Conductivity of an electron gas in a quantizing magnetic field with scattering by point defects

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The transverse and longitudinal conductivities of an electron gas in a quantizing magnetic field with electron scattering induced by randomly distributed zero-radius potentials are calculated using the exact expression for the scattering amplitude. We use an approximation that is linear in the electric field strength and impurity concentration (which is common practice in such calculations). Both degenerate and nondegenerate gases of charge carriers are examined. In the quasiclassical limit the transverse conductivity is shown to have sharp, deep minima, and the longitudinal conductivity, sharp and high maxima at intersections of the Landau levels with the Fermi level. These extrema are shown to be related to the singularities in the scattering amplitude rather than to those in the density of states. © *1995 American Institute of Physics.*

1. INTRODUCTION

Studies of the conductivity of charge carriers in a quantizing magnetic field with scattering by the short-range potential of impurities have been carried out in Refs. 1–4. Adams and Holstein[1] allowed for scattering in the Born approximation. To remove the divergences appearing in this approximation, Skobov[5] suggested a method of calculating the scattering amplitude based on ideas lying outside the scope of the Born approximation. Using this method, he calculated[2] the transverse conductivity of the electron gas in terms of the scattering amplitude of a single center in the absence of a a magnetic field. Note that in finding the scattering amplitude, Skobov[5] used an approximate expression for the Green's function obtained by truncating the perturbation series according to the number of the Landau level. As noted in Ref. 6, the scattering potential used in Refs. 2 and 5 is a zero-radius potential. The exact expression for the electron scattering amplitude in a magnetic field and a zero-radius potential, allowing for the sum of all terms in the series for the Green's function, has been found by Demkov and Drukarev.[7] Since the use of the exact expression of the Green's function for point potentials in a magnetic field may lead to physically interesting consequences,[8–12] it would appear to be worthwhile to find a formula for the conductivity of the electron gas that allows for the exact expression for the scattering amplitude. As demonstrated below, using such an expression leads to significant differences from some of the results of Ref. 2. Specifically, it appears that for $\mu \gg \hbar\omega_c$, the transverse conductivity exhibits sharp, deep minima, while the longitudinal conductivity exhibits sharp, high maxima at the points where the Landau levels cross the Fermi level; these extrema are related to singularities in the scattering amplitude rather than in the density of states.

The goal of the present investigation is to find the transverse and longitudinal conductivities of an electron gas in a quantizing magnetic field H, where the electrons are scattered by randomly distributed zero-radius potentials, by incorporating the exact expression for the scattering amplitude. We consider the linear approximation in the electric field strength and the impurity concentration, which is the standard approach in such calculations. Both degenerate and nondegenerate gases of charge carriers are examined.

2. ELECTRON TRANSITION PROBABILITY

The exact expression for the electron scattering amplitude of a zero-radius potential in a magnetic field[7] is

$$t(E)=\frac{2\sqrt{2}\pi l_H^3 \hbar\omega_c\alpha}{1+\alpha\zeta(1/2,\ 1/2-E/\hbar\omega_c)}, \tag{1}$$

where $E=\hbar\omega_c(n+1/2)+\hbar^2k_z^2/2m^*$ is the Landau-level energy, $\alpha=\lambda/\sqrt{2}l_H$ is the pseudopotential coupling constant, λ is the scattering length, and l_H is the magnetic length. Introducing a_0, the zero-energy scattering amplitude for a zero-radius potential in the absence of a magnetic field, we have

$$t(E)=\frac{a_0}{1+\alpha\zeta(1/2,\ 1/2-E/\hbar\omega_c)}\ . \tag{2}$$

Here $\zeta(s,v)$ is the generalized Riemann zeta function, and $a_0=2\pi\hbar^2\lambda/m^*$.

