Spectrum of the surface magnetic polaritons of a ferromagnetic plate

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The spectrum of the electromagnetic oscillations of a ferromagnetic plate magnetized parallel to its surfaces has been analyzed. The dispersion relation for waves (magnetic polaritons) propagating parallel to the magnetic field and the magnetization has been calculated. It has been shown that three types of waves exist: fast waves with normal dispersion, slow waves with anomalous dispersion, which transform into the familiar type of magnetostatic waves in the limit of large values of the wave vector, and special waves. An analysis with comparison to the case of a wave propagating perpendicular to the magnetic field has been performed.

1. INTRODUCTION

In Ref. 1, we analyzed the dispersion relation for electromagnetic waves propagating perpendicular to a magnetic field **H** and its associated parallel magnetization **M** in a ferromagnetic plate of thickness $2d$. The amplitudes of the waves (surface magnetic polaritons) decay exponentially with increasing distance from the plate. Solving Maxwell's equations with standard boundary conditions and taking into account the temporal dispersion of the magnetic susceptibility (a consequence of the Landau-Lifshitz equations), we can find the relation between the frequency ω and the wave vector k over the entire range for the existence of magnetic polaritons ($k \geqslant \omega/c$). In this case (when $\mathbf{k} \perp \mathbf{H}$) an irreversible Damon-Eshbach surface wave[2] exists on the boundary of the ferromagnetic half-space in the quasiclassical limit ($k \gg \omega/c$), so that the results in Ref. 1 may be regarded as a generalization of the results in Ref. 2.

The plots of $\omega = \omega(k)$ found in Ref. 1 are schematically depicted in Fig. 1 in order to be able to compare the results obtained here with the results in Ref. 1.

The case $\mathbf{k} \perp \mathbf{H}$ is interesting due to the existence of a special branch of oscillations in the interval $[\sqrt{\omega_0(\omega_0+\omega_M)},\ \omega_0 + 1/2\,\omega_M]$, which transforms into a Damon–Eshbach wave[2] when $kd \gg 1$ and $c \to \infty$:[1)]

$$\omega = \omega_{DE} \equiv \omega_0 + \frac{1}{2}\,\omega_M. \tag{1}$$

Unlike the waves of the upper and lower groups, the electromagnetic fields in this group are superpositions of hyperbolic, rather than trigonometric, functions. The velocity of the special branch vanishes at the limits of the frequency range and reaches a value $\sim c$ at the maximum. The curve describing the special branch begins to the right of the straight line $\omega = ck$ [the coordinates of its origin are $kc = \sqrt{(\omega_0/\omega_M)}(\omega_0 + \omega_M)$, $\omega = \sqrt{\omega_0(\omega_0+\omega_M)}$]. At these values of the frequency and the wave vector, the magnetic field and the magnetization are concentrated near the boundaries of the plate (the penetration depth of the field and the magnetization at this point is formally equal to zero; see, however, Ref. 1). When $\omega \to \omega_{DE}$, the penetration depth approximates the wavelength $\lambda = 2\pi/k$.

2. PROPAGATION OF A WAVE IN A PLATE PARALLEL TO THE MAGNETIC FIELD AND THE MAGNETIZATION (k||H)

Let a ferromagnetic plate of thickness $2d$ be magnetized parallel to its surfaces and let a wave propagate in the plate parallel to the magnetic field **H** and the magnetization **M**. As the distance from the plate increases, the electromagnetic field decays exponentially with a logarithmic decay constant

$$\gamma_0 = \sqrt{k^2 - \omega^2/c^2}. \tag{2}$$

Hence there is a restriction on the frequencies ω and wave vectors k that may be considered: $\omega \leqslant ck$.

For waves within the plate, the component of the wave vector that is normal to the plane of the plate is determined by Maxwell's equations, supplemented by the constitutive relations (**e** and **h** are the variable electric and magnetic fields, and **d** and **b** are the electric and magnetic induction:

