Apparent anisotropy in the orientation of extragalactic objects due to the curvature of spacetime

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We show that anisotopic curvature of spacetime leads to an apparent anisotropy in the orientation of distant galaxies. We have calculated this effect to first order in the curvature for the most general Petrov type I spaces. The probability density distribution for the orientation of galactic normals can be described by a triaxial ellipsoid whose axes lie along the unit vectors of the canonical tetrad, while the amplitude is proportional to the square of the distance and is given by the real parts of the stationary curvatures of the Weyl tensor. We estimate deviations of the geometry of the universe from Friedmann models using anisotropy data derived from observations at scales of order 100 Mpc. Other testable consequences of the bending of light rays are examined. We show that this does not provide an explanation for the so-called Birch effect—a systematic deviation of the plane of polarization of radio sources from the direction of their major axes. It can therefore not be ascribed to the rotation of the universe.

1. INTRODUCTION

Over the past 30 years, astronomers have produced a number of large-scale catalogs of galaxies which, as a group, cover almost the entire celestial sphere.[1–3] In every one of them, the distribution of galactic position angle (p in Fig. 1, the angle between a galaxy's major axis and the celestial meridian) displays a statistically significant anisotropy. This is a weak effect, and it shows up only when data from tens of thousands of galaxies are processed. A new approach to the study of anisotropy[4] has made it possible to determine the distribution of galactic normals over the celestial sphere in the quadrupole approximation, and the result can be described as a triaxial ellipsoid. The axes determined from data in three independent catalogs of galaxies are highly consistent with one another. Subsequently, a similar anisotropy was discovered by analyzing the disposition of pairs of galaxies (where the line joining the pair was used instead of the major axes). In this case as well, the anisotropy ellipsoid exhibited approximately the same characteristics.[1]

The investigation of anisotropy in the orientation of pairs of galaxies has divulged an important fact: this anisotropy shows up both in physical gravitationally interacting pairs and in optical doubles that appear to be near each other in projection on the sky, but that are actually at substantially different distances. Anisotropy in the orientation of optical pairs of galaxies cannot be accounted for by tidal forces or other physical mechanisms that might single out a preferred direction.

In what follows, we show that such anisotropy might result from distortion of light rays as they propagate through curved spacetime. As a consequence, the apparent orientation of galaxies can differ somewhat from the original orientation. The relationship (1) between the apparent and original position angles is nonlinear, and ensures that preferred directions of apparent orientation will exist, even when the "true" distribution is perfectly isotropic.

The resultant type of anisotropy and the direction and amplitude of its maximum and minimum are governed by the curvature tensor of spacetime, averaged over the region of space in which we reside, out to a radius of order 100 Mpc. In the present paper, we relate these quantities to the eigenvalues and eigenvectors of Weyl's conformal curvature tensor for the most general Petrov type I spaces.[5] It should be pointed out here that this phenomenon provides us with a unique opportunity to determine directly the extent to which the actual geometry of the universe deviates from the Friedmann models, for which the Weyl tensor vanishes identically. The magnitude of these deviations may well furnish a test of cosmological models and scenarios for the evolution of inhomogeneities.

The problem of the isotropy and anisotropy of galactic orientations is an important element of the theory of galaxy formation. Introducing a global anisotropic component of the spacetime curvature, which is most likely due to nonuniformities in the distribution of matter, makes it possible to compare the real randomness of galactic orientations with the observed anisotropy. Based solely on a single set of data on galactic orientations, it would be impossible to elucidate the scale of the phenomenon and to draw up a scenario for its origin and development. Anisotropy of the non-Friedmann component of the curvature might also result from local or global rotation, or from some stupendous gravitational wave, but the latter alternatives should probably be thrown out on account of their inconsistency with the observed isotropy of the microwave background.

In this paper, we report on a preliminary assessment of deviations from Friedmann models. More detailed evaluations would require allowance for a number of strictly astronomical factors, such as absorption in the Milky Way, selection effects, systematic catalog errors, and so on. Our

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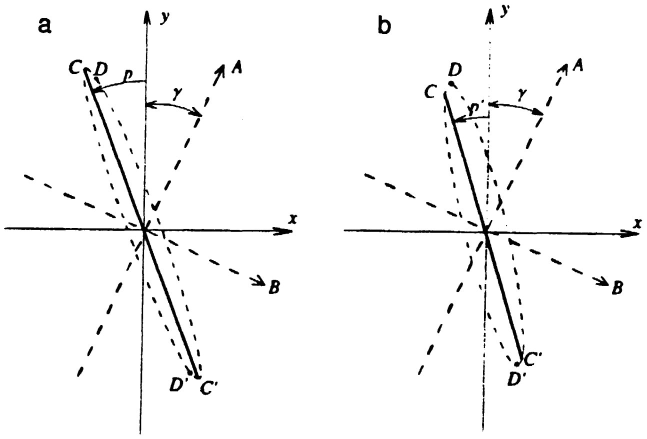

