

# Nonlinear dynamics of domain walls in the field of an acoustic wave

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We study the drift of a 180-degree domain wall in a weak ferromagnet in the elastic-stress field generated by an acoustic wave, find the dependence of the drift velocity on the frequency, amplitude, and polarization of the acoustic wave. The possible drift of a stripe domain structure is predicted. © 1994 American Institute of Physics.

## 1. INTRODUCTION

An enormous amount of work, both theoretical and experimental, has been done in studying the magnetoelastic interaction in magnetically ordered crystals. Attention has been focused on linear magnetoelastic waves of various types (e.g., analysis of the magnetoacoustic resonance and the magnetoelastic gap in the spin-wave spectrum), the influence of magnetostriction effects on the nature of the magnetic anisotropy of crystals, and the magnetostriction blocking of nuclei in phase transitions. As for the effect of this interaction on the dynamics of nonlinear excitations, a significant number of papers have been devoted to studying the phonon stopping of domain walls and the scattering of sound in magnetic substances with a domain structure. The direct interaction of domain walls (DW) with the field of an external acoustic wave has been studied to a lesser extent, even though such interaction leads to several interesting effects,<sup>1–5</sup> among which are (1) the reorientation of stationary DWs and domain structures with respect to the direction of propagation and polarization of sound,<sup>3</sup> (2) vibrational DW motion with the sound frequency,<sup>1,2</sup> and (3) the DW drift (i.e., the appearance of a constant DW velocity component) in an oscillating external field.<sup>4</sup> The third effect is the most interesting and the least studied. Denisov<sup>4</sup> investigated it theoretically by applying an averaging method in solving the approximate Slonczewski equations. Vlasko-Vlasov and Tikhomirov<sup>5</sup> observed the drift of DWs and Bloch lines directly.

DW drift in an external variable magnetic field was predicted theoretically by Schlomann and Milne,<sup>6,7</sup> and the most consistent theory for ferromagnets was proposed by Bar'yakhtar, Gorobets, and Denisov<sup>8</sup> and for two-sublattice weak ferromagnets (WFM) by the present authors.<sup>9</sup> In this paper we study the drift of a 180-degree magnetic DW in an acoustic-wave field using the example of the two-sublattice WFM model, which describes, among other things, the magnetic subsystem of rare-earth orthoferrites. In explaining the DW drift in the acoustic-wave field we use not the approximate Slonczewski equations, which was Denisov's approach in Ref. 4, but a more consistent method based on describing the nonlinear dynamics of the magnetic substance in terms of the effective Lagrangian.<sup>10,11</sup>

## 2. EQUATIONS OF MOTION

The nonlinear microscopic dynamics of a two-sublattice WFM can be described on the basis of the Lagrangian density  $L$  expressed in terms of the antiferromagnetism unit vector  $\mathbf{l}$ , with  $\mathbf{l}^2 = 1$  (Refs. 10 and 11). Allowing for the magnetoelastic interaction, we can write the Lagrangian density  $L\{\mathbf{l}\}$  for a rare-earth orthoferrite WFM, characterized by a  $2_x^- 2_z^-$  symmetry (the Cartesian  $x$ ,  $y$ , and  $z$  axes are oriented, respectively, along the  $a$ ,  $b$ , and  $c$  axes of the crystal):

$$L = M_0^2 \left\{ \frac{\alpha}{2c^2} \dot{\mathbf{l}}^2 - \frac{\alpha}{2} (\nabla \cdot \mathbf{l})^2 - \frac{\beta_1}{2} l_z^2 - \frac{\beta_2}{2} l_y^2 - \gamma u_{ik} l_i l_k \right\}, \quad (1)$$

where the dot denotes a derivative with respect to time,  $M_0$  is the length of the sublattice-magnetization vector,  $c = \frac{1}{2} g M_0 \sqrt{\alpha \delta}$  is the characteristic velocity, coinciding with the minimum spin-wave phase velocity,  $\delta$  and  $\alpha$ , respectively, are the homogeneous- and inhomogeneous-exchange coupling constants,  $g$  is the gyromagnetic ratio,  $\beta_1$  and  $\beta_2$  are the effective anisotropy constants,  $u_{ij}$  is the elastic strain tensor, and  $\gamma$  is the magnetoelastic constant. The term describing the energy of the elastic subsystem proper is not written here because in what follows we consider the acoustic wave as a fixed external field and ignore the inverse effect of the magnetic subsystem on the elastic.

