Potential influence of pre-exponential factors on the temperature dependence of variable-range hopping conductivity

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We have analyzed how temperature-dependent pre-exponential factors affect the temperature dependence of conductivity due to variable-range hopping in the range of temperatures where crossover occurs from the Mott to the Efros–Shklovskii law. We show that the temperature dependence introduced by these factors can significantly affect the details of the crossover, and in some cases can even mimic it (if the temperature dependence of the resistance is inferred without taking these factors into account). In our analysis of several sample crystals of doped CdTe over the temperature range 0.4 to 20 K, we found that by including pre-exponential factors we eliminated an apparent crossover for the lowest-resistance samples, so that the conductivity was described by the Mott law alone. For higher-resistance samples we were unable to represent the temperature dependence of the resistivity by a Mott law over the entire temperature range, even when pre-exponential factors were included; at low temperatures, these samples exhibit a tendency towards a transition to the Efros–Shklovskii law. In all cases, inclusion of these factors leads to significant reestimates of the parameters that characterize the closeness of the sample to the metal–insulator transition (localization radius, dielectric constant). These conclusions are verified by measurements of the positive magnetoresistance.

1. INTRODUCTION

In recent years there has been considerable interest in studying the crossover from variable-range hopping conductivity of Mott type (for which the density of states at the Fermi level $N(\varepsilon)=\text{const}$) to conductivity via states of a Coulomb gap (for which $N(\varepsilon)\propto\varepsilon^2$). This phenomenon, which should be observable as the temperature decreases, has been the subject of a series of experimental and theoretical papers.[1–6] Especially noteworthy among these are the paper by Aharony *et al.*,[2] where an attempt was made to obtain a single universal analytic expression for the temperature dependence of the conductivity $\sigma(T)$ in the crossover region, and that of Shlimak *et al.*,[5] in which experimental data were used to fit the behavior of the density of states analytically in the crossover region. A common feature of most of these references is their simplistic treatment of pre-exponential factors in the function $\sigma(T)$, which are assumed to be temperature-independent and the same for both regimes. In the theoretical analysis of variable-range hopping conductivity, neglect of these factors is almost a tradition. This fact underlines the importance of the work of Mansfield,[7] who demonstrated that ignoring the temperature dependence of pre-exponential factors can lead to significant errors in determining the parameters of variable-range hopping conductivity, especially for low-resistance samples that are close to the metal–insulator transition.

In this paper we attempt to analyze the role of pre-exponential factors in determining the temperature dependence of the conductivity in the region where it changes over from the Mott law to the Efros–Shklovskii law. We show that the temperature behavior of pre-exponential factors can affect the details of the crossover in a significant way, and in certain cases can even mimic crossover when the experimental data are inadequately processed.

Our study of doped crystals of CdTe reveals that for low-resistance samples, the inclusion of these factors eliminates cases of questionable crossover, so that the conductivity is of the Mott type everywhere. For higher-resistance samples, we found that it was not possible to fit the function $\sigma(T)$ with only one of these laws over the entire temperature range under study, even when pre-exponential factors are taken into account, and we showed that the temperature dependence tends to change over to the Efros–Shklovskii law at low temperatures. In this case, however, the crossover temperature turns out to be lower than that estimated without including these pre-exponential factors. In both cases this leads to a significant change in estimates of the parameters that characterize how close the sample is to the metal–insulator transition (e.g., the localization radius a, the density of states at the Fermi level $N(\varepsilon_F)$, and the dielectric constant κ).

2. MODEL

In view of the great diversity of theoretical results that emerge from calculations of the pre-exponential factors in the expressions

$$\rho(T)=\rho_0\exp\left(\frac{T_0}{T}\right)^{1/4}, \tag{1}$$

$$\rho(T)=\rho_1\exp\left(\frac{T_1}{T}\right)^{1/2}, \tag{2}$$

corresponding to the Mott and the Efros–Shklovskii laws, respectively, we present a concise analysis of the problem here.