We introduce the notation $E=\hbar\omega_c(N+\delta+1/2)$, with N an integer and $0\leq\delta<1$. Then the exact expression (2) can be compared with Skobov's result[5] if we use the shift theorem for $\zeta(s,v)$:

$$\zeta(1/2,\ -N-\delta)=\zeta(1/2,\ 1-\delta)+i\sum_{n=0}^{N}\frac{1}{\sqrt{N+\delta-n}}. \tag{3}$$

In Ref. 5 the exact expression (3) is actually replaced by $iK(E)$, where

$$iK(E)=\frac{1}{\sqrt{1-\delta}}+i\sum_{n=0}^{N}\frac{1}{\sqrt{N+\delta-n}}. \tag{4}$$

From Hermite's formula for the generalized ζ function[13] one can easily obtain[6]

$$\zeta(1/2,1-\delta)\simeq\frac{1}{\sqrt{1-\delta}}+\frac{1}{2\sqrt{2-\delta}}-2\sqrt{2-\delta}+\frac{1}{24}(2-\delta)^{-3/2}. \tag{5}$$

Comparing (3) and (4) and allowing for (5), we see that the last three terms in (5) are absent from (4). The second and third terms in (5) play an important role for values of δ not too close to unity.

To find the electron transition probability (transition rate), we use the Lippmann formula[14] relating the probability to the scattering amplitude:

$$W_{ab}=\frac{2\pi}{\hbar}|T_{ab}|^2\delta(\varepsilon_a-\varepsilon_b), \tag{6}$$

the scattering matrix elements in our case being

$$T_{ab}=\frac{l_H t(\varepsilon_a)}{V}\varphi_n\left(\frac{y_a^0}{l_H}\right)\varphi_m\left(\frac{y_b^0}{l_H}\right). \tag{7}$$

Here V is the normalization volume, φ_k is the oscillator function, and y_a^0 locates the center of the cyclotron orbit. Combining (6) and (7), we obtain

$$W_{ab}=\frac{2\pi}{\hbar}|t(\varepsilon_a)|^2\frac{l_H^2}{V^2}\varphi_n^2\left(\frac{y_a^0}{l_H}\right)\varphi_m^2\left(\frac{y_b^0}{l_H}\right), \tag{8}$$

where a and b label the initial and final states in an electron transition.

3. TRANSVERSE CONDUCTIVITY

To calculate the transverse conductivity σ_{xx} we use the well-known formula[2,4] obtained in the linear approximation in the electric field and the impurity concentration:

$$\sigma_{xx}=e^2n_i\sum_{a,b}\left(-\frac{\partial f_0}{\partial\varepsilon_a}\right)\frac{(y_a^0-y_b^0)^2}{2}W_{ab}. \tag{9}$$

Here $|a\rangle=|n,k_x,k_z\rangle$, $f_0(\varepsilon_a)$ is the equilibrium distribution function, and n_i is the impurity concentration.

Substituting (8) into (9), some simple manipulations yield

$$\sigma_{xx}=\frac{e^2\hbar n_i}{\pi m^*}\sum_{n,m}(n+m+1)\int_0^\infty\left(-\frac{\partial f_0}{\partial\varepsilon}\right)\times\frac{|\zeta(1/2,-\varepsilon/\hbar\omega_c-n)+\alpha^{-1}|^{-2}d\varepsilon}{\sqrt{\varepsilon[\varepsilon+\hbar\omega_c(n-m)]}}. \tag{10}$$

Let us first examine a Boltzmann gas. Here we know that

$$f_0(\varepsilon)=\frac{8\pi^2\hbar^2n_e}{m^*\omega_c\sqrt{2\pi m^*T}}\sinh\left(\frac{\hbar\omega_c}{2T}\right)\times\exp\left\{-\frac{\varepsilon+\hbar\omega_c(n+1/2)}{T}\right\}, \tag{11}$$

where n_e is the electron concentration.

