Large relativistic corrections to the positronium decay probability

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We find the relativistic corrections to the positronium decay probability. Our calculations are based on noncovariant perturbation theory. The relative corrections to the total probability are found to amount to $40(\alpha/\pi)^2$ for the singlet state and $46(\alpha/\pi)^2$ for the triplet.

1. INTRODUCTION

The large difference between the experimental value of the orthopositronium decay probability[1]

$$\Gamma^{o}_{\mathrm{expt}} = 7.0482 \pm 16\ \mu\mathrm{s}^{-1} \tag{1}$$

and the theoretical value of this quantity incorporating relative-order corrections α and $\alpha^2\log(1/\alpha)$ (see Refs. 2–5)

$$\Gamma^{o}_{\mathrm{theor}} = m\alpha^6 \frac{2(\pi^2-9)}{9\pi}\left[1 - 10.28\frac{\alpha}{\pi} - \tfrac{1}{3}\alpha^2\log\frac{1}{\alpha}\right] = 7.038\,30\ \mu\mathrm{s}^{-1} \tag{2}$$

poses a serious problem for modern quantum electrodynamics (QED). To resolve this contradiction within QED, corrections of the order of $(\alpha/\pi)^2$, which have yet to be calculated, must be included in the theoretical result (2) with a numerical factor of 250 ± 40. Such a value of the factor may seem unreasonably large.

There are, however, arguments that this might occur.[6] The large factor -10.28 in the term representing the correction of order α/π to the orthopositronium decay probability [see Eq. (2)] means that the typical value of the factor in the correction of order α/π to the decay amplitude is roughly equal to five. Accordingly, the square of this correction contributes a term of order $25(\alpha/\pi)^2$ to the decay probability. A direct numerical calculation of this contribution[7] gives 28.8 ± 0.2 for the value of the coefficient of $(\alpha/\pi)^2$. It is natural to expect, then, that the interference of the second-order correction in α to the amplitude of the process and the zeroth-order amplitude gives a correction twice as large as the square of the first-order amplitude. In other words, the natural scale for the total second-order correction to the orthopositronium decay probability is[8]

$$100\left(\frac{\alpha}{\pi}\right)^2. \tag{3}$$

Samuel and Li[9] recently arrived at a similar conclusion by basing the reasoning on the Padé approximation.

Relativistic corrections can provide an even larger contribution to the positronium decay probability. A simple argument in their favor is that the corresponding expansion parameter $(v/c)^2 \sim \alpha^2$ is $\pi^2 \sim 10$ times larger than the similar parameter $(\alpha/\pi)^2$ of ordinary second-order radiative corrections.

In this paper we obtain the relativistic corrections to the orthopositronium and parapositronium decay probabilities. Earlier this problem was considered in Refs. 10 and 11. Our approach differs from that used in Refs. 10 and 11. What is more important, however, is that the results and conclusions differ too. The origin of this discrepancy is explained below.

We started our calculations with the relativistic correction to the parapositronium decay probability, considering them only as a warming-up exercise before solving the more difficult problem of orthopositronium. We found, however, that the corrections obtained for the singlet state are large and very close in accuracy to the data obtained in recent experiments.[12]

2. CORRECTIONS TO THE PARAPOSITRONIUM DECAY PROBABILITY

Relativistic corrections to positronium decay originate in both the annihilation kernel and the wave function of the bound state. We begin with the corrections caused by the expansion of the annihilation kernel. The corrections caused by the wave function of the bound state will be considered later.

The central point of our work consists in the following. When calculating the decay amplitude in the positronium rest frame, we must integrate the product of the annihilation kernel and the positronium wave function over the three-dimensional momentum $\mathbf{p}$ of the electron and positron. To lowest order in v/c the annihilation kernels of parapositronium and orthopositronium are momentum-independent. Therefore, what remains is an integral with respect to $\mathbf{p}$ of the nonrelativistic wave function in the momentum representation, which is equivalent to $\psi(r=0)$ in the coordinate representation. The main difficulty here is that allowing for a correction of the order $(p/m)^2$ in the annihilation kernel leads to the following integral over momenta:

$$\int d^3p\left(\frac{p}{m}\right)^2\psi(\mathbf{p}) = \int d^3p\left(\frac{p}{m}\right)^2\frac{8\sqrt{\pi a^3}}{(p^2a^2+1)^2}, \tag{4}$$

which diverges linearly as $p\to\infty$ (here $a = 2/m\alpha$ is the Bohr radius of positronium). These difficulties are resolved in the following way. The exact expression for the annihilation kernel does not grow as $p\to\infty$, in contrast to its expansion in powers of p/m. Hence, the integral of the product of the kernel and $\psi(p)$ does indeed converge. The integral with respect to $|\mathbf{p}|$ can be transformed into an integral from $-\infty$ to $+\infty$ and the integration contour can be shifted into the upper half-plane. Here we must allow for the pole of the

