Excitation of the active medium of a micromaser by light from a sub-Poissonian optical laser

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The operation of a micromaser under conditions such that injected active atoms are excited in the last stage by light from an auxiliary sub-Poissonian laser has been analyzed theoretically. It has been shown that special choice of the physical parameters permits the realization of two limiting regimes: totally random excitation in which the quantum properties of the exciting light are not manifested in any way, and strictly regular excitation in which the always present photon noise of the laser becomes insignificant. Here it is important that the noise spectrum of the exciting light is inhomogeneous, since its low-frequency portion is suppressed. This light stimulates occupation of the upper maser level, also with suppression of the low-frequency noise. This is sufficient for the laser output to become sub-Poissonian, as when population fluctuations are totally absent.

1. INTRODUCTION

The working medium of a micromaser has the form of an atomic beam, which is gradually excited as it approaches the micromaser cavity. Here we assume that the last step, after which the atoms are in the upper maser level, is effected by the light from an auxiliary sub-Poissonian laser. This guarantees that elements of regularity, which, we recall, cause effective sub-Poissonian lasing in the case of optical lasers, are introduced into the atomic excitation system.[1] A decisive role was played here by the fact that the shot noise for the excitation of the atoms was suppressed at low frequencies, distinguishing this problem from the problems previously solved,[2] in which the noise spectrum for excitation of the working medium was assumed to be homogeneous.

The stationary states of the intracavity field oscillator of a micromaser may be classical with Poissonian photon statistics or quantum with sub-Poissonian statistics.[3] Here we shall endeavor to ascertain how the statistical properties of micromaser radiation are transformed in the specific mode that we have selected to excite the active medium when the physical parameters of the exciting and excited systems vary over a broad range.

2. PHYSICAL MODEL

Figure 1 is a schematic representation of the physical situation we wish to discuss here. A random atomic beam moves toward the micromaser cavity, being gradually excited to all the higher energy levels in a random manner. In the last stage, right before the entrance, the beam passes through the intracavity space of an auxiliary sub-Poissonian laser with resultant excitation of the atoms to the upper maser level to create the population inversion needed to overcome the threshold.

We assume that sub-Poissonian lasing at a frequency ω_L is realized in the context of the same model as in Ref. 4. The active atoms have the energy structure shown on the left in Fig. 2. Regular pumping to the $|2\rangle_L$ state from the $|0\rangle_L$ state takes place at a rate r_L. The maser atoms randomly enter the intracavity space at a mean rate r_M and are then in the $|3\rangle_M$ state (on the right in Fig. 2). Interacting with the laser field, they make a transition to the upper laser level $|2\rangle_M$ and enter the microcavity.

We shall consider the optical laser and the micromaser theoretically as a single compound system with a compound cavity and a two-component medium (one in the atomic beam, which interacts both with the micromaser output and with the optical laser output, and another in the optical cavity, which interacts only with the optical laser output). A similar approach makes it possible to automatically introduce into the discussion all the details of the statistical picture of the exciting light which are characteristic of the emission of the sub-Poissonian laser. For example, previously, when the exciting light taken to be purely external,[2] it was possible to ignore the inhomogeneous course of the spectral line of the noise of the exciting light. Here we have exciting light which is created by an assigned physical system. In particular, it is known[4] that its spectral components at zero frequency are suppressed to a considerable degree, and this, as we shall see, is decisive in shaping the statistical properties of the maser radiation.

The following relation may be written for the two-mode density matrix describing both the output field of the optical laser and the output field of the micromaser:

$$\dot{\rho}=(\dot{\rho})_L+(\dot{\rho})_M+(\dot{\rho})_D. \tag{1}$$

It indicates that the two-mode output field density matrix varies with time due to interactions with the laser medium (L) and the micromaser medium (M) and due to damping of the field from the cavities of both types (D).

The expression for $(\dot{\rho})_L$ can be taken from our earlier work:[4]

$$(\dot{\rho})_L=r_L\left(\hat{L}-\frac{1}{2}\hat{L}^2\right)\rho, \tag{2}$$

where the operator $\hat{L}$ has the following form

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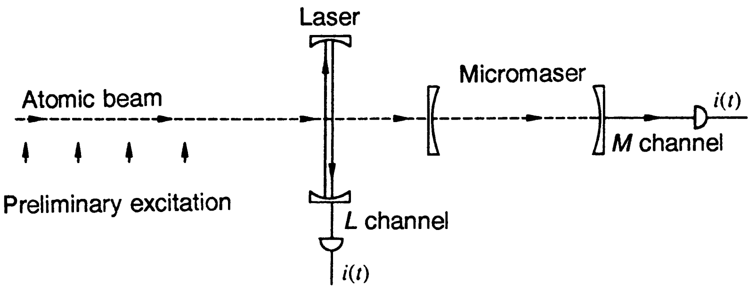

FIG. 1.