It is convenient to introduce two angular variables  $\theta$  and  $\varphi$  that parametrize the unit vector  $\mathbf{l}$ ,

$$l_x + i l_z = \cos \theta \exp(i\varphi), \quad l_y = \sin \theta, \quad (2)$$

in terms of which the Lagrangian density (1) assumes the form

$$L\{\theta, \varphi\} = M_0^2 \left\{ \frac{\alpha}{2c^2} (\dot{\theta}^2 + \sin^2 \theta \dot{\varphi}^2) - \frac{\alpha}{2} [(\text{grad } \theta)^2 + \sin^2 \theta (\text{grad } \varphi)^2] - \frac{\beta_1}{2} \sin^2 \theta \sin^2 \varphi \right. \\ \left. \times \left\{ -\frac{\beta_2}{2} \cos^2 \theta - \gamma \times [\sin^2 \theta (u_{xy} \cos \varphi + u_{yz} \sin \varphi) + u_{yy} \cos^2 \theta + \sin^2 \theta (u_{xx} \cos^2 \theta + u_{xz} \sin^2 \varphi) + u_{zz} \sin^2 \varphi] \right\} \right\}. \quad (3)$$

The dynamic stopping of DWs caused by various dissipative processes will be taken into account by using the dissipative function

$$Q = \frac{\lambda M_0}{2g} \dot{\theta}^2 = \frac{\lambda M_0}{2g} (\dot{\theta}^2 + \sin^2 \theta \dot{\varphi}^2), \quad (4)$$

where  $\lambda$  is a phenomenological relaxation constant.

The equations of motion in terms of the angular variables  $\theta$  and  $\varphi$  take the form

$$\alpha \left( \nabla^2 \theta - \frac{\alpha}{c^2} \ddot{\theta} \right) + \sin \theta \cos \theta \left[ \alpha \left( \frac{1}{c^2} \dot{\varphi}^2 - (\text{grad } \varphi)^2 \right) - \beta_1 \sin^2 \varphi + \beta_2 \right] - \gamma [\sin 2\theta (u_{xx} \cos^2 \theta + u_{xz} \sin 2\varphi) + u_{zz} \sin^2 \varphi - y_{yy}] + 2 \cos 2\theta (u_{xy} \cos \varphi + u_{yz} \sin \varphi) \\ = \frac{\lambda}{g M_0} \dot{\theta}, \quad (5)$$

$$\alpha \text{div} [\sin^2 \theta (\text{grad } \varphi)] - \frac{\alpha}{c^2} \sin^2 \theta \dot{\varphi}^2 - \beta_1 \sin^2 \theta \sin \varphi \cos \varphi + \gamma [\sin 2\theta (u_{xx} \sin 2\varphi - 2u_{xz} \cos 2\varphi - u_{zz} \sin 2\varphi) + \sin 2\theta (u_{xy} \sin \varphi - u_{yz} \cos \varphi)] = \frac{\lambda}{g M_0} \sin^2 \theta \dot{\varphi}. \quad (6)$$

For  $0 < \beta_1 < \beta_2$  in the absence of external fields the equilibrium orientation of vector  $\mathbf{l}$  is along the  $X$  axis, and far from the spin-reorientation region the stable DW is a 180-degree DW with vector  $\mathbf{l}$  rotating in the  $XZ$  plane.<sup>10</sup> This DW corresponds to  $\theta = \theta_0 = \frac{1}{2} \pi$  and an angle variable  $\varphi = \varphi_0(y)$  satisfying the equation

$$\alpha \varphi_0'' - \beta_1 \sin \varphi_0 \cos \varphi_0 = 0 \quad (7)$$

(we assume that magnetization in the DW is distributed non-uniformly along the  $Y$  axis; a prime denotes differentiation with respect to this coordinate). A static 180-degree DW in which  $\varphi_0(y)$  satisfies the boundary conditions  $\varphi_0(-\infty) = 0$ ,  $\varphi_0(+\infty) = \pi$ , and  $\varphi_0'(\pm\infty) = 0$  is described by the following relations:

$$\varphi_0' = \frac{1}{y_0} \sin \varphi_0 = \frac{1}{y_0} \cosh^{-1} \left( \frac{y}{y_0} \right), \quad (8)$$

$$\cos \varphi_0(y_0) = -\tanh \frac{y}{y_0},$$

where  $y_0 = \sqrt{\alpha/\beta_1}$  has the meaning of DW thickness.