FIG. 1. A flat galaxy observed edge-on as a line segment CC'. In Minkowski space, its apparent position angle would correspond to the original angle p (a). When a light beam propagates through an anisotropically curved spacetime, it undergoes shear deformation, which ends up elongating the image by a factor K along the A axis and compressing it by a factor of K along the B axis. As a result, the galaxy CC' is observed in position angle $p' = \arctan[K^{-2}\mathrm{tg}(p+\gamma)]-\gamma$ (b). For an isotropic distribution of p, the distribution of p' becomes anisotropic, with its maximum along A and its minimum along B. For a galaxy that is not quite so flat (dashed ellipse), the effect is more pronounced, due to the fact that after shear deformation the apparent major axis of the ellipse is DD' rather than CC'. The observed galactic axial ratio changes as well.

principal goal here is to construct a theory of the origin of the apparent anisotropy in the orientation of distant extragalactic objects engendered by the distortion of light rays as they propagate through curved space, and to estimate the parameters of the curvature on the basis of the observed anisotropy.

We calculate to first order in the small anisotropy. It would be straightforward to proceed to the next approximation, but it would also be premature, in view of the low accuracy of our data on anisotropy in the orientation of distant galaxies.

2. THE DISTORTION OF LIGHT RAYS AND ANISOTROPY

We know that light rays are deformed in three ways as they propagate through the curved space of general relativity—they undergo broadening, rotation, and shear.[6,7] The first two affect a circular beam by altering its radius and causing it to rotate about its axis, but neither can produce anisotropy, since neither distorts the distribution in position angle. Shear is a different issue, however: it turns a circular beam into an elliptical one as the beam is compressed and elongated along two perpendicular axes.

We direct our attention to a flat galaxy viewed edge-on, treating it as a line segment (CC' in Fig. 1). The original position angle p is reckoned in the positive direction from the celestial meridian. As a result of shear, which elongates a beam by a factor $K>1$ along the A axis and compresses it by a factor K along the B axis (Fig. 1), we see the galaxy with position angle p', which is related to p by

$$\tan(p'+\gamma)=K^{-2}\tan(p+\gamma), \tag{1}$$

where γ is the angle between the elongation axis and the celestial meridian. The distribution of apparent galactic position angles p' becomes anisotropic. The function

$$F(\alpha_0,\delta_0)=\frac{4}{N}\sum_{i=1}^{N}\cos(2p_i) \tag{2}$$

has been used by Parnovskiĭ *et al.*[4] to study anisotropy in galactic orientations; here N is the number of galaxies considered and p_i is the position angle of the ith galaxy relative to the direction of the point (α_0,δ_0). From here on we use α_e and δ_e for standard astronomical equatorial coordinates, and we use α and δ for coordinates in some other spherical coordinate system whose axis points toward $\alpha_e=\alpha_0,\delta_e=\delta_0$ (the *celestial pole* in astronomy). Here p_i—the position angle of the galaxy in the (α,δ) coordinate system—also depends on α_0 and δ_0.

In the quadrupole approximation, the function $F(\alpha_0,\delta_0)$ is the relative anisotropy of the probability density for the orientation of galactic normals, taken with the negative of the sign used in Ref. 4.

From Eq. (1), we find that when the original orientation of the individual galaxies is isotropically distributed, the mean value

$$\langle\cos(1p')\rangle =\frac{1}{\pi}\int_0^{\pi}\cos\{2[\mathrm{arctg}(K^{-2}\,\mathrm{tg}(p+\gamma))-\gamma]\}dp =\frac{K^2-1}{K^2+1}\cos(2\gamma) \tag{3}$$

is nonzero, i.e., the resulting distribution is anisotropic. If one observes a galaxy of elliptical cross section (shown dashed in Fig. 1) with axes a and $b<a$, then the images of the points C and C' after shearing will no longer be on the major axis of the ellipse.