$$\hat{L}=2\underset{\rightarrow}{a_L^+}\underset{\leftarrow}{a_L}\hat{Q}^{-1}-1, \tag{3}$$

$$\hat{Q}=[\underset{\rightarrow}{a_L}\underset{\rightarrow}{a_L^+}+\underset{\leftarrow}{a_L}\underset{\leftarrow}{a_L^+}]+\frac{1}{2}\beta_{\mathrm{L}}[\underset{\rightarrow}{a_L}\underset{\rightarrow}{a_L^+}-\underset{\leftarrow}{a_L}\underset{\leftarrow}{a_L^+}]^2.$$

Here r_{L} is the mean excitation rate of the upper laser level; a_{L} and a_{L}^+ are the photon operators for the laser mode ($[a_{\mathrm{L}},a_{\mathrm{L}}^+]=1$); and β_{L} is the saturation parameter of the atomic transition of the optical laser. In Eq. (2) the operator $1/2\hat{L}^2$ appears under the condition that regular pumping of the upper laser level is provided. If the pumping is random, this term is simply absent. The arrows under the operators specify the direction from which they act on the expressions following them.

The damping terms are represented in the form

$$(\dot{\rho})_{\mathrm{D}}=-\gamma_{\mathrm{L}}\hat{R}_{\mathrm{L}}\rho-\gamma_{\mathrm{M}}\hat{R}_{\mathrm{M}}\rho, \tag{4}$$

where

$$\hat{R}_{\mathrm{L}}=\underset{\rightarrow}{a_L}\underset{\leftarrow}{a_L^+}-\frac{1}{2}[\underset{\rightarrow}{a_L^+}\underset{\rightarrow}{a_L}+\underset{\leftarrow}{a_L^+}\underset{\leftarrow}{a_L}] \tag{5}$$

and

$$\hat{R}_{\mathrm{M}}=(1+n_{\mathrm{b}})\left[\underset{\rightarrow}{a_M}\underset{\leftarrow}{a_M^+}-\frac{1}{2}[\underset{\rightarrow}{a_M^+}\underset{\rightarrow}{a_M}+\underset{\leftarrow}{a_M^+}\underset{\leftarrow}{a_M}]\right]$$
$$+n_{\mathrm{b}}\left[\underset{\rightarrow}{a_M^+}\underset{\leftarrow}{a_M}-\frac{1}{2}[\underset{\rightarrow}{a_M}\underset{\rightarrow}{a_M^+}+\underset{\leftarrow}{a_M^+}\underset{\leftarrow}{a_M}]\right], \tag{6}$$

γ_{L}^{-1} and γ_{M}^{-1} are the lifetimes of the photons in the laser and maser cavities due to their finite Q factors, a_{M} and a_{M}^+ are the maser photon operators $[a_{\mathrm{M}},a_{\mathrm{M}}^+]=1$, and n_{b} takes into account the temperature phenomena in the microcavity.

3. MASTER KINETIC EQUATION

To write a master kinetic equation, we must calculate $(\dot{\rho})_M$. It appears as a result of interactions with maser atoms which were already excited by the field of the optical laser. We follow the thinking in the work of Scully and Lamb[5] and first construct the increment of the two-mode field density matrix due to the interaction of the two-mode field with only one atom.

Let F be the density matrix of the two-mode field and a single atom from the atomic beam, which interacts with the ω_{L} mode during the time interval $[t,t+\tau_{\mathrm{L}}]$ and then with the maser ω_{M} mode during the time interval $[t+\tau_{\mathrm{L}},t+\tau_{\mathrm{L}}+\tau_{\mathrm{M}}]$. As a starting equation we take

$$\dot{\hat{F}}=-i[\hat{V},\hat{F}], \tag{7}$$

where the Hamiltonian of the interaction in the resonant dipole approximation is written in the form

$$\hat{V}=g_{23}^*a_{\mathrm{L}}\hat{J}_{23}+g_{21}^*a_{\mathrm{M}}\hat{J}_{21}+\mathrm{h.c.}. \tag{8}$$

The atomic operators act in the following manner:

$$\hat{J}_{21}|1\rangle_{\mathrm{M}}=|2\rangle_{\mathrm{M}},\quad \hat{J}_{23}|3\rangle_{\mathrm{M}}=|2\rangle_{\mathrm{M}}. \tag{9}$$

The relaxation processes of the atoms are not taken into account in (7), since the time intervals τ_1 and τ_2 are assumed to be short compared with characteristic atomic times.

