The following three articles were originally published in the October 1994 issue. They are reprinted here with all the necessary changes and corrections.

Imaginary part of the electron mass operator in a static uniform magnetic field at finite temperature and nonzero chemical potential

V. Ch. Zhukovskiĭ, P. G. Midodashvili, and P. A. Éminov

Moscow State Institute of Electronics and Mathematics, 109028 Moscow, Russia
(Submitted 25 November 1993; resubmitted 29 April 1994)
Zh. Eksp. Teor. Fiz. **106**, 929–935 (October 1994)

The correspondence between the real and imaginary parts of mass operators at finite temperature and nonzero chemical potential is discussed using the real- and imaginary-time representations. Within the single-loop approximation, the imaginary part of the electron energy shift in a static uniform magnetic field at finite temperature and nonzero chemical potential is calculated and its physical interpretation is given. In the limiting cases, asymptotic expressions for the imaginary part of the electron energy shift are obtained, its zero-field finite-T value being zero. © *1994 American Institute of Physics.*

1. INTRODUCTION

In a number of recent studies the effects of finite temperature and nonzero chemical potential on the probabilities of various processes are discussed.[1–3] The most effective technique for calculating the corresponding physical quantities is the Green's function method.[4–10] In particular, the calculation of the energy spectrum of a system in quantum field theory at finite T and nonzero chemical potential reduces to finding the poles of the retarded (or advanced) Green's functions $G^{\mathrm{r}}(G^{\mathrm{a}})$, which are related to the time-dependent Green's function G by

$$\left.\begin{matrix} G^{\mathrm{r}}(\omega,\mathbf{p}) \\ G^{\mathrm{a}}(\omega,\mathbf{p}) \end{matrix}\right\} = \mathrm{Re}\, G(\omega,\mathbf{p}) \pm \mathrm{cth}\, \frac{\omega-\mu}{2T}\, \mathrm{Im}\, G(\omega,\mathbf{p}) \tag{1}$$

for a Fermi system and by

$$\left.\begin{matrix} G^{\mathrm{r}}(\omega,\mathbf{p}) \\ G^{\mathrm{a}}(\omega,\mathbf{p}) \end{matrix}\right\} = \mathrm{Re}\, G(\omega,\mathbf{p}) \pm \mathrm{th}\, \frac{\omega-\mu}{2T}\, \mathrm{Im}\, G(\omega,\mathbf{p}) \tag{2}$$

in the case of a Bose system.

In the real-time representation, in order to calculate the time-dependent Green's function of a system of interacting particles one uses either the Keldysh diagram technique[6] or the equivalent thermo-field dynamics method.[7] In some studies (e.g., Refs. 11 and 12), the elementary excitation spectrum is calculated by the method of temperature Green's functions. In this case, in order to find the energy spectrum of the system one must analytically continue the appropriate temperature Green's function to the retarded (advanced) Green's function.

From what has been said there follows a relation between the real and imaginary parts of the mass operators in the real-time (Σ) and imaginary-time (Σ^{r}) formalisms:

$$\mathrm{Re} \overset{\mathrm{r}}{\Sigma} = \mathrm{Re} \Sigma\,, \tag{3}$$

$$\mathrm{Im} \overset{\mathrm{r}}{\Sigma} = \mathrm{cth}\, \frac{\omega-\mu}{2T}\, \mathrm{Im} \Sigma \tag{4}$$

for fermions, and

$$\mathrm{Re} \overset{\mathrm{r}}{\Sigma} = \mathrm{Re} \Sigma\,, \tag{5}$$

$$\mathrm{Im} \overset{\mathrm{r}}{\Sigma} = \mathrm{th}\, \frac{\omega-\mu}{2T}\, \mathrm{Im} \Sigma \tag{6}$$

for bosons.

The relations (3)–(6) are exact and follow from first principles.[4,5] In this connection, it should be noted that Eqs. (4) and (6), the correspondence between the imaginary parts of the real-time and imaginary-time self-energy diagrams, not only were derived anew but also verified by direct single-loop calculation in a series of studies.[13–16]

In the present work the single-loop approximation is employed to calculate the imaginary part of the electron energy shift in a static uniform magnetic field at finite temperature and nonzero chemical potential.

In Sec. 2 the general structure of the imaginary part of the electron energy shift in the real-time and imaginary-time representations is examined. The result is expressed in terms of the probabilities of the corresponding external-field-induced processes. It is shown that physical significance resides in the imaginary part of the retarded mass operator. In Sec. 3 asymptotic expressions for the imaginary part of the electron energy shift for certain limiting cases are obtained. Section 4 discusses the results.

