Theory of mode synchronization with a coherent absorber: transient processes and stability analysis

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The stability of the mode synchronization regime is studied in a laser with a coherent absorber (absorption line width much less than the width of the amplification line). The solution, a 2π self-induced-transparency pulse, is shown to be stable against perturbations in the form of small amplitude and phase modulation (fast perturbations). The parameter s serves as a control parameter; it is equal to the square of the ratio of the Rabi frequency of the amplifying medium to that of the absorbing medium, and at the same time plays the role of the absorption coefficient of the amplifier. It is found that the condition for stable lasing requires that the inequality $s \geqslant 1$ hold, which corresponds to strong absorption of the active medium. A calculation is also presented of the absorber density needed to ensure that the regime of soliton-like pulses is stable against the occurrence of continuous generation (slow perturbations). The problem is solved taking into account the effects of an optical filter, which limits the amplification band and without which stability against slow perturbations cannot be achieved. Two time scales are identified in this model and are used to analyze the transient evolution of the field in the cavity. A critical value is found for the energy of a "seed" pulse necessary for lasing to occur.

1. INTRODUCTION

The present work is a logical continuation of previous work,[1] where a system of equations was introduced to describe mode synchronization in a laser with a coherent absorber. The analytical solutions were also found in the form of soliton-like 2π pulses. The energy and width of the pulses are determined uniquely by saturation of the gain, linear losses in the cavity, and phase relaxation processes in the absorber. This property distinguishes the solutions in question from the classical 2π pulses of McCall and Hahn,[2,3] the energy and width of which are determined by the initial conditions and transient processes in the medium.

Below results are reported on the stability of the solution in the form of a 2π pulse against the onset of the continuous generation regime and against perturbations localized within the pulse itself. The transient processes in this laser system have also been analyzed.

The chief complication in achieving stable mode synchronization lies in the suppression of the continuous generation regime. For this it is necessary to ensure that the overall gain coefficient of a weak signal be negative throughout the entire frequency band at the leading and trailing edges of the pulse. This condition is hard to satisfy, since the width of a 2π pulse must satisfy a double inequality: $T_{2g} \ll \tau_p \ll T_{2p}$ (here T_{2g} and T_{2p} are the phase relaxation times of the amplifier and the absorber, respectively). The narrower the spectral profile of the absorber, the higher the density of absorbing atoms required to avoid continuous generation at the edges of the amplification profile. The rise in the absorber density increases energy losses for the 2π pulse, and when the absorber density exceeds a critical value (see Ref. 1), the mode synchronization regime is terminated. In Ref. 1 it was shown that a satisfactory intermediate absorber density can be found only when an optical filter is present which cuts off the edge of the amplification contour and thereby ensures that the 2π pulses are stabilized for low densities of the absorbing atoms.

If the interaction between the field of the pulse and the absorber is coherent, another instability mechanism can develop. This is instability with respect to small amplitude modulation. It is well known that the generation of short pulses in a laser with a slow absorber[4] depends sensitively on the magnitude of the stability parameter,[5] which is equal to the ratio of the saturation energy density of the amplifier to that of the absorber, $s_0 = \varepsilon_0 \sigma_p / \sigma_g$, where σ_g and σ_p are the amplifier and absorber cross sections, respectively; here ε_0 is the ratio of the area of the beam in the amplifier to that in the absorber. The necessary condition for lasing is that the absorber saturate faster than the amplifier, $s_0 > 1$. In the case of passive mode synchronization with a coherent absorber, the interaction between the pulse and the absorbing medium results from different mechanisms (coherent and incoherent). Now the stability parameter is determined by twice the ratio of the square of the Rabi frequency of the amplifier to that of the absorber:

$$s = \varepsilon_0 \left(2 \frac{R_g}{R_p} \right)^2, \quad R_g = \frac{2d_g}{\hbar} \mathscr{E}, \quad R_p = \frac{2d_p}{\hbar} \mathscr{E}. \tag{1}$$

For $\mathscr{E}$ here we can take the amplitude of the pulse; this has no great significance, since the ratio of the Rabi frequencies enters into (1) and the field cancels out. Everywhere in what follows we will assume that the parameter ε_0 is equal to unity, and when it is necessary to take into account the difference in the areas of the beam in the amplifier and in the absorber a simple renormalization of the dipole moments can be carried out. Stable production of self-induced transparency (SIT) pulses requires that the condition $s \geqslant 1$ hold; in the opposite limit the amplitude modulation destroys the time-independent form of the pulse. In the regime in which 2π pulses are generated the parameter s plays the role of the saturation coefficient of the amplifier, and so directly affects the propagation speed of the pulse. Increasing s speeds up the saturation of the amplification profile, and consequently causes the leading edge of the pulse to grow preferentially. Then the trailing edge experiences substantially less amplification. Ultimately the propagation speed of the pulse increases due to a shift in the center of mass of the pulse forward along the direction of propagation. The effect of linear dispersion in the absorbing medium does not depend on the magnitude of s, but instead causes the velocity to decrease. For $s=1$ the two competing processes cancel out. For a 2π pulse to propagate stably it is necessary that the amplifier as a whole accelerate the pulse, i.e., $s\geqslant 1$. Then the conditions for a perturbation developing on the trailing edge of the 2π pulse to grow exponentially no longer hold.

We have shown[1] that for $\tau_p \ll T_{2p}$ the formation of the pulse takes place primarily under the influence of the absorber. We use this fact in treating the transient processes by which a steady 2π pulse is established. It will be shown that the number of passes required for the pulse to evolve to a steady state due to coherent processes in the absorber is much less than the number required for a steady value of the energy to develop on account of the action of the amplifier. As the energy changes the shape of the pulse is adjusted almost instantaneously in response to these variations. This separation of processes into fast and slow allows us to obtain an analytical estimate for the threshold conditions associated with the onset of lasing and to discuss the basic laws governing this transition. Depending on the density of the absorbing atoms in the cavity, there is a change in the threshold value of the energy of a "seed" pulse required to initiate lasing and the transition to the steady 2π-pulse state.