The equation of this ellipse in the (A,B) coordinate system is

$$A(\lambda)=a\cos(p+\gamma)\cos\lambda+b\sin(p+\gamma)\sin\lambda,$$
$$B(\lambda)=b\cos(p+\gamma)\sin\lambda+a\sin(p+\gamma)\cos\lambda. \tag{4}$$

Here $\lambda=0$ corresponds to the point C. After shearing, the squared modulus of the radius vector of the apparent ellipse, $\rho^2=K^2A^2+K^{-2}B^2$, reaches a maximum at $\lambda=\lambda_0$, where

$$\mathrm{tg}(2\lambda_0) =\frac{2q\sin(2(p+\gamma))(K^4-1)}{(q^2+1)(K^2-1)\cos(1(p+\gamma))+(q^2-1)(K^4+1)},$$
$$q\equiv a/b>1. \tag{5}$$

The major axis of the apparent ellipse joins the points corresponding to D and D' in Fig. 1. The apparent position angle can then be obtained from

$$\mathrm{tg}(p'+\gamma)=-K^{-2}\frac{B(\lambda_0)}{A(\lambda_0)}, \tag{6}$$

with A, B, and λ_0 taken from (4) and (5). The expression for $\langle\cos(2p')\rangle$ obtained from these equations can be reduced to elliptic integrals. We are interested, however, in the case $K-1\ll1$, where Eq. (6) acquires the same form as Eq. (1) with the replacement of K by $\tilde{K}$, where

$$\tilde{K}-1=\Lambda(K-1),\quad \Lambda=\frac{q^2+1}{q^2-1}=\frac{a^2+b^2}{a^2-b^2}, \tag{7}$$

which effectively describes a Λ-fold magnification of the shear modulus $K-1$.

We can therefore limit our calculations to flat galaxies viewed edge-on, multiplying the resulting function by $\langle\Lambda\rangle$, which is the mean value of Λ for the sample of galaxies. Note that this is not the first derivation of such a factor.[8] Λ diverges as $b\to a$, but no special consideration is required. As a rule, the position angle of a galaxy with an approximately circular cross section is simply undefined. Moreover, the sample of galaxies used in Ref. 4 to study anisotropy had $q\geq1.5$.

We now turn to Eq. (3), in which K and γ depend on α and δ. Furthermore, the shear modulus $K-1$ depends on the distance r to the galaxy (quadratically, as will be shown below). Taking $K-1\ll1$, we find that for galaxies uniformly distributed in space, (2) and (3) yield the mean value

$$F(\alpha_0,\delta_0)\approx\langle\Lambda\rangle\pi^{-1}\int_0^{2\pi}\int_{-\pi/2}^{\pi/2}d\alpha\cos\delta d\delta \times[K(\alpha,\delta,\alpha_0,\delta_0,\langle r^2\rangle)-1] \times\cos(2\gamma(\alpha,\delta,\alpha_0,\delta_0)). \tag{8}$$

The mean-squared value $\langle r^2\rangle$ can be related to the depth of the sample R, i.e., to an estimated maximum distance of galaxies in the given catalog:

$$\langle r^2\rangle=\int_0^R r^4dr\bigg/\int_0^R r^2dr=\frac{3R^2}{5}.$$

Getting down to specifics, we now use Eq. (8) to find the direction and magnitude of shear in general relativity.

3. DETERMINATION OF SHEAR PARAMETERS FROM THE EQUATION FOR THE DEVIATION OF ISOTROPIC GEODESICS

We can find the alteration in the form of a beam of light as it propagates through curved spacetime by solving the equation of geodesic deviation,

$$\frac{D^2d^i}{dr^2}=R^i_{klm}u^ku^ld^m, \tag{9}$$

where u^k is the tangent vector of the isotropic geodesic corresponding to the center of the beam, r is the affine parameter along the geodesic, and d^i is the deviation vector. After transforming to tetrad components $d^{(i)}=h^{(i)}_k d^k$ with respect to the parallel transported frame $h^{(i)}_k$, the solution takes the form:[9]

$$d^{(i)}=C^{(i)}_{(k)}d^{(k)}_0+S^{(i)}_{(k)}v^{(k)}_0 r. \tag{10}$$

Here $d^{(k)}_0$ and $v^{(k)}_0$ are the initial deviation of the beam and its rate of change, and $C^{(i)}_{(k)}$ and $S^{(i)}_{(k)}$ are fundamental solution matrices. For propagation of a parallel beam at the initial point we have $v^{(k)}_0=0$, and the second term in (10) vanishes. For a beam that diverges at the initial point $d^{(k)}_0=0$, and the first term vanishes.

In the present case a light beam converges either to one's eye or the objective of the observer's telescope. Time-reversing this process, we obtain a beam leaving the observer. At small r, the shape of that beam is dictated by the vector $\mathbf{d}$ from Eq. (10) with the first term omitted, and it corresponds to the apparent shape of the galaxy. The "true" shape that would be observed in flat spacetime is given by the vector $v^{(k)}_0 r$.