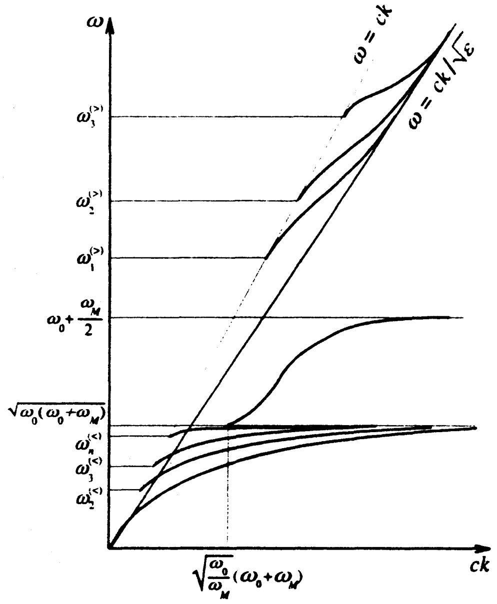

FIG. 1. Schematic dependence of the frequency on the wave vector in the case of wave propagation perpendicular to the magnetic field in a plate ($\mathbf{k} \perp \mathbf{H}$).

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$$b_i=\mu_{ik}(\omega)h_k, \quad d_i=\varepsilon e_i.$$

Here $\mu_{ik}=\mu_{ik}(\omega)$ is the magnetic susceptibility tensor,

$$\hat{\mu}=\begin{pmatrix} \mu & i\mu' & 0 \\ -i\mu' & \mu & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mu=1+\frac{\omega_M\omega_0}{\omega_0^2-\omega^2},$$

$$\mu'=\frac{\omega_M\omega}{\omega_0^2-\omega^2} \tag{3}$$

[see, for example Eq. (9.2.1) in Ref. 3].

Since the atomic frequencies determining the dispersion of the dielectric constant ε are considerably greater than the frequencies of interest to us here ($\omega\sim\omega_0,\omega_M$), ε may be regarded as a frequency-independent constant. When the spectrum of natural oscillations is considered, dissipative processes may, of course, be neglected. This means that presumably $\omega\tau\gg1$, where τ is the spin (magnetic) relaxation time.

In an infinite space, two circularly polarized waves (two magnetic polaritons) propagate parallel to the magnetization. Their dispersion relations are:[3]

$$k^2=\frac{\omega^2\varepsilon}{c^2}(\mu\pm\mu'),$$

or

$$k_+^2=\frac{\omega^2\varepsilon}{c^2}\,\frac{\omega_0+\omega_M-\omega}{\omega_0-\omega}, \quad k_-^2=\frac{\omega^2\varepsilon}{c^2}\,\frac{\omega_0+\omega_M+\omega}{\omega_0+\omega}. \tag{4}$$

It can be seen that the specific frequency dispersion of the magnetic permeability due to ferromagnetic resonance is manifested by the existence of a range of opacity ($\omega_0<\omega<\omega_0+\omega_M$) and a slow quasistatic wave ($ck\gg\omega_0$) with a group velocity tending to zero as $\omega\rightarrow\omega_0$ from the low-frequency side

$$v_{gr}=\frac{2c}{\omega_0}\sqrt{\frac{(\omega_0-\omega)^3}{\varepsilon\omega_M}}.$$

These properties are characteristic of a polariton with a wave vector k_+ (Fig. 2).

In our formulation of the problem, the electromagnetic field in the plate depends on two coordinates. We use the letter q to denote the component of the field vector perpendicular to the plate surface. Consistent with the existence of waves with two polarizations [see (4)], we have

$$q_\pm^2=\frac{1}{2\mu}\{(\omega^2\varepsilon/c^2)(\mu^2-\mu'^2+\mu)-k^2(\mu+1)\pm\sqrt{[(\omega^2\varepsilon/c^2)(\mu^2-\mu'^2-\mu)-(\mu-1)k^2]^2+4(\omega^2\varepsilon/c^2)\mu'^2k^2}\}. \tag{5}$$

After substituting the expressions for μ and μ', we have

$$q_\pm^2=\frac{k^2\left[\omega^2-\omega_0\left(\omega_0+\frac{1}{2}\omega_M\right)\right]-(\omega^2\varepsilon/c^2)\left[\omega^2-\left(\omega_0+\frac{1}{2}\omega_M\right)(\omega_0+\omega_M)\right]\pm\frac{1}{2}\omega_M\sqrt{D(k,\omega)}}{\omega_0(\omega_0+\omega_M)-\omega^2} \tag{5'}$$

Here $D(k,\omega) = [(\omega^2\varepsilon/c^2)(\omega_0+\omega_M)-k^2\omega_0]^2+4k^2\omega^2(\omega^2\varepsilon^2/c^2)>0$. Due to the cumbersome nature of expressions (5) and (5′), it is difficult to determine the signs of q_+^2 and q_-^2, i.e., to show when a wave is the result of the superposition of trigonometric functions and when it is the result of the superposition of hyperbolic functions. The values of $q_\pm(\omega,k)$ are henceforth calculated at characteristic points that are important for a qualitative analysis. It turns out that in most cases the electromagnetic field in the plate results from the superposition of both trigonometric and hyperbolic functions (see below).