Representing σ_{xx} as $\sigma_{xx}=e^2n_e/m^*\omega_c^2\tau$ and combining (10) with (11), we have

$$\tau^{-1}=\frac{8\pi^2\hbar^2n_i\hbar\omega_c}{m^*T\sqrt{2\pi m^*T}}\exp\left\{-\frac{\hbar\omega_c}{2T}\right\}\sinh\left(\frac{\hbar\omega_c}{2T}\right)\times\sum_{n,m=0}^{\infty}J_{nm}(\alpha)\exp\left\{-\frac{\hbar\omega_c n}{T}\right\}, \tag{12}$$

where

$$J_{nm}(\alpha)=\int_0^\infty\frac{dx\exp\{-x\}}{\sqrt{x[x+\hbar\omega_c/T(n-m)]}}\left|\zeta\left(1/2,-\frac{xT}{\hbar\omega_c}\right)+\alpha^{-1}\right|^{-2}. \tag{13}$$

Next we calculate σ_{xx} in the ultraquantum limit $n=m=0$. In this case $\hbar\omega_c/T\gg1$ and hence

$$\sinh\left(\frac{\hbar\omega_c}{2T}\right)\exp\left\{-\frac{\hbar\omega_c}{2T}\right\}\simeq\frac{1}{2},$$

which yields

$$\tau^{-1}\simeq\frac{(2\pi\hbar)^2n_i\hbar\omega_c}{\sqrt{2\pi(m^*)^3T^3}}J_{00}(\alpha). \tag{14}$$

In estimating the integral in (14), we take into account the fact that because of the exponential factor in the integrand, the important range of variation of x is $x\leqslant1$, and hence $xT/\hbar\omega_c\ll1$. Then (5) yields

$$\zeta\left(\frac{1}{2},-\frac{xT}{\hbar\omega_c}\right)\simeq\frac{i}{\sqrt{xT/\hbar\omega_c}}-\frac{3}{2}. \tag{15}$$

Using this estimate, we obtain

$$J_{00}(\alpha)\simeq\frac{4\alpha^2}{(2-3\alpha)^2}\mathrm{Ei}\left[\frac{4\hbar\omega_c\alpha^2}{(2-3\alpha)^2T}\right]. \tag{16}$$

Then

$$\sigma_{xx}\simeq\frac{8\hbar^2\lambda n_in_ee^2}{\sqrt{2(m^*)^3}T^{3/2}}\mathrm{Ei}\left[\frac{\lambda^2}{l_H^4}\frac{2\hbar^2}{(2-3\alpha)^2m^*T}\right]. \tag{17}$$

For a degenerate gas in the ultraquantum limit at $T=0$, the range of the Fermi level is determined by $1/2\leqslant\mu/\hbar\omega_c\leqslant3/2$, so that Eq. (10) yields

$$\sigma_{xx}=\frac{e^2n_i\hbar}{\pi m^*(\mu-\hbar\omega_c/2)}\left|\zeta\left(\frac{1}{2},\frac{1}{2}-\frac{\mu}{\hbar\omega_c}\right)+\alpha^{-1}\right|^{-2}. \tag{18}$$

Using the estimate (5) and the expression for the transverse conductivity (19), and introducing the chemical potential μ_0 of the electron gas in the absence of a magnetic field, we can easily show that

$$\sigma_{xx}\simeq\frac{e^2n_i}{\pi m^*\omega_c}\frac{b^2}{(a^2+b^{-2})}, \tag{19}$$

where

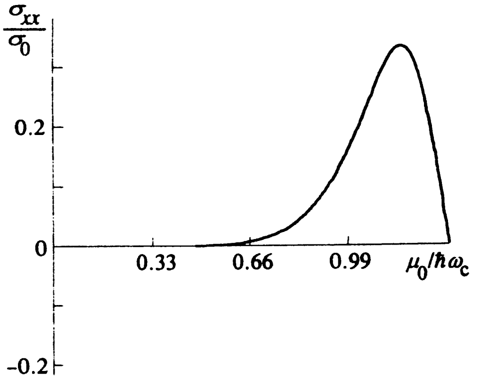

FIG. 1. The dependence of transverse conductivity on $\mu_0/\hbar\omega_c$ in the ultraquantum limit at $\sigma_0=e^2\hbar n_i/\pi m^*\mu_0$.