We can nail down the choice of moving frame $h^{(i)}_k$ by expressing it in terms of the unit vectors $\mathbf{e}_r$, $\mathbf{e}_\alpha$, $\mathbf{e}_\delta$ of a spherical coordinate system that is parallel-transported from the observer along each ray belonging to the beam. We choose the tangent vector $\mathbf{u}=(1,\mathbf{e}_r)/\sqrt{2}$ to be $\mathbf{h}_{(0)}$. The second isotropic vector is $\mathbf{h}_{(1)}=(1,-\mathbf{e}_r)/\sqrt{2}$, and the two mutually conjugate complex vectors in the plane orthogonal to the beam are $\mathbf{h}_{(2,3)}=(0,\mathbf{e}_\alpha\mp i\mathbf{e}_\delta)/\sqrt{2}$. The initial deviation takes place in the plane of the sky $(\mathbf{e}_\alpha,\mathbf{e}_\delta)$, i.e., $\mathbf{v}_{(0)}=w\mathbf{h}_{(2)}+w^*\mathbf{h}_{(3)}$, where the asterisk denotes complex conjugation. The argument of w determines the apparent position angle of the galaxy.

With this choice of moving frame, the affine parameter r is the distance to the galaxy, and the required component $d^{(2)}$ of the solution (10) for the divergent beam becomes

$$d^{(2)}=r(S^{(2)}_{(2)}w+S^{(2)}_{(3)}w^*),\quad d^{(3)}=d^{(2)*}. \tag{11}$$

Aleksandrov[9] has obtained a power series in the curvature tensor and its derivatives for the components $S^{(i)}_{(k)}$ in the parallel-transport frame. In the region of space under consideration here, $z\leq0.05$, and the anisotropy can be considered small. The deviation of S from the unit matrix is then small, and we can retain just the two leading terms of the expansion:

$$S^{(2)}_{(2)}=1+\frac{1}{3}r^2\Phi_{00},\quad S^{(2)}_{(3)}=\frac{1}{3}r^2\Psi^*_0, \tag{12}$$

where

$$r^2|\Psi_0|\ll1,\quad r^2\Phi_{00}\ll1. \tag{13}$$

Here $\Psi_0=R_{iklm}h^i_{(0)}h^k_{(2)}h^l_{(0)}h^m_{(2)}$ is a component of the Weyl spinor, and $\Phi_{00}=R_{iklm}h^i_{(0)}h^k_{(2)}h^l_{(0)}h^m_{(2)}$ is the spinor component of the traceless part of the Ricci tensor (see Ref. 10) Similar expressions can also easily be obtained from the Sachs equations.[6,7]

Subsequent terms in the expansion of the matrix S contain two types of corrections. The first type consists of nonlinear terms in $r^2\Psi_a$ and $r^2\Phi_{ab}$, which we can always neglect, and the second type consists of terms that are multiple covariant derivatives of the curvature tensor multiplied by corresponding powers of r. The latter describe the variation of the curvature tensor in the region in question. A term of this kind from the covariant derivative is at least of order r^3. It is possible to discard it if the extent to which spacetime deviates from the Friedmann models, which is characterized by Ψ_0, varies little over the scales considered here ($\sim$100 Mpc). We assume that this is the case, and that Ψ_0 and Φ_{00} do not vary along the path of the ray.

Noting that Φ_{00} is real, the relationship between the original and apparent position angles that follows from (11) can be brought to the form (1) via the parameters

$$\gamma=-\frac{1}{2}\arg\Psi_0,\quad K^2=1+\frac{2}{3}r^2|\Psi_0|. \tag{14}$$

We also have $K-1\ll 1$ by virtue of (13). The expression in the integrand of (8) is then

$$(K-1)\cos 2\gamma\approx\frac{r^2}{3}\,\mathrm{Re}(\Psi_0). \tag{15}$$

This effect is clearly independent of Φ_{00}—i.e., of the presence of matter—and it vanishes in Friedman models, for which $\Psi_0=0$. With (15), Eq. (8) then takes the form

$$F(\alpha_0,\delta_0)\approx\frac{\langle\Lambda\rangle R^2}{5\pi}\int_0^{2\pi}\int_{-\pi/2}^{\pi/2}\times\mathrm{Re}(\Psi_0(\alpha,\delta,\alpha_0,\delta_0))d\alpha\cos\delta d\delta. \tag{16}$$

The next step is to determine the function Ψ_0 for a petrov type I space.

4. RELATIONSHIP OF ANISOTROPY TO INVARIANTS OF THE WEYL TENSOR

We now express Ψ_0 in terms of the eigenvalues and eigenvectors of the Weyl tensor, noting that the latter belong to the more general type I in Petrov's classification scheme.[5] It is worthwhile to carry out the calculations in the spinor formalism, making use of the spinor equivalent of the conformal curvature tensor, namely the Weyl spinor Ψ_{ABCD}. The fact that there exists a spin vector basis (o^A,ι^A) in which it takes the form[10]

$$\Psi_{ABCD}=\mu(o_Ao_Bo_Co_D+\iota_A\iota_B\iota_C\iota_D)+6\rho o(A^oB^{\iota}C^{\iota}D) \tag{17}$$

means that it is Petrov-Penrose type I spinor. Here μ and ρ are complex functions of the coordinates that are related to the stationary Petrov curvatures.