We rewrite Eq. (7) in terms of matrix elements according to the atomic indices. For two time intervals of length τ_{L} and τ_{M} we have, respectively, two different systems of equations:

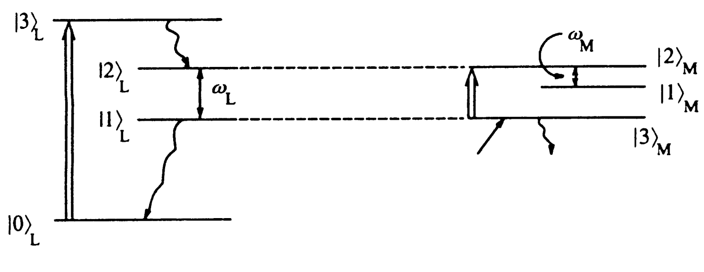

FIG. 2.

$$\dot{\hat{F}}_{11}=\dot{\hat{F}}_{12}=\dot{\hat{F}}_{21}=\dot{\hat{F}}_{13}=\dot{\hat{F}}_{31}=0,$$

$$\dot{\hat{F}}_{22}=-ig^*_{23}a_{\mathrm{L}}\hat{F}_{32}+ig_{23}\hat{F}_{23}a^+_{\mathrm{L}},$$

$$\dot{\hat{F}}_{33}=-ig_{23}a^+_{\mathrm{L}}\hat{F}_{23}+ig^*_{23}\hat{F}_{32}a_{\mathrm{L}},$$

$$\dot{\hat{F}}_{23}=-ig^*_{23}a_{\mathrm{L}}\hat{F}_{33}+ig^*_{23}\hat{F}_{22}a_{\mathrm{L}};$$

$$\dot{\hat{F}}_{32}=-ig_{23}a^+_{L}\hat{F}_{22}+ig_{23}\hat{F}_{33}a^+_{L}, \qquad (10)$$

$$\dot{\hat{F}}_{33}=\dot{\hat{F}}_{23}=\dot{\hat{F}}_{32}=\dot{\hat{F}}_{13}=\dot{\hat{F}}_{31}=0,$$

$$\dot{\hat{F}}_{11}=-ig_{21}a^+_{\mathrm{M}}\hat{F}_{21}+ig^*_{21}\hat{F}_{12}a_{\mathrm{M}},$$

$$\dot{\hat{F}}_{22}=-ig^*_{21}a_{\mathrm{M}}\hat{F}_{12}+ig_{21}\hat{F}_{21}a^+_{\mathrm{M}},$$

$$\dot{\hat{F}}_{12}=-ig_{21}a^+_{\mathrm{M}}\hat{F}_{22}+ig_{21}\hat{F}_{11}a^+_{\mathrm{M}},$$

$$\dot{\hat{F}}_{21}=-ig^*_{21}a_{\mathrm{M}}\hat{F}_{11}+ig^*_{21}\hat{F}_{22}a_{\mathrm{M}}. \qquad (11)$$

We now go over to the Laplace representation, which defined by

$$\hat{F}_s=\int_0^\infty \hat{F}(t)e^{-st}dt. \qquad (12)$$

All the derivatives on the right should be replaced according to the rule $\dot{\hat{F}}\rightarrow sF_s$. The two matrix elements $\hat{F}_{33}$ and $\hat{F}_{22}$ are exceptions, since they, unlike the other matrix elements, are not equal to zero at the initial times t and $t+\tau_{\mathrm{L}}$, respectively, according to the set-up of the problem, and we should bear in mind the rules

$$\dot{F}_{33}(t)\rightarrow sF_{33s}-F_{33}(t),$$

$$\dot{F}_{22}(t)\rightarrow sF_{22s}-F_{22}(t). \qquad (13)$$

Now, instead of two systems of differential equations, we have two systems of algebraic equations, which can be solved relatively simply. The solution of the first system can be written in the following form:

$$F_{22s}=2|g_{23}|^2\underset{\rightarrow}{a_L}\underset{\leftarrow}{a^+_L}\,\frac{s}{(s-\hat{s}_1)(s-\hat{s}_2)(s-\hat{s}_3)(s-\hat{s}_4)}\,F_{33}(t),$$

$$F_{33s}=[s^2+|g_{23}|^2[\underset{\rightarrow}{a^+_L}\underset{\rightarrow}{a_L}+\underset{\leftarrow}{a^+_L}\underset{\leftarrow}{a_L}]]$$

$$\times\frac{s}{(s-\hat{s}_1)(s-\hat{s}_2)(s-\hat{s}_3)(s-\hat{s}_4)}F_{33}(t), \qquad (14)$$

where

$$\hat{s}_{1,2,3,4}=\pm i|g_{23}|\Big(\sqrt{\underset{\rightarrow}{a^+_L}\underset{\rightarrow}{a_L}}\pm\sqrt{\underset{\leftarrow}{a^+_L}\underset{\leftarrow}{a_L}}\Big). \qquad (15)$$

Analogous expressions are written for the second system of equations:

$$\hat{F}_{11s}=2|g_{21}|^2\underset{\rightarrow}{a^+_M}\underset{\leftarrow}{a_M}$$

$$\times\frac{s}{(s-\hat{s}_1)(s-\hat{s}_2)(s-\hat{s}_3)(s-\hat{s}_4)}F_{22}(t+\tau_1),$$

$$F_{22s}=[s^2+|g_{21}|^2[\underset{\rightarrow}{a_M}\underset{\rightarrow}{a^+_M}+\underset{\leftarrow}{a_M}\underset{\leftarrow}{a^+_M}]]$$

$$\times\frac{s}{(s-\hat{s}_1)(s-\hat{s}_2)(s-\hat{s}_3)(s-\hat{s}_4)}F_{22}(t+\tau_1), \qquad (16)$$

where we now have

$$\hat{s}_{1,2,3,4}=\pm i|g_{21}|\Big(\sqrt{\underset{\rightarrow}{a_M}\underset{\rightarrow}{a^+_M}}\pm\sqrt{\underset{\leftarrow}{a_M}\underset{\leftarrow}{a^+_M}}\Big). \qquad (17)$$