The material in this paper is organized as follows. In Sec. 2 the basic system of model equations is presented without derivation; the solution derived from them is the SIT pulse (the derivation is given in Ref. 1). Section 3 is devoted to studying the energetic stability of 2π pulses and analyzing the properties of the transient stage. In Sec. 4 we derive the stability conditions for 2π pulses against fast perturbations localized within the pulse itself. In Sec. 5 the stability of the mode synchronization regime is studied with respect to the onset of continuous generation.

2. MODEL EQUATIONS AND SOLUTIONS IN THE FORM OF SIT PULSES

A pulse propagating inside the cavity passes successively through the amplifier and the absorber and experiences losses at the mirrors. If the relative changes in the field are small in traversing each of the cavity elements, then we can look for a solution of the problem in the form of a stationary pulse that depends on the wave coordinate $u=(t-z/v_p)/\tau_p$, where v_p and τ_p are the velocity and width of the pulse. The amplifying medium is assumed to be slow, i.e., the pulse width τ_p satisfies the inequality $T_{2g}\ll\tau_p\ll T_{1g}$ (here T_{2g} and T_{1g} are the transverse and longitudinal relaxation times of the amplifier). It is assumed that the populations of the amplifying and absorbing media reach their steady values in the time required for a pulse to traverse the cavity. We will also assume that the profiles of both media are homogeneously broadened and that they have the same center.

The evolution of the field is described by the wave equation in the approximation of slowly varying phases and amplitudes. The field is divided into real amplitude and phase, while the polarization is divided into synphase and quadrature parts:

$$\left\{\left(\frac{\partial}{\partial z}-\frac{v_p^{-1}-c^{-1}}{\tau_p}\frac{\partial}{\partial u}\right)+L_{BW}\right\}E=-\frac{L_g}{T_{2g}}Q_g - \frac{L_p}{T_{2p}}\mu^{-1}Q_p, \tag{2a}$$

$$E\left(\frac{\partial}{\partial z}-\frac{v_p^{-1}-c^{-1}}{\tau_p}\frac{\partial}{\partial u}\right)\varphi=\frac{L_g}{T_{2g}}P_g+\frac{L_p}{T_{2p}}\mu^{-1}P_p. \tag{2b}$$

The total field and polarization take the form

$$\mathscr{E}(z,t)=\frac{\hbar}{d_g}E(u,z)\exp\{i[\varphi(z,u)-(\omega_0 t-kz)]\},$$

$$\mathscr{P}(z,t)=\{d_g n_g[P_g(z,u)+iQ_g(z,u)]+d_p n_p[P_p(z,u)+iQ_p(z,u)]\}\exp\{i[\varphi(z,u)-(\omega_0 t-kz)]\}. \tag{3}$$

Here ω_0 represents the central frequency of the amplifier and absorber transmissions; d_g and d_p are the transition dipole moments of the amplifier and absorber; and $L_{BW}(\omega)$ is the frequency-dependent linear loss introduced by the optical filter. We have also written

$$L_g=\frac{2\pi\omega_0 d_g^2 n_g}{c\hbar}T_{2g},\quad L_p=\frac{2\pi\omega_0 d_p^2 n_p}{c\hbar}T_{2p},\quad \mu=\frac{d_p}{d_g}.$$

We write down the transmission function L_{BW} of the filter in the time representation, assuming that it is centered on the frequency ω_0 and has a parabolic dependence on the frequency ω:

$$L_{BW}(t)=L_{cav}\left[1-\frac{1}{\Delta\omega_p^2}\frac{\partial^2}{\partial t^2}\right]. \tag{4}$$

The polarization of the absorber is described by the system of Bloch equation, in which processes of spontaneous inversion relaxation are not treated, in accordance with the specified model.

$$\frac{\partial}{\partial u}P_p=\frac{\partial}{\partial u}\varphi Q_p-P_p/(T\kappa),$$

$$\frac{\partial}{\partial u}Q_p=-\frac{\partial}{\partial u}\varphi P_p-Q_p/(T\kappa)-\tau_p\mu E N_p,$$

$$\frac{\partial}{\partial u}N_p=\tau_p\mu E Q_p. \tag{5}$$

Here we have written

$$T=T_{2g}/\tau_p,\quad \kappa=T_{2p}/T_{2g}. \tag{6}$$

The Bloch equations for an amplifier can be simplified if we use the small parameter T, i.e., if we assume that the pulse spectrum is much narrower than the spectrum of the amplification line. The expressions for the components of the Bloch vector are found in Ref. 1:

$$N_g(z,u)=1-T_{2g}J(z,u)+\frac{1}{2}T_{2g}^2[J^2(z,u)+E^2(z,u)]$$
$$-T_{2g}^3\left[\frac{1}{6}J^3(z,u)+\int_{-\infty}^{t}E^4(z,u)d\bar{t}\right.$$
$$\left.-\int_{-\infty}^{t}\left(\frac{\partial E(z,u)}{\partial \bar{t}}\right)^2 d\bar{t}\right], \quad (7)$$

$$\frac{1}{T_{2g}}Q_g(z,u)=-E+T\left[\frac{\partial}{\partial u}E+\tau_p JE\right]-T^2\left[\frac{1}{2}\tau_p^2J^2E\right.$$
$$+\frac{3}{2}\tau_p^2E^3+\tau_pJ\frac{\partial}{\partial u}E+\frac{\partial^2}{\partial u^2}E-E$$
$$\left.\times\left(\frac{\partial\varphi}{\partial u}\right)^2\right], \quad (8)$$

$$\frac{1}{T_{2g}}P_g(z,u)=-TE\frac{\partial\varphi}{\partial u}+T^2\left[\tau_pJE\frac{\partial\varphi}{\partial u}+E\frac{\partial^2\varphi}{\partial u^2}\right.$$
$$\left.+2\frac{\partial E}{\partial u}\frac{\partial\varphi}{\partial u}\right]. \quad (9)$$