### 3. PERTURBATION THEORY. LINEAR APPROXIMATION

To analyze the DW motion in an acoustic-wave field, we follow Refs. 8 and 9 and use a version of perturbation theory for solitons, assuming the acoustic-field amplitude small. We define a collective variable  $Y(t)$  as the DW center at an arbitrary time  $t$  and seek the solution to the equations of motion in the form

$$\theta(y, t) = \frac{1}{2} \pi + \vartheta_1(\xi, t) + \vartheta_2(\xi, t) + \dots, \quad (9)$$

$$\varphi(y, t) = \varphi_0(\xi) + \psi_1(\xi, t) + \xi_2(\xi, t) + \dots,$$

with  $\xi = y - Y(t)$ ; the subscripts  $n = 1, 2, \dots$  denote the order of the quantity relative to the acoustic-wave amplitude. The function  $\varphi_0(\xi)$  describes the motion of an undistorted DW, with the structure of the function the same as that of  $\varphi_0(y)$  in the static solution (8). The terms  $\vartheta_n$  and  $\psi_n$ ,  $n = 1, 2, \dots$ , give the distortions of the DW shape and the excitation of spin waves caused by interaction with the acoustic wave.

The wall drift velocity is defined as the instantaneous DW velocity  $V(t) = \dot{Y}(t)$  averaged over the oscillation period, or  $V_{\text{dr}} = \bar{V}(t)$  (the bar denotes averaging over the acoustic-wave period).

We consider now a characteristic situation in which a monochromatic wave of frequency  $\omega$  impinges on the DW plane perpendicular to the plane. Assuming that  $V = V_1 + V_2 + \dots$ , we can write the perturbation equation in the first order of the acoustic-wave amplitude  $u_{0i}$  as

$$\left( \hat{L} + \frac{1}{\omega_0^2} \frac{d^2}{dt^2} + \frac{\omega_r}{\omega_0} \frac{d}{dt} \right) \psi_1 = \frac{\sin \varphi_0(\xi)}{\beta_1 y_0} \left( \frac{\alpha \dot{V}_1}{c^2} + \frac{\lambda V_1}{g M_0} \right), \quad (10)$$

$$\left( \hat{L} + \sigma + \frac{1}{\omega_0^2} \frac{d^2}{dt^2} + \frac{\omega_r}{\omega_0} \frac{d}{dt} \right) \vartheta_1 = \frac{i \gamma k e^{ik(\xi+Y) - i\omega t}}{\beta_1} (u_{0x} \cos \varphi_0 + u_{0z} \sin \varphi_0), \quad (11)$$

where the following notation has been adopted:  $\sigma = (\beta_2 - \beta_1)/\beta_1$ ,  $\omega_0 = c/y_0$  is the activation frequency of the lower spin-wave mode,  $\omega_r = \frac{10}{1} \lambda \delta g M_0$  is the characteristic relaxation frequency,  $k = \omega/s$  is the wave vector, and  $s$  is the velocity of sound.

The operator  $\hat{L}$  is the Schrödinger operator with a non-reflecting potential:

$$\hat{L} = -y_0^2 \frac{d^2}{d\xi^2} + 1 - \frac{2}{\cosh^2(\xi/y_0)}. \quad (12)$$

The spectrum and the wave functions of  $\hat{L}$  (12) are well known. The spectrum consists of one discrete level with the eigenvalue  $\lambda_0 = 0$  corresponding to the localized wave function

$$f_0(\xi) = \frac{1}{\sqrt{2y_0}} \cosh^{-1} \left( \frac{\xi}{y_0} \right), \quad (13)$$

and a continuous part  $\lambda_p = 1 + p^2 y_0^2$  with the following eigenfunctions:

$$f_p(\xi) = \frac{1}{b_p \sqrt{L}} \left( \tanh \frac{\xi}{y_0} - ipy_0 \right) e^{ip\xi}, \quad (14)$$

with  $b_p = \sqrt{1 + p^2 y_0^2}$ , where  $L$  is the crystal length.