2. GENERAL STRUCTURE OF THE IMAGINARY PART

In Ref. 17 it is shown that the single-loop finite-density finite-T contribution to the electron energy shift in an electron–positron plasma, in equilibrium at T in an external magnetic field $\mathbf{H}\uparrow\uparrow OZ$, is given by

 1063-7761/95/020356-04$10.00

$$\Delta E_n = \Delta E_n^{\mathrm{B}} + \Delta E_n^{\mathrm{F}} + \Delta E_n^{\mathrm{F-B}}, \qquad (7)$$

where

$$\Delta E_n^{\mathrm{B}} = -\frac{\alpha}{4\pi^2}\int d^4k \sum_{n',\varepsilon,s'} (J_\mu^+ J^\mu) \times \frac{\delta(k^2) n_{\mathrm{B}}(|\mathbf{k}|)}{k_0 - E_n + \varepsilon E_{n'}(1-i\delta)},$$

$$\Delta E_n^{\mathrm{F}} = -\frac{\alpha}{4\pi^2}\int d^4k \sum_{n',\varepsilon,s'} \varepsilon (J_\mu^+ J^\mu) \times \frac{\delta(E_n - \varepsilon E_{n'} - k_0) n_{\mathrm{F}}(E_{n'})}{k^2 + i\delta}, \qquad (8)$$

$$\Delta E_n^{\mathrm{F-B}} = \frac{i\alpha}{2\pi}\int d^4k \sum_{n',\varepsilon,s'} \varepsilon (J_\mu^+ J^\mu) \times \delta(k^2)\delta(E_n - \varepsilon E_{n'} - k_0) n_{\mathrm{B}}(|\mathbf{k}|) n_{\mathrm{F}}(E_{n'})$$

and $n(|\mathbf{k}|)$ and $n(E)$ are the Bose and Fermi distribution functions:

$$n_{\mathrm{B}}(|\mathbf{k}|) = \frac{1}{\exp(|\mathbf{k}|/T) - 1},$$

$$n_{\mathrm{F}}(E_{n'}) = \left[\exp\left(\frac{E_{n'} - \varepsilon\mu}{T}\right) + 1\right]^{-1}. \qquad (9)$$

Equation (8) sums over all the quantum numbers of the intermediate states (here plasma particle states; $\varepsilon = \pm 1$ is the sign of the energy and $s' = \pm 1$ the spin projection).

The unperturbed energy levels of an electron in a static uniform magnetic field are given by[18]

$$E_n = \sqrt{m^2 + 2eHn + p_3^2}, \qquad (10)$$

where $n = 0,1,\ldots$ is the principal quantum number, $-\infty < p_3 < +\infty$, and the explicit form of the transition current

$$J_\mu = \int \bar{\Psi}_{n'}^{(\varepsilon)}(\mathbf{x}) \gamma_\mu \exp(-ik\mathbf{x}) \Psi_n^{(\varepsilon)}(\mathbf{x}) d^3x \qquad (11)$$

where $\Psi_n(\mathbf{x})$ is the coordinate part of the electron wave function in a static uniform magnetic field, is here omitted as overly complicated.[18]

The real part of Eqs. (7)–(8) is treated in Refs. 17 and 19. Using the Sokhotsky formula we separate the imaginary part of the electron energy shift which, after integrating over k_0 and summing over $\varepsilon = \pm 1$, becomes

$$\mathrm{Im}\,(\delta E_n) = -\frac{\alpha}{8\pi}\int \frac{d^3k}{|\mathbf{k}|} \sum_{n',s'} [F(\varepsilon=+1)\delta(|\mathbf{k}| - E_n + E_{n'}) + F(\varepsilon=1)\delta(-|\mathbf{k}| - E_n + E_{n'}) - F(\varepsilon=-1)\delta(|\mathbf{k}| - En - E_{n'}) - F(\varepsilon=-1)\delta \times (-|\mathbf{k}| - E_n - E_{n'})][n_{\mathrm{B}}(1-n_{\mathrm{F}}) - n_{\mathrm{F}}(1+n_{\mathrm{B}})], \qquad (12)$$

with

$$F = (J_\mu^+ J^\mu). \qquad (13)$$

According to the optical theorem the imaginary part of the quantity δE_n in the case $T=0, \mu=0$ is proportional to the decay probability of a given state,

$$\mathrm{Im}\,(\delta E_n) = -\tfrac{1}{2} w_n, \qquad (14)$$

where w_n is the total scattering probability.