Here

$$J(u,z)\equiv\int_{-\infty}^{t}E^2(\bar{t},z)d\bar{t}=\tau_p\int_{-\infty}^{u}E^2(\bar{u},z)d\bar{u} \quad (10)$$

is the energy which passes through the amplifier at a specified time t. In expressions (7)–(9) we have taken the initial condition for inversion:

$$N_g(u=-\infty,z)=+1.$$

In Ref. 1 an analytical solution of Eqs. (2) and (5) together with (8) and (9) was found, taking into account the terms in the expansion of the amplifier polarization through T^1. Since the line centers of the two media are the same, the steady solution in the form of a pulse is also centered at the frequency ω_0 and has no phase modulation:

$$E_0(u)=A_0\mathrm{sech}u. \quad (11)$$

The pulse parameters A_0, τ_p, and v_p satisfy the relations

$$(A_0\tau_p)^2=s, \quad (12)$$

$$\frac{c}{v_p}=1+\frac{3(\Omega_pT_{2p})^2}{3(\kappa T)^2+4\kappa T+1}+(1-s)(\Omega_gT_{2g})^2, \quad (13)$$

where

$$\Omega_p^2=\frac{2\pi\omega_0d_p^2n_p}{\hbar},\quad \Omega_g^2=\frac{2\pi\omega_0d_g^2n_g}{\hbar}$$

are the squares of the cooperative absorber and amplifier frequencies. Here

$$\left[\eta-\frac{1}{2}(T_{2g}J_0)\right](3T^2+4\kappa^{-1}T+\kappa^{-2})=\rho\kappa^{-2},$$

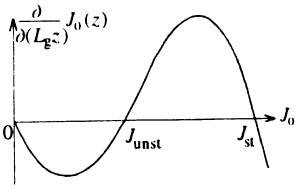

FIG. 1. Schematic representation of the evolution of the energy of a "seed" pulse toward its stationary value $J=0$ or $J=J_{st}$, depending on the initial value of the energy.

$$\eta=\frac{L_g-L_{cav}}{L_g},\quad \rho=L_p/L_g, \quad (14)$$

$$J_0=\int_{-\infty}^{\infty}E^2dt=J(+\infty)=2A_0^2\tau_p \quad (15)$$

is the total energy of the pulse. Condition (12) implies that the area of the pulse in the absorber does not depend on the parameters of the problem and is equal to 2π. Equation (13) determines the velocity of the pulse. Equation (14) determines the energy (and hence the width) of the pulse. The behavior of T as a function of ρ is shown in Fig. 1 of Ref. 1. The analysis of Eq. (14) simplifies if we consider sufficiently short pulses: $\tau_p\ll T_{2p}$. For $\rho=0$ the quantity T assumes its maximum value $T_{max}=\eta/s$; as ρ increases T falls monotonically and reaches its minimum value $T_{min}=(2/3)\eta/s$ at $\rho=\rho_{cr}=\eta[(2/3)\eta(\kappa/s)]^2$. Equation (14) has a second solution for T that grows as a function of ρ. This solution is unphysical and we will show below that it is unstable.

In the Introduction we indicated that for a 2π pulse to be stable against the onset of the continuous regime the concentration of the absorber should be greater than some value ρ_{min}. Below we show that this quantity is equal to

$$\rho_{min}=\left(\frac{1}{2}\eta\frac{\kappa}{\kappa_f}\right)^2,\quad \text{for } \kappa_f^2=1+\frac{1-\eta}{(T_{2g}\Delta\omega_f)^2}\gg1. \quad (16)$$

In Ref. 1 it was pointed out that the absorber density can be chosen so as to satisfy $\rho_{min}<\rho<\rho_{cr}$ only if an optical filter is present: $\kappa_f^2>1$.

3. ANALYSIS OF THE TRANSITIONAL EVOLUTION

We have already noted that the solution of Eq. (14) for the pulse width is not unique; this is reflected in the presence of two branches of $T(\rho)$ in Fig. 1 of Ref. 1. One solution corresponds to increasing power as a function of the absorber density and the other to decreasing power. We show that the first of these is unstable. Integrating Eq. (2a) with respect to time from minus to plus infinity, we find an equation for the energy of a pulse propagated inside the cavity:

$$\frac{\partial}{\partial(L_gz)}J_0(z)=(\eta-1)\left[J_0(z)+\frac{1}{\Delta\omega_f^2}\int_{-\infty}^{\infty}\left(\frac{\partial E}{\partial t}\right)^2dt\right]$$

$$-T_{2g}^{-1}N_g(z)\Big|_{-\infty}^{\infty}-\frac{\rho}{\mu^2T_{2p}}N_p(z)\Big|_{-\infty}^{\infty}. \tag{17}$$