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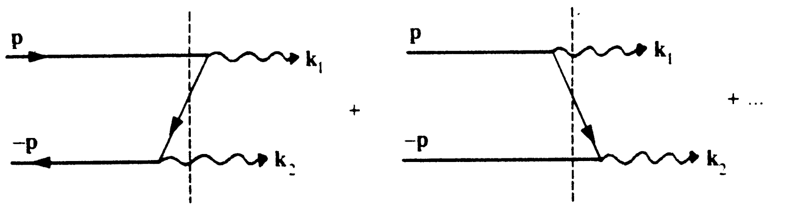

FIG. 1. The diagrams of noncovariant perturbation theory for the parapositronium annihilation kernel. The dashed lines stand for intermediate states. The diagrams differing in photon permutation are not shown.

wave function at $p=\frac{1}{2}\,im\alpha$ and the branching originating at point $p=im$ and related to the annihilation amplitude. Obviously it is this pole that corresponds to the desired annihilation corrections. The contribution of the pole can easily be calculated. This gives us the following procedure for calculating the relativistic corrections. After expanding the amplitude in powers of $\mathbf{p}/m$ and averaging over the angles we must perform the substitution

$$\left(\frac{p}{m}\right)^2=v^2\rightarrow-\frac{3}{4}\alpha^2. \tag{5}$$

The surprise is the "minus" in this expression. One must bear in mind, however, that the cut also contributes to the integral. This contribution is determined by the relativistic momenta $|p|\sim m$ and corresponds to the ordinary radiative corrections, which have a relative order of roughly α/π and are much larger than the effect we are interested in ($\sim\alpha^2$).

The next idea important to our discussion is the use of noncovariant perturbation theory (see, e.g., Ref. 13), which simplifies calculations considerably and allows the binding energy of positronium to be considered easily. The annihilation amplitude is described by the diagrams shown in Fig. 1 and has the form

$$M=4\pi\alpha V^{\dagger}(\mathbf{e}_2\boldsymbol{\alpha})\,\frac{\Lambda_+(\mathbf{p}-\mathbf{k}_1)-\Lambda_-(\mathbf{p}-\mathbf{k}_1)}{E-\omega-\epsilon(\mathbf{p}-\mathbf{k}_1)-\epsilon(p)}\,(\mathbf{e}_2\boldsymbol{\alpha})U+(1\leftrightarrow2) \tag{6}$$

$$V=\sqrt{\frac{\epsilon(p)+m}{2\epsilon(p)}}\,(1-\boldsymbol{\alpha}\mathbf{p}/\epsilon(p)+m)\begin{pmatrix}0\\ \chi\end{pmatrix},$$

$$U=\sqrt{\frac{\epsilon(p)+m}{2\epsilon(p)}}\,(1+\boldsymbol{\alpha}\mathbf{p}\epsilon(p)+m)\begin{pmatrix}\phi\\ 0\end{pmatrix}.$$

Here χ and ψ are nonrelativistic spinors, $E=2m-\frac{1}{4}m\alpha^4$ is the total positronium energy, $\mathbf{e}_{1,2}$ and $\mathbf{k}_{1,2}$ are the photon polarization vectors and momenta, $\omega_1=\omega_2=\omega=\frac{1}{2}E$ is the photon energy, we have written $\epsilon(p)=\sqrt{m^2+p^2}$, and

$$\Lambda_{\pm}(\mathbf{p})=\tfrac{1}{2}[1\pm\boldsymbol{\alpha}\mathbf{p}+\beta m/\epsilon(p)]$$

are the projection operators, respectively, on the positive and negative frequency states of an electron with momentum $\mathbf{p}$. The Coulomb interaction in an intermediate state is unimportant because the momentum of one of the particles in it is close to m.