The boundary conditions, i.e., the continuity of the tangential components of the magnetic field and the electric field in the wave, permit the derivation of a system of homogeneous equations, and the vanishing of the determinant of the system serves as a dispersion relation, i.e., it establishes the relation between the frequency ω and the wave vector k. The dispersion relation breaks down into two equations:

$$\frac{\mu(\omega^2\varepsilon/c^2)-k^2-q_+^2}{(\omega^2\varepsilon/c^2)-q_+^2}[1+(\gamma_0/q_+)\mathrm{tg}(q_+d)]$$
$$\times[1+(\gamma_0\varepsilon/q_-)\mathrm{tg}(q_-d)]-\frac{\mu(\omega^2\varepsilon/c^2)-k^2q_-^2}{(\omega^2\varepsilon/c^2)-q_-^2}$$
$$\times[1+(\gamma_0\varepsilon/q_+)\mathrm{tg}(q_+d)][1+(\gamma_0/q_-)\mathrm{tg}(q_-d)]=0, \tag{6}$$

$$\frac{\mu(\omega^2\varepsilon/c^2)-k^2-q_+^2}{(\omega^2\varepsilon/c^2)-q_+^2}[1-(q_+/\gamma_0)\mathrm{tg}(q_+d)]$$
$$\times[1-(q_-/\gamma_0\varepsilon)\mathrm{tg}(q_-d)]-\frac{\mu(\omega^2\varepsilon/c^2)-k^2-q_-^2}{(\omega^2\varepsilon/c^2)-q_-^2}$$
$$\times[1-(q_+/\gamma_0\varepsilon)\mathrm{tg}(q_+d)][1-(q_-/\gamma_0)\mathrm{tg}(q_-d)]=0. \tag{6'}$$

The logarithmic decay rate γ_0 is defined by Eq. (2), and the magnetic permeability μ is defined by Eq. (3).

The boundary conditions mix the two polarizations [+ and −, see (6) and (6′)], and the breakdown into two equations reveals some internal symmetry in the problem, which we have not analyzed. The n-th branch of Eq. (6) will hence-

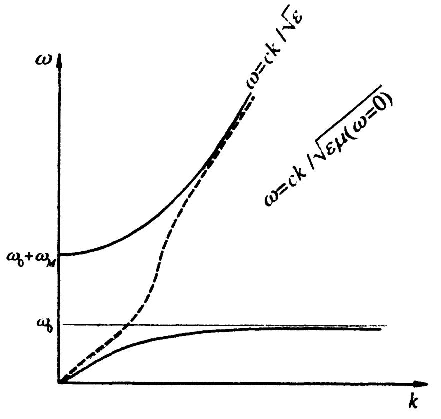

FIG. 2. Schematic dependence of the frequency on the wave vector of a magnetic polariton propagating in an unrestricted space. The solid line is a plot of $\omega=\omega_+(k)$, and the dashed line is plot of $\omega=\omega_-(k)$.

forth be denoted by $\omega_n(k)$, and accordingly the n-th branch of Eq. (6′) will be denoted by $\omega_n'(k)$. As will be shown below, the form of the dispersion dependence is highly dependent on the value of $\xi=\pi c/2\sqrt{2}\omega_0 d$ [see (8)]. A schematic plot of the dispersion curves for $\xi>1$ is shown in Fig. 3a. Figure 3b shows the special case for some value $\xi<1$ (the number of branches originating below $\omega=\omega_0$ and intersecting other branches varies: the smaller the value of ξ, the greater the number; for further details see below). Let us consider the simpler case of $\xi>1$, which is depicted in Fig. 3a, in greater detail. The branches corresponding to roots of Eqs. (6) and (6′) are depicted differently. It is seen that with the exception of the single branch emerging from the origin (from the point $\omega=0$, $k=0$), which intersects an infinite number of branches (see below), the solutions of (6) and (6′) alternate.