In deriving Eq. (17) we have assumed that the inversion of the absorber and the amplifier and the field E have an additional dependence on the longitudinal position z. The difficulty in finding an analytical solution of Eq. (17) is associated with the presence of information about the changes in the pulse shape during the transitional process. We used the fact, noted earlier, that the absorber plays the dominant role in the formation of the pulse shape. If the pulse is originally launched into the laser cavity with a smooth envelope and width $\tau_p<T_{2p}$, then after traversing a distance equal to several linear absorption lengths $L_pz\approx 2$–3 it acquires the shape of a hyperbolic secant and its area becomes equal to 2π (Ref. 3). This means that the pulse shape and its area become established after $N=(2-3)/(L_pl_{cav})$ passes through the cavity [we assume that all the absorption (gain) coefficients L_p, L_g, L_{cav} for weak signals are scaled by the cavity length l_{cav}]. On the other hand, in order to get an idea about the number of traversals of the cavity required to allow the energy to reach a steady value we use the following estimate:

$$M\approx[(L_g-L_{cav})l_{cav}]^{-1}.$$

If the ratio

$$\frac{M}{N}>\eta\frac{\kappa^2}{\kappa_f^2}$$

is large (here we have used $L_p/L_g=\rho_{min}$), the transient process can be clearly divided into two stages. The first stage is related to the conversion of the original field profile into a 2π pulse with losses of a small part of the energy. The evolution of the pulse in the second, longer stage is due to the action of the amplifying medium and is associated with the gradual increase in the energy of the 2π pulse to its time-independent value. The role of the absorber in the second stage reduces to keeping the area of the pulse and the shape of the envelope unchanged, and also to the adiabatic tracking of the changes in the field due to the amplifier processes.

Using this hierarchy of the processes by which a time-independent pulse shape develops we can study Eq. (17) analytically. For the field $E(u,z)$ we assume a pulse with amplitude, velocity, and width which depend on the longitudinal position z:

$$E(u,z)=A_0(z)\operatorname{sech}\frac{t-z/v_p(z)}{\tau_p(z)},$$

$$A_0(z)\tau_p(z)=\sqrt{s},\quad \frac{c}{v_p}=1+(\Omega_p\tau_p)^2. \tag{18}$$

Substituting the solution in the form (18) into Eq. (17) and using the condition that the area under the envelope is constant, we obtain the desired equation for the evolution of the energy:

$$\frac{\partial}{\partial(L_gz)}J_0(z)=J_0(z)\left\{\eta-\frac{1}{2}T_{2g}J_0(z)\right.$$

$$-\frac{\kappa_f^2-2s-2s^2}{12s^2}[T_{2g}J_0(z)]^2$$

$$\left.-\frac{4s^2}{3\kappa^2}\rho[T_{2g}J_0(z)]^{-2}\right\}. \tag{19}$$

In deriving Eq. (19) we have used the following expressions; for their derivation see Ref. 1:

$$N_p\Big|_{-\infty}^{\infty}=\frac{8}{3}(\kappa T)^{-1}, \tag{20}$$

$$N_g\Big|_{-\infty}^{\infty}=-(T_{2g}J_0)\left\{1-\frac{1}{2}T_{2g}J_0+\frac{1}{6}(T_{2g}J_0)^2\right.$$

$$\left.+\frac{2}{3}sT^2-\frac{1}{3}T^2\right\}. \tag{21}$$

Here J_0 and T are functions of z. The important step in the derivation of the control equation (19) is representing the field in the form (18), and hence the energy losses in the first stage of the transition process, i.e., in the formation of a 2π pulse from the initial field profile, are not treated. An estimate of the size of this correction is given at the end of the section.

For real nonnegative values of $J_0(z)$ Eq. (19) determines three time-independent solutions. They are shown in graphical form in Fig. 1. The solution with largest energy J_{st} corresponds to the upper branch in Fig. 1 of Ref. 1, and the solution J_{unst} with the smallest energy corresponds to the lower branch in Fig. 1 of Ref. 1. The picture of the evolution of the pulse looks different, depending on the initial conditions. If the energy of the "seed" pulse is less than J_{unst}, then there is no lasing—after several passes through the resonator the energy of the seed pulse decreases to zero. The value $J=0$ is stable within its region of attraction: $0\leqslant J\leqslant J_{unst}$. If the original pulse energy is larger than J_{unst}, then after the transition process lasing occurs in the mode synchronization regime with energy J_{st} in a single pulse. The stable point $J=J_{st}$ has a region of attraction $J>J_{unst}$.

We have shown that the regime in which 2π pulses are stably generated is possible for a single value $J=J_{st}$ of the energy. In addition, the value J_{unst} can be regarded as an energetic threshold, i.e., a seed spike of the field must be larger than this (under the condition that the duration of the spike is shorter than the phase relaxation time T_{2p}) so that the necessary conditions for generation of 2π pulses hold.

Returning to the analysis of the first stage in the formation of SIT pulses we note that energy losses have been neglected in the first stage of the evolution toward the steady profile. In reality, as a result of the process of transition to a 2π pulse part of the energy is scattered in the absorbing medium (even in the absence of relaxation). If we know how the pulse area $\theta(z)$ changes we can estimate the "coherent" energy losses quantitatively:

$$\Delta J_{coh}(z)=\frac{\rho}{\mu^2T_{2p}}N_p^{sol}\Big|_{-\infty}^{\infty}=\frac{\rho}{\mu^2T_{2p}}$$

$$\times(1-\cos[\mu\tau_p\theta(z)]). \tag{22}$$

Here the quantity N_p^{sol} has been derived from Eq. (5) under the assumption $T_{2p}\to\infty$. Expression (22) should be added to the right-hand side of Eq. (19). Now the corrected threshold value for the seed energy of the pulse can be written

$$J_{th}=J_{unst}+\int_{-\infty}^{\infty}\Delta J_{coh}(z)dz.$$

For example, for an arriving pulse with area $2\pi<\theta_0<3\pi$ the energy losses associated with the change in the shape amount to approximately 10% of the input energy.[6] Thus we obtain the estimate

$$J_{th}\approx\frac{11}{10}J_{unst}.$$

If the shape of the input pulse is the same as that of the 2π pulse, the coherent energy losses vanish.