We represent the amplitude M as $M=M_0+\delta M$, where M_0 is the amplitude in the lowest order. Expanding M in powers of $\mathbf{p}/m$, averaging over the directions of $\mathbf{p}$ (recall that we are examining the S-state), and employing (5), we get

$$\delta M=\alpha^2\left(\frac{1}{2}+\frac{\sqrt{2}}{8}\right)M_0. \tag{7}$$

The respective relative correction to the parapositronium decay probability caused by the expansion of the annihilation kernel is

$$\frac{\delta\Gamma_1^p}{\Gamma_0^p}=\alpha^2\left(1+\frac{\sqrt{2}}{4}\right)=1.35\alpha^2. \tag{8}$$

3. CORRECTIONS TO THE ORTHOPOSITRONIUM DECAY PROBABILITY

We now consider the relativistic correction to the orthopositronium decay probability caused by the expansion of the annihilation kernel. This problem is much more involved than in the case of parapositronium. The diagrams of the noncovariant perturbation theory contributing to the three-

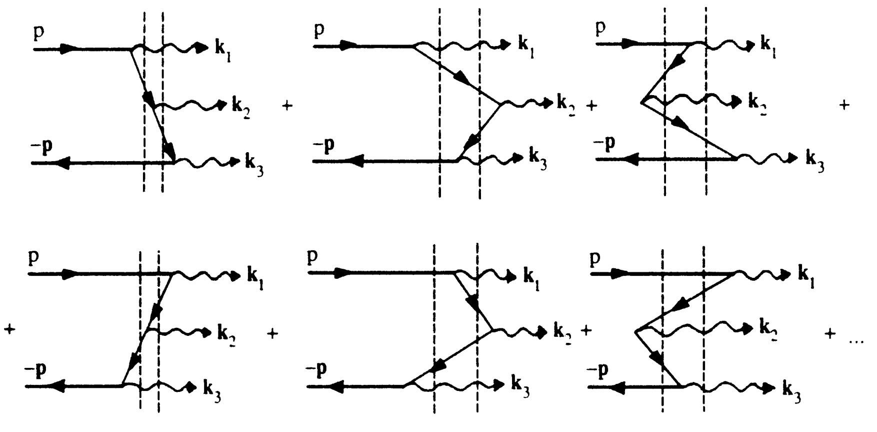

FIG. 2. The diagrams of noncovariant perturbation theory for the orthopositronium annihilation kernel.

photon annihilation kernel are shown in Fig. 2. It has proved convenient to transform the amplitude in the following manner so that further calculations become considerably simpler. We write the energy E of the initial state, which enters into the perturbation-theory denominators, as

$$E=E-2\epsilon(p)+2\epsilon(p)$$

and expand the amplitude in powers of the difference

$$W-2\epsilon(p)=m\alpha^2$$

[here we use the substitution (5) for p^2/m^2]. The zeroth term in the series expansion in powers of this difference transforms into the ordinary covariant Feynman amplitude of three-photon annihilation of an electron and a positron with 4-momenta $(\epsilon(p),\pm\mathbf{p})$. Expanding this "covariant" part of the annihilation amplitude in powers of $\mathbf{p}/m$ up to second-order terms, averaging the terms quadratic in $\mathbf{p}/m$ over the directions of $\mathbf{p}$, and employing the substitution rule (5), we get

$$\begin{aligned}M_c=\tfrac{5}{8}\alpha^2M_0-\frac{(4\pi\alpha)^{3/2}\alpha^2}{8m^2}\chi^\dagger\Big\{&[(\mathbf{e}_2\mathbf{e}_3)(\mathbf{n}_1\mathbf{n}_3+\mathbf{n}_1\mathbf{n}_2-\mathbf{n}_2\mathbf{n}_3)\\&-(\mathbf{n}_2\mathbf{n}_3)(\mathbf{h}_2\mathbf{h}_3)]\boldsymbol{\sigma}\mathbf{e}_1+[(\mathbf{e}_2\mathbf{h}_3)(\mathbf{n}_1\mathbf{n}_3)+(\mathbf{e}_3\mathbf{h}_2)\\&\times(\mathbf{n}_1\mathbf{n}_2)]\boldsymbol{\sigma}\mathbf{h}_1+\frac{4m^2}{\omega_2\omega_3}(\mathbf{e}_2\mathbf{e}_3)\boldsymbol{\sigma}\mathbf{e}_1+\frac{2m}{\omega_1}[(\mathbf{e}_2\mathbf{e}_3\\&+\mathbf{h}_2\mathbf{h}_3)\boldsymbol{\sigma}\mathbf{e}_1+(\mathbf{e}_1\mathbf{n}_3)\times(\mathbf{e}_2\times\mathbf{h}_3)\boldsymbol{\sigma}+(\mathbf{e}_1\mathbf{n}_2)(\mathbf{e}_3\times\mathbf{h}_2)\boldsymbol{\sigma}]\\&+(1\to2)+(1\to3)\Big\}\phi.\end{aligned}\qquad(9)$$