Performing inverse Laplace transformation, we obtain the following equations

$$F_{11}(t+\tau_{\mathrm{L}}+\tau_{\mathrm{M}})$$

$$=\beta_L\underset{\rightarrow}{a_{\mathrm{L}}}\underset{\leftarrow}{a^+_L}\underset{\rightarrow}{a^+_M}\underset{\leftarrow}{a_M}\frac{\sin\sqrt{\beta_{\mathrm{M}}\underset{\rightarrow}{a_M}\underset{\rightarrow}{a^+_M}}}{\sqrt{\underset{\rightarrow}{a_M}\underset{\rightarrow}{a^+_M}}}\,\frac{\sin\sqrt{\beta_{\mathrm{M}}\underset{\leftarrow}{a_M}\underset{\leftarrow}{a^+_M}}}{\sqrt{\underset{\leftarrow}{a_M}\underset{\leftarrow}{a^+_M}}}$$

$$\times F_{33}(t), \qquad (18)$$

$$F_{22}(t+\tau_{\mathrm{L}}+\tau_{\mathrm{M}})$$

$$=\beta_{\mathrm{L}}\underset{\rightarrow}{a_L}\underset{\leftarrow}{a^+_L}\cos\sqrt{\beta_{\mathrm{M}}\underset{\rightarrow}{a_M}\underset{\rightarrow}{a^+_M}}\cos\sqrt{\beta_{\mathrm{M}}\underset{\leftarrow}{a_M}\underset{\leftarrow}{a^+_M}}F_{33}(t), \qquad (19)$$

$$F_{33}(t+\tau_{\mathrm{L}}+\tau_{\mathrm{M}})=F_{33}(t+\tau_{\mathrm{L}})$$

$$=\left[1-\frac{1}{2}\beta_L[\underset{\rightarrow}{a^+_L}\underset{\rightarrow}{a_L}+\underset{\leftarrow}{a^+_L}\underset{\leftarrow}{a_L}]\right]F_{33}(t). \qquad (20)$$

Here

$$\beta_{\mathrm{L}}=|g_{23}|^2\tau^2_{\mathrm{L}},\quad \beta_{\mathrm{M}}=|g_{21}|^2\tau^2_{\mathrm{M}} \qquad (21)$$

are saturation parameters characteristic of this problem. Equations (18)–(20) were written under the assumption that the interaction between the laser field and the maser atom (the $|2\rangle_{\mathrm{M}}\rightarrow|3\rangle_{\mathrm{M}}$ transition) is weak and we can restrict ourselves to the first nonvanishing order of perturbation theory with respect to the exciting laser action.

Now, having the matrix elements in explicit forms at our disposal, we can write the increment of the field density matrix, which is expressed in terms of these matrix elements in the form

$$\rho(t+\tau_{\mathrm{L}}+\tau_{\mathrm{M}})-\rho(t)=\sum_{i=1,2,3}F_{ii}(t+\tau_{\mathrm{L}}+\tau_{\mathrm{M}})-F_{33}(t). \qquad (22)$$

This increment is produced by only one atom. However, $r_{\mathrm{M}}\tau$ atoms enter the microcavity during the time $\tau=\tau_{\mathrm{L}}+\tau_{\mathrm{M}}$. Recalling that the entry of the atoms is totally random, we can obtain the total increment of the field form the increment due to a single atom by simple multiplication by the number of atoms. Going over to a "coarse" time scale, we obtain our master equation in the following form:

$$\dot{\rho}=r_{\mathrm{L}}\left(\hat{L}-\frac{1}{2}\hat{L}^2\right)\rho-\Gamma_{\mathrm{L}}\hat{R}_{\mathrm{L}}\rho+r_{\mathrm{M}}\hat{M}\rho-\gamma_{\mathrm{M}}\hat{R}_{\mathrm{M}}\rho. \qquad (23)$$

For the laser mode the following new damping rate of the laser field from the optical cavity appears instead of γ_L due to additional absorption in the maser medium:

$$\Gamma_L = \gamma_L + r_M \beta_L . \tag{24}$$

The operator $\hat{M}$ has the form

$$\hat{M} = \beta_L \underset{\rightarrow}{a_L} \underset{\leftarrow}{a_L^+} \left[\underset{\rightarrow}{a_M^+} \underset{\leftarrow}{a_M} \frac{\sin \sqrt{\beta_M \underset{\rightarrow}{a_M} \underset{\rightarrow}{a_M^+}}}{\sqrt{\underset{\rightarrow}{a_M} \underset{\rightarrow}{a_M^+}}} \frac{\sin \sqrt{\beta_M \underset{\leftarrow}{a_M} \underset{\leftarrow}{a_M^+}}}{\sqrt{\underset{\leftarrow}{a_M} \underset{\leftarrow}{a_M^+}}} \right.$$

$$\left. + \cos \sqrt{\beta_M \underset{\rightarrow}{a_M} \underset{\rightarrow}{a_M^+}} \cos \sqrt{\beta_M \underset{\leftarrow}{a_M} \underset{\leftarrow}{a_M^+}} - 1 \right] . \tag{25}$$