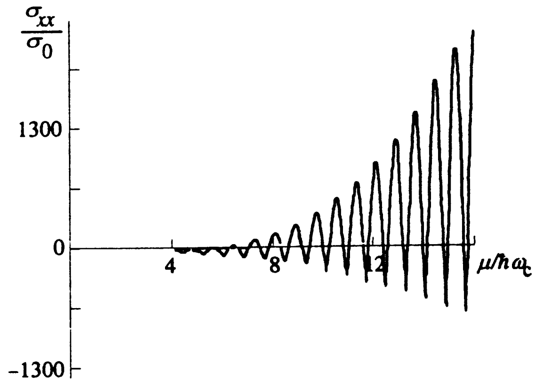

FIG. 2. The dependence of transverse conductivity on $\mu/\hbar\omega_c$ at $\sigma_0=(16/3\pi)e^2\hbar n_i/m^*\mu$ and $\mu/T=50$.

$$b=\frac{2}{3}\left(\frac{\mu_0}{\hbar\omega_c}\right)^{3/2},$$

$$a\simeq\frac{1}{\sqrt{1-b^2}}+\frac{1}{2}\frac{1}{2\sqrt{2-b^2}}-2\sqrt{2-b^2}+\frac{1}{24}(2-b^2)^{-3/2}+\alpha^{-1}.$$

Figure 1 shows σ_{xx} as a function of $\mu_0/\hbar\omega_c$.

When $\hbar\omega_c\ll\mu$, it is convenient to write Eq. (10) as

$$\sigma_{xx}=2\int_{\hbar\omega_c/2}^{\infty}dEA(E)\frac{\partial f_0}{\partial\mu}\sum_{m=0}^{N}\frac{1}{\sqrt{N+\delta-m}}\times\sum_{n=0}^{N}\frac{n+1/2}{\sqrt{N+\delta-n}}, \tag{20}$$

where

$$A(E)=\frac{e^2n_i}{\pi m^*\omega_c}\left|\zeta\left(\frac{1}{2},\frac{1}{2}-\frac{E}{\hbar\omega_c}\right)+\alpha^{-1}\right|^{-2}. \tag{21}$$

Evaluating the sums in (20) via Poisson's summation formula and allowing, via (5), for the asymptotic behavior of the generalized ζ function as $\delta\to0$ in the product of Fourier series, we obtain after lengthy but fairly simple calculations

$$\sigma_{xx}=\frac{16l_H^2e^2n_i}{3\pi\hbar}\left(\frac{\mu}{\hbar\omega_c}\right)^2\left\{\left|\zeta\left(\frac{1}{2},\frac{1}{2}-\frac{\mu}{\hbar\omega_c}\right)+\alpha^{-1}\right|^{-2}\times\left[1+\sum_{k=1}^{\infty}A_k\cos\left(\frac{2\pi k\mu}{\hbar\omega_c}-\frac{\pi}{4}\right)\right]+\left\{\left[\frac{3\hbar\omega_c}{8\mu}\ln\left(\frac{2-3\alpha}{2\alpha}\right)+\sum_{k=1}^{\infty}B_k\cos\left(\frac{2\pi k\mu}{\hbar\omega_c}\right)\right]\frac{8\alpha^2}{(2-3\alpha)^2}\right\}, \tag{22}$$

where

$$A_k=(-1)^k\frac{5}{2}\sqrt{\frac{\hbar\omega_c}{\mu}}\frac{1}{\sqrt{2k}}\frac{2\pi^2kT}{\hbar\omega_c\sinh(2\pi^2kT/\hbar\omega_c)},$$

$$B_k=(-1)^{k+1}\mathrm{Ci}\left[\frac{8\pi k\alpha^2}{(2-3\alpha)^2}\right]\frac{3\hbar\omega_c}{8\mu}\frac{2\pi^2kT}{\hbar\omega_c\sinh(2\pi^2kT/\hbar\omega_c)}.$$

Figure 2 shows σ_{xx} as a function of $\mu/\hbar\omega_c$.