Let us compare Fig. 3a with Fig. 1. In Fig. 1 the characteristic frequencies are $\sqrt{\omega_0(\omega_0+\omega_M)}$ and $\omega_{DE}=\omega_0+1/2\omega_M$, and in Fig. 3a they are ω_0 and $\sqrt{\omega_0(\omega_0+\omega_M)}$. In both cases (Figs. 1 and 3), the dispersion relations have an infinite number of solutions, i.e., the spectrum has an infinite number of branches. In both cases the branches can be divided into two distinctly different groups [the solutions of Eq. (6) and of Eq. (6′) can be divided into two groups]. In one group the frequencies are greater than the largest of the characteristic frequencies, and in the other group they are smaller. However, while in the first case ($\mathbf{k}\perp\mathbf{H}$, Fig. 1) all the frequencies of the lower group are smaller than both characteristic frequencies, in the latter case ($\mathbf{k}\|\mathbf{H}$, Fig. 3a) the frequencies of the lower group fall in the range $(\omega_0, \sqrt{\omega_0(\omega_0+\omega_M)})$. Some special branches not conforming to this rule will be described below. The frequencies of the branches belonging to the upper group will be identified by the superscript (>): $[\omega_n^{(>)}(k), \omega_n^{(>)'}(k)]$, and the frequencies of the branches belong to the lowering group will be identified by the superscript (<): $[\omega_n^{(<)}(k), \omega_n^{(<)'}(k)]$.

The lowest characteristic frequency [$\sqrt{\omega_0(\omega_0+\omega_M)}$ in Fig. 1, ω_0 in Fig. 3a] is the limit point (as $k\to\infty$) of the branches of the lower group; however, in the former case (Fig. 1) all the branches of the lower group have normal dispersion and approach the limit point from below, while in the latter case (Fig. 3a) the branches have anomalous dispersion and approach the limit point from above. All the branches begin on the straight line $\omega=ck$ (where γ_0 vanishes). The origins of the branches belonging to the lower group have a limit point: in both cases it is $\omega=\sqrt{\omega_0(\omega_0+\omega_M)}$ (Figs. 1 and 3a).

The branches of the upper group are outwardly similar: both when $\mathbf{k}\perp\mathbf{H}$ and when $\mathbf{k}\|\mathbf{H}$ they begin on the ray $\omega=ck$ and asymptotically approach the straight line $\omega=ck/\sqrt{\varepsilon}$ (as $k\to\infty$). There is, however, a significant difference. As was shown in Ref. 1, when $\mathbf{k}\perp\mathbf{H}$, the upper group of branches does not "survive," i.e., the frequencies of all the branches go to infinity, as $\varepsilon\to 1$. When $\mathbf{k}\|\mathbf{H}$, the frequencies of all the branches increase but do not reach infinity as $\varepsilon\to 1$. In fact, an accurate calculation of the equation of the straight line which the branches of the upper group asymptotically approach as $k\to\infty$ ($\mathbf{k}\|\mathbf{H}$) gives