The functions  $\{f_0, f_p\}$  form a complete orthonormalized set, and it is natural to seek the solutions to the first-order approximation equations (10) and (11) in the form of expansions in this set:

$$\vartheta_1(\xi, t) = \text{Re} \left\{ \left[ \sum_p c_p f_p(\xi) + c_0 f_0(\xi) \right] e^{ikY - i\omega t} \right\}, \quad (15)$$

$$\psi_1(\xi, t) = \text{Re} \left\{ \left[ \sum_p d_p f_p(\xi) + d_0 f_0(\xi) \right] e^{ikY - i\omega t} \right\}. \quad (16)$$

A remark is in order. The first-approximation equations (10) and (11) describe excitation of linear spin waves superposed on DWs. The last term in the expansion of the function  $\psi_1(\xi, t)$  corresponds to the shear (Goldstone) mode, i.e., to DW motion as a whole. However, the corresponding degree of freedom of the system has already been taken into account by introducing the collective coordinate  $Y(t)$  into the definition of the variable  $\xi$ . Hence the shear mode must be left out of the expansion (16), i.e., we must put  $d_0 = 0$  (for a detailed description see Rajaraman<sup>12</sup>).

The other expansion coefficients in (15) and (16) are found by the standard method of multiplying the right-hand sides of Eqs. (10) and (11) by  $f_p^*$  and  $f_0^*$  and integrating the products with respect to  $\xi$ .

For a monochromatic acoustic wave of frequency  $\omega$ , i.e.,  $u = u_0 \cos(\omega t - ky)$ , we obtain from Eqs. (10) and (11) the following:

$$\begin{aligned} \psi_1(\xi, t) = 0, \quad \vartheta_1(\xi, t) &= \frac{\gamma \pi k^2 y_0}{4\beta_1} \text{Re} \{ B(\xi) e^{ikY - i\omega t} \}, \\ B(\xi) &= -\frac{y_0}{\pi} [u_{0z} D_1(\xi) + i u_{0x} D_2(\xi)] \\ &+ b_1(t) \sin \varphi_0(\xi) + b_2(t) f_k(\xi). \end{aligned} \quad (17)$$

Here we have introduced the notation

$$b_1(t) = \frac{1}{\sigma - q_1 - iq_2} \left[ \frac{u_{0x}}{\cosh(\pi k y_0 / 2)} + \frac{i u_{0z}}{\sinh(\pi k y_0 / 2)} \right],$$

$$b_2(t) = -\frac{2i\sqrt{L} u_{0x}}{\pi k y_0 b_k (\lambda_k + \sigma - q_1 - iq_2)},$$

$$D_1(\xi) = \sqrt{L} \int_{-\infty}^{+\infty} dp \frac{f_p(\xi)}{b_p (\lambda_p - q_1 - iq_2) \sinh[\pi y_0 (p - k) / 2]},$$

$$D_2(\xi) = \sqrt{L} \int_{-\infty}^{+\infty} dp \frac{f_p(\xi)}{b_p (\lambda_p - q_1 - iq_2) \cosh[\pi y_0 (p - k) / 2]},$$

with  $q_1 = (\omega / \omega_0)^2$ , and  $q_2 = \omega \omega_t / \omega_0^2$ .

The condition  $d_0 = 0$  is equivalent to requiring the right-hand side of Eq. (10) to be orthogonal to the function  $f_0$ , which in turn determines the equation for the DW velocity  $V_1(t)$  to first order in the field,

$$\dot{V}_1 + \frac{\lambda c^2}{\alpha g M_0} V_1 = 0, \quad (18)$$

which has only a trivial solution (we are interested only in the forced solutions of the equations of motion).

Thus, in the geometry of the problem considered here (normal incidence of the acoustic wave on a DW) and to first order in the wave amplitude, sound does not generate DW motion, in contrast to the DW dynamics caused by an oscillating magnetic field and studied in Ref. 9; rather, it excites localized and unlocalized spin waves, described by (17). Note that only the terms related to transverse acoustic vibrations contribute to (17).