This operator coincides exactly with the standard maser operator,[3] if the operator multiplier in front of the square brackets $\underset{\rightarrow}{a_L} \underset{\leftarrow}{a_L^+}$ is replaced by the c number n_L and the quantity $r_M \beta_L n_L$ is called the mean rate of the injection of active atoms in the upper maser state.

4. CALCULATION OF AVERAGES

We recall a rule which enables us to simplify the calculation of averages:[6] for a stationary light flux in the most general case

$$g_L(t) = \langle a_L^+ a_L^+(t) a_L(t) a_L \rangle = \langle a_L^+ a_L \rangle_{st} \langle a_L^+ a_L \rangle_t' ;$$

here $\langle a_L^+ a_L \rangle_t'$ is the solution of the differential equation for the average number of photons with an initial condition of the special form

$$\langle a_L^+ a_L \rangle_{t=0} = \frac{\langle a_L^+ a_L^+ a_L a_L \rangle_{st}}{\langle a_L^+ a_L \rangle_{st}} ,$$

where $\langle a_L^+ a_L \rangle_{st}$ and $\langle a_L^+ a_L^+ a_L a_L \rangle_{st}$ are the solutions of the corresponding stationary problems.
Of course, precisely the same rule can be formulated for the average

$$g_M(t) = \langle a_M^+ a_M^+(t) a_M(t) a_M \rangle .$$

The master kinetic equation (23) makes it possible, in principle, to write all the necessary expressions, which have the following forms in the present case:

$$\langle \dot{a_L^+ a_L} \rangle = -\Gamma_L \langle a_L^+ a_L \rangle + r_L = D , \tag{26}$$

$$\langle \dot{a_M^+ a_M} \rangle = -\gamma_M (\langle a_M^+ a_M \rangle - n_b) + r_M \beta_L \langle a_L^+ a_L \sin^2 \sqrt{\beta_M a_M a_M^+} \rangle , \tag{27}$$

$$\langle \dot{a_L^+ a_L^+ a_L a_L} \rangle = -2\Gamma_L \langle a_L^+ a_L^+ a_L a_L \rangle + 2 r_L \langle a_L^+ a_L \rangle - r_L = 0 , \tag{28}$$

$$\langle \dot{a_M^+ a_M^+ a_M a_M} \rangle = -2\gamma_M \langle a_M^+ a_M^+ a_M a_M \rangle + 2 r_M \beta_L \langle a_L^+ a_L a_M^+ a_M \sin^2 \sqrt{\beta_M a_M a_M^+} \rangle = 0 , \tag{29}$$

$$\langle \dot{a_L^+ a_L a_M^+ a_M} \rangle = -(\Gamma_L + \gamma_M) \langle a_L^+ a_L a_M^+ a_M \rangle + r_L \langle a_M^+ a_M \rangle + r_M \beta_L \langle a_L^+ a_L^+ a_L a_L \sin^2 \sqrt{\beta_M a_M a_M^+} \rangle = 0 . \tag{30}$$

This system of differential equations is not closed; therefore, it cannot be solved exactly. As usual, help is provided by an assumption that the photon number fluctuations are small, which makes it possible to break up the chains of equations and linearize them with respect to a small parameter. It is convenient to introduce the operators of the photon number fluctuations $\hat{\varepsilon}_L$ and $\hat{\varepsilon}_M$:

$$a_L^+ a_L = n_L + \hat{\varepsilon}_L , \quad a_M^+ a_M = n_M + \hat{\varepsilon}_M , \tag{31}$$

where n_L and n_M coincide with good accuracy with the stationary semiclassical (without consideration of the fluctuations) solutions of the laser-maser problem:

$$n_L = \frac{r_L}{\Gamma_L} , \quad \frac{n_M}{n_L} = \frac{r_M \beta_L}{\gamma_M} \sin^2 \sqrt{\beta_M n_M} . \tag{32}$$

These equations follow from Eqs. (26) and (27) when the operators are replaced by deterministic c numbers, provided the number of thermal photons in the maser mode is small compared with the total number of photons ($n_b \ll n_M$).