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The factors defined above can be estimated most simply from the following considerations (compare with Shklovskii *et al.*[8]). By changing the ratio of the energies of lattice sites, an applied electric field makes hopping in the direction of the field somewhat more preferable, which also leads to the appearance of a current. The factor that characterizes the relative number of "directed" hops obviously can be obtained by expanding the exponential that describes the hopping probability in powers of the asymmetry of the site energy produced by the electric field ($\sim eER$, where R is the characteristic hopping length), which gives a term $\sim eER/T$. The fact that the hopping process translates an electron by a distance R in a time $\sim\nu^{-1}$ (where ν is the hopping probability) leads to the following expression for the conductivity:

$$\sigma\approx\frac{e^2\nu R^2}{T}n, \tag{3}$$

where n is the total concentration of carriers that participate in the hopping transport,

$$\nu=\nu_0\exp\left(-\frac{2R}{a}-\frac{W}{T}\right),$$

and W is a typical hopping energy. Thus, the temperature dependence of these factors is determined by the corresponding temperature dependences of ν_0, R, and n. To estimate the last of these quantities, Mott[9] assumed that most of the mobile carriers come from an energy band of width $\sim T$ in the vicinity of the Fermi level:

$$n\approx 2\int_{\varepsilon_F}^{\varepsilon_F+T}d\varepsilon N(\varepsilon), \tag{4}$$

where N is the density of states. We note that although this assumption (which we will verify below) leads to a correct order-of-magnitude expression for the pre-exponential factor in the Mott regime, it is not easy to reconcile with the existence of an effective bandwidth of energies W, and cannot be used to obtain the correct expression in the Efros–Shklovskii regime.

In their subsequent paper, Allen and Adkins[10] assumed that all the states in the band of energies W participate in variable-range hopping:

$$n\approx n_W=2\int_{\varepsilon_F}^{\varepsilon_F+W}d\varepsilon N(\varepsilon). \tag{5}$$

We note that this assumption is deficient in that it does not include the exponentially broad scatter of frequencies ν for hopping between various pairs of lattice sites, which ensures that carriers can contribute to the conductivity only if their characteristic hopping radius exceeds a certain "threshold" value (implying that the effective value of n should be smaller than the total concentration of sites in the band W).

A rigorous analysis of the problem within the framework of percolation theory (see, e.g., Ref. 8) leads to an expression for σ that is essentially analogous to (3), with the replacement

$$n=n_W\xi_c^{-\gamma},\quad \xi_c=W/T, \tag{6}$$

where γ is the critical index for percolation theory; for the three-dimensional case this is approximately 0.9. The approach we follow here is essentially that of percolation theory.

However, it is worth noting that the factor ν_0 is model-sensitive and depends on the specific features of the material. When the characteristic wavelength of those phonons responsible for activation is smaller than R (which is typical of the situation under study), we have

$$\nu_0=\frac{\Lambda^2}{\pi\rho s^5\hbar^4}WI^2 \tag{7}$$

(see Ref. 9). Here Λ is the deformation potential, s and ρ are the velocity of sound and the density of the material, and I is the energy overlap integral of the wave functions at the lattice sites. This latter quantity can be sensitive to the locations of the centers as specified by a given realization. For hydrogenic centers with Coulomb interactions

$$I=\frac{2e^2}{3\kappa a}\frac{R}{a}\exp\left(-\frac{2R}{a}\right). \tag{8}$$

In this case we are led to the expressions ρ_1:

$$\rho_0=C\frac{ea^4}{N_F}\left(\frac{4\pi N_F}{3B_c}\right)^{3/2}\sqrt{\frac{3a}{2}}+\sqrt{T}\left(\frac{T_0}{T}\right)^{\gamma/4}$$

$$\approx 10.5Ca^3\left(\frac{T}{T_0}\right)^{s_0}, \tag{9}$$

$$\rho_1=C\frac{a^4}{N_0}\left(\frac{4\pi N_0}{9B_c}\right)^{4/3}T\left(\frac{T_1}{T}\right)^{\gamma/2}\approx 0.4Ca^3\left(\frac{T}{T_1}\right)^{s_1}, \tag{10}$$

$$s_0=\frac{2-\gamma}{4}\approx\frac{1}{4},\quad s_1=2s_0. \tag{11}$$

Here C is a constant determined by the parameters of the electron–phonon interaction, $B_c\approx 2.7$, N_F and N_0 are respectively the density of states at the Fermi level for the Mott model and the constant in the function $N(\varepsilon)=N_0\varepsilon^2$ for the Efros–Shklovskii model (in which $N_0=1.15\kappa^3/e^6$, $T_0=\beta_0/N_Fa^3$, and $T_1\approx\beta_1e^2/\kappa a$; β_0 and β_1 are numerical coefficients). In the paper by Mansfield,[7] $\beta_0=49$, $\beta_1=10$, whereas Shklovskii *et al.*[8] give the values $\beta_0=16$, $\beta_1=2.8$, based on their numerical modeling.