In conclusion, we note that in estimating the number N of passes needed to reach a steady envelope profile we have used an expression for the absorption coefficient L_p of a weak signal. However, if the width of the seed pulse satisfies

$$\tau_{in}<T_{2p},$$

then the characteristic length in the absorber increases: $(L_p\tau_{in}/T_{2p})^{-1}$. Hence the ratio M/N which we have obtained is a slight overestimate, and in more detailed calculations it should be replaced according to $L_p\to L_p\tau_{in}/T_{2p}$.

4. STABILITY OF 2π PULSES AGAINST AMPLITUDE MODULATION

The perturbations that distort the time-independent shape (11) of the pulse can be divided into two classes, slow and fast. Fast perturbations, which are located within the pulse itself, are equivalent to exponentially growing amplitude modulation of the envelope and ultimately lead to destruction of the steady shape of the pulse. Our problem is to find the range of parameters within which 2π soliton-like pulses are produced which are stable against perturbations located within the envelope.

When the centers of the amplifier, absorber, and filter lines coincide, spectrally bounded pulses of the form (17) are generated at this same frequency. Then it can easily be shown that the equations for studying phase and amplitude stability decouple and can be studied separately. This separation takes place on account of the procedure for linearizing Eqs. (2) and (4) with respect to small perturbations of the phase and amplitude. Terms of the form $(\partial\varphi/\partial u)Q$ and $(\partial\varphi/\partial u)P$ couple the amplitude and phase perturbations, but after linearization this coupling is gone.

We start the investigation with the amplitude stability and write down the field as a sum of the time-independent solution and a small correction:

$$E(u,z)=E_0(u)+E_{pert}(u)\,\exp(\lambda L_g z). \quad (23)$$

Here $E_0(u)$ is the time-independent solution in the form of a 2π pulse. Treating the field perturbation in the linear approximation enables us to look for a solution in factored form, i.e., in the form of a product $E_{pert}(u)f(z)$.[1)] The equations of the coefficient obtained in the process of linearizing Eqs. (2) and (5) evidently do not depend on z; we can seek a solution by replacing $f(z)$ with an exponential, as in Eq. (23). The quantity λ is called the growth rate of the perturbation. The time-independent solution (11) is unstable if the real part of λ assumes positive values.

We write down the linearized equations for the perturbation $E_{pert}(u)$ in the form

$$\frac{v_p^{-1}-c^{-1}}{L_g\tau_p}\ddot{e}_1-\lambda\dot{e}_1+\frac{\rho}{\kappa T}(2\,\mathrm{sech}^2u-1)e_1=(1-\eta)\dot{e}_1+\frac{Q_g(e_1)}{T_{2g}}+\frac{\rho}{\mu T_{2p}}Q_p^{loss}(e_1). \quad (24)$$

Here we have introduced the notation

$$Q_p^{loss}(e_1)=-\frac{\mu T_{2g}}{\kappa T^2}\left(1-\frac{2}{\mathrm{ch}^2u}\right)\left\{\int_{-\infty}^{u}e_1(u)du+8\int_{-\infty}^{u}\left[\frac{1}{\mathrm{ch}\,u(1-2/\mathrm{ch}^2u)^2}\times\int_{-\infty}^{u}\left(\frac{2}{\mathrm{ch}^2u}-1\right)\frac{\mathrm{sh}u}{\mathrm{ch}^2u}e_1(u)du\right]du\right\},$$

$$e_1(u)=\int_{-\infty}^{u}E_{pert}(u)du. \quad (25)$$

The expression for $Q_g(e_1)/T_{2g}$ is derived from (8) by substituting the field $E(u,z)$ in the form (23). In writing expressions (25) we have used the fact, established previously, that the right-hand side of Eq. (24) plays a secondary role in the formation of the field profile and can be treated in the next order of perturbation theory. We rewrite the left-hand side of Eq. (24) in self-adjoint form, using the relations between the parameters of the classical 2π pulse:

$$(A_0\tau_p)^2=s;\quad c/v_p=1+(\Omega_p\tau_p)^2, \quad (26)$$

$$\ddot{\mathscr{E}}_1-\left(\lambda_0^2+1-\frac{2}{\mathrm{ch}^2u}\right)\mathscr{E}_1=\frac{\kappa T}{\rho}\left\{(1-\eta)(\dot{\mathscr{E}}_1+\lambda_0\mathscr{E}_1)+\frac{Q_g(\mathscr{E}_1)}{T_{2g}}+\frac{\rho}{\mu T_{2p}}Q_p(\mathscr{E}_1^{loss})\right\}. \quad (27)$$

Here we have written

$$\mathscr{E}_1(u)=e_1(u)e^{(-\lambda_0 u)},\quad \lambda_0=\frac{\kappa T}{2\rho}\lambda.$$

In the first step we solve the problem for the eigenvalues and eigenfunctions of Eq. (27), setting the right-hand side equal to zero. The eigenvalues in the potential $U=1-2/\mathrm{ch}^2u$ satisfy the inequality $\mathrm{Re}(\lambda_0)\leq 0$. The discrete spectrum of the homogeneous equation (27) consists of the number $\lambda_{00}=0$. The eigenfunction corresponding to $\lambda_{00}=0$ is $\mathscr{E}_1(u)=\mathrm{sech}\,u$, which describes the neutral stability of a 2π pulse against longitudinal perturbation. The right-hand side of Eq. (27) is small and can give rise only to a slight change in the eigenvalue $\lambda_{00}=0$. In accordance with our perturbation treatment, we neglect terms on the right-hand side proportional to λ_0. The condition for solubility of the inhomogeneous equation