Here

$$\begin{aligned}M_0=-\frac{(4\pi\alpha)^{3/2}}{2m^2}\chi^\dagger[&(\mathbf{e}_2\mathbf{e}_3-\mathbf{h}_2\mathbf{h}_3)\boldsymbol{\sigma}\mathbf{e}_1\\&+(\mathbf{e}_2\mathbf{h}_3+\mathbf{e}_3\mathbf{h}_2)\boldsymbol{\sigma}\mathbf{h}_1\\&+(1\to2)+(1\to3)]\phi\end{aligned}\qquad(10)$$

is the zeroth-order amplitude in v/c, $\mathbf{n}_i=\mathbf{k}_i/\omega_i$, and $\mathbf{h}_i=\mathbf{n}_i\mathbf{e}_i$.

The interference of M_c and M_0 after summation over the photon polarizations and integration over the final states yields the following relative correction to the orthopositronium decay probability:

$$\frac{\delta\Gamma_1^o}{\Gamma_0^o}=\alpha^2\frac{31\pi^2-240}{16(\pi^2-9)}.\qquad(11)$$

Next we must allow for the correction caused by the change in the phase volume due to the binding energy $E-2m=-\frac{1}{4}m\alpha^2$. We can easily show that this correction is

$$\frac{\delta\Gamma_2^o}{\Gamma_0^o}=-\alpha^2\frac{1}{4}.\qquad(12)$$

Thus, the total "covariant" correction to the orthopositronium decay probability is

$$\frac{\delta\Gamma_1^o+\delta\Gamma_2^o}{\Gamma_0^o}=\alpha^2\frac{27\pi^2-204}{16(\pi^2-9)}.\qquad(13)$$

If in (10) we replace α^2 with $-\frac{4}{3}v^2$ in accordance with rule (5) and replace $E-2m$ with mv^2 when allowing for the change in the phase volume, the result coincides with the v2-correction to the probability of three-photon annihilation of a free electron and positron in the $^3S_1$ state obtained by Kuraev *et al.*[10] (see also Ref. 11).

Now let us turn to the "noncovariant" part of the correction to the annihilation amplitude [i.e., terms proportional to $E-2\epsilon(p)=\frac{1}{2}m\alpha^2$]:

$$\begin{aligned}M_{nc}=\frac{(4\pi\alpha)^{3/2}\alpha^2}{8m}\chi^\dagger\Big\{&[(\mathbf{e}_1\mathbf{e}_3)\boldsymbol{\sigma}\mathbf{e}_2+(\mathbf{e}_1\mathbf{e}_2)\boldsymbol{\sigma}\mathbf{e}_3-(\mathbf{e}_2\mathbf{e}_3)\boldsymbol{\sigma}\mathbf{e}_1]\\&\times\left[\frac{(m+\omega_2-\epsilon_2)(m+\omega_3-\epsilon_3)}{2\epsilon_2\epsilon_3(\epsilon_2+\epsilon_3-\omega_1)}+\frac{\epsilon_2-\omega_2}{m\epsilon_2}+\frac{\epsilon_3-\omega_3}{m\epsilon_3}\right]\\&+[(\mathbf{e}_1\mathbf{h}_3)\boldsymbol{\sigma}\mathbf{h}_2+(\mathbf{e}_1\mathbf{h}_2)\boldsymbol{\sigma}\mathbf{h}_3-(\mathbf{h}_2\mathbf{h}_3)\boldsymbol{\sigma}\mathbf{e}_1]\\&\times\left[\frac{(m-\omega_2+\epsilon_2)(m-\omega_3+\epsilon_3)}{2\epsilon_2\epsilon_3(\epsilon_2+\epsilon_3-\omega_1)}+\frac{m-\omega_2+\epsilon_2}{\epsilon_2(m-\omega_2-\epsilon_2)}\right.\\&\left.+\frac{m-\omega_3+\epsilon_3}{\epsilon_3(m-\omega_3-\epsilon_3)}\right]+(1\to2)+(1\to3)\Big\}\phi.\end{aligned}\qquad(14)$$