We note that the estimate $s_0=1/2$, $s_1=1$ given by Mansfield[7] stems from work based on the assumptions of Allen and Adkins (see above). In contrast, Mott[9] used the estimated value $I=I_0\exp(-2R/a)$, where $I_0=$const; in the final analysis, this estimate is the origin of the value $s_0=-1/4$, which is often cited in the literature.

In light of this model sensitivity, we will treat the quantities s_0 and s_1 as unknown phenomenological parameters in certain expressions, despite our clear prejudice in favor of the results of percolation theory $s_0\approx 1/4$, $s_1\approx 1/2$.

Above all, note that if we define the crossover temperature from the condition that the characteristic energy for Mott variable-range hopping ($\approx T^{3/4}(N_Fa^3)^{-1/4}$) be equal to the maximum Coulomb gap ($\Delta\approx N_F^{1/2}e^3/\kappa^{3/2}$), i.e.,

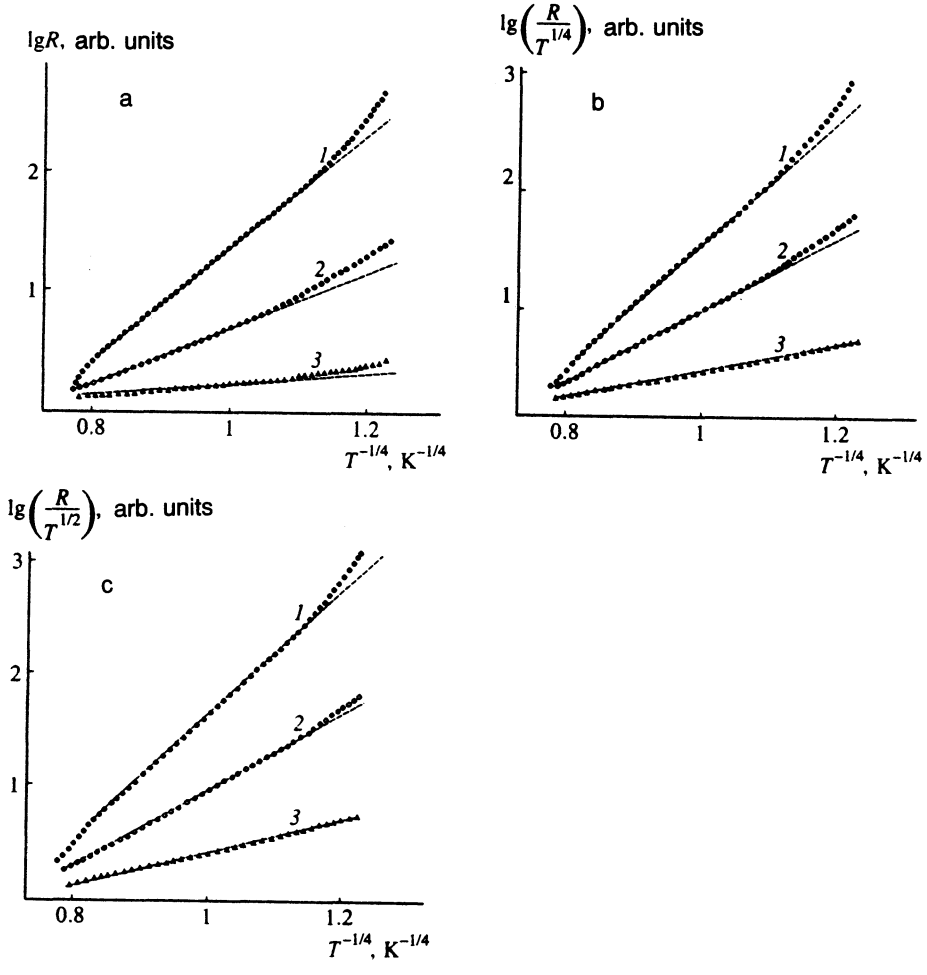

FIG. 1. Temperature dependence of the resistance of three samples, in which $\log(R/T^{s})$ is plotted against $T^{-1/4}$ over the temperature range 0.4 to 3 K; labeling of the curves corresponds to the sample labels in Table I. a) $s_0=0$, b) $s_0=1/4$, c) $s_0=1/2$.

