaneously in the symmetric direction. It interesting to trace the consistency of the number of emerging surface-wave solutions with the existing requirements of the general theory.[13,16] If the medium is nonpiezoactive ($m=0$), in the geometry considered there should always be a single branch of surface waves:[5,16] Rayleigh waves when $f>0$ and quasibulk waves when $f<0$. The "activation" of piezomagnetism (when $\gamma=0$) alters the situation. In this case the number of solutions depends on the type of boundary conditions:[13,17] no more than one solution for surface waves on the boundary with a superconductor or a superdiamagnetic material [boundary conditions (3a) and (3b)], and no more than two on a boundary with a metal or an insulator [boundary conditions (3c) and (3d)]. In particular, when $f>0$ and the existence of the Rayleigh branch in the subsonic velocity range is ensured, this corresponds to the absence of localization in (6), (7), and (10) [$p(\varphi)\leq 0$ when $\gamma=0$] and its presence in (6) and (8) ($p>0$ when $\gamma=0$). According to Ref. 13, the "activation" of an additional magnetoelectric interaction also allows two surface-wave solutions in cases (3a) and (3b). Along with the Rayleigh branch, this pair is also formed by the solution of (6), (7), and (10), but only within sector (20) (Fig. 1a). Only one branch of surface waves is realized in the subsonic range outside of the sector when $f>0$ and over the entire neighborhood of the transversely isotropic direction when $f<0$ (see Figs. 1a and 1b).

It should be noted in conclusion that the surface waves considered above are very deeply penetrating due to the weakness of the magnetoelectric interaction. Their penetration depth $h\sim(m^2\gamma^2 k)^{-1}$ is four orders of magnitude greater than the wavelength even at record values of the parameters ($m^2\sim 10^{-1}$ and $\gamma\sim 10^{-2}$),[17] and it amounts to several millimeters for waves with a frequency ~ 10 GHz. On the other hand, in composite materials containing piezoelectric and piezomagnetic substances, γ can be larger: $\gamma\sim 10^{-1}$ (in accordance with the results of Ref. 18). This reduces our estimate of the localization depth by two orders of magnitude.

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[1)]A small change in the displacement vector **u**, as well as small additions associated with other localized partial waves, are not significant for our ensuing discussion.

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