Having in mind the effects of finite temperature and nonzero chemical potential, the imaginary part of the electron energy shift can be understood as follows: The first term in (12) corresponds to the probability of synchrotron emission by the initial electron with energy $E_n(e^- \to e^{-\prime} + \gamma)$ in a magnetic field, with statistical weight $(1+n_{\mathrm{B}})(1 - n_{\mathrm{F}})$, minus the probability for the inverse process, with weight $n_{\mathrm{F}} n_{\mathrm{B}}$.

Including the contribution from spontaneous transitions at $T=0$ then reduces to the following replacement in the first term of (12):

$$Q = n_{\mathrm{B}}(1-n_{\mathrm{F}}) - (n_{\mathrm{B}}+1)n_{\mathrm{F}} \to Q + 1. \qquad (15)$$

The second term in (12) corresponds to the difference in probability between the excitation of the initial electron by absorbing a photon, with statistical weight $n_{\mathrm{B}}(1 - n_{\mathrm{F}})$, and the inverse process, with $n_{\mathrm{F}}(1+n_{\mathrm{B}})$.

The third term in Eq. (12) corresponds to the probability difference between the one-photon annihilation of the initial electron with a plasma positron, with weight $n_{\mathrm{F}}(1+n_{\mathrm{B}})$, and the inverse process of creation of an electron–positron pair by a photon, with weight $n_{\mathrm{B}}(1 - n_{\mathrm{F}})$.

The last term is unphysical and does not contribute to the imaginary part of the electron energy shift.

Let Γ_{d} denote the sum of the probabilities of all those processes transferring the electron to other states,

$$\Gamma_{\mathrm{d}} = w(e^- \to e^{-\prime} + \gamma) + w(e^- + \gamma \to e^{-\prime}) + w(e^- + e^{+\prime} \to \gamma). \qquad (16)$$

Further, let Γ_{i} be the sum of the probabilities of all the inverse processes,

$$\Gamma_{\mathrm{i}} = w(e^{-\prime} + \gamma \to e^-) + w(e^{-\prime} \to e^- + \gamma) + w(\gamma \to e^- + e^{+\prime}). \qquad (17)$$

The probabilities enter in Eqs. (16) and (17) statistically weighted [see Eq. (12) and the discussion that follows]. But then the imaginary part of δE_n in the real-time representation may be written

$$\mathrm{Im}\,(\Delta E_n) = -\tfrac{1}{2}(\Gamma_{\mathrm{d}} - \Gamma_{\mathrm{i}}), \qquad (18)$$

whereas in the imaginary-time representation, from Eqs. (18) and (4),

$$\mathrm{Im}\,(\Delta E_n^{\Gamma}) = -\tfrac{1}{2}(\Gamma_d - \Gamma_i), \qquad (19)$$

Let us show that it is the imaginary part of the mass operator which is physically meaningful. Let $f(\omega,t)$ be the nonequilibrium distribution function of particle energies. Then the following kinetic equation can be written down:

$$\frac{\partial f}{\partial t}=-f\Gamma_d+(1+\sigma f)\Gamma_i, \tag{20}$$

where by the Pauli principle $\sigma=+1$ for the boson case and $\sigma=-1$ for the fermion case.

For small departures of the system from its equilibrium state, we find from Eq. (20)

$$f(\omega,t)=f_0(\omega)+c(\omega)\exp(-\Gamma t), \tag{21}$$

where

$$\Gamma=\Gamma_d-\sigma\Gamma_i, \quad f_0(\omega)=\Gamma_i/\Gamma. \tag{22}$$

From Eqs. (21) and (22) it is seen that in the fermion case the quantity $\Gamma_d+\Gamma_i$, which is determined by the imaginary part of the retarded mass operator, Eq. (19), determines the relaxation time of the system to its equilibrium state.

3. CALCULATION OF THE QUANTITY $\mathrm{Im}(\delta E_N)$

In this section we carry out an explicit calculation of the imaginary part of the electron energy shift in certain limiting cases. Consider the charge-symmetrical case, in which the chemical potential is zero. This means that at finite T an equal number of positrons and electrons are excited.[20] For the ground state of the electron, the first term in Eq. (12) does not contribute to $\mathrm{Im}(\delta E_n)$. In the low temperature limit which we discuss below and in which $T\ll m$ holds, where m is the electron mass, the contributions from the third term in (12) and from the process $e^{-\prime}\to e^-+\gamma$ will be suppressed exponentially.