4. LONGITUDINAL CONDUCTIVITY

To calculate the longitudinal conductivity σ_{zz}, we use[4]

$$\sigma_{zz}=\frac{4\sqrt{2}e^2}{(2\pi l_H)^2\hbar\sqrt{m^*}}\sum_n\int dE\left(-\frac{\partial f_0}{\partial E}\right)\sqrt{E}\tau(E)\times\left[\frac{E}{\hbar\omega_c}-n-\frac{1}{2}\right]^{-1/2}, \tag{23}$$

where

$$\tau^{-1}(E)=\frac{(2\pi)^2\sqrt{2}m^*n_i}{(2\pi\hbar)^3l_H}|t(E)|^2\sum_m\left[\frac{E}{\hbar\omega_c}-m-\frac{1}{2}\right]^{1/2}. \tag{24}$$

Combining (23) and (24), we obtain

$$\sigma_{zz}=\frac{2e^2\hbar^2}{\pi l_H(m^*)^{3/2}a_0^2n_i}\int_{\hbar\omega_c/2}^{\infty}dE\sqrt{E}\left(-\frac{\partial f_0}{\partial E}\right)\left|1+\alpha\zeta\left(\frac{1}{2},\frac{1}{2}-\frac{E}{\hbar\omega_c}\right)\right|^2\times\frac{\sum_n[E/\hbar\omega_c-n-1/2]^{1/2}}{\sum_m[E/\hbar\omega_c-m-1/2]^{-1/2}}. \tag{25}$$

For a nondegenerate gas in the ultraquantum limit with $\hbar\omega_c\gg T$, Eq. (25) yields

$$\sigma_{zz}=\frac{4\sqrt{2\pi}e^2\hbar l_H^3n_eT}{a_0^2m^*n_i}J(\alpha), \tag{26}$$

where

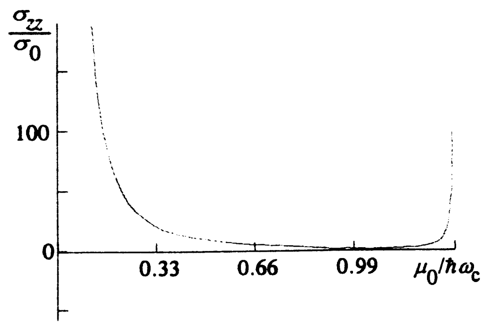

FIG. 3. The dependence of longitudinal conductivity on $\mu_0/\hbar\omega_c$ in the ultraquantum limit at $\sigma_0=e^2m^{*2}\ \mu_0^2/4\pi^3 n_i\hbar^5$.

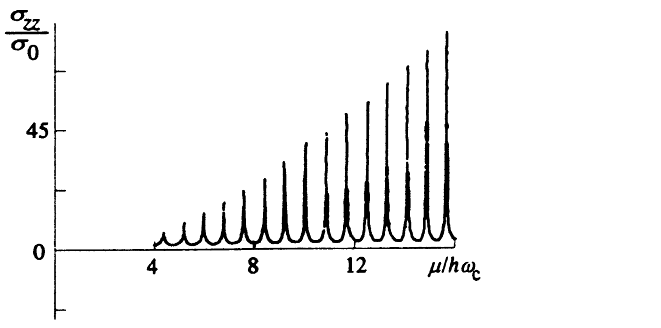

FIG. 4. The dependence of longitudinal conductivity on $\mu/\hbar\omega_c$ at $\sigma_0=8e^2\hbar\mu/3\pi m^* a_0^2 n_i$ and $\mu/T=50$.