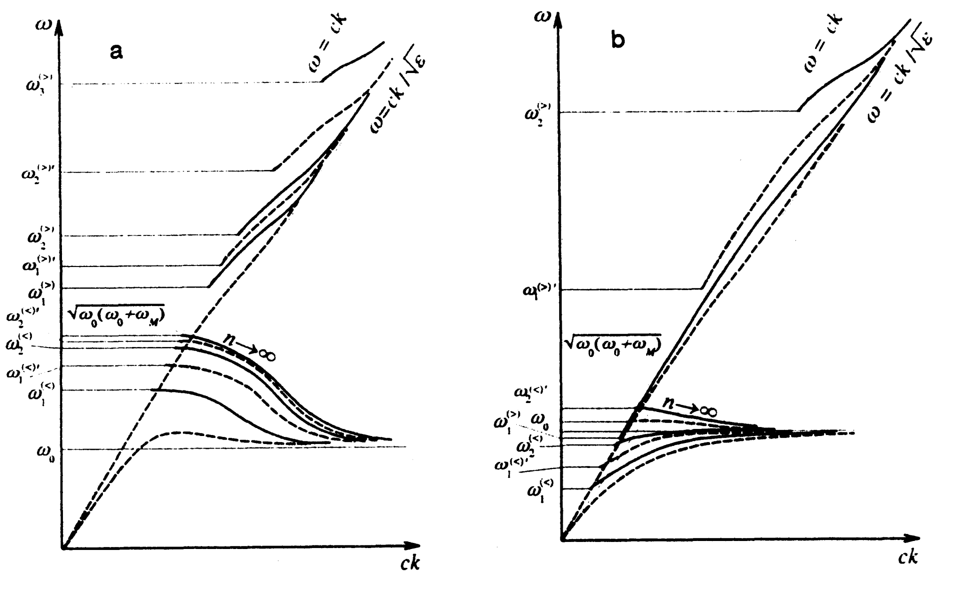

FIG. 3. Schematic dependence of the frequency on the wave vector in the case of wave propagation parallel to the magnetic field in a plate ($\mathbf{k}\|\mathbf{H}$). The solid lines are solutions of Eq. (6), and the dashed lines are solutions of Eq. (6′). a) $\xi>1$; b) $\xi<1$.

$$\omega=\frac{ck}{\sqrt{\varepsilon}}-\frac{\omega_M}{\sqrt{\varepsilon}}.$$

Therefore, when $\varepsilon=1$, both straight lines (the straight line on which the initial points are located, and the asymptote of the branches of the upper group) are parallel to one another and do not merge. The branches of the dispersion curves are located between them when $\varepsilon=1$.

Let us now consider the special branches appearing in the case depicted in Fig. 3a, i.e., when $\mathbf{k}\|\mathbf{H}$ and $\xi>1$ (for a description of the special branch appearing when $\mathbf{k}\perp\mathbf{H}$, see above). Two branches emerge from the origin: $\omega_0^{(>)'}(k)$ and $\omega_0^{(<)'}(k)$. When $k\to 0$, the branches are very close to one another and to the straight line $\omega=ck$:

$$ck-\omega_0^{(\lessgtr)'}(k)=\frac{1}{2}(kd)^2\begin{cases}\left(\dfrac{\omega_M}{\omega_0+\omega_M}\right)^2\\ \left(\dfrac{\omega_M}{\omega_0}\right)^2\end{cases}.$$

As $k\to\infty$, the frequency of one wave $\omega=\omega_0^{(<)'}(k)$ tends to its limiting value $(\omega\to\omega_0)$.

$$\omega_0^{(<)'}(k)\approx\omega_0+\frac{\omega_M}{2(ck)^2}\left[\left(\frac{\pi c}{2d}\right)^2-2\omega_0^2\right],\quad k\to\infty,\tag{7}$$

and its group velocity is

$$v_{gr}=\frac{c\omega_M}{(ck)^3}\left[\left(\frac{\pi c}{2d}\right)^2-2\omega_0^2\right]$$
$$=-2\sqrt{2}c\sqrt{\frac{(\omega-\omega_0)^3}{\omega_M\left[\left(\frac{\pi c}{2d}\right)^2-2\omega_0^2\right]}}.\tag{7'}$$

The latter equations reveal why the ratio

$$\xi=\frac{\pi c}{2\sqrt{2}\omega_0 d}\approx\frac{2\cdot 10^3}{H[\mathrm{Oe}]d[\mathrm{cm}]},\quad H\gg\beta M\tag{8}$$

plays an important role. When $\zeta>1$, the plot of $\omega_0^{(<)'}(k)$ approaches the straight line $\omega=\omega_0$ from above, and as $k\to\infty$ it has anomalous dispersion, and therefore intersects the straight line $\omega=\omega_0$, reaching a certain maximum point $k=k_{\max}$ $(d\omega_0^{(<)'}(k)/dk|_{k=k_{\max}}=0)$. Unfortunately, the complexity of the dispersion relation precludes deriving an analytic expression for $k_{\max}$.

The branch describing the wave $\omega_0^{(>)'}(k)$ "violates the rules," since it intersects all the curves of the lower group and asymptotically approaches the straight line $\omega=ck/\sqrt{\varepsilon}$ as $k\to\infty$, as do all the curves of the upper group of branches.

Let us now consider the case $\xi<1$ (Fig. 3b). First, we note at once that the number of $\omega_n^{(>)}$ and $\omega_n^{(>)'}$ branches exhibiting special behavior (intersecting other branches) depends on the value of ξ (while in the case $\xi>1$ the picture is qualitatively identical for any ξ), just as does the number of $\omega_n^{(<)}$ and $\omega_n^{(<)'}$ branches having normal dispersion over the entire range of wave vectors, i.e., approaching $\omega=\omega_0$ from below (there are four such branches in our figure). Even when ξ is infinitesimally small, their number is restricted, and an infinite number of the $\omega_n^{(<)}$ and $\omega_n^{(<)'}$ approach ω_0 from above. The number of $\omega_n^{(>)}$ and $\omega_n^{(>)'}$ branches intersecting other branches is also always restricted, and an infinite number of these branches lies above the value $\omega=\sqrt{\omega_0(\omega_0+\omega_M)}$ when ξ is infinitesimally small.