Retaining the terms of lowest order in $\hat{\varepsilon}/n$ in the average, we obtain

$$\langle \hat{\varepsilon}_L \hat{\varepsilon}_M \rangle_{st} = -\tfrac{1}{2} n_M \frac{\gamma_M}{\Gamma_L + \Gamma_M} , \tag{33}$$

$$\Gamma_M = \gamma_M (1 + y) , \quad y = \sqrt{\beta_M n_M} |\cot \sqrt{\beta_M n_M}| , \tag{34}$$

$$\langle \hat{\varepsilon}_L^2 \rangle_{st} = \tfrac{1}{2} n_L , \tag{35}$$

$$\langle \hat{\varepsilon}_M^2 \rangle_{st} = \frac{n_M}{1+y} \left(1 + 2 n_b - \frac{1}{2} \frac{n_M}{n_L} \frac{\gamma_M}{\Gamma_M + \Gamma_L} \right) . \tag{36}$$

The following expressions may be written for the statistical Mandel parameters: for the maser output

$$\xi_M = \frac{\langle \hat{\varepsilon}_M^2 \rangle}{n_M} - 1 = \xi_0 - \xi_1 , \quad \xi_0 = \frac{2 n_b - y}{1 + y} ,$$

$$\xi_1 = \frac{1}{2} \frac{n_M}{n_L} \frac{\Gamma_M}{\Gamma_M + \Gamma_L} \frac{1}{(1+y)^2} , \tag{37}$$

and for the laser output

$$\xi_L = \frac{\langle \hat{\varepsilon}_L^2 \rangle}{n_L} - 1 = -\frac{1}{2} . \tag{38}$$

The Mandel parameter for the laser output is the same as when there is no active maser medium. This is perfectly natural, since the maser medium determines only the level of losses of the laser field from the cavity, on which the mean output power, but not its statistical properties under saturation conditions, depends. This, of course, is a direct consequence of the fact that we used the lowest approximation in the interaction of the laser field with the maser medium.

The time-dependent solution of Eq. (26) has the following form:

$$\langle a_L^+ a_L\rangle_t = \langle a_L^+ a_L\rangle_{t=0} e^{-\Gamma_L t} + \frac{r_L}{\Gamma_L}(1-e^{-\Gamma_L t}). \tag{39}$$

Equation (27) must be linearized:

$$\langle \dot{\hat{\varepsilon}}_M\rangle_t = -\Gamma_M\langle\hat{\varepsilon}_M\rangle + \gamma_M \frac{n_M}{n_L}\langle\hat{\varepsilon}_L\rangle. \tag{40}$$

Now, with consideration of (38) it is not difficult to derive the expression

$$\langle\hat{\varepsilon}_M\rangle_t = \langle\hat{\varepsilon}_M\rangle_{t=0} e^{-\Gamma_M t} + \gamma_M \frac{n_M}{n_L}\langle\hat{\varepsilon}_L\rangle_{t=0} \frac{e^{-\Gamma_L t}-e^{-\Gamma_M t}}{\Gamma_M-\Gamma_L}. \tag{41}$$

According to the rule which we formulated at the beginning of this section, we should require

$$\langle\hat{\varepsilon}_M\rangle_{t=0} = \langle a_M^+ a_M\rangle'_{t=0} - n_M = \frac{\langle a_M^+ a_M^+ a_M a_M\rangle_{st}}{n_M} - n_M = \xi_M \tag{42}$$

and

$$\langle\hat{\varepsilon}_L\rangle_{t=0} = \langle a_L^+ a_L\rangle'_{t=0} - n_L = \frac{\langle a_L^+ a_L a_M^+ a_M\rangle_{st}}{n_M} - n_L = \frac{\langle\hat{\varepsilon}_L\hat{\varepsilon}_M\rangle_{st}}{n_M}. \tag{43}$$

Now, according to the same rule we can write the explicit equations

$$g_L(t) = = n_L^2 - \tfrac{1}{2} n_L e^{-\Gamma_L t},$$

$$g_M(t) = n_M^2 + n_M \xi_0 e^{-\Gamma_M t} - n_M \xi_1 \frac{\Gamma_M e^{-\Gamma_L t} - \Gamma_L e^{-\Gamma_M t}}{\Gamma_M - \Gamma_L}. \tag{44}$$

The former expression coincides with the expression known from the theory of a sub-Poissonian laser.[4]

5. PHOTOCURRENT SPECTRUM WHEN THE MICROMASER EMISSION IS RECORDED

The emission can be recorded along two channels: the laser channel (L) and the maser channel (M) (see Fig. 2). When, for example, the maser emission is recorded, the formula for the photocurrent spectrum takes the form[6,7]

$$i^{(2)}_{M\omega} = i^{(2)}_{M,\text{shot}}\left\{1 + 2q_M \frac{1}{n_M}\gamma_M \operatorname{Re}\int_0^\infty g_M(t) e^{i\omega t} dt\right\}, \tag{45}$$

where q_M is the quantum efficiency of the photodetector in the M channel.

When the laser emission is recorded in the L channel, precisely the same equation can be written if we replace the subscript M by L everywhere.