#### 4. SECOND APPROXIMATION: DW DRIFT

We now analyze the equations to second order in the amplitude of the acoustic wave. Allowing for the solutions of the first-order approximation (7), we can write the second-order equation for  $\psi_2(\xi, t)$  as

$$\begin{aligned} \hat{L} \psi_2 &= \left( \frac{\alpha \dot{V}_2}{c^2} + \frac{\lambda V_2}{g M_0} \right) \frac{\sin \varphi_0}{\beta_1 y_0} - \frac{V_1^2}{2c^2} \sin 2\varphi_0 \\ &- 2y_0 \vartheta_1(\xi, t) \vartheta_1'(\xi, t) \sin \varphi_0 \\ &- \frac{i \gamma k}{\beta_1} (u_{0x} \sin \varphi_0 \\ &- u_{0z} \cos \varphi_0) \vartheta_1(\xi, t) e^{ik(\xi + Y) - i\omega t}. \end{aligned} \quad (19)$$

The second-approximation equation for the function  $\vartheta_2(\xi, t)$  contains no second-order term in the expansion of the DW velocity ( $V_2$ ) and is, therefore, of no interest to us.

The solution to Eq. (19) can again be sought as an expansion in the eigenfunctions of operator  $\hat{L}$  similar to (16). Here we must require, just as in the first-order equation, that the expansion coefficient  $d_0^{(2)}$  (corresponding to the shear mode) vanish. Hence when calculating the DW velocity we need not find the complete solution to Eq. (19); it is enough to calculate  $d_0^{(2)}$  and equate it to zero. As a result we arrive at an equation for the velocity  $V_2$ , which after (17) is taken into account assumes the form

$$\dot{V}_2 + \omega_t V_2 = N + \bar{N} \cos 2\omega t, \quad (20)$$

where

$$\begin{aligned} N &= \omega_0^2 \int_{-\infty}^{+\infty} d\xi B(\xi) \sin \varphi_0 \left\{ y_0 B'(\xi) \sin \varphi_0 \right. \\ &+ \left. \frac{i \gamma k}{2\beta_1} e^{ik\xi} (u_{0x} \sin \varphi_0 - u_{0z} \cos \varphi_0) \right\}, \end{aligned}$$

with  $B(\xi)$  defined in (17). The expression for the coefficient  $\bar{N}$  has a similar structure but is much more cumbersome, and so is not given here.

The integration of Eq. (20) is elementary:

$$V_2(t) = \frac{N}{\omega_r} + \frac{\tilde{N}(\omega_r \cos 2\omega t + 2\omega \sin 2\omega t)}{\omega_r^2 + 4\omega^2}. \quad (21)$$

This solution describes the DW dynamics in an acoustic-wave field and contains both periodic time-dependent terms, corresponding to DW oscillations, and time-independent terms, which determine the desired drift velocity  $V_{dr} = \bar{V}_2$ :

$$V_{dr} = \mu_{xx}(k)(ku_{0x})^2 + \mu_{xz}(k)(ku_{0x})(ku_{0z}) + \mu_{zz}(k)(ku_{0z})^2, \quad (22)$$

where the  $\mu_{ij}(k)$  are the nonlinear DW mobilities in the acoustic-wave field.

$$S_1(q) = \frac{y_0}{\sigma - q_1 - iq_2} \int_{-\infty}^{+\infty} \frac{\lambda_p dp}{(\lambda_p + \sigma - q_1 + iq_2) \cosh[\pi y_0(p-k)/2] \cosh(\pi p y_0/2)}$$

$$S_2(q) = \int_{-\infty}^{+\infty} \frac{y_0(\lambda_p - k^2 y_0^2) dp}{\lambda_p(\lambda_p - q_1 + iq_2) \cosh^2[\pi y_0(p-k)/2]}.$$

The other two nonlinear mobilities,  $\mu_{xx}$  and  $\mu_{xz}$ , have a similar structure and for this reason are not given here in general form.

In the case most interesting from the experimental view, i.e., the long-wave approximation ( $ky_0 \ll 1$ ), which corresponds to the frequency range  $\omega = sk \ll 10^1 \text{ s}^{-1}$ , the expressions for the nonlinear mobilities  $\mu_{ik}$  simplify considerably:

$$\mu_{xx} = \frac{\mu_0 k y_0 q_2}{(1 + \sigma)^2}, \quad \mu_{zz} = -\frac{\mu_0 q_2}{\sigma^2}, \quad \mu_{xz} = -\frac{2\mu_0}{\sigma(1 + \sigma)}, \quad (24)$$

where  $m_0 = \nu_0 \delta(\gamma M_0)^2 / 4\beta_1$ .