So far, we have been interested only in the dispersion relation (the dependence of the frequency of the branches on the wave vector). The structure of the electromagnetic field depends on the signs of q_+^2 and q_-^2 (see above). It can be shown that when ω, $k\to 0$ the fields of the waves with both polarizations result from the superposition of trigonometric functions, since $q_\pm^2>0$. In fact, according to (5), when $\omega\to 0$ and $k\to 0$, we have

$$q_+^2\approx\frac{\omega^2}{c^2}\frac{\omega_M}{\omega_0},\quad q_-^2\approx\frac{\omega^2}{c^2}\frac{\omega_M}{\omega_0+\omega_M}.$$

As $k\to 0$, the fields of all the waves result from the superposition of trigonometric and hyperbolic functions (this statement is true when $\omega\to\omega_0$ and when $\omega\to\infty$). A similar statement can be made for the frequencies $\omega_n^{(>)}$ and $\omega_n^{(>)'}$ lying on the straight line $\omega=ck$. With respect to the lower group of waves $(\omega_0<\omega<\sqrt{\omega_0(\omega_0+\omega_M)})$, the fields at the points of intersection of the curves $\omega=\omega_n^{(<)}(k)$ and $\omega=\omega_n^{(<)'}(k)$ with the straight line $\omega=ck$ result from the superposition of trigonometric functions. The wave fields of the upper group apparently result from the superposition of trigonometric and hyperbolic functions at all values of the wave vector k; the remaining waves must have finite values $k=k_{\lim}^n$, at which "conversion" occurs, i.e., q^2, q_+^2, or q_-^2 changes sign. Unfortunately, it seems impossible to us to calculate $k_{\lim}^n$ without using numerical methods.

It is, of course, impossible to obtain expressions for $\omega=\omega_n^{(\lessgtr)}(k)$ and $\omega=\omega_n^{(\lessgtr)'}(k)$ by analytic methods. However, the values of the frequencies and group velocities at a series of characteristic points can be calculated.[2)]

We begin with the initial frequencies of the lower group of branches. The values of the frequencies $\omega_n^{(<)}$ on the straight line $\omega=ck$ are given by

$$(\omega_n^{(<)})^2=\frac{1}{2}A_n(A_n+2\omega_0+\omega_M$$
$$-\sqrt{A_n^2+2(2\omega_0+\omega_M)A_n+\omega_M^2}),$$
$$A_n=[(\pi c)^2/\omega_M](n-1/2)^2/d^2,\quad n=1,2.\tag{9}$$

For $\omega_n^{(<)'}$, $n-1/2$ must be replaced by n in the expression for A_n. At large n (i.e., when $A_n\gg\omega_0+\omega_M$)

$$(\omega_n^{<})^2\approx\omega_0(\omega_0+\omega_M)\left(1-\frac{2\omega_0+\omega_M}{A_n}\right)\tag{9'}$$

in accordance with the assertion that $\sqrt{\omega_0(\omega_0+\omega_M)}$ is the convergence point. At small values of n, the group velocities of the waves of the lower branches are of the order of the speed of light when $k=\omega_n^{(<)}/c$ and $k=\omega_n^{(<)'}/c$, but they decrease with increasing n:

$$|v_n^{(<)}| \approx \frac{(\omega_0+\omega_M)(2\omega_0+\omega_M)d^2}{(\pi n)^2 c} \to 0 \quad \text{as } n\to\infty.$$

The limiting curves ($n\to\infty$) begin with a quadratic dependence

$$\omega_\infty^{(<)}(k) \approx \sqrt{\omega_0(\omega_0+\omega_M)}\{1-\beta_\infty(k-k_\infty)^2\},$$

$$k_\infty = \frac{\sqrt{\omega_0(\omega_0+\omega_M)}}{c}.$$

The dependence for $\omega_\infty^{(<)'}(k)$ is similar. It is difficult to determine β_∞. Its dimensions are cm^2, and most probably $\beta_\infty \sim d^2$.