In view of the fact that we are integrating over a specified region of space in Eqs. (8) and (16), we can replace the field of the spin tensor Ψ_{ABCD} with its effective value, i.e., we can assume that μ and ρ are complex constants, and that (o^A,ι^A) is a parallel spin basis.

The spinor basis (o^A,ι^A) is related to the Petrov canonical tetrad $\mathbf{g}_{(i)}$ as follows:

$$\mathbf{g}_{(0)}=(o^Ao^{A'}+\iota^A\iota^{A'})/\sqrt{2};\quad \mathbf{g}_{(1)}=(o^A\iota^{A'}+o^{A'}\iota^A)/\sqrt{2};$$
$$\mathbf{g}_{(2)}=(o^A\iota^{A'}-o^{A'}\iota^A)/i\sqrt{2};\quad \mathbf{g}_{(3)}=(o^Ao^{A'}-\iota^A\iota^{A'})/\sqrt{2}. \tag{18}$$

We can assume the timelike vector $\mathbf{g}_{(0)}$ is tangent to the observer's time axis in the present approximation, since in the region of space under consideration, with $R\approx 100$ Mpc, the relative velocities are gravitating bodies and the observer are much less than the speed of light, and special relativistic effects can be neglected.

Let $\mathbf{w}_{(i)}$ denote the unit vectors of the Cartesian coordinate system corresponding to the spherical coordinate (r,α,δ). We specify the relative orientation of the triads $\mathbf{g}_{(k)}$ and $\mathbf{w}_{(i)}$ $(i,k=1,2,3)$ via the Euler angles ψ, ϑ, φ, with $\mathbf{w}_{(i)}=M_{(i)}^{(k)}\mathbf{g}_{(k)}$, where

$$M=\begin{pmatrix}\cos\psi\cos\varphi-\sin\psi\sin\varphi\cos\vartheta & -\cos\psi\sin\varphi-\sin\psi\cos\varphi\cos\vartheta & \sin\psi\sin\vartheta\\ \sin\psi\cos\varphi+\cos\psi\sin\varphi\cos\vartheta & -\sin\psi\sin\varphi+\cos\psi\cos\varphi\cos\vartheta & -\cos\psi\sin\vartheta\\ \sin\varphi\sin\vartheta & \cos\varphi\sin\vartheta & \cos\vartheta\end{pmatrix}. \tag{19}$$

To calculate Ψ_0, it is now sufficient to note that relative to the Cartesian axes $\mathbf{w}_{(i)}$, the unit vectors $\mathbf{e}_{(i)}$ are given by $\mathbf{e}_{(i)}=N_{(i)}^{(k)}\mathbf{w}_{(k)}$, with

$$N=\begin{pmatrix}\cos\alpha\cos\delta & \sin\alpha\cos\delta & \sin\delta\\ -\sin\alpha & \cos\alpha & 0\\ -\cos\alpha\sin\delta & -\sin\alpha\sin\delta & \cos\delta\end{pmatrix}. \tag{20}$$

We now introduce the spin basis (ξ^A,η^A) corresponding to the unit vectors $\mathbf{e}_{(i)}$. The compound rotation $\mathbf{e}=NMg$ induces a unitary spin transformation of the form

$$\begin{pmatrix}\xi\\ \eta\end{pmatrix}=\begin{pmatrix}u & v\\ -v^* & u^*\end{pmatrix}\begin{pmatrix}o\\ \iota\end{pmatrix}, \tag{21}$$

where

$$u=e^{-i\varphi/2}[\cos(\theta/2)\cos(\tilde\theta/2)e^{-i\tilde\psi/2}-\sin(\theta/2)\sin(\tilde\theta/2)e^{i\tilde\psi/2}]$$
$$v=-ie^{i\varphi/2}[\sin(\theta/2)\cos(\tilde\theta/2)e^{-i\tilde\psi/2}+\cos(\theta/2)\sin(\tilde\theta/2)e^{i\tilde\psi/2}]. \tag{22}$$

Here, $\tilde\theta=\delta-\pi/2$, $\tilde\psi-\alpha-\pi/2$.