$$J(\alpha)=\int_0^\infty dx\left|1+\alpha\zeta\left(\frac{1}{2},-\frac{xT}{\hbar\omega_c}\right)\right|^2 x\sqrt{x+\frac{\hbar\omega_c}{2T}}\times\exp(-x).$$

Evaluating $J(\alpha)$ via (5), we obtain

$$J(\alpha)\simeq\left(\frac{\hbar\omega_c}{T}\right)^{5/2}\left[\psi\left(2,\frac{7}{2},\frac{\hbar\omega_c}{2T}\right)+8\alpha^2\psi\left(1,\frac{5}{2},\frac{\hbar\omega_c}{2T}\right)\right], \quad (27)$$

where $\psi(a,b,x)$ is a confluent hypergeometric function.[13] Bearing in mind the asymptotic behavior of $\psi(a,b,x)$ for large values of x and combining (26) with (27), we obtain

$$\sigma_{zz}\simeq\frac{4\sqrt{2\pi}e^2\hbar l_H^3 n_e T}{a_0^2 m^* n_i}\sqrt{\frac{\hbar\omega_c}{2T}}\left[\left(\frac{3\alpha-2}{2}\right)^2+2\alpha^2\left(\frac{\hbar\omega_c}{2T}\right)\right]. \quad (28)$$

For a degenerate gas with $n=m=0$ and $T=0$, Eq. (23) yields

$$\sigma_{zz}=\frac{2e^2\hbar^3\alpha^2}{\pi l_H^2(m^*)^2 a_0^2 h_i}\left(\frac{\mu}{\hbar\omega_c}\right)^{3/2}\left|\alpha^{-1}+\zeta\left(\frac{1}{2},\frac{1}{2}-\frac{\mu}{\hbar\omega_c}\right)\right|^2. \quad (29)$$

Making use of (5) and introducing μ_0, Eq. (29) yields

$$\sigma_{zz}\simeq\frac{e^2}{4\pi^3\hbar l_H^4 n_i}\left(\frac{\mu_0}{\hbar\omega_c}\right)^{3/2}(a^2+b^{-2}), \quad (30)$$

where a and b were defined earlier. Figure 3 shows σ_{zz} as a function of $\mu_0/\hbar\omega_c$.

We now examine the Shubnikov–de Haas effect in longitudinal conductivity. We use Poisson's summation formula to evaluate the sum in (25):

$$\frac{\Sigma_n(N-n+\delta)^{1/2}}{\Sigma_m(N-m+\delta)^{-1/2}}\simeq\frac{N+\delta+1/2}{1+(N+\delta+1/2)^{-1/2}\sum_{k=1}^{\infty}[\cos(2\pi k\delta-\pi/4)/\sqrt{2k}]}. \quad (31)$$

Multiplying the numerator and denominator in (31) by

$$1-(N+\delta+1/2)^{-1/2}\sum_{k=1}^{\infty}\frac{\cos(2\pi k\delta-\pi/4)}{\sqrt{2k}},$$

we have

$$\frac{\Sigma_n(N-n+\delta)^{1/2}}{\Sigma_m(N-m+\delta)^{-1/2}}\simeq\frac{1}{3}(N+\delta+1/2)\times\frac{1-(N+\delta+1/2)^{-1/2}\Sigma_{k=1}^{\infty}[\cos(2\pi k\delta-\pi/4)/\sqrt{2k}]}{1-(N+\delta+1/2)^{-1}/4\delta}. \quad (32)$$

Combining (5) with (32), we obtain

$$\sigma_{zz}=\frac{8e^2\hbar^2\mu^{3/2}}{3\pi l_H(m^*)^{3/2}a_0^2 n_i\hbar\omega_c}\times\frac{1-\frac{\hbar\omega_c}{\mu}\sum_{k=1}^{\infty}(-1)^k D_k\cos\left(\frac{2\pi k\mu}{\hbar\omega_c}-\frac{\pi}{4}\right)}{\alpha^2\frac{\hbar\omega_c}{\mu}+4\left|1+\alpha\zeta\left(\frac{1}{2},\frac{1}{2}-\frac{\mu}{\hbar\omega_c}\right)\right|^{-2}}, \quad (33)$$

where

$$D_k=\frac{2\pi^2 kT}{\hbar\omega_c\sinh(2\pi^2 kT/\hbar\omega_c)}\frac{1}{\sqrt{2k}}.$$

Figure 4 shows σ_{zz} as a function of $\mu/\hbar\omega_c$.