As we have already noted, $\omega_n^{(<)}(k)$ and $\omega_n^{(<)'}(k)$ tend to ω_0 as $k\to\infty$ (for all n). The asymptotic values of $\omega_n^{(<)}(k)$ for $\xi > 1$ are

$$\omega_n^{(<)}(k) \approx \omega_0 + \frac{\omega_M}{2(kd_n)^2}, \quad \frac{1}{d_n^2} = \left(\frac{\pi n}{d}\right)^2 - \frac{2\omega_0^2}{c^2},$$

$$n=1,2,\ldots \quad k\to\infty. \tag{10}$$

Upon the transition to $\omega_n^{(<)'}(k)$, n must be replaced by $n+1/2$ in the expression for $1/d_n^2$. The group velocities of these waves naturally tend to zero as $\omega\to\omega_0$:

$$v_n^{(<)} \approx -\sqrt{\frac{2(\omega-\omega_0)^{3/2}}{\omega_n}}\, d_n. \tag{10'}$$

Equations (10) and (10′), like Eqs. (9) and (9′), were calculated for the case $\varepsilon=1$. In addition, it is assumed that $\xi > 1$ [see (8)]. When $\xi < 1$, the analysis becomes very complicated due to the fact that the plots of $\omega=\omega_n^{(<)}(k)$ and $\omega = \omega_n^{(<)'}(k)$ behave differently for different n. However, at any value of ξ, there are values of n for which Eqs. (10) and (10′) hold. Values of the minimal frequencies $\omega_n^{(>)}$ of the upper group of branches can also be presented. For this purpose the sign in front of the radical in Eq. (9) must be reversed. The group velocities for $k=\omega_n^{(>)}/c$ as $n\to\infty$ are $d\omega_n^{(>)}/dk = c/[1+3/4(d\omega_M/c)^2]$.

The problem allows transition to the quasistatic limit ($c\to\infty$). Here the branches of the lower group of both equations "survive." We introduce the notation $\omega_n(k)|_{c\to\infty} = \omega_n^{qs}(k)$. According to the results in Ref. 4 we have

$$[\omega_n^{qs}(k)]^2 = \omega_0(\omega_0+\omega_M)\,\frac{x_n^2+\omega_0(kd)^2/(\omega_0+\omega_M)}{x_n^2+(kd)^2}, \tag{11}$$

where x_n is the nth root of one of two transcendental equations: $x \operatorname{ctg} x = -kd$ or $x \tan x = kd$. The former equation refers to the "solid" branches, and the latter equation refers to the "dashed" branches. A comparison of Fig. 3 with Fig. 4, which shows the spectrum in the quasiclassical limit, reveals the role of electrodynamic processes (the finite nature of the speed of light).

3. CONCLUDING REMARKS

In our opinion, we have thoroughly characterized the spectrum of surface magnons propagating parallel to the magnetization **M**. A comparison with the spectrum of magnons propagating perpendicular to **M** reveals that the spectra observed in these two limiting cases differ significantly (this is especially true of the lower and special branches). This raises the problem of the transition from one type of spectrum to the other as the wave propagation direction varies. In Ref. 4 we showed that the frequency ω_{DE} of a Damon-Eshbach wave (an analog of the special wave in Fig. 1) depends on the propagation direction

$$\omega_{DE} = \frac{\omega_0+(\omega_0+\omega_M)\sin^2\theta}{2\sin\theta}, \quad \cos\theta = \frac{\mathbf{kM}}{kM}, \tag{12}$$

and exists only at values of θ greater than the critical value ($\theta \geq \theta_{cr}$), where

$$\theta_{cr} = \arcsin\sqrt{\frac{\omega_0}{\omega_0+\omega_M}}. \tag{12'}$$

It would be interesting to ascertain the critical angle for the special wave in Fig. 1. This problem can probably only be solved numerically. It seems obvious that the critical angle for the existence of a special wave θ_{cr} must depend not only on the parameters of the problem (ω_0, ω_M, ε), but also on the wavelength (the magnitude of the wave vector **k**). Of course, as $k\to\infty$, the value of $\theta_{cr}(k)$ should become identical to expression (12′) for θ_{cr}.

The existence of a group of waves with anomalous dispersion over a broad range of wave vectors (wavelengths) should be regarded as a unique property of the spectrum considered here. It should also be noted that the frequency range for their existence $[\omega_0, \sqrt{\omega_0(\omega_0+\omega_M)}]$ can easily be adjusted by varying the magnetic field and/or the temperature (the magnetization, and therefore the frequency ω_M, depend on the temperature; see footnote 1).