Bearing in mind (44), we obtain: in the M channel

$$i^{(2)}_{M\omega} = i^{(2)}_{M,\text{shot}}\left\{\left(1 + 2q_M\left(\xi_0 + \xi_1 \frac{\Gamma_L}{\Gamma_M-\Gamma_L}\right)\frac{\gamma_M\Gamma_M}{\Gamma_M^2+\omega^2}\right.\right.$$
$$\left. - 2q_M\xi_1 \frac{\Gamma_M}{\Gamma_M-\Gamma_L}\frac{\gamma_M\Gamma_L}{\Gamma_L^2+\omega^2}\right\} \tag{46}$$

and in the L channel

$$i^{(2)}_{L\omega} = i^{(2)}_{L,\text{shot}}\left\{1 - q_L \frac{\gamma_L}{\Gamma_L}\frac{\Gamma_L^2}{\Gamma_L^2+\omega^2}\right\}. \tag{47}$$

As we see, when the laser emission is recorded in the absence of a maser medium and the quantum efficiency of the photodetector is $q_L=1$, the dip in the noise spectrum at zero frequency extends down to zero. When a maser medium is present, the depth of the dip becomes equal to

$$\delta_L = \frac{\gamma_L}{\Gamma_L} = \frac{\gamma_L}{\gamma_L + r_M\beta_L}. \tag{48}$$

Thus, if the efficiency of the interaction of the laser radiation with the maser medium is weak (if the absorption of the laser radiation in the maser medium is small), i.e., if $r_M\beta_L \ll \gamma_L$, no appreciable changes in the measured statistical properties of the auxiliary sub-Poissonian laser occur. At the same time, if the absorption of the laser radiation in the maser medium is so great that it determines the main losses from the cavity, i.e., if $r_M\beta_L \gg \gamma_L$, the depth of the dip in the photocurrent spectrum becomes negligibly small, and the laser emission becomes practically Poissonian. These results may be interpreted by assuming that a new quantum efficiency of the photodetector appears here:

$$\tilde{g}_L = q_L\delta_L. \tag{49}$$

This is the usual conclusion when, as in the present case, there are uncontrollable losses.

Now let us discuss recording in the M channel. Just as was previously done in Ref. 6, it is convenient to consider two characteristics. One of them

$$\xi_M = \xi_0 - \xi_1 \tag{50}$$

characterizes the integral photon number fluctuations: $\overline{\Delta n_M^2} = n_M(1 + \xi_M)$. The other is equal to the depth of the dip or the height of the peak, depending on the sign of the equation in the photocurrent spectrum (46)

$$\delta_M = \frac{i^{(2)}_{M,\omega=0}}{i^{(2)}_{M,\text{shot}}} - 1 \tag{51}$$

and thereby determines the suppression or enhancement of the shot noise in the observation. It is not difficult to obtain the expression

$$\delta_M = 2\frac{\gamma_M}{\Gamma_M}\left(\xi_0 - \xi_1 \frac{\Gamma_M+\Gamma_L}{\Gamma_L}\right). \tag{52}$$

Now, taking into account the explicit expressions for the quantities appearing here, we can write

$$\xi_M = \frac{2n_b - y - (1/2)A_1 \sin^2\sqrt{\beta_M n_M}}{1+y}, \tag{53}$$

$$\delta_M = 2\,\frac{2n_b - y - (1/2)\,A_2 \sin^2\sqrt{\beta_M n_M}}{(1+y)^2}, \tag{54}$$

where

$$A_1 = \frac{r_M\beta_L}{\gamma_M(1+y) + \gamma_L + r_M\beta_L}, \quad A_2 = \frac{r_M\beta_L}{\gamma_L + r_M\beta_L}.$$

For comparison we present the analogous expressions for the case of the regular injection of atoms:[6]

$$i_\omega^{(2)} = i_{\text{shot}}^{(2)}\left(1 + 2q_M\xi\,\frac{\gamma_M\Gamma_M}{\Gamma_M^2 + \omega^2}\right), \tag{55}$$

$$\xi = \frac{2n_b - y - (1/2)\sin^2\sqrt{\beta_M n_M}}{1+y}, \tag{56}$$

$$\delta = 2\,\frac{2n_b - y - (1/2)\sin^2\sqrt{\beta_M n_M}}{(1+y)^2}. \tag{57}$$

6. QUANTITATIVE DISCUSSION OF THE PHOTOCURRENT SPECTRUM

We recall the difference between the physical situation under consideration here and the case of the regular injection of active atoms within a micromaser. When there is regular injection, a strictly identical number of active atoms enters the cavity during each unit of time: $\overline{\Delta N^2} = \overline{N^2} - \bar{N}^2 = 0$. In our case the best situation, under which $\overline{\Delta N^2} = 1/2\bar{N}$, arises when each laser photon produces an active atom, since the statistics of the photons in a sub-Poissonian laser are specified by the relation $\overline{\Delta n_L^2} = (1/2)n_L$. Therefore, here we cannot expect effective suppression of the quantum noise.[2] However, our formal approach automatically also takes into account the fact that the excitation noise spectrum of the medium is inhomogeneous, and the noise is completely suppressed at near-zero frequencies. We expect that this fact may be important in determining the quantum properties of light. In any case, it was so in the case of an optical laser when the active medium was excited by light from an auxiliary sub-Poissonian laser.[1] We ascertained there that the noise of the exciting laser, in which the low frequencies are suppressed, cause analogous noise in the active medium of the secondary system and, accordingly, the low-frequency noise in the secondary laser output is also suppressed.