Finally, we obtain an expression for the desired components of the Weyl spinor:

$$\Psi_0=\Psi_{ABCD}\xi^A\xi^B\xi^C\xi^D\equiv\mu(u^4+v^4)+6\rho u^2v^2$$
$$=\mu\{\cos(2\psi-2\alpha)[-\tfrac{1}{4}(1+\cos^2\theta)(1+\sin^2\delta)\cos 2\varphi+i\sin\delta\cos\theta\sin 2\varphi]$$
$$+\tfrac{1}{2}\sin(2\psi-2\alpha)[(1+\sin^2\delta)\cos\theta\sin 2\varphi+i\sin\delta(1+\cos^2\theta)\cos 2\varphi]$$
$$-\sin\theta\cos\delta[\sin(\psi-\alpha)(i\sin 2\varphi-\cos\theta\sin\delta\cos 2\varphi)-\cos(\psi-\alpha)$$
$$-(i\cos\theta\cos 2\varphi\pm\sin\delta\sin 2\varphi)]$$
$$+\tfrac{3}{4}\sin^2\theta\cos^2\delta\cos 2\varphi\}+\tfrac{3}{4}\rho\{\sin^2\theta[(1+\sin^2\delta)\cos(2\psi-2\alpha)-2i\sin\delta\sin(2\psi-2\alpha)]$$
$$+\sin 2\theta[2i\cos\delta\cos(\psi-\alpha)+\sin 2\delta\sin(\psi-\alpha)]+\cos^2\delta(1-3\cos^2\theta)\}. \quad (23)$$

This expression depends on the coordinates α and δ, the Euler angles ψ, θ, and φ, which specify the orientation of the Petrov tetrad, and the two complex invariants $\mu=\mu_1+i\mu_2$ and $\rho=\rho_1+i\rho_2$, which characterize the deviation from Friedmann models in the present development.

Substituting (23) into (16) and integrating over the celestial sphere yields

$$F(\alpha_0,\delta_0)=\frac{2}{5}\langle\Lambda\rangle R^2Q(\theta,\varphi);$$
$$Q(\theta,\varphi)=\mu_1\cos 2\varphi\sin^2\theta+\rho_1(1-3\cos^2\theta). \quad (24)$$

The dependence on α_0 and δ_0 enters into this expression implicitly. It is straightforward to find the explicit dependence of φ and θ on α_0 and δ_0, and on the Euler angles ψ_e, θ_e, and φ_e with respect to the standard Cartesian coordinate system:

$$\mathrm{tg}(\varphi-\varphi_e)=\frac{\cos\delta_0\cos(\psi_e-\alpha_0)}{\sin\theta_e\sin\delta_0-\cos\theta_3\cos\delta_0\sin(\psi_e-\alpha_0)}$$
$$\cos\theta=\cos\theta_e\sin\delta_0+\sin\theta_e\cos\delta_0\sin(\psi_e-\alpha_0). \quad (25)$$

Substituting these equations into (24) yields the desired dependence, which, however, will not be needed further.

It is immediately clear from (24) that the maxima and minima of the function F occur in the directions of the unit vectors of the Petrov moving frame. Calculating Q for these directions, we obtain

$$Q(0,0)=-2\rho_1;\quad Q(\pi/2,0)=\rho_1+\mu_1;$$
$$Q(\pi/2,0)=\rho_1-\mu_1. \quad (26)$$

These expressions are the same as the real parts of the time-independent Petrov curvatures for the Weyl tensor. We denote these curvatures by β_1, β_2, and β_3, and assume that $\beta_1\geqslant\beta_2\geqslant\beta_3$. Noting that $\beta_1+\beta_2+\beta_3=0$, we have $\beta_1>0$ and $\beta_3<0$. From (24) and (26),

$$\beta_1\cong\frac{5}{2\langle\Lambda\rangle}R^{-2}F_{\max};\quad \beta_3\cong\frac{5}{2\langle\Lambda\rangle}R^{-2}F_{\min};$$
$$\beta_2\cong\frac{5}{2\langle\Lambda\rangle}R^{-2}(|F_{\min}|-F_{\max}). \quad (27)$$

The distribution density of galactic normals can be obtained in the quadrupole approximation by subtracting the function $F(\alpha_0,\delta_0)/4\pi$ from the mean density $1/4\pi$.[4] We see from (24) that the probability density of the normals can be described by a triaxial ellipsoid whose axes are parallel to the Petrov unit vectors. If $\mu_1=0$ or $9\rho_1^2=\mu_1^2$, it becomes an ellipsoid of revolution. Note that this is inevitable if a type I space degenerates into a Petrov type D space.

5. ESTIMATED PARAMETERS OF THE CURVATURE TENSOR AVERAGED OVER ANISOTROPY DATA

We now return to the data on the extrema of F obtained in Ref. 4. If we construct for all of the galaxies listed in catalogs 1–3, we find that in all three catalogs it peaks essentially at the celestial pole, and it has a minimum in the neighborhood of $\alpha=30–110°$, with $\delta=-20–30°$. For the more accurate catalogs,[2,3] $F_{\max}$ is 16–18% and $F_{\min}\approx-8–11\%$. Similar results are obtained when one investigates anisotropy in the orientation of galaxy pairs.