To obtain a numerical estimate for the above expressions, we use the parameters of the typical and well-studied weak ferromagnet YFeO<sub>3</sub> (see Ref. 13):  $M_0 \approx 10^3 \text{ Oe}$ ,  $g = 1.76 \times 10^7 \text{ s}^{-1} \text{ Oe}^{-1}$ ,  $\sigma \approx 2$ ,  $\gamma M_0^2 \approx 10^7 \text{ erg cm}^{-3}$ ,  $\omega \approx 2 \times 10^{12} \text{ s}^{-1}$ , and  $y_0 \approx 10^{-6} \text{ cm}$ . The values of  $\nu_0$  and  $\omega_r$  for YFeO<sub>3</sub> are taken from Ref. 9:  $\nu_0 \approx 3.5 \times 10^{-2} \text{ cm s}^{-1} \text{ Oe}^{-2}$ , and  $\omega_r \approx 0.7 \times 10^{10} \text{ s}^{-1}$ . It also happens that for all reasonable frequencies the parameter  $q_2 = \omega \omega_r / \omega_0^2$  is much less than unity and, as (24) implies, the nonlinear mobilities  $\mu_{xx}$  and  $\mu_{zz}$  are proportional to  $q_2$  and small, too. Hence the main contribution to the DW drift velocity is provided by the term in (22) related to the off-diagonal nonlinear mobility  $\mu_{xz}$ . Using the above values of the WFM parameters, we get the following estimate for the DW drift velocity at low frequencies:

$$V_{dr} \sim 10^{-1} (\omega u_0)^2 \text{ cm s}^{-1}. \quad (25)$$

Thus, in this frequency range the drift velocity is proportional to the square of the frequency. We also note that the minus in the expression for the nonlinear mobility  $\mu_{xz}$ , which provides the dominant contribution to the DW drift

Generally the expressions for  $\mu_{ij}(k)$  are involved. For instance, the nonlinear mobility  $\mu_{zz}$  has the form

$$\mu_{zz} = \frac{\nu_0 \pi \delta k y_0 \gamma^2 M_0^2}{8\beta_1} \text{Im} \left\{ \frac{2}{(\sigma - q_1 + iq_2) \sin \pi k y_0} + \frac{k y_0 S_1(q)}{4 \sinh(\pi k y_0/2)} + \frac{S_2(q)}{\pi} \right\}. \quad (23)$$

Here  $\nu_0 = \pi y_0 g^2 / 4\omega_r$ , introduced in Ref. 9, is the characteristic nonlinear DW mobility in a WFM placed in a variable magnetic field, and the functions  $S_1(q)$  and  $S_2(q)$  are given by the following expressions:

velocity, is an indication that the direction of the drift DW motion is opposite to the direction in which the acoustic wave propagates.

One-sublattice ferromagnets exhibit a similar quadratic dependence of the drift velocity on the frequency in the low-frequency range,<sup>4</sup> but the corresponding proportionality factor is entirely different. This is because the dynamic equations have different structure and because in Ref. 4 only one polarization of the acoustic wave is considered (the one perpendicular to the easy-magnetization axis), while in our case, as noted earlier, the contribution to the drift velocity is related to the off-diagonal nonlinear mobility  $\mu_{xz}$ .

As is well known, the experimental possibilities of exciting an acoustic wave in a crystal are limited not by the wave amplitude but by the size of the strain tensor  $u_{ij} \sim ku_0$ , which for WFMs cannot exceed  $10^{-5}$  (at higher values of the strain tensor the crystal disintegrates). Hence we write the estimate (25) in the form

$$V_{dr} \sim 10^{-1} s^2 (ku_0)^2 \text{ cm s}^{-1}. \quad (25')$$

This implies that for a sound velocity  $s \sim 3 \times 10^5 \text{ cm s}^{-1}$  characteristic of WFMs and the maximum admissible value of the strain tensor  $ku_0 \sim 10^{-5}$ , the DW drift velocity amounts to  $1 \text{ cm s}^{-1}$ .