Here $\epsilon_i=\sqrt{\omega_i^2+m^2}$. The correction to the decay probability related to the amplitude $M_{\rm nc}$ was found numerically and amounted to

$$\frac{\delta\Gamma_3^o}{\Gamma_0^o}=0.807\alpha^2.\qquad(15)$$

Note that the exceptional term with $\sqrt{2}$ in the correction to the singlet-state decay probability [see Eqs. (7) and (8)] has the same "noncovariant" origin.

Returning to the decay of the triplet state, we note that our results for the α2-correction differ from those obtained in Refs. 10 and 11. It is not so much a matter of the "noncovariant" correction (14) omitted in Refs. 10 and 11: this correction is moderate numerically. The main difference lies in the correspondence between v^2 and α^2. Kuraev *et al.*[10] assumed that $v^2\to\alpha^2$, which leads to an erroneous sign of the α2-correction (but it still yields a reasonable absolute value). On the other hand, the assumption that $v^2\to-\frac{1}{4}\alpha^2$ used by Labelle *et al.*[11] yields a correct sign but strongly understates the value of the correction. Labelle *et al.*[11] explicitly state, however, that they have obtained only a fraction of the relativistic corrections. Therefore, there is no real contradiction between our results and those obtained in Ref. 11.

4. CORRECTIONS TO THE DECAY PROBABILITY CAUSED BY THE CHANGE IN THE POSITRONIUM WAVE FUNCTION

We now examine effects related to the relativistic corrections to the positronium wave function $\psi(\mathbf{r})$. To this end we use the Breit equation as is done in Ref. 6. Here parapositronium and orthopositronium can be considered along parallel lines.

The part of the Breit Hamiltonian (BH) corresponding to the relativistic corrections to the dispersion law and the Coulomb interaction,

$$8V_C=-\frac{p^4}{4m^3}+\frac{\pi\alpha}{m^2}\delta(\mathbf{r}), \tag{16}$$

can easily be transformed into

$$V_C=\frac{\alpha^3}{8r}. \tag{17}$$

Here and in what follows we drop the constant terms in the perturbation (obviously, they do not give rise to a change in the wave function) and substitute $-\frac{1}{2}m\alpha$ for ∂_r, which acts on the ground state of positronium.

The next spin-independent term in BH,

$$V_m=-\frac{\alpha}{2m^2r}\left(p^2+\frac{1}{r^2}\mathbf{r}(\mathbf{rp})\mathbf{p}\right), \tag{18}$$

describes the magnetic interaction of the electron and positron due to orbital motion. For the ground state this operator transforms into

$$V_m=\frac{\alpha^3}{4r}-\frac{\alpha^2}{2mr^2}. \tag{19}$$

The last term in BH yielding corrections we are interested in is the contact spin–spin interaction

$$V_s=\frac{\pi\alpha}{m^2}A\,\delta(\mathbf{r}),\quad A=\tfrac{7}{3}S(S+1)-2. \tag{20}$$

A convenient way to write it is

$$V_s=\frac{A}{4m}\left[H,\frac{\alpha}{r}\right]+A\frac{\alpha^2}{4mr^2},\quad H=\frac{p^2}{m}-\frac{\alpha}{r}. \tag{21}$$

The terms $\alpha^3/8r$ and $\alpha^3/4r$ in Eqs. (16) and (18) taken together lead, obviously, to a shift in the coupling constant: $\alpha\to\alpha(1-\frac{3}{8}\alpha^2)$. This in turn yields the following relative correction to $|\psi(0)|^2$ and, respectively, to the decay probability

$$\frac{\delta\Gamma_{\psi1}}{\Gamma_0}=-\frac{9\alpha^2}{8}, \tag{22}$$

which is the same for orthopositronium and parapositronium.