$$T_c \approx \left(\frac{\beta_0}{\beta_1}\right)\frac{T_1^2}{T_0}, \tag{12}$$

then not only are the exponents in the laws (1), (2) comparable, but so are the values of the pre-exponential factors (9), (10), up to a numerical factor of order unity. In our view, this indicates that the approaches used to calculate the exponents and pre-exponential factors are consistent. The lack of exact numerical agreement between laws (1), (2) at the crossover point is most likely associated with the smoothness of the transition, i.e., with a presence of a transition region whose width is no smaller than T. In this case it is important to note that even when the exponential terms match asymptotically at the point $T_c^* = (T_1^2/T_0)$, the change of functional dependences at the transition is not accompanied by an appreciable change in the temperature dependence. In fact, as the temperature is decreased by $\delta T = T - T_c^*$, the deviation of $\rho(T)$ from the function determined by the dependence (2) due to the exponential factor is

$$\Delta\rho(T) \sim \exp\left[\left(\frac{T_0}{T_c^*+\delta T}\right)^{1/4} - \left(\frac{T_1}{T_c^*+\delta T}\right)^{1/2}\right]$$
$$\sim \exp\left[4\sqrt{\frac{T_0}{T_1}\frac{\delta T}{T_c^*}}\right]^{-1}, \tag{13}$$

so that even for $T_0/T_1 \sim 100$, the deviation is not too large, even for $\delta T \sim T_c^*$. In our view, this circumstance clearly indicates that it is necessary to include the pre-exponential factor, especially in a detailed analysis of the behavior of the density of states in the vicinity of the edge of the Coulomb gap.[5]

Let us now discuss the possible role of temperature dependences of ρ_0 and ρ_1. It is obvious that such dependences

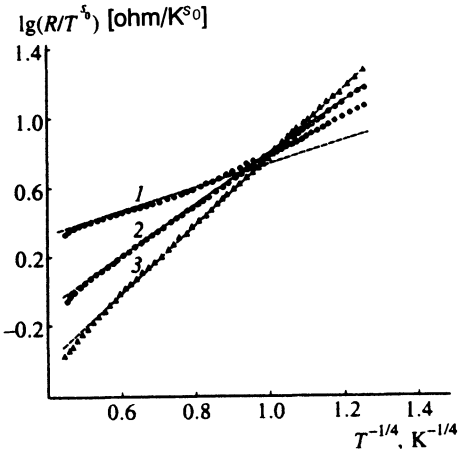

FIG. 2. Temperature dependence of the resistivity of sample No. 3 in which $\log(R/T^{s_0})$ is plotted against $T^{-1/4}$ over the temperature range 0.4–10 K: *1*—$s_0=0$; *2*—$s_0=1/4$; *3*—$s_0=1/2$.

TABLE I.

№	n_{300K}, cm^{-3}	N_F, cm^{-3}/eV	T_0, K			a, Å		
			$s_0 = 0$	$s_0 = 1/4$	$s_0 = 1/2$	$s_0 = 0$	$s_0 = 1/4$	$s_0 = 1/2$
1	$8 \cdot 10^{16}$	$5 \cdot 10^{18}$	$1.2 \cdot 10^4$	$1.7 \cdot 10^4$	$2.8 \cdot 10^4$	170	140	120
2	$9 \cdot 10^{16}$	$1 \cdot 10^{19}$	$1 \cdot 10^3$	$2.2 \cdot 10^3$	$4 \cdot 10^3$	290	220	180
3	$1.2 \cdot 10^{17}$	$4 \cdot 10^{19}$	5	85	350	1050	410	260

will lead to a deviation of the curves $\log[R(T)]$ (here and in what follows R is the resistance of the sample) from the corresponding linear functions that result when these curves are plotted in the "rectifying coordinates" $(1/T)^{1/2}$ or $(1/T)^{1/4}$. This deviation will be especially marked when the range of variation of T is sizable. Thus, in coordinates $x = \sqrt{T_c/T}$ the function (2) becomes