In the short-wave approximation ( $ky_0 \gg 1$ ), corresponding to the hypersonic frequencies  $\omega \gg 10^{11} \text{ s}^{-1}$ , the nonlinear mobilities  $\mu_{ik}(k)$  decrease as the frequency grows ( $\omega = ks$ ):

$$\begin{aligned} \mu_{xx} &= \mu_0 \eta_1 q_2 \left( \frac{\omega y_0}{s} \right)^{-5} \sim \omega^{-4}, \\ \mu_{xz} &= \mu_0 \eta_2 \left( \frac{\omega y_0}{s} \right)^{-4} \sim \omega^{-4}, \\ \mu_{zz} &= \mu_0 \eta_3 \frac{q_2}{q_1} \exp\left(-\frac{\pi \omega y_0}{s}\right) \sim \omega^{-3} \exp\left(-\frac{\pi \omega y_0}{s}\right), \end{aligned} \quad (26)$$

where the  $\eta_i$  ( $i=1,2,3$ ) are numerical factors of the order of unity.

It can be seen that at low frequencies the main contribution to the DW drift velocity is provided by the nonlinear mobility  $\mu_{xz}$ , and for  $V_{dr}$  the following estimate holds:

$$V_{dr} \sim 10^{-5} (u_0 \omega)^2 \left( \frac{\omega_0}{\omega} \right)^4 \text{ cm s}^{-1}. \quad (27)$$

Thus, at high frequencies  $V_{dr}$  is inversely proportional to the square of the frequency, which agrees with the results of Ref. 3.

## 5. DRIFT OF A STRIPE-DOMAIN STRUCTURE

We consider now the possibility of drift in an acoustic-wave field of a plane-parallel, or stripe, domain structure (SDS) consisting of 180-degree DWs. Here it must be borne in mind that neighboring DWs in a domain structure have opposite topological charges determined by the boundary conditions imposed on Eq. (7). Moreover, the rotation of vector  $\mathbf{l}$  in various DWs can be about the positive or the negative direction of the  $Z$  axis. These two factors determine the DW drift direction in a field of fixed frequency  $\omega$ . SDS drift is possible, naturally, only if neighboring DWs move in the same direction.

We define the topological DW charge  $R = \pm 1$  and the parameter  $\rho = \pm 1$ , which describes the direction of rotation of vector  $\mathbf{l}$  in a DW, as follows:

$$l_x(\pm\infty) = \mp R, l_z(y=0) = \pm \rho.$$

The DWs described in the previous sections and having a magnetization distribution (8) correspond to  $R = \rho = \pm 1$ . Generally, instead of (8) we get

$$\varphi'_0 = \frac{1}{y_0} R \sin \varphi_0 = \frac{1}{y_0} R \rho \cosh^{-1} \left( \frac{y}{y_0} \right), \quad (28)$$

$$\cos \varphi_0 = -R \tanh \frac{y}{y_0}.$$

Analysis shows that in the general case the drift velocity of a DW with given values of parameters  $R$  and  $\rho$  is determined by a formula similar to (22):

$$V_{dr} = \mu_{xx}(k)(ku_{0x})^2 + R\rho\mu_{xz}(k)(ku_{0x})(ku_{0z}) + \mu_{zz}(k)(ku_{0z})^2. \quad (29)$$

In a SDS the topological charges  $R$  of neighboring DWs are always "unlike." Since the dominant contribution to the drift velocity is provided by the off-diagonal nonlinear mobility

$\mu_{xz}$ , for the corresponding term in (29) to be the same for all DWs and for all the DWs in the structure to drift in the same direction, the parameter  $\rho$  in the neighboring DWs must also be "unlike," i.e., the vectors  $\mathbf{l}$  in neighboring DWs must rotate in opposite directions. If this condition is not met, no SDS drift is possible. An exception is when the acoustic wave is polarized exactly along the  $X$  or  $Z$  axis. Then only one term on the right-hand side of (29) that is independent of the DW polarizations is nonzero (either the first or the third), and all the DWs in the SDS drift in the same direction. Here, however, the drift velocity determined by these terms is much smaller than the one due to the nonlinear polarization  $\mu_{xz}$ .

Direct comparison of the theory developed in this paper with the experimental data in Ref. 5 has no meaning because Vlasko-Vlasov and Tikhomirov<sup>5</sup> studied DW drift in a one-sublattice ferromagnet and, besides, the geometry of their experiment was different (they chose an acoustic wave propagating in the DW plane, while in our calculations the wave was incident on the DW at right angles to the DW plane).

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