(27) is that the right-hand side be orthogonal to the solution of the homogeneous equation for the eigenvalue $\lambda_{00}=0$. Thus we find an expression for a new more accurate value of λ_0:

$$\lambda_0^2=-\frac{\kappa T}{\rho}\int_{-\infty}^{\infty}\mathscr{E}_1(u)\,\frac{Q_g(\varepsilon_1)}{T_{2g}}\,du, \tag{28}$$

and finally obtain

$$\lambda_0^2=-\frac{2}{3}\frac{\kappa T^2}{\rho}(s-1). \tag{29}$$

In deriving (29) we have restricted consideration to the first two terms in the expansion $Q_g(\mathscr{E}_1)$ [Eq. (8)]. For $s<1$ the quadratic equation has two real roots which differ in absolute value and have opposite signs. The perturbation correction to the field of the time-independent pulse has its own characteristic values of the pulse width and velocity. They can be determined by analyzing the behavior of the increase in the leading and trailing edges of the perturbation:

$$\pm\exp(-u)\exp(\lambda_0 u)\exp\left\{\frac{2\rho}{\kappa T}\lambda_0 L_g z\right\}$$

$$=\pm\exp\left\{-\frac{t-z/v_{\text{pert}}}{\tau_{\text{pert}}}\right\},$$

$$\tau_{\text{pert}}=\tau_p(1+\lambda_0),\quad v_{\text{pert}}=v_p\left(1-2\lambda_0\left(1-\frac{v_p}{c}\right)\right). \tag{30}$$

For $\lambda_0>0$ the localized perturbation moves with a smaller velocity than the original pulse, $v_{\text{pert}}<v_p$, and grows exponentially in amplitude. Ultimately the trailing edge of the pulse is deformed and the time-independent shape of the field is destroyed. For $\lambda_0<0$ the perturbation leads the main pulse and causes it to damp at the leading edge. For $s>1$ there are two complex conjugate purely imaginary eigenvalues. In this case the perturbation pulsates, with the velocity and pulse length changing periodically about their mean values v_p and τ_p. The amplitude of a local perturbation also oscillates about its original value, first growing and then decaying. In summary, for $s>1$ small changes in the parameters of a 2π pulse do not destroy its shape.

The explicit dependence of the stability of a 2π pulse on the magnitude of the parameter s is a consequence of the role of s as the saturation coefficient of the amplifier. Increasing s causes rapid saturation of the gain profile and hence preferential growth of the leading edge of the pulse. Hence the trailing edge suffers and is amplified less. Then the center of mass of the pulse shifts forward and its velocity increases [see Eq. (13)]. This is the explanation for the suppression of instability growth when the remaining amplification becomes insufficient for exponential growth of the perturbation at the trailing edge of the pulse. The competition between the two processes (delay of the pulse due to linear dispersion of the group velocity[7] and acceleration due to the saturation effect) determines the stability criterion. Stability of 2π pulses results if the latter process dominates and the action of the amplifier ultimately results in acceleration of the pulses, i.e., $s\geqslant 1$.

Now we turn to consideration of the phase stability. The corresponding field perturbations will be sought in the form

$$\mathscr{E}_{\text{ph}}=\Phi(u)\exp(\lambda L_g z)=\varphi(u)E(u)\exp(\lambda L_g z). \tag{31}$$

Linearizing the equation for the perturbation yields

$$\ddot{\Phi}-2\lambda_0\dot{\Phi}+\left(\frac{2}{\text{ch}^2 u}-1\right)\Phi$$

$$=\frac{T}{\rho}\left\{\left(1-\eta+T_{2g}^{-1}\frac{Q_g(E_0)}{E_0}\right)(\dot{\Phi}+\Phi\,\text{th}u)-T_{2g}^{-1}\dot{P}_g(\Phi)\right.$$

$$\left.+\frac{1}{\kappa T}[\lambda\Phi-\rho T^{-1}(\dot{\Phi}+\Phi\,\text{th}u)-T_{2g}^{-1}P_g(\Phi)]\right\},$$

$$\lambda_0=\frac{1}{2}\frac{T}{\rho}\lambda. \tag{32}$$

Here $Q_g(E_0)$ is determined by Eq. (8) and the function $P_g(\Phi)$ can be written

$$\frac{1}{T_{2g}}P_g(\Phi)=[T-T^2 s(1+\text{th}u)](\dot{\Phi}+\Phi\,\text{th}u)-T^2$$

$$\times\left[\left(\frac{2}{\text{ch}^2 u}-1\right)\Phi+\ddot{\Phi}\right]. \tag{33}$$

In deriving Eqs. (32) and (33) we have used the relation between the parameters (26) for the steady solution. The substitution

$$\Phi(u)=\Psi(u)\exp(\lambda_0 u) \tag{34}$$

reduces the left-hand side of Eq. (32) to self-adjoint form:

$$\ddot{\Psi}-\left(\lambda_0^2+1-\frac{2}{\text{ch}^2 u}\right)\Psi$$

$$=\frac{T}{\rho}\left\{\left(1-\eta+T_{2g}^{-1}\frac{Q_g(E_0)}{E_0}\right)(\dot{\Psi}+\Psi\,\text{th}u)-T_{2g}^{-1}\dot{P}_g(\Phi)\right.$$

$$\left.-\frac{1}{\kappa T}[\rho T^{-1}(\dot{\Psi}+\Psi\,\text{th}u)+T_{2g}^{-1}P_g(\Phi)]\right\}. \tag{35}$$

Equation (35) is similar in form to Eq. (27), which we have already solved. The discrete spectrum of the homogeneous equation consists of the eigenvalue $\lambda_{00}=0$ and the corresponding eigenfunction $\Psi=\text{sech}\,u$. The correction to λ_{00} is found from the condition that the right-hand side be orthogonal to the solution of the homogeneous equation. It is found that the correction vanishes and the pulse remains neutrally stable against various perturbations. That is, for a 2π pulse the location of the zero of phase makes no difference.