$$\ln \rho = \sqrt{\frac{T_c}{T_1}}\, x - 2s_1 \ln x + \text{const}. \tag{14}$$

It is clear that for $s_1 \approx 1/2$ the departure of $\ln[\rho(T)]$ from the corresponding linear dependence reaches a value of 1 over an interval of variation in which x changes by a factor of 3. In this case, since $d^2(\ln \rho)/dx^2 > 0$, as x decreases (T increases) the deviation takes place in the direction of larger resistances (compared to the linear law) and is more marked for smaller x. Therefore, if we do not take into account the pre-exponential factor, the temperature dependence of the resistance can mimic a transition to another regime (i.e., the crossover behavior under discussion).

Likewise, for the regime of Mott variable-range hopping, in the coordinate $y = (T_c/T)^{1/4}$ we have

$$\ln \rho = \sqrt{\frac{T_0}{T_1}}\, y - 4s_0 \ln y + \text{const}, \tag{15}$$

which formally coincides with (14). Accordingly, in this case the departure from linearity as T decreases (increasing y) also takes place in the direction of larger resistances, and also can imitate crossover.

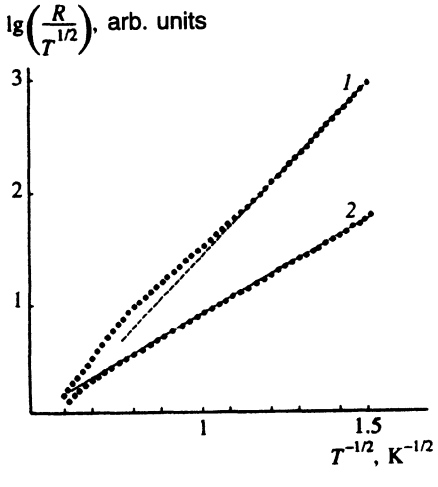

FIG. 3. Temperature dependence of the resistances of samples Nos. 1 and 2, in which $\log(R/T^s)$ is plotted as a function of $T^{-1/2}$ ($s_1 = 1/2$).

Finally, let us discuss the possible roles of the functions $\rho_0(T)$ and $\rho_1(T)$ for a case where crossover has really been observed. Note that in most papers, crossover is defined as the point where the function $\ln[\rho(x)]$ first departs from linearity in coordinates $x = 1/\sqrt{T}$ as x decreases. However, as we noted above, because of the presence of a pre-exponential term, the slope of the function $\ln[\rho(x)]$ is not, generally speaking, a constant even for large x (which leads to a certain arbitrariness in this "graphical" definition of crossover). Therefore, we will estimate the deviation $\Delta/n\rho$ of the function $\ln[\rho(x)]$ from a straight line for decreasing x ($x<1$), where $x = \sqrt{T_c^*/T}$, due to a transition to the Mott regime by taking the slope of the line to correspond to the function (1), including the pre-exponential factor at the point $x=1$:

$$\Delta \ln[\rho(x)] = \sqrt{\frac{T_0}{T_1}}\,(\sqrt{x} - x) + 2s_1(x-1) - 2s_0 \ln x. \tag{16}$$

It is apparent that the fact that s_0 and s_1 differ from zero greatly alters the temperature variation compared to the situation discussed in Refs. 5, 6: since the maximum value of $(\sqrt{x} - x)$ for $x<1$ is only 1/4, even when $T_0/T_1 = 10^2$ all the terms in (9) turn out to be quantities of the same order. In particular, it is clear in this case that for sufficiently large values of the difference $s_1 - s_0$ the sign of the deviation can, in principle, be negative as well (in the direction of smaller resistances); as was shown in Ref. 5, this cannot happen if we assume that $\rho_0 = \rho_1 = \text{const}$.

3. EXPERIMENT

We measured the temperature-dependent resistance of three samples of CdTe doped with shallow Cl donors having ionization energies 0.014 eV over the temperature range 0.4 to 20 K. At 300 K, the concentration of electrons n varied within the limits $(0.8–1.2) \cdot 10^{17}$ cm^{-3}; this is close to the critical value $n_c = 1.5 \cdot 10^{17}$ cm^{-3} for a metal–insulator transition in uncompensated CdTe. The concentration of donors was estimated to be $(2–5) \cdot 10^{17}$ cm^{-3}, so that the fact that the samples were on the insulator side of the transition indicates the presence of disorder caused by compensation. In Ref. 11 the temperature-dependent resistance was measured for these samples, and $\log R(T)$ was plotted as a function of $T^{-1/4}$. For all the samples, as T decreased we observed a departure from linearity in the direction of higher resistance, which was interpreted as evidence of crossover from conductivity with variable-range hopping corresponding to the Mott law to variable-range hopping via the states of the Coulomb gap (Efros–Shklovskii hopping).