If we assume that the observed anisotropy is entirely due to the mechanism discussed above, we can estimate the real parts of the time-independent Petrov curvatures. Taking the depth of the sample in Ref. 3 to be $R\approx100$ Mpc, we have

$$\beta_1\approx(160\ \mathrm{Mpc})^{-2},\quad \beta_2\approx-(250\ \mathrm{Mpc})^{-2},$$
$$\beta_3\approx-(200\ \mathrm{Mpc})^{-2}. \quad (28)$$

Since all three curvatures are much greater than the background curvature for the Friedmann models, the curvature tensor will be essentially identical with the Weyl tensor. This comes as no surprise. If for clarity we represent the Friedmann model as a sphere, and deviations as ripples on its surface, then at small scales the curvature of the resulting wrinkled globe will be dictated precisely by the "bumps" and "ruts." Small deviations will be those with

$$|\beta_\lambda|R^2<1, \quad (29)$$

which holds for the present case. We then find that the existence of an anisotropic component of the curvature is not at variance with the observed quasi-isotropy of the Hubble expansion.

One questionable aspect of the anisotropy ellipsoid considered here is the coincidence of the maximum of F with the celestial pole–a preferred direction for spurious anisotropy that can result from measurement errors or unrecognized selection effects in the data of Ref. 4. Galaxies with the smallest angular size are the most susceptible to such errors. If we resample catalogs 1–3 so as to select only galaxies larger than some angular size, the maximum of F moves away from the pole to $\alpha=180–260°$, $\delta=30–70°$, and the minimum is relocated to $\alpha=30–110°$, $\delta=10–50°$.[4] $F_{\max}$ ranges accordingly from about 20 to 25%, and $F_{\min}\approx-20–-25\%$. The depth of the sample becomes $R\approx50$ Mpc Equation (27) then yields

$$\beta_1\approx-\beta_3\approx(80\ \mathrm{Mpc})^{-2}. \quad (30)$$

We also find that (29) is satisfied at $R=50$ Mpc, but not at $R=100$ Mpc.

As we noted above, our present goal is to construct a theory of the onset of apparent anisotropy. The estimates (28) and (30) are at best tentative—improvements will come not so much from further development of the theory as from more accurate determinations of the parameters of the anisotropy ellipsoid. We have recently undertaken to separate out of the observed anisotropy in galactic orientations that part due to measurement errors and selection effects. The residual anisotropy $F_{\max}$ and $|F_{\min}|$ is at most 10% [unpublished results of S. L. Parnovskiĭ, I. D. Karachentsev, and V. E. Karachentseva], yielding

$$|\beta_i| \leq (200 \text{ Mpc})^{-2}. \tag{31}$$

6. TESTING THE HYPOTHESIS

We now suggest a number of implications that are amenable to test. The present phenomenon does not lead to a difference in the galactic angular momentum probability distribution for opposite directions on the celestial sphere (dipole anisotropy). Unfortunately, the direction of rotation is known for just a few galaxies in the Local Supercluster, and we thus have no solid statistical basis for accepting or rejecting the existence of dipole anisotropy.

A second implication relates to changes in the apparent axial ratio of a galaxy under shear. Instead of the true ratio q, we would observe an ellipse with ratio q', where

$$\ln q' = \frac{1}{2} \ln\left(\frac{K^2A^2(\lambda_0)+K^{-2}B^2(\lambda_0)}{K^2A^2(\lambda_0+\pi/2)+K^{-2}B^2(\lambda_0+\pi/2)} \right)$$
$$\approx \ln\left\{ q\left[1+2(K-1)\cos(2(p+\gamma)\right]\right\} \approx \ln(q)$$
$$+\frac{2}{3} r^2\left(\text{Re } \Psi_0 \cos 2p - \text{Im } \Psi_0 \sin 2p\right). \tag{32}$$

Here we have made use of Eqs. (4), (5), (12), and (13). For a sample consisting of isotropically oriented galaxies distributed uniformly in space, we obtain the expectation values

$$\langle \ln q' \cos 2p \rangle \approx \frac{R^2}{5} \text{Re } \Psi_0, \tag{33}$$

$$\langle \ln q' \sin 2p \rangle \approx -\frac{R^2}{5} \text{Im } \Psi_0. \tag{34}$$

To first order in $K-1$, these quantities do not change when p is replaced by the apparent position angle p'. Equations (23) and (33) make it possible to check the estimated parameters μ_1 and ρ_1, as well as the eigenvector orientations of the Weyl tensor. On the other hand, with (23), (34) enables one to estimate μ_2 and ρ_2 and check the determination of the eigenvector directions. It should be noted, however, that astronomical measurements of the galactic axial ratio are of much lower precision than those of position angles, which are used in Eq. (2).