Now, let us compare the equations for the two physical situations. We first compare the general form of the photocurrent spectrum i.e., Eqs. (46) and (55). As we see, in the general case they differ fairly strongly. In the case of regular pumping, we have a dip (or a peak) of Lorentzian shape at zero frequency. In our case the structure at zero frequency is formed by two Lorentzian features, one of which depends only on the properties of the exciting radiation.

As for the parameters ξ_M and δ_M, a comparison of Eqs. (53) and (54) with Eqs. (56) and (57) reveals that they differ from ξ and δ for the regular injection of atoms by the coefficients A_1 and A_2, which can run through values from zero to unity. Below we shall consider several limiting cases, in which a very interesting and physically obvious situation arises.

Let the efficiency of the interaction of the laser emission with the maser medium be low:

$$r_M\beta_L \ll \gamma_M, \gamma_L. \tag{58}$$

It is not difficult to see that we then have $A_1, A_2 \ll 1$ and that we can obtain precisely the same expressions for ξ and δ as in the case of random injection:[6]

$$\xi_M = \frac{2n_b - y}{1+y} = \xi_0, \quad \delta_M = 2\,\frac{2n_b - y}{(1+y)^2}. \tag{59}$$

The photocurrent spectrum also takes on the usual form:

$$i_{M\omega}^{(2)} = i_{M,\text{shot}}^{(2)}\left\{1 + 2q_M\xi_M\,\frac{\gamma_M\Gamma_M}{\Gamma_M^2 + \omega^2}\right\}. \tag{60}$$

Thus, we conclude that under condition (58) excitation of the active laser medium to the upper maser level is a random process. From the physical point of view, this is a perfectly natural result, since only when the laser radiation interacts with the maser medium is efficiently and each laser photon produces an atom in the upper maser level can we expect that the statistics of the atoms would duplicate the statistics of the laser photons and thus turn out to have no noise at low frequencies.

In the directly opposite case, in which

$$r_M\beta_L \gg \Gamma_M, \Gamma_L, \tag{61}$$

$A_1 = A_2 = 1$, and Eqs. (53) and (54) coincide exactly with Eqs. (56) and (57). The photocurrent spectrum also takes on the usual form (60). In this case we obtain the same results as for the regular injection of active atoms.[6]

The intermediate case in which

$$\gamma_L \ll r_M\beta_L \ll \gamma_M \tag{62}$$

is physically consistent. Here we have $A_1 \ll 1$ and $A_2 = 1$, i.e., ξ_M is the same as for random injection (59), and δ coincides with (57), where injection is strictly regular. At the same time, the photocurrent spectrum has the form

$$i_{M\omega}^{(2)} = i_{M,\text{shot}}^{(2)}\left\{1 + 2q_M\xi_0\,\frac{\gamma_M\Gamma_M}{\Gamma_M^2 + \omega^2} - 2q_M\xi_1\,\frac{\gamma_M\Gamma_L}{\Gamma_L^2 + \omega^2}\right\}. \tag{63}$$

As we see, it contains a contribution from an excitation noise spectrum with a spectral width Γ_L. Moreover, this term becomes the main term for the stationary states of a micromaser oscillator for which $\sin^2\sqrt{\beta_M n_M} \approx 1 (y \approx 0)$. In this case the photocurrent noise spectrum contains precisely the same dip as in the case of the exciting laser light.

Here it must be understood how the observed quantum manifestations can correspond to a Poissonian (or even super-Poissonian) state of the intracavity field oscillator. It is not surprising when the reverse situation occurs, i.e., when, for example, Poissonian noise in the photocurrent corresponds to a Fock state of the intracavity field oscillator. Everything here is physically obvious: the photons leaving the cavity in this case are totally uncorrelated with one another due to the infinitely fast decay of the fluctuations within the cavity. Furthermore, in this case we can say that the observed classical Poissonian light (Poissonian stream of photoelectrons) corresponds to a quantum state of the intracavity oscillator. But what should we do now? Should we say that the observed quantum light corresponds to a classical Poissonian state of the oscillator? However, this is against common sense, since it is impossible to imagine that light would be transformed from "bad" to "good" as a result of leaving the cavity. In our opinion, it can only be assumed that, generally, the Mandel parameter ξ, which characterized the stationary state of the oscillator, is not the only characteristic that can signal the appearance of nonclassical states in the field oscillator. There is still some dynamics, which determines the decay law of the fluctuations appearing in the system. This dynamics can be completely quantum and ensure integral photon number fluctuations at the $0 \leqslant \xi$ level. In fact, if the low frequencies are completely suppressed in the intensity noise spectrum, while some high frequencies are enhanced to the same extent, this light will be quantum on the one hand, since classical light with noise below the shot level does not exist, and Poissonian on the other hand. Just this situation arose in the example we analyzed of a micromaser with excitation by light from a sub-Poissonian laser in case (62).

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