TABLE II.

№	T_1, K			κ		a, Å
	$s_1=0$	$s_1=1/2$	$s_1=1$	$s_0=1/4,\ s_1=1/2$	$s_0=1/2,\ s_1=1$	($s_0=1/4,\ s_1=1/2$)
1	35	50	66	60	50	125
2	6	20	25	115	105	170

In this paper we investigated how the temperature dependence of the pre-exponential factors $\rho_0(T)$ and $\rho_1(T)$ affected the behavior of the temperature dependence of the resistance for these samples. In Figs. 1(a,b,c) we plot the dependence of $\log(R/T^{s_0})$ on $T^{-1/4}$ for the three samples, using Eqs. (1) and (9) for $s_0=0$, 1/4, and 1/2. In Fig. 2 we use an expanded scale to plot the curves for sample 3 with $s_0=0$, 1/4, and 1/2. It is clear that whereas for $s_0=0$ we observe a considerable deviation from a straight line in the low-temperature region (which previously was interpreted as a manifestation of crossover), for the choices $s_0=1/4$ or $s_0=1/2$ the curve is, in fact, "rectified" on this scale for the entire temperature interval. This resolves the question of whether crossover occurs, and leads to a considerable reestimate of the parameter T_0.

In our two other samples, the deviation from linearity is preserved even when we used a nonzero value of s_0, both for $s_0=1/4$ and $s_0=1/2$; however, this deviation is observed at lower temperatures than it is when the curves are plotted assuming $s_0=0$. In our view, the presence of these deviations is evidence that crossover occurs in samples Nos. 1 and 2 with decreasing temperature. Unfortunately, the limited temperature range does not allow us to observe the regions where the Efros–Shklovskii law is clearly in evidence.

The values of the parameter T_0 obtained for various values of s_0 are shown in Table I. It is clear that these values differ markedly, especially for samples in which the resistivity is only weakly temperature-dependent. Using these values of T_0, we also estimated the localization radius a, taking for the density of states at the Fermi level the estimate $N_F \sim N_D/\varepsilon_1$, where N_D is the concentration of donors and ε_1 is the activation energy, which is determined from the high-temperature portion of the function $R(T)$. We note that this is a rather inaccurate way to calculate a, for the following reasons. First of all, it is difficult to estimate the donor concentration unless the doping is light (as in sample No. 1), in which case it is possible to investigate and interpret the temperature dependence of the Hall effect. For the two other samples, we estimated the values of N_D by assuming that N_D increases in proportion to the number of Cl atoms introduced. On the other hand, the estimate $N_F \sim N_D/\varepsilon_1$, which in itself assumes constancy of $N(\varepsilon)$ over the corresponding interval of energies, is, generally speaking, questionable. Despite all this, we still regard this analysis as useful, since it allows us to make at least an order-of-magnitude estimate of a.

In order to estimate the parameter T_1 based on the low-temperature behavior of the resistance, in Fig. 3 we plot $\log(R/T^{s_1})$ versus $T^{1/2}$ for $s_1=1/2$. The estimated values of T_1 taken from the low-temperature linear portions are listed in Table II; they turn out to be considerably larger than the corresponding values obtained without including the pre-exponential factor.

The conclusions we have formulated above are based on analysis of data on the temperature behavior of the conductivity. In order to bolster our arguments in favor of these conclusions, we measured the magnetoresistance in the range of magnetic fields 0 to 6 T. In weak magnetic fields, the dependence of the positive magnetoresistance on magnetic field for variable-range hopping conductivity is given by the expression[7]

$$\ln \frac{R(H)}{R(0)} = \left(\frac{H}{B}\right)^2, \tag{17}$$

where H is the magnetic field and B is a coefficient that depends on the temperature, the localization radius, and the conductivity mechanism (either the Mott or Efros–Shklovskii law):

$$B^2 = \frac{c^2\hbar^2}{C_0 e^2 a^4}\left(\frac{T}{T_0}\right)^{3/4} \tag{18}$$

or

$$B^2 = \frac{c^2\hbar^2}{C_1 e^2 a^4}\left(\frac{T}{T_1}\right)^{3/2}. \tag{19}$$