Then, calculating by the method of Ref. 12 we arrive at the result

$$\mathrm{Im}\,(\delta E_0)=-\frac{4}{3}\,\alpha m\left(\frac{T}{m}\right)^2\left(\frac{eH}{mT}\right)^2\left[\exp\left(\frac{eH}{mT}\right)-1\right]^{-1}. \tag{23}$$

In the limiting cases of relatively weak ($eH\ll mT$) and strong ($eH\gg mT$) magnetic fields, it follows from (23) that

$$\mathrm{Im}\,(\delta E_0)=\begin{cases}-\frac{4}{3}\,\alpha m\left(\frac{T}{m}\right)^2\frac{eH}{mT}, & eH\ll mT,\\ -\frac{4}{3}\,\alpha m\left(\frac{T}{m}\right)^2\left(\frac{eH}{mT}\right)^2\exp\left(-\frac{eH}{mT}\right), & eH\gg mT.\end{cases} \tag{24}$$

The exponential suppression of the quantity $\mathrm{Im}(\delta E_0)$ in the limiting case $eH\gg mT$ can be explained by the fact that increasing the magnetic field strength increases the energy gap between the ground and first excited energy levels of an electron in an external magnetic field. Therefore the excitation of the electron requires high energy photons, but according to the Planck black-body distribution, few of these exist. From Eq. (23) it follows that in the absence of a field the probability of electron excitation from the ground state by absorption of one photon is zero, as it must be. This disagrees with the corresponding result of Ref. 21, which is probably wrong. Note that the result (23) corresponds to the stimulated dipole transition $n=0\to n=1$ in a magnetic field by absorbing a photon with a frequency equal to the cyclotron frequency $\omega=eH/m$. Suppose further that the magnetic field strength and the electron energy satisfy

$$H\ll H_0=\frac{m^2}{e}=4.41\cdot 10^{13}\mathrm{G}, \quad E_n\gg m. \tag{25}$$

These conditions secure the quasiclassical nature of electron motion in the external magnetic field.[17,22]

Then in the region of relatively low temperatures

$$T\ll m, \tag{26}$$

along with the usual synchrotron emission, stimulated emission and absorption processes are a significant source of the energy loss of a relativistic electron, whose power, under the condition (25), is found to be

$$W^{\mathrm{emit}}_{\mathrm{stim}}=-\frac{\alpha}{\pi}\,m^2\int_{(2n)^{-1}}^{\infty}\frac{u\,du}{(u+1)^3}\left[\exp\left(\frac{E_n u}{T(u+1)}\right)-1\right]^{-1}\times\left[\int_{x_1}^{\infty}\Phi(t)dt+\left(\frac{\chi}{u}\right)^{2/3}\frac{u^2+2u+2}{u+1}\,\Phi'(x_1)\right], \tag{27}$$

$$W^{\mathrm{absopb}}_{\mathrm{stim}}=-\frac{\alpha}{\pi}\,m^2\int_{(2n)^{-1}}^{\infty}\frac{u\,du}{\exp(E_n u/T)-1}\left[\int_{x_2}^{\infty}\Phi(t)dt+\left(\frac{\chi(u+1)}{u}\right)^{2/3}\frac{u^2+2u+2}{u+1}\,\Phi'(x_2)\right]. \tag{28}$$

The arguments of the Airy functions[22] in Eqs. (27) and (28) are

$$x_1=\left(\frac{u}{\chi}\right)^{2/3}, \quad x_2=\left(\frac{u}{\chi(u+1)}\right)^{2/3}, \tag{29}$$

where χ is the characteristic synchrotron emission parameter

$$\chi=\frac{H}{H_0}\frac{E_n}{m}. \tag{30}$$

In the relatively low-temperature range (26), from Eqs. (27) and (28)

$$W^{\mathrm{emit}}_{\mathrm{stim}}=W^{\mathrm{absorb}}_{\mathrm{stim}}=-2\,\frac{\alpha}{\pi}\,m^2\left(\frac{T}{E_n\chi}\right)^{4/3}\chi^2\Gamma\left(\frac{4}{3}\right)\zeta\left(\frac{4}{3}\right)\Phi'(0), \tag{31}$$

where

$$\Phi'(0)=-\frac{3^{1/6}}{2\sqrt{\pi}}\,\Gamma(2/3)$$

is the value of the derivative of the Airy function at zero, and $\zeta\,(x)$ is the Riemann zeta function.