It is equally easy to calculate the correction related to the commutator in (20):

$$\frac{\delta\Gamma_{\psi2}}{\Gamma_0}=A\frac{\alpha^2}{2}. \tag{23}$$

Finally, we examine the singular term of the transformed Breit Hamiltonian:

$$\tilde{V}_2=\frac{\lambda}{mr^2},\quad \lambda=\alpha^2\left(\frac{A}{4}-\frac{1}{2}\right). \tag{24}$$

The normalized solution to the radial wave equation

$$\left(\frac{1}{r}\frac{d^2}{dr^2}r-\frac{\lambda}{r^2}+\frac{m\alpha}{r}+m\tilde{E}\right)R=0 \tag{25}$$

has the form

$$R=2\left(\frac{m\alpha}{2}\right)^{3/2}[1-\lambda(3-C)](m\alpha r)^\lambda \times\exp\left[-\frac{(1-\lambda)m\alpha r}{2}\right], \tag{26}$$

where $C\approx0.577$ is Euler's constant. The fact that the eigenvalue $\tilde{E}$ differs from $-\frac{1}{4}m\alpha^2$ is unimportant for our purposes. From (25) we can easily find the corresponding relative correction to $|\psi(0)|^2$ and the positronium decay probability:

$$\frac{\delta\Gamma_{\psi3}}{\Gamma_0}=-2\lambda[(3-C)-\log(m\alpha r_0)], \tag{27}$$

where $r_0\sim 1/m$ is the distance over which annihilation takes place. The logarithmically strengthened part of this correction,

$$\alpha^2\log(1/\alpha)\begin{cases}2 & \text{if } S=0,\\ -1/3 & \text{if } S=1,\end{cases} \tag{28}$$

was obtained earlier for the triplet case [see Eq. (2)] and the singlet case in Refs. 4 and 6, respectively. So we omitted it in (26). Thus, we arrive at the following expression for the total relativistic correction caused by the modification of the ψ-function:

$$\frac{\delta\Gamma_\psi}{\Gamma_0}=\alpha^2\begin{cases}31/8-2C-2\log(mr_0) & \text{if } S=0,\\ -19/24+(1/3)C+(1/3)\log(mr_0) & \text{if } S=1.\end{cases} \tag{29}$$

We assume that ±1 serves as a reasonable estimate of the interval of possible values of $\log(mr_0)$; this spread appears because of the uncertainty in the cutoff of r_0 at small distances. On the other hand, in our treatment the cutoff of the logarithmic contribution at atomic distances has been done exactly.

Our result for the relativistic correction caused by the orthopositronium wave function, $[-19/24+(1/3)C]\alpha^2=-0.6\alpha^2$ differs from that obtained by Labelle *et al.*[11] At present we have no explanation for this discrepancy, since the work of Labelle *et al.* gave only the numerical result for this correction: $1.16\alpha^2$.

5. CONCLUSION

Summing all the contributions, we arrive at the expressions for the relativistic correction to the decay probability for parapositronium and orthopositronium, respectively:

$$\frac{\delta\Gamma^p}{\Gamma_0^p}=4.1\alpha^2=40\left(\frac{\alpha}{\pi}\right)^2, \tag{30}$$

$$\frac{\delta\Gamma^o}{\Gamma_0^o}=4.7\alpha^2=46\left(\frac{\alpha}{\pi}\right)^2 \tag{31}$$

(apparently, it is also useful to give these results in ordinary "radiation" units of α/π). The omission of the terms with $\log(mr_0)$ leads to an error that we estimate at

$$\pm2\alpha^2=\pm20\left(\frac{\alpha}{\pi}\right)^2 \quad \text{if} \quad S=0, \tag{32}$$

$$\pm\frac{1}{3}\,\alpha^2=\pm 3\left(\frac{\alpha}{\pi}\right)^2 \quad \text{if} \quad S=1. \tag{33}$$

For orthopositronium decay our correction (30) with the correction found by Burichenko[7] substantially lowers the difference between the result predicted by theory and that given by experiment from $(250\pm 40)(\alpha/\pi)^2$ to $(175\pm 40)\times(\alpha/\pi)^2$.

For parapositronium the value of the calculated correction is close to the attained experimental accuracy.[12] Here there is ground to believe that ordinary radiative correction will prove to be large. Hence, the measuring of the effect appears fairly realistic.

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