The waves investigated here have another property (it is characteristic of both $\mathbf{k}\perp\mathbf{M}$ and $\mathbf{k}\|\mathbf{M}$): the same wave has totally different values of the group velocity at different values of the wave vector k. We present one example. A special wave plotted as a function of the wave vector ω

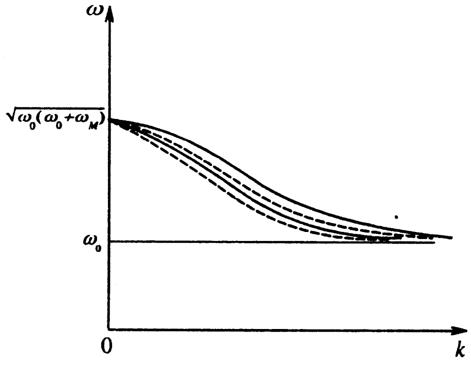

FIG. 4. Spectrum of magnetostatic oscillations in a plate. The solid and dashed lines are branches to which the branches $\omega_n^{(<)}(k)$ and $\omega_n^{(<)'}(k)$, respectively, tend in the limit $c\to\infty$; these alternate. The lower branch near $k=0$, $\omega = \sqrt{\omega_0(\omega_0+\omega_n)}$ has a linear dependence on $\omega=\omega(k)$, and the remaining branches have a quadratic branch.

$= \omega_0^{(<)'}(k)$ emerges from the origin and asymptotically approaches ω_0. At small values of k, it has $v_{gr} \sim c$, and as $\omega \to \omega_0$ ($k \to \infty$), its group velocity tends to zero [see (7′)].

The existence of limit points implies that the identification of each dispersion dependence near these points requires satisfaction of some very rigid requirements imposed on the relaxation time τ. The relaxation time must satisfy the following condition: $\Delta\omega\tau \gg 1$, where $\Delta\omega$ is the difference between the frequencies of neighboring branches. Since $\Delta\omega \to 0$ as the limit points are approached, the latter inequality demarcates (at a fixed temperature and quality of the relaxation time value) the boundaries for the validity of our treatment:

$$\Delta\omega \gg \frac{1}{\tau}.$$

In addition, the hypothetical nature of the limiting transition must be taken into account.

If the frequencies of the branches ω_n tend to infinity as $k \to \infty$, the entire treatment should be restricted so as to ensure fulfillment of the condition formulated in the introduction $\omega \ll \omega_{\rm opt}$, where $\omega_{\rm opt}$ denotes the characteristic atomic frequencies (they determine the frequency dispersion of the dielectric constant ε).

If the frequencies ω_n tend to finite limiting values as $k \to \infty$, the restriction on k is associated with the neglect (in our treatment) of the inhomogeneous exchange interaction. Both k and q must satisfy the conditions

$$ak \ll \sqrt{\hbar\omega_{\rm char}/I}, \quad aq \ll \sqrt{\hbar\omega_{\rm char}/I}, \tag{13}$$

where $\omega_{\rm char}$ is the characteristic (magnetic) frequency (ω_0, ω_M), I is the exchange integral (as a rule, $I \sim T_C$, where T_C is the Curie temperature), and a is the crystal-lattice parameter. Although usually $\hbar\omega_{\rm char} \ll I$, condition (13) does not impose excessively severe restrictions for macroscopic waves.

In closing, we thank A. S. Semenov, who at our request computed the function $\omega = \omega_n(k)$ for $\varepsilon = 1$ over a range of ξ with $\omega_0 = \omega_M/2$ ($n = 1,2,\ldots,6$). The agreement between the numerical and analytic results has augmented our confidence in the validity of the present conclusions.

1) We use the notation adopted in Ref. 1: $\omega_0 = gH_{\rm eff}$, $H_{\rm eff} = H + \beta M$, β is the anisotropy constant ($\beta > 0$), g is the gyromagnetic ratio, and $\omega_M = 4\pi gM$.

2) Performing even such an "abridged" program, we restrict ourselves to the case $\varepsilon = 1$, in which Eqs. (6) and (6′) reduce to relations equating the expressions in square brackets to zero. The results obtained do not qualitatively differ from the general case $\varepsilon > 1$. As can easily be shown, the multiplier in front of the expression in square brackets does not vanish.

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