Expression (18) corresponds to the Mott law, for which the coefficient $C_0=0.0025$. Expression (19) corresponds to conductivity via states of the Coulomb gap; two values of the coefficient C_1 are found in the literature, 0.0015 and 0.0035.[5] It should be noted that the region of positive magnetoresis-

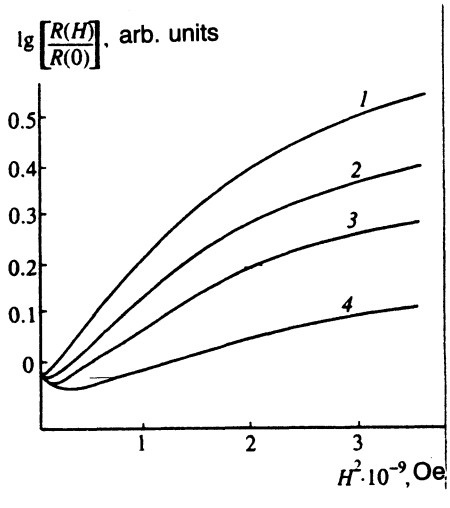

FIG. 4. Magnetic field dependence (H) of the resistance of sample No. 2 at various temperatures: *1*—0.43 K; *2*—0.6 K; *3*—0.85 K; *4*—1.6 K.

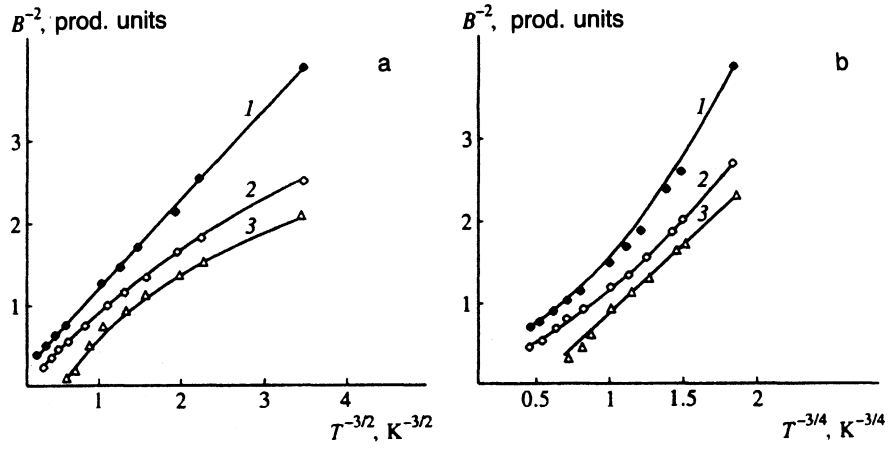

FIG. 5. Temperature dependence of the parameter B^{-2}; curves (a) and (b) correspond to Eqs. (19) and (18), respectively. Labeling of the curves corresponds to the sample labels in Table I.

tance has been rather well-studied in a large number of experimental and theoretical papers, in which good agreement is observed between the temperature dependence of the slope of $\log[R(H)/R(0)]$ versus H^2 and Eqs. (18) or (19), depending on which law the temperature behavior of the conductivity follows at $H=0$.