5. STABILITY OF 2π PULSES AGAINST SLOW PERTURBATIONS

Of the special importance in the operation of a laser in the passive mode-synchronization regime is the task of ensuring that 2π pulses are stable against the onset of continuous generation. This is related to the fact that the total gain coefficient of a weak signal in the cavity should be negative over the entire frequency band, and the narrower the absorber line the larger its density must be in order to achieve this. Below we find the minimum value of the absorber density at which the threshold for the onset of the single-mode regime is nowhere reached over the entire amplification profile.

The problem reduces to the investigation of the amplification of small field fluctuations occurring in that part of the cavity where there is no pulse at a particular time. That is, from the formal standpoint we will concern ourselves with verifying the stability of the trivial solution. The system of equations (2), (5) is linearized about the value $E=0$, and we find the condition for a small field perturbation in the cavity to be damped:

$$L_{\text{cav}}\left[1+\left(\frac{\Delta}{T_{2g}\Delta\omega_f}\right)^2\right]+\frac{L_p}{1+\kappa^2\Delta^2}-\frac{L_g}{1+\Delta^2}>0,$$
$$\Delta=\delta T_{2g}. \tag{36}$$

In deriving (36) we have assumed that the centers of the amplifier, absorber, and filter lines coincide; here δ denotes the magnitude of the mismatch in the carrier frequency of the pulse with respect to the line centers. It is necessary that inequalities (36) hold for all values of the mismatch in order that 2π pulses be stable. There is a mismatch $\Delta^2=\Delta^2_{\max}$, depending on the relationship with the parameters, for which the expression on the left-hand side of (36) takes on the largest value. Thus, it suffices to verify that the inequality (36) holds for $\Delta^2_{\max}$, and then it will certainly hold for all the other values. Introducing the quantity κ_f^2 from Eq. (16) and setting $\kappa_f^2\gg 1$ we find $\Delta^2_{\max}$:

$$\Delta^2_{\max}=\frac{\sqrt{\rho}}{\kappa\kappa_f}. \tag{37}$$

Substituting (37) into the inequality (36), we find an expression for the minimum absorber density:

$$\rho\geqslant\rho_{\min}=\left(\frac{1}{2}\,\eta\,\frac{\kappa}{\kappa_f}\right)^2. \tag{38}$$

Note that if the spectral width of the filter is large, i.e., $\kappa_f^2\approx 1$, then the expression for $\Delta^2_{\max}$ will change:

$$\Delta^2_{\max}=\frac{\sqrt{\rho\kappa^2}-1}{\kappa^2-\sqrt{\rho\kappa^2}}\quad\text{for } 1<\sqrt{\rho\kappa^2}<\kappa^2, \tag{39}$$

and consequently the lower bound for ρ will change:

$$\rho\geqslant\kappa^2(1-\sqrt{1-\eta}). \tag{40}$$

The condition $\kappa_f^2\approx 1$ means that we can disregard the finite spectral width of the linear losses. But in Ref. 1 we showed that when the filter is too wide, 2π pulses cannot be generated because of the upper critical value (ρ_{cr}) for the absorber density. The physical interpretation of this restriction is very simple: the lower bound on ρ rises so far that the amplification in the laser becomes insufficient to overcome the increased losses of the field. Consequently, the filter in this model fills an important role, overcoming the onset of continuous generation at the edges of the amplification profile and thereby lowering the threshold $\rho_{\min}$.

We have treated the conditions necessary to suppress slow fluctuations at the leading edge of a pulse, i.e., when the difference in the populations of the absorbing and amplifying media agree with their time-independent values. After the pulse has passed the states of the media at its trailing edge change: the interaction with the field removes the inversion of the amplifier, while some of the atoms of the absorber are found in the upper excited state. The characteristic feature of a 2π pulse (after passing through it leaves the absorbing media in the ground state) is slightly transformed in this model. Because of the relaxation processes part of the energy of the pulse goes into exciting the medium, which causes the populations N_p of the absorber to differ from unity by the small quantity $\Delta N_p=8/3(\kappa T)^{-1}\ll 1$, [Eq. (20)] at the trailing edge of a 2π pulse. Then the inversion of the amplifier is reduced by ΔN_g, given by Eq. (21):

$$\Delta N_g=T_{2g}J_0\left(1-\frac{1}{2}\,T_{2g}J_0\right). \tag{41}$$

Now we can write down the condition for a small perturbation at the trailing edge of the pulse to be damped:

$$L_{\text{cav}}\left[1+\left(\frac{\Delta}{T_{2g}\Delta\omega_f}\right)^2\right]+\frac{L_p(1-\Delta N_p)}{1+\kappa^2\Delta^2}-\frac{L_g(1-\Delta N_g)}{1+\Delta^2}>0. \tag{42}$$