5. DISCUSSION

Comparison of the results obtained above for the transverse conductivity σ_{xx} in the ultraquantum limit and the formulas of Ref. 2 suggests that in the case of a nondegenerate gas only the argument of the function $\mathrm{Ei}(x)$ in (17) is changed by the factor $2/(2-3\alpha)^2$. In a degenerate gas, the dependence of σ_{xx} on the field strength H and the scattering length, Eq. (19), is more complicated than that obtained in Ref. 2, and is determined by the factor $(a^2+b^{-2})^{-1}$. For magnetic field strengths $\mu\gg\hbar\omega_c$, a situation emerges that is quite different from that discussed in Ref. 2, where the H dependence of the scattering amplitude is taken into account only by a term $\sim\hbar\omega_c/\mu$, which is small compared to the leading term. This small term corresponds to the product of series. Furthermore, as Eq. (22) suggests, the leading term contains the factor $|\alpha^{-1}+\zeta(1/2,1/2-\mu/\hbar\omega_c)|^{-2}$, which vanishes at points where $\mu=\hbar\omega_c(n+1/2)$ (since the generalized zeta function has a root singularity at those points). Hence the conductivity near these points is determined only by the second term, corresponding to a product of Fourier series. This suggests that allowing for the H dependence of $t(E)$ is important when studying the Shubnikov–de Haas effect in the vicinity of the region where a Landau level crosses the Fermi level.

Note that the behavior of σ_{zz} near the points where $\mu=\hbar\omega_c(n+1/2)$ is quite different, as Eq. (33) implies. At those points the denominator of the second fraction in (33) becomes small ($\sim\alpha^2\hbar\omega_c/\mu$), and hence the conductivity σ_{zz} at those points is high. The height of the peaks in σ_{zz} can easily be estimated. If we assume that the energy of the bound state in a delta-like well formed by a point potential is equal to that of the second level of the hydrogen atom,[15] $\alpha\sim0.1$ for $H\sim10$ kOe. Then at the maxima determined by the singularities of the zeta function, the values of σ_{zz} increase by two orders of magnitude for $\mu/\hbar\omega_c\sim10$. The depth of the minima in σ_{xx} are of the same order, as Eq. (22) implies.

These estimates and Eqs. (22) and (23) (Figs. 2 and 4) suggest that allowing for the exact scattering amplitude in a zero-radius potential drastically changes the physical picture of the Shubnikov–de Haas effect in both the transverse conductivity (σ_{xx}) and the longitudinal conductivity (σ_{zz}). In either case, the curves are nonsinusoidal. In σ_{xx} (Fig. 2) the curve near the maxima is sinusoidal, while the minima are extremely sharp. The behavior of $\sigma_{xx}(\mu/\hbar\omega_c)$ is asymmetric with respect to the horizontal axis, since the height of the peaks grows faster ($\propto(\mu/\hbar\omega_c)^3$) than the depth of the minima ($\propto(\mu/\hbar\omega_c)^2$). In $\sigma_{zz}(\mu/\hbar\omega_c)$ (Fig. 4), the sharp, high peaks are separated by broad minima, where the conductivity is low. The maxima in σ_{zz} and the minima in σ_{xx} are equidistant from one another at points where $\mu=\hbar\omega_c(n+1/2)$, and are determined by the singularities of the scattering amplitude $t(E)$ at those points.

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