A third effect crops up in the clustering of galaxies in deep samples of selected regions of the sky. The direction of cluster axes should in this case be preferentially orthogonal to the shear axes.

A number of other effects also exist, such as a weak variation in apparent galactic magnitudes, but these are too small to be detectable in the near future.

Finally, we mention the rotation of the polarization angle of radio waves emitted by distant sources, an effect that we discuss in the next section.

7. THE BIRCH EFFECT

Birch reported[11] in 1982 that had detected a systematic deviation of radio polarization of distant sources from their apparent position angles p'. If p_1 denotes the position angle corresponding to the polarization, then the quantity $\Delta = p_1 - p'$ is, as a rule, positive for sources in the celestial hemisphere centered at $\alpha \approx 45°$, $\delta \approx 35°$, and negative for sources in the opposite hemisphere. The means of Δ for the two hemispheres, $\langle\Delta\rangle_1 = -31.6 \pm 8°$ and $\langle\Delta\rangle_2 = 12.3 \pm 7.3°$, are nonzero at a high confidence level. Birch maintained that this provides evidence for the rotation at an angular velocity $\omega \sim 10^{-13}$ rad/yr of the universe or a large part of it, about the axis joining the centers of these two hemispheres. No substantiation was provided for this hypothesis, nor was any given for the estimated angular velocity.

Subsequently, a number of attempts were made to explain the Birch effect in terms of Faraday rotation in the Galactic magnetic field, but the original supposition of rotation of the universe was retained. Panov and Sbytov[8] calculated the angle Δ for a specific spacetime model with rotation.

What could lead to a difference between p_1 and p'? Since there is no rotation in a convergent light beam, the effect must be related to shear. Also, since the polarization vector is parallel-transported along the beam axis, we have $p_1 = p$ when the polarization direction of the beam is initially along the source's major axis, i.e., the direction of polarization corresponds to the true orientation of the radio source. In the weak-anisotropy approximation, we obtain from (1), (7), and (12)

$$\Delta = p - p' \approx -(K-1)\sin(2(p+\gamma))$$
$$= -\frac{r^2}{3}\left(\text{Re}(\Psi_0)\sin(2p) + \text{Im}(\Psi_0)\cos(2p)\right), \tag{35}$$

which taken together with (23) yields Δ as a function of all of the parameters. The model considered in Ref. 8 pertains to Petrov type D, for which (in the notation of Ref. 8)

$$\rho_1 = km^2/[6(p+k)], \quad \mu_1 = 0.$$

Furthermore, the rotation axis points along the polar axis of the spherical coordinate system, and the Euler angles ψ, θ, and φ vanish. Substituting these quantities into (23) and (35), we obtain the equation for Δ in this special case.

In general, we see that the reason for noncoincidence of the radio source orientation and the polarization may be more than just rotation. It may also be any other deviation from a conformally flat Friedmann spacetime—in particular, irregularities in the distribution of matter.

Nevertheless, we can suggest two reasons why the observations described in Ref. 11 may not result from general relativistic effects in the propagation of radio waves. First, the magnitude and sign of Δ are sensitive functions of the source position angle. If they are randomly distributed, we will no longer observe two celestial hemispheres with different signs of Δ. Second, as can be seen from (35), the effect is quadratic in the distance. Birch claims, however, that the effect is the same for samples spanning redshifts from $z=0.3$ to $z\approx1.5$. Nevertheless, an investigation of Δ as a function of position angle and location on the celestial sphere may confirm the theory proposed here, which predicts the behavior implied by (23) and (35)

Note that other factors can influence Δ, including the previously mentioned Galactic magnetic field. Observable radio source, however, can be located much farther away than galaxies, and observational data on those sources make it possible to ascertain the actual large-scale geometry of the universe. We cannot rule out the possibility that the present approximation (14) may not hold in that situation.

8. CONCLUSION

We have shown that the deviation of the curvature of spacetime from that given by Friedmann models results in an observable anisotropy (possibly optical) in the orientation of galaxies and galaxy pairs. For weak anisotropy, the probability density function for the orientation of normals on the celestial sphere is given by a triaxial ellipsoid whose axes are aligned with the canonical Petrov tetrad, and whose parameters are related to the real parts of the characteristic curvatures of the Weyl tensor. This endows us for the first time with the ability to determine the geometry of the universe on scales of 50–100 Mpc. At larger distances one can observe the deviation of the plane of polarization of radio waves from the major axis of distant radio sources. To test this hypothesis, we can take advantage of the correlation between the axial ratio q of a galaxy and its position angle p, as given by (33), plus other deductions listed in Sec. 6.

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