In Eq. (42) the symbols L_g and L_p refer to their time-independent values, which have not been perturbed by the passage of the pulse. The signs in front of the corrections ΔN_g and ΔN_p to the time-independent values are chosen so that the corrections themselves are positive. The inequality (42) is satisfied for all values of pumps when condition (38) holds. We show this, dividing the proof into two parts. For pumps which are not too large ($\eta\leqslant 1/2$) we can estimate the quantity ΔN_g by using the solution (14) for T: $\Delta N_g\geqslant\eta$. The amplification is found to be below the level of the linear losses, i.e., inequality (42) is satisfied regardless of the value of the absorber density. If the pump is sufficiently strong, $\eta\geqslant 1/2$, so that $\Delta N_g<\eta$ holds, we can write the following condition:

$$1-\frac{\Delta N_g}{\eta}\leqslant\sqrt{\left(1+\frac{\Delta\kappa_f^2}{\kappa_f^2}\right)(1-\Delta N_g)(1-\Delta N_p)},$$
$$\Delta\kappa_f^2=\frac{\Delta N_g}{(T_{2g}\Delta\omega_f)^2}. \tag{43}$$

When it holds the threshold which is determined by inequality (42) lies below $\rho_{\min}$ [see Eq. (38)]. In order to prove (43) we use the energy conservation law (17) for the steady state: $\partial/\partial z=0$, substituting the expression for $\rho_{\min}$ in place of ρ, and find $\Delta N_g\gg\Delta N_p$. If the small quantity ΔN_p on the right-hand side of (3) is replaced by the larger one ΔN_g, the inequality becomes stronger:

$$1-\frac{\Delta N_g}{\eta}\leqslant(1-\Delta N_g)\sqrt{1+\frac{\Delta\kappa_f^2}{\kappa_f^2}}, \tag{44}$$

which always holds. Thus, we arrive at the conclusion that the inequality (42) is a weaker condition than (36), and the lower bound $\rho_{\min}$ on the absorber density is not exceeded [see Eq. (38)].

In this section we have analyzed the stability of a 2π pulse against the onset of continuous generation at the leading and trailing edges of the pulse. The stability criterion is expressed by the inequality (38). We have also shown that if the condition for the suppression of a small field fluctuation holds at the leading edge of the pulse, then the same condition is automatically satisfied at the trailing edge of the pulse also. However, the approach developed in the foregoing discussion does not include processes by which the time-independent values of the absorption and amplification coefficients are established, since in the relevant equations (5) and (7) for the inversion we have set $T_{1p}, T_{1g} \to \infty$. In the real case the finite time for T_{1p} and T_{1g} to be reached causes the coefficients in inequality (42) to be time-dependent:

$$\Delta N_g \exp(-t/T_{1g}) \text{ and } \Delta N_p \exp(-t/T_{1p}).$$

We will not dwell in detail on the changes introduced by relaxation effects in the expression for the threshold density of the absorber. We note only that for $T_{1g} > T_{1p}$ the value of the threshold does not change. In the opposite case $T_{1g} < T_{1p}$, after the pulse has passed, a temporal window can develop within which conditions hold for a weak signal to be amplified. It remains an open question as to how this affects the generation of 2π pulses.

6. DISCUSSION

The present work, together with Ref. 1, constitutes a complete study of the basic processes by which laser radiation interacts with a wide-band amplifier and coherent absorber inside the cavity. On the basis of these calculations we can make specific recommendations in order to experimentally produce self-induced transparency pulses. The most promising possibilities for stable mode synchronization appear in connection with the use of solid-state active and passive media. Aside from the obvious advantages associated with the developments of these materials as radiation sources for fiber-optic communication systems, there is one additional important factor. It is well known that the coherent interaction of the strong field of a pulse with an absorber results in beam instability against transverse perturbations,[8–10] which ultimately leads to self-focusing of the beam as a whole or causes it to break up into filaments.[11–14] The instability mechanism is related to the dependence of the beam velocity on the field strength, which causes the outside of the beam to lag behind the central part. Consequently, the depth to which a pulse penetrates into a medium without undergoing substantial distortion is limited to $(10–20)L_p^{-1} l_p$ (here l_p is the length of the absorber cell). It has recently been shown[15,16] that this instability can be eliminated by making the transverse field profile of the pulse consistent with the transverse density profile of the absorbing atoms. Making the velocity with which the field propagates the same in all parts of the beam suppresses the instability.[17] In fact, there has been discussion about creating a new structure, a resonant optical waveguide. Its technological implementation is directly related to the technology for preparing ordinary optical waveguides with various resonant impurities (e.g., with erbium ions). The key requirement, which is determined by the coherent effects of propagation, is that the density of the absorbing atoms have the right profile.

There are bright prospects for employing optical waveguides with resonant impurities to study coherent phenomena: the propagation of π pulses in an amplifier and 2π pulses in an absorber, which have been demonstrated experimentally.[18,19] References 18 and 19 observed all of the manifestations of SIT in their "pure form," uncomplicated by hard-to-control spatial variations.

In Ref. 20 a program for creating a source of SIT solitons was proposed, using as an active medium a length of fiber with erbium ions at room temperature and as an absorber a section of waveguide with erbium impurities at 4.2 K.

The reasons given above are convincing evidence that it makes sense to shift the main thrust of research toward solid media in which it is relatively easy to create the conditions for suppressing transverse instability mechanisms. Here questions immediately arise which require attention: the effect of the nonlinearity in the index of refraction and group-velocity dispersion on the processes by which SIT solitons form; the effect of the profile of the index of refraction of nonresonant atoms on the conditions that the transverse field and absorbing atom densities have consistent profiles; the choice of optimum parameters for the absorber, amplifier, and optical filter, from the standpoint of obtaining the shortest possible light pulses. Continuation of these studies will lead to development of a new mechanism for forming solitons, distinct from the Schrödinger mechanism.

[1)]It is obvious that small corrections to the solution $E_0(u)$ which arise due to the action of amplification and absorption processes (found in the linear approximation) can in no way effect this perturbation theory. As a result of the linearization procedure the terms containing the corrections vanish identically.

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