For our samples, we observed a negative magnetoresistance in the range of weak magnetic fields $H<2$T, followed by a region of quadratic positive magnetoresistance which for $H>4$T is replaced by a region where the positive magnetoresistance is a weak function of $H(\log[R(H)/R(0)]\propto H^{2/3})$, see Fig. 4. The latter (along with the negative magnetoresistance) is probably associated with effects due to intermediate scatterers.[12] Figure 5 shows the slope of the positive magnetoresistance function versus H for the three samples at temperatures in the range 0.4 to 3 K. It is clear that for sample No. 1 this function is well-described by Eq. (19), which corresponds to variable-range hopping via states of the Coulomb gap over the entire temperature range. On the other hand, the temperature dependence of the conductivity for this sample indicates that the transition to this type of variable-range hopping conductivity takes place at lower temperatures, $T<1$ K. The reason for this behavior is that the presence of a magnetic field, which corresponds to the region of quadratic positive magnetoresistance, can increase the crossover temperature, a circumstance that has been observed in a number of experimental papers.[13] For sample No. 2, the temperature dependence of the slope of the positive magnetoresistance indicates the existence of a transition from variable-range hopping of Mott type to hopping via states of the Coulomb gap for $T<1$ K. Finally, for the lowest-resistance sample, the corresponding temperature dependences of the positive magnetoresistance are rectified when we use a scale corresponding to the Mott law over the entire range of temperatures, which is consistent with the conclusion arrived at by analysis of the temperature dependence of the conductivity, taking into account $R_0(T)$. Thus, the qualitative temperature behavior of the quadratic positive magnetoresistance and the temperature behavior of the conductivity are in agreement.

In order to estimate quantitatively how close the samples are to the metal–insulator transition, we estimated the radius of localization from the expression for the slope of $\ln[R(H)/R(0)]$ vs H^2. In this case, we used Eq. (19) for sample No. 1 and Eq. (18) for samples Nos. 2 and 3. Our results are shown in Table III.

By comparing the values of a obtained in this way with previous estimates obtained directly from the values of T_0 under various assumptions about the values of s_0 (Table I), we arrive at the following conclusions. When we assume that $s_0=0$, we cannot reconcile the results of the two independent approaches mentioned above. However, if we assume that $s_0=1/4$ and $1/2$, we achieve reasonable agreement, better for $s_0=1/4$. We note that the values of a increase significantly for the low-resistance samples, which reflects the divergence of the value of a as we approach the metal–insulator transition. However, when we assume $s_0=1/4$, $1/2$, the estimates obtained by starting from values of T_0 indicate that the divergence is not as significant as when $s_0=0$ (see Ref. 11).

For samples in which crossover is observed, we also used our estimates of T_1 to determine the dielectric constant κ (Table II) under the conditions $s_0=1/4$, $s_1=1/2$ and $s_0=1/2$, $s_1=1$. It is clear that the values of κ estimated in this way are considerably higher than the dielectric constant of the lattice κ_0, which also reflects the divergence of this parameter as we approach the metal–insulator transition. If we assume that the corresponding critical index is twice that for the parameter a, as the theory indicates,[14] these consider-

TABLE III. Localization radii computed using Eqs. (18), (19) from the dependence of the resistance on magnetic field in the region of quadratic positive magnetoresistance.

№	a,Å ($s_0=s_1=0$)	a,Å ($s_0=1/4$, $s_1=1/2$)	a,Å ($s_0=1/2$, $s_1=1$)
1	200	165	150
2	390	240	210
3	700	400	310

ations lead us to yet another independent estimate of the parameter a based on the expression $a=a_0\sqrt{z}$, where $z=\kappa/\kappa_0$, $a_0\approx 50$ Å, $\kappa_0\approx 10$ (Table II). It is clear that the values of a obtained in this way are in reasonable agreement with estimates obtained from values of the positive magnetoresistance for the same s_0. For $s_0=s_1=0$, no such agreement is observed.

4. CONCLUSION

Thus, by comparing the theoretical results with experimental data for CdTe crystals, we have demonstrated that a first-principles analysis of the behavior of hopping conductivity requires inclusion of the temperature dependence of the preexponential factor, especially in the region of crossover from Mott conductivity to conductivity via states of the Coulomb gap. Neglect of this dependence can lead to an inaccurate determination of the crossover point, and even to an apparent crossover in a temperature range where it does not, in fact, occur. Likewise, unless the pre-exponential factor is included, it turns out to be impossible to obtain reasonable estimates of the parameters that characterize the closeness of a material to the metal–insulator transition from data on variable-range hopping. In particular, by choosing various values of s for the temperature dependence of the preexponential factor, we find that it is not possible to reconcile estimates of the localization radius using different independent methods when $s_0=s_1=0$, whereas the choice $s_0=1/4$, $s_1=1/2$ leads the best agreement. We note that the lack of self-consistency of estimates of this kind for $s_0=s_1=0$ was demonstrated previously by Shklovskii *et al.*[11], whose results revealed a significant divergence of a as the metal–insulator transition is approached without any detectable divergence of κ, which is inconsistent with the theory.

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