4. DISCUSSION

Let us summarize the basic results of this study. In the single-loop approximation, a general structure of the imaginary part of the electron energy shift in the real- and imaginary-time representations is established in terms of the probabilities of the corresponding external-field-induced processes. It is shown that real physical significance is carried by the imaginary part of the retarded mass operator. Calculation of the imaginary part of the electron energy shift shows the result to be finite and not infrared divergent. In the charge symmetrical case, the power of the stimulated electron emission and of stimulated absorption are calculated.

In summary, processes in the absence of a field, as compared with those induced by external fields, are of higher order in the fine structure constant and exhibit a "poor" infrared behavior. The corresponding calculations are therefore extremely laborious.[23–25] It is important however that, by comparison, externally induced processes may contribute crucially to the energy loss rate of particles propagatingthrough a plasma in intensive fields.

We are grateful to I. V. Tyutin for discussions on the results of this study.

[1] S. Varma, Phys. Rev. **D46**, 3640 (1992).
[2] Yu. K. Zemtsov and A. N. Starostin, Zh. Eksp. Teor. Fiz. **103**, 345 (1993) [JETP **76**, 186 (1993)].
[3] S. Masood, Phys. Rev. **D44**, 3943 (1991).
[4] V. L. Bonch-Bruevich and S. V. Tyablikov, *Method of Green's Functions in Statistical Physics* [in Russian], GIFML, Moscow (1961).
[5] E. M. Lifshitz and L. P. Pitaevskiĭ, *Statistical Physics*, Pergamon, New York (1980), Vol. 9, Part 2.
[6] E. M. Lifshitz and L. P. Pitaevskiĭ, *Physical Kinetics* Pergamon, Oxford (1981).
[7] H. Umezava, M. Matsumoto, and M. Tachiki, *Thermo Field Dynamics and Condensed States*, North Holland, Amsterdam (1980).
[8] E. S. Fradkin, Trudy FIAN **29**, 7 (1965).
[9] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskii, *Methods of Quantum Field Theory in Statistical Physics*, Prentice–Hall, Englewood Cliffs, New Jersey (1963).
[10] S. P. Gavrilov, D. N. Gitman, and E. S. Fradkin, Trudy FIAN **198**, 208, (1989).
[11] O. K. Kalashinikov and V. V. Klimov, Yad. Fiz. **31**, 1357 (1980) [Sov. J. Nucl. Phys. **31**, 699 (1980)].
[12] V. V. Klimov, Yad. Fiz. **33**, 1734 (1981) [Sov. J. Nucl. Phys. **33**, 934 (1981)].
[13] A. Weldon, Phys. Rev. **D28**, 2007 (1983).
[14] Y. Fujimoto and M. Morikava, Phys. Rev. **D33**, 590 (1986).
[15] R. Jackiv and V. P. Nair, Columbia Preprint CU-TP-594 (1993).
[16] R. L. Kobes and Q. W. Semenoff, Nucl. Phys. **B260**, 714 (1985).
[17] A. A. Sokolov and I. M. Ternov, *The Relativistic Electron* [in Russian], Nauka, Moscow (1983).
[18] I. M. Ternov, V. Ch. Zhukovskiĭ, P. G. Midodashvili and P. A. Éminov, Yad. Fiz. **43**, 764 (1986) [Sov. J. Nucl. Phys. **43**, 485 (1986)].
[19] V. Ch. Zhukovskiĭ, T. L. Shoniya and P. A. Éminov, Yad. Fiz. **57**, 1437 (1994) [J. Nucl. Phys. **57**, 1365 (1994)].
[20] A. E. Shabad, Trudy FIAN **192**, 1 (1988).
[21] P. H. Cox, W. S. Hellman, and A. Yildiz, Ann. Phys. **154**, 211 (1984).
[22] V. I. Ritus, Trudy FIAN **111**, 1 (1979).
[23] M. Thoma and M. Gyalassy, Nucl. Phys. **B351**, 491 (1991).
[24] E. Braaten and T. Yuan, Phys. Rev. Lett. **66**, 2183 (1991).
[25] T. Altherr, E. Petitgirard, and T. Gaztelurruita, Phys. Rev. **D47**, 703 (1993).

Translated